Measuring Interdependence of Inflation Uncertainty

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Co-movement of inflation

1. Inflation tends to co-move closely across countries (Monacelli and Sala, 2009; Ciccarelli and Mojon, 2010; Mumtaz and Surico, 2012; Bäurle et al., 2021)
2. A large number of studies have analyzed the potential channels of the co-movement in inflation:
   - Common macroeconomic shocks (Henriksen et al., 2013)
   - Trade openness (Melitz and Ottaviano, 2008)
   - Labor market channel through migration (Bentolila et al., 2008)
   - Exchange rate regimes (Calvo and Reinhart, 2002)
   - Financial integration (Rey, 2016)

Co-movement of inflation uncertainty

1. Ball (1992) proposed a model where high inflation produces high uncertainty around future inflation through monetary policy channel.
2. Since then, the relationship between the level of inflation and inflation uncertainty (volatility) has been extensively studied (Grier and Perry, 2000; Kontonikas, 2004).
3. Uncertainty about inflation of a country can be transmitted to other countries.
   - The transmission of inflation uncertainty shocks may exhibit different patterns over the various stages of global macro-financial cycles.

Research questions

1. The objective of this paper is to explore the various empirical methodologies to measure the strength of the interdependence of inflation uncertainty.
    - In particular, this paper showcase the empirical framework by examining the case of the euro area and the UK with close economic, trade and financial integration.
2. A probability model using copulas
    - Probability models assume that changes in the probability of the two events occurring together reflect the strength of the dependence of those two events.
    - Copula: a flexible tool to construct a multivariate conditional distribution by combining marginal densities estimated separately.
3. Identification through heteroskedasticity
    - Potential endogeneity of the probability model estimates.
    - Propose a simple structural framework and estimate by exploiting heteroskedasticity in the data.

Related literature

1. The measures of inflation uncertainty
   - GARCH models: Grier and Perry (2000), Kontonikas (2004), Caporale et al. (2012)
   - Survey-based disagreement measures: Holland (1995), Giordani and Söderlind (2003), Clements and Harvey (2011), Wright (2011), Binder (2017)
   - *ex post* unpredictability measures: Friedman (1977), Stock and Watson (2007).
2. At risk approach
   - Adrian et al. (2019), López-Salido and Loria (2020), Sokol (2021)
   - This paper considers a two-country model rather than a variable conditional on other macroeconomic or financial variables.
3. Estimation of the strength of interdependence
   - Pesaran and Pick (2007) illustrate a two-country framework and show that the correlation-based tests of contagion could be biased due to endogeneity.
   - They define contagion as a situation whereby a crisis in one country increases the probability of a crisis in another country over and above what would be implied by the interdependence in non-crisis periods.
   - This paper exploits a broader definition of interdependence, that encompasses crises and non-crises periods alike, for identification through heteroskedasticity.

Measuring inflation uncertainty by *ex post* forecast errors

- Inflation uncertainty is estimated by *ex post* forecast errors from a bivariate VAR BEKK GARCH (1,1) model.

$$U_{t,h} = \Sigma_{t,h}^{1/2} \Sigma_{t|t-h}^{-1/2} e_{t|t-h} = \Sigma_{t,h}^{1/2} \Sigma_{t|t-h}^{-1/2} (\pi_t - \pi_{t|t-h}) \quad (1)$$

  - $e_{t|t-h}$: the h-period ahead forecast error conditional on the information available at the period of the prediction.
  - $\Sigma_{t,h}$: the unconditional variance-covariance matrix of e_t.
  - $\Sigma_{t|t-h}$: the conditional variance-covariance matrix of $e_{t|t-h}$.

- Data: Monthly inflation of the UK and the euro area (Jan 1997-March 2016).
  - Main considerations are the Global Financial Crisis and European Debt Crisis, macroeconomic shocks.
  - Exclude the episode of Brexit referendum that may have arisen due to political factors.

Measuring inflation uncertainty by *ex post* forecast errors

Figure 1: Inflation uncertainty

Measuring independence by a probability model

1. Marginal density functions
   - Two Piece Normal (TPN) distribution (Wallis, 2004)

$$f_{TPN}(x;\sigma_1,\sigma_2,\mu) = \begin{cases} A\exp\{-(x-\mu)^2/2\sigma_1^2\} & \text{if } x \le \mu \\ A\exp\{-(x-\mu)^2/2\sigma_2^2\} & \text{if } x > \mu \end{cases} \quad (2)$$

   where $A = (\sqrt{2\pi}(\sigma_1+\sigma_2)/2)^{-1}$.

   - Weighted Skew Normal (WSN) distribution (Makarova, 2018)

$$U = \underbrace{X}_{\text{baseline forecast error}} + \underbrace{\alpha \cdot Y \cdot I_{Y>m} + \beta \cdot Y \cdot I_{Y<k}}_{\text{Signal part based on revised forecast error}} \quad (3)$$

   where X and Y are bivariate $N(0,\sigma^2)$ with correlation coefficient, ρ.

2. Simulated minimum distance estimation

Estimation

Measuring independence by a probability model

1. Conditional density function: Copulas

$$\hat{\theta}_1 = \arg \max_{\theta_1} \sum_{t=1}^{T} \ln \left(f_1(U_{1t};\theta_1) \right) \tag{4}$$

$$\hat{\theta}_2 = \arg \max_{\theta_2} \sum_{t=1}^{T} \ln \left(f_2(U_{2t};\theta_2) \right) \tag{5}$$

$$\hat{\gamma} = \arg \max_{\gamma} \sum_{t=1}^{T} \ln \left(c(F_1(U_1;\hat{\theta}_1), F_2(U_2;\hat{\theta}_2);\gamma) \right) \tag{6}$$

- γ: copula parameter; $\hat{\theta}_1, \hat{\theta}_2$: estimated marginal densities.
- Frank copula allows for asymmetric dependence structures without favoring either upper or lower tail dependence.

Measuring independence by a probability model

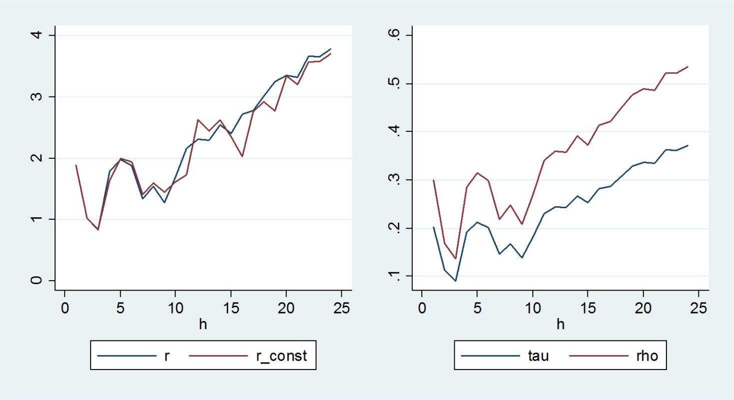

Figure 2: Copula parameters and rank correlation

Endogenous model of interdependence

1. An endogenous model of interdependence (Rigobon, 2019):

$$A\begin{bmatrix}U_1\\U_2\end{bmatrix}=\begin{bmatrix}\eta\\\varepsilon\end{bmatrix}, \text{with } A=\begin{bmatrix}1 & -\alpha\\-\beta & 1\end{bmatrix} \text{ and } \Omega=\begin{bmatrix}V_1 & C_{12}\\C_{12} & V_2\end{bmatrix} \tag{7}$$

   - U_1 and U_2: inflation uncertainty of the UK and the euro area.
   - η and ε: structural shocks, independent Normal distribution.
   - Ω: var-cov matrix of $[U_1\,U_2]'$.
   - α and β: the coefficients capturing interdependence of uncertainty.

2. A potential bias in reduced form if endogeneity is not properly addressed.

$$U_1=\frac{1}{(1-\alpha\beta)}(\eta+\alpha\varepsilon),\ U_2=\frac{1}{(1-\alpha\beta)}(\beta\eta+\varepsilon) \tag{8}$$

$$\Omega=\frac{1}{(1-\alpha\beta)^2}\begin{bmatrix}\alpha^2\sigma_\varepsilon^2+\sigma_\eta^2 & \alpha\sigma_\varepsilon^2+\beta\sigma_\eta^2\\\alpha\sigma_\varepsilon^2+\beta\sigma_\eta^2 & \sigma_\varepsilon^2+\beta^2\sigma_\eta^2\end{bmatrix} \tag{9}$$

Empirical evidence of endogeneity

Quantile regressions (Koenker, 2005) can provide empirical evidence of endogeneity.

- The model for the τ-th conditional quantile of U_1 on U_2, and vice versa, can be written as follows.

$$Q_{U_1}(\tau) = U_2\beta_1(\tau) \tag{10}$$
$$Q_{U_2}(\tau) = U_1\beta_2(\tau) \tag{11}$$

- The linear quantile regression model can misspecify the true relation of U_1 and U_2 if the true model is non-linear and/or the data have heavier tails.
  - This could lead to more variability in the coefficients in the extreme quantiles, resulting in potential sudden jumps in the slope estimates.
- Therefore, non-flat quantile treatment effects indicate potential non-linearity and/or non-Gaussianity of the data conditional on the quantile.

Empirical evidence of endogeneity

The quantile regressions of the UK inflation uncertainty on the euro inflation uncertainty.

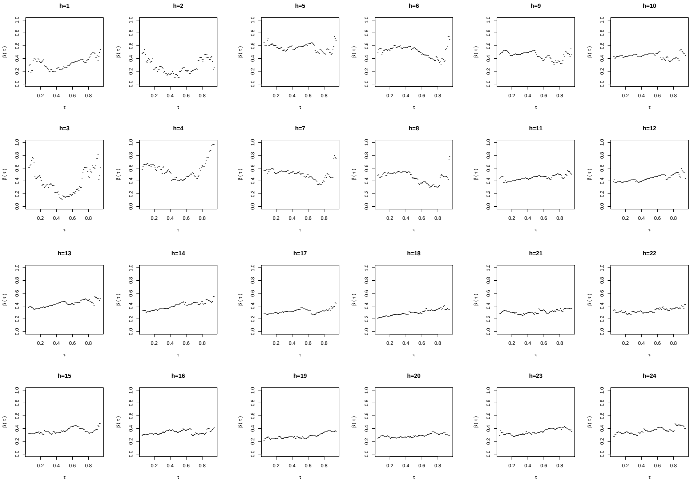

Figure 3: The estimated slope coefficients (β) of linear quantile regressions (I)

Empirical evidence of endogeneity

The quantile regressions of the euro inflation uncertainty on the UK inflation uncertainty.

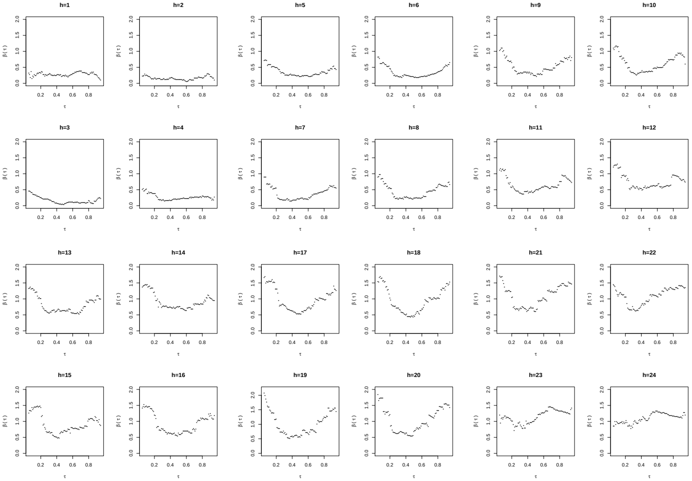

Figure 4: The estimated slope coefficients (β) of linear quantile regressions (II)

Empirical evidence of endogeneity

Simulations of quantile regressions with *iid* errors to confirm whether the quantile slope coefficients are flat across τ's when there is no endogeneity.

1. Assume a model of random variables X and Y: $Y = 0.5X + \epsilon$.
   - The relationship between X and Y is uniquely defined by an *iid* error terms, exogenous to X.
2. Draw a random variable X from various distributions.
   - $X \sim N(2, 1)$; $\epsilon \sim iidN(0, 1)$.
   - $X \sim t(5) + 2$; $\epsilon \sim iidN(0, 1)$.
   - $X \sim t(5) + 2$; $\epsilon \sim iidt(5)$.
   - $X \sim N(1, 1) + 0.25\chi^2(4)$; $\epsilon \sim iidN(0, 1)$.
3. Y is determined by drawing an error term, ϵ's.
4. This allows for different symmetric and asymmetric distributions while maintaining *iid* assumptions for error terms.
5. The sample size is $n = 1001$ for drawing X and the error terms are drawn for $r = 100$ times.

Empirical evidence of endogeneity

To contrast with the model of no endogeneity, a simple representation of the classical linear simultaneous equation model is constructed for the case of endogeneity.

1. Assume a model of random variables X and Y:
   $Y = 0.5X + 0.3X_2 + \epsilon_1$, $X = X_2 + X_3 + \epsilon_2$.
   - X and Y are endogenous in that both are determined within the model by errors and the exogenous variables, X_2 and X_3.
2. 4 combinations of underlying DGP for exogenous variables and error terms.
   - $X_2 \sim N(1,1)$; $X_3 \sim N(1,1)$; $\epsilon_1 \sim iidN(0,1)$; $\epsilon_2 \sim iidN(0,1)$.
   - $X_2 \sim t(5)+1$; $X_3 \sim t(5)+1$; $\epsilon_1 \sim iidN(0,1)$; $\epsilon_2 \sim iidN(0,1)$.
   - $X_2 \sim t(5)+1$; $X_3 \sim t(5)+1$; $\epsilon_1 \sim iidt(5)$; $\epsilon_2 \sim iidt(5)$.
   - $X_2 \sim N(1,1)$; $X_3 \sim 0.25\chi^2(4)$; $\epsilon_1 \sim iidN(0,1)$; $\epsilon_2 \sim iidN(0,1)$.

Empirical evidence of endogeneity

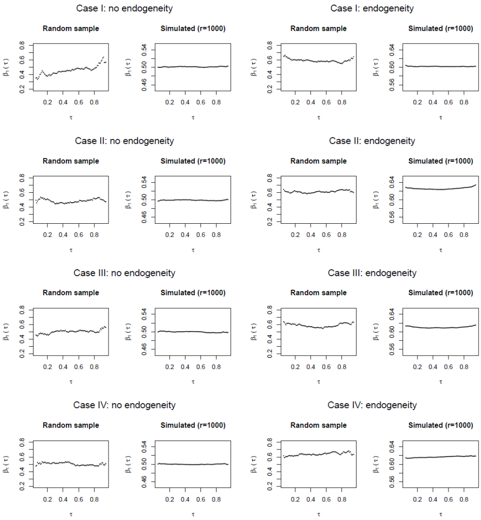

Figure 5: The slope coefficients of linear quantile regression: $\beta_1(\tau)$, simulation

Empirical evidence of endogeneity

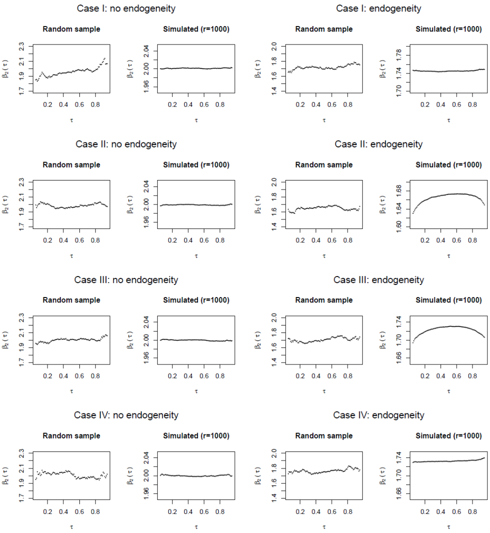

Figure 6: The slope coefficients of linear quantile regression: $\beta_2(\tau)$, simulation

Measuring independence by identification through heteroskedasticity

1. The variance-covariance matrix of the structural model:

$$A\Omega A^T == \begin{bmatrix} \cdot & -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta) \\ -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta) & \cdot \end{bmatrix} \quad (12)$$

2. Optimization problem: the off-diagonal terms equal to zero.

$$\min_{\alpha,\beta} f(\alpha, \beta; V_1, V_2, C_{12}) = -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta) \quad (13)$$

3. Assume parameter stability and heteroskedasticity.
   - Define *RH* if $\theta > median(\theta)$ and *RL* otherwise.

$$\theta = \frac{V_1}{V_2} = m(p; h, rw) \quad (14)$$

     - $p \in \{1, 2, 3\}$ with 1: pre-crisis period, 2: the Global Financial Crisis period, 3: post-crisis period. $h = 1, 2, \ldots, 24$ and $rw = 12$.

4. α and β is estimated by minimum distance using V_1, V_2, and C_{12}.

Measuring independence by identification through heteroskedasticity

- Crisis period:β exceeds 1 for longer term horizons $\rightarrow$ amplifying effects of the surprises in the UK inflation on the euro area inflation.
- Pre- and post-crisis period: the range of the estimates lies [-1, 1], mostly close to zero $\rightarrow$ interdependence is statistically insignificant.

Figure 7: Interdependece: identification through heteroskedasticity

Key findings and policy implications

1. Probability model: The popular measures of interdependence, such as correlations and copula parameters, can be biased if there is endogeneity.
2. Endogenous model: The strength of the propagation of inflation uncertainty intensifies during the GFS period while it dampens during the post-crisis period.
3. Monetary policy credibility becomes more crucial in times of great uncertainty with high degrees of contagion.

Thank you!

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The estimated parameters of marginal densities

TABLE 2 The estimated parameters of marginal densities

		h=6		h=12		h=18		h=24	
		UK	Euro	UK	Euro	UK	Euro	UK	Euro
WSN	α	-1.81	-3.61	-1.47	-3.19	-0.84	-3.19	-1.00	-0.96
		(0.36)	(1.29)	(0.42)	(0.54)	(0.90)	(0.49)	(0.37)	(0.01)
	β	-0.98	-2.72	-1.38	-0.21	-0.95	0.00	-1.10	-0.01
		(0.46)	(1.01)	(0.69)	(0.17)	(1.54)	(0.00)	(1.09)	(0.02)
	σ	0.99	0.56	1.22	1.47	1.13	1.74	1.07	2.28
		(0.08)	(0.29)	(0.29)	(0.08)	(0.51)	(0.10)	(0.31)	(0.12)
	MD	**1.99**	**13.56**	**14.13**	**22.70**	12.64	**46.18**	6.65	24.86
TPN	σ_1	1.70	0.56	1.58	1.61	3.91	1.07	0.54	1.84
		(0.71)	(0.27)	(0.42)	(1.51)	(0.27)	(0.31)	(0.67)	(0.75)
	σ_2	1.03	2.76	1.78	1.83	0.23	3.99	1.78	2.60
		(0.21)	(1.12)	(0.47)	(0.71)	(0.20)	(0.03)	(0.03)	(0.89)
	μ	0.35	-1.12	-0.26	-0.32	-2.59	-1.18	-0.96	-1.38
		(0.40)	(2.02)	(0.70)	(1.51)	(0.62)	(0.83)	(0.51)	(1.20)
	MD	4.64	39.19	15.37	39.97	**6.56**	55.71	**3.05**	**18.38**
Sample size		146		140		134		128	

Notes: MD denotes the minimum distance statistics for the equiprobable null hypothesis against the alternative hypothesis of bumps or dips in the probability. Under the null hypothesis, the MD statistic has an asymptotic χ^2 distribution (Cressie and Read, 1984).

main

The goodness-of-fit of marginal densities

Figure 8: Box plot of probability integral transformation

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