

International Comparison Program (ICP) Technical Advisory Group (TAG)

Similarity Linking vs. Multilateral Indices in ICP

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Introduction

- **What are similarity measures?**
- **What are minimum spanning trees?**
- **Building a price index based on MST**
- **Traditional indices in multilateral comparison (GEKS and CCD)**
- **Regional fixity principle and linking**

Similarity indices used

- For similarity linking two measures were used:
- A modified version of Diewert measure (2021, formula (218)): for countries 1 and 2, it is:

- $$\Delta_1^2 = \sum s_1^i \left(1 - \frac{p_2^i/p_1^i}{LA_1^2}\right)^2 + \sum s_2^i \left(1 - \frac{p_1^i/p_2^i}{LA_2^1}\right)^2 \quad (1)$$

- Where LA is the Laspeyres index.
- And Diewert (2009):

- $$\Delta_1^2 = \sum \frac{s_1^i + s_2^i}{2} \left[\ln\left(\frac{p_2^i}{p_1^i} / T_1^2\right) \right]^2 \quad (2)$$

- Where T is the Törnqvist index.
- The main difference between formulae (1) and (2) is weights. Formula (1) is more punishing for deviations/errors in larger items. Also, compare the squared first term of Taylor expansion of $\left[\ln\left(\frac{p_2^i}{p_1^i} / T_1^2\right) \right]$ around 1, which evaluates to $\left(1 - \frac{p_2^i/p_1^i}{T_1^2}\right)^2$, to formula (1).

Similarity indices used (cont.)

- Formula (1) was derived from the original version of the formula (Diewert (2021, formula (218))) in the following way:
- $$\Delta_1^2 = \sum \left(s_1^i - \frac{p_2^i q_1^i}{\sum p_2^i q_1^i} \right)^2 + \sum \left(s_2^i - \frac{p_1^i q_2^i}{\sum p_1^i q_2^i} \right)^2 = \sum \left(s_1^i - \frac{(p_2^i/p_1^i)p_1^i q_1^i}{\sum (p_2^i/p_1^i)p_1^i q_1^i} \right)^2 + \sum \left(s_2^i - \frac{(p_1^i/p_2^i)p_2^i q_2^i}{\sum (p_1^i/p_2^i)p_2^i q_2^i} \right)^2 = \sum \left(s_1^i - \frac{(p_2^i/p_1^i)p_1^i q_1^i / \sum p_1^i q_1^i}{\sum (p_2^i/p_1^i)p_1^i q_1^i / \sum p_1^i q_1^i} \right)^2 + \sum \left(s_2^i - \frac{(p_1^i/p_2^i)p_2^i q_2^i / \sum p_2^i q_2^i}{\sum (p_1^i/p_2^i)p_2^i q_2^i / \sum p_2^i q_2^i} \right)^2 = \sum \left(s_1^i - \frac{(p_2^i/p_1^i)s_1^i}{\sum (p_2^i/p_1^i)s_1^i} \right)^2 + \sum \left(s_2^i - \frac{(p_1^i/p_2^i)s_2^i}{\sum (p_1^i/p_2^i)s_2^i} \right)^2 = \sum \left(s_1^i - \frac{(p_2^i/p_1^i)s_1^i}{LA_1^2} \right)^2 + \sum \left(s_2^i - \frac{(p_1^i/p_2^i)s_2^i}{LA_2^1} \right)^2 = \sum s_1^{i^2} \left(1 - \frac{p_2^i/p_1^i}{LA_1^2} \right)^2 + \sum s_2^{i^2} \left(1 - \frac{p_1^i/p_2^i}{LA_2^1} \right)^2$$
- The original formulation has the advantage that it accommodates missing prices. There are no missing BH PPPs in the ICP, however, thus in this setting the two formulations are equivalent.

Similarity indices used (cont.)

MSTs [*minimum spanning trees*] were constructed for 154 countries that took part in the 2021 ICP exercise [PCE without Net Purchases Abroad], based on measures (1) and (2).

The MSTs are presented in the next two pages below.

The countries are color-coded, according to the ICP regions:

Africa – green

Asia – red

OECD – yellow

LAC – grey

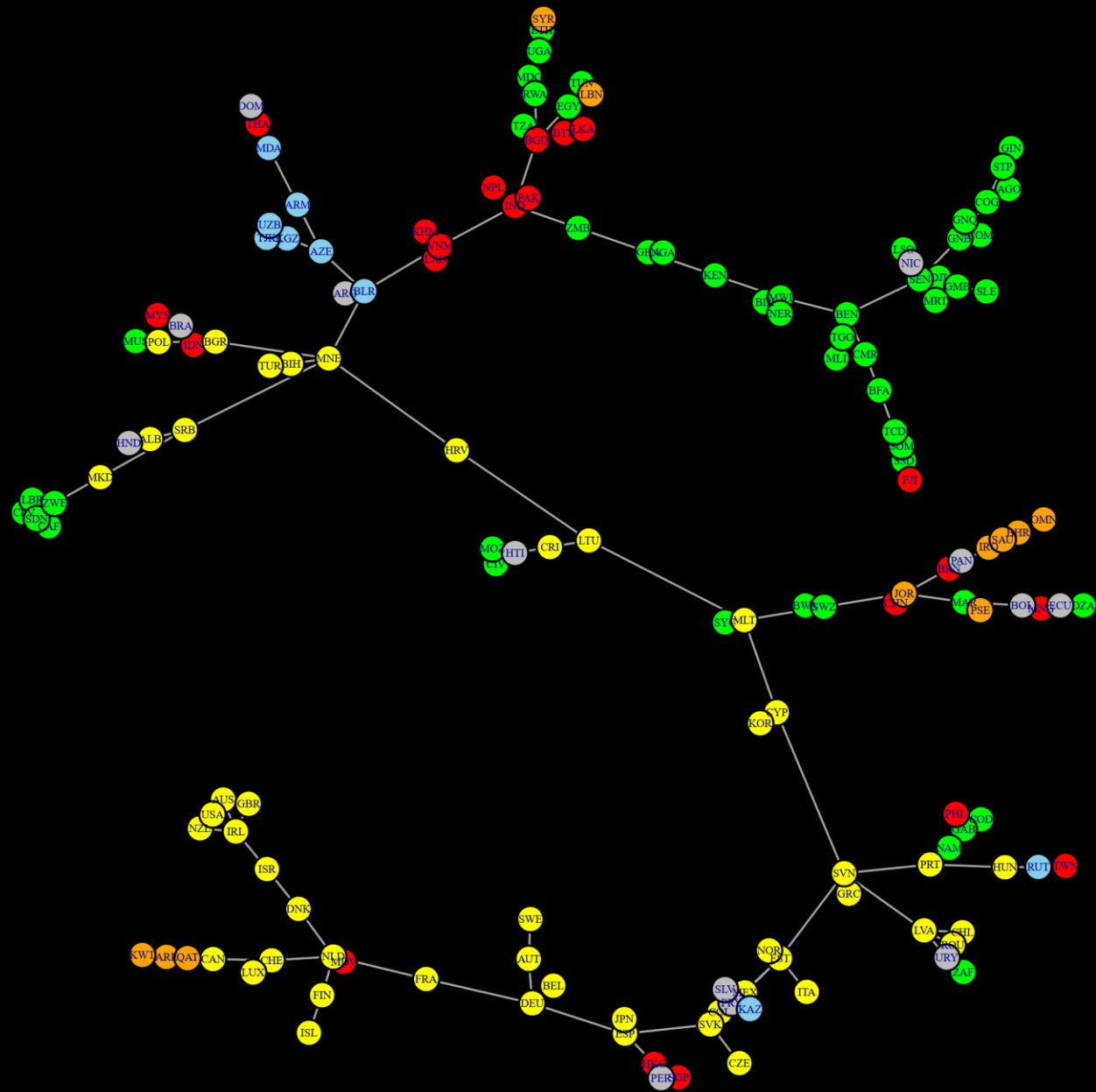
WAS – orange

CIS - cyan

II. Global MSTs

Figure 1. Global
MST, formula (1)

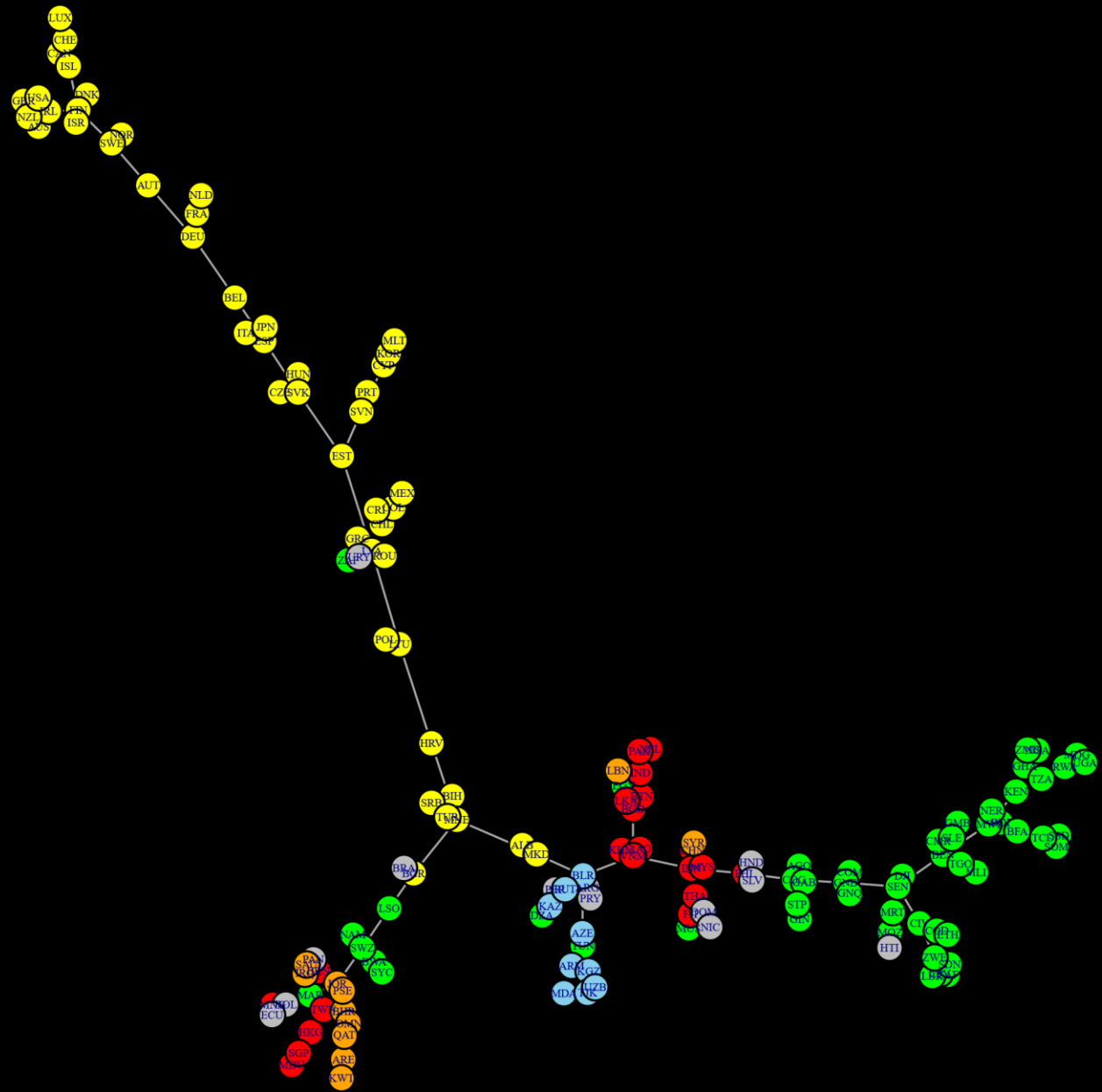
$$\Delta_1^2 = \sum s_1^{i^2} \left(1 - \frac{p_2^i/p_1^i}{LA_1^2} \right)^2 + \sum s_2^{i^2} \left(1 - \frac{p_1^i/p_2^i}{LA_2^1} \right)^2$$



II. Global MSTs

**Figure 2. Global
MST, formula (2)**

$$\Delta_1^2 = \sum \frac{s_1^i + s_2^i}{2} \left[\ln \left(\frac{p_2^i}{p_1^i} / T_1^2 \right) \right]^2$$



Global MSTs

From Figures (1) and (2) we deduce that Figure (2) retains regional clusters better than Figure (1) and is topologically simpler. For example, the CIS cluster (in blue) is broken up and is appearing in three places in Figure (1), while it is all in one cluster in Figure (2). Some unexpected links appear in Figure (1) as well, such as MDV-NLD, PER-HKG, IDN-BGR, etc. The same is happening with Asia where Asian countries are in tighter clusters in Figure (2) than in Figure (1) [look at the position of CHN and FJI in Figure (1), for example], and it is similar with other regions, e.g., Africa which is being broken up into multiple clusters, with some of the links to countries from other regions being quite unexpected.

As a background, the global ICP is build up from regional comparisons, using a core item list to link them. Ideally, the item quality should be comparable across all regions. However, the prices are validated in each region separately. Thus, some degree of “regionalization” of the global items is being introduced. And formula (2) is picking that up better than formula (1).

[This is not to say that the abovementioned price “regionalization” is a good thing. The discussion here is about various indices, and how sensitive they are to reflect structures already embedded in the data.]

With these different topologies, one could expect significantly different PPPs from implementing formula (1) vs. formula (2), which is reflected in Figure 3.

Global MSTs

Figure (3) plots ratio of the two MST-linked indices (both are Tornqvist based).

Countries are grouped by region, going from Africa on the left to CIS on the right.

One can see that the differences between the two indices range from minus 8% [FJI] to plus 18% [LKA] (USA = 1).

However, the differences are not random: the OECD region tends to be a couple of percentage higher on average when using formula (1), and the CIS region is almost uniformly around 5% higher.

Thus, we conclude that the MSTs and, consequently, the MST-linked indices, are highly dependent on the similarity measure used and may result in significantly different PPPs.

Global MSTs

Figure 3. MST-Linked Tornqvist Indices (ratio of formula (2) to formula (1))



Multi-lateral indices (CCD and GEKS), one- and two-stage

Now, compare the results of the CAR method (CCD(two-stage) and GEKS(two-stage)) – this is the ICP standard linking method used during 2011-2021 comparisons, and PPPs from the unrestricted global aggregations with 154 countries (CCD (one-stage) and EKS (one-stage)). The results in terms of relatives are presented in Figure 4.

One can observe significant deviations, especially between CCD (one-stage) and EKS (one-stage).

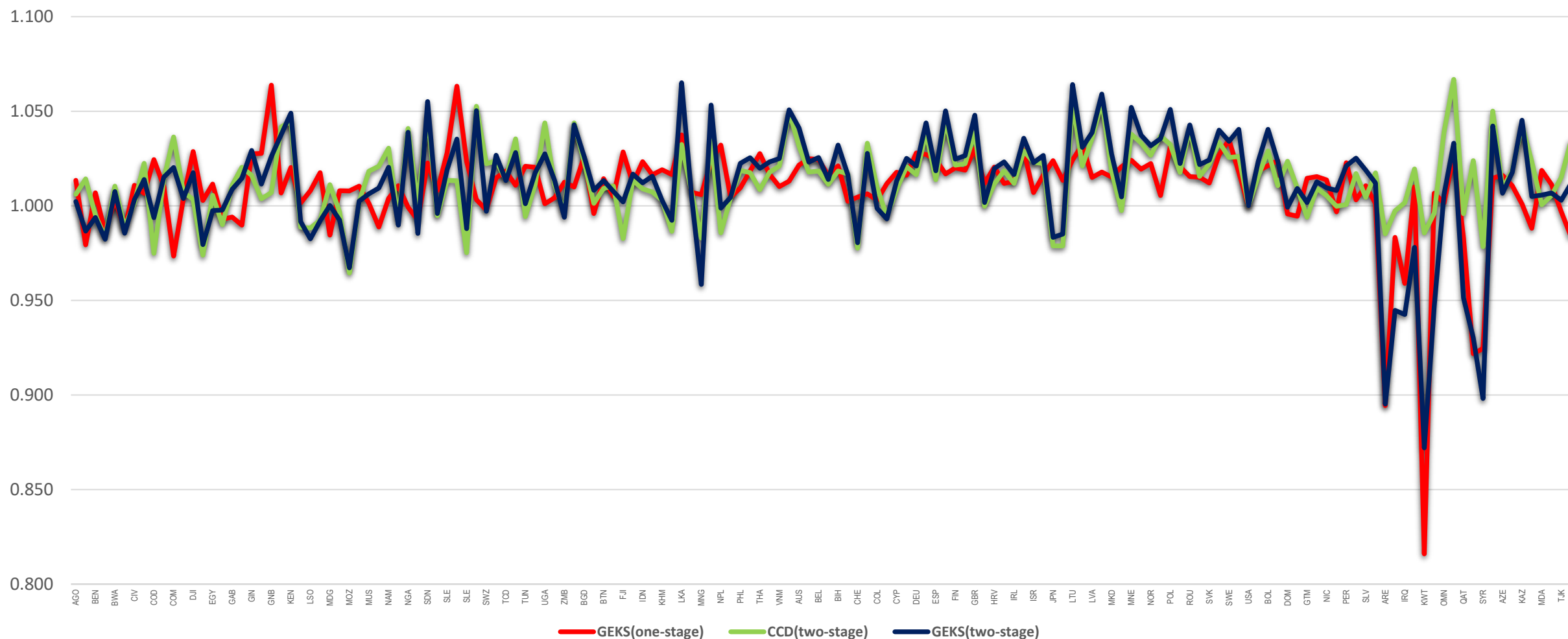
Note that the benchmark against which all other indices are compared is the CCD index.

The reason for that is that GEKS is built on binary Fishers and CCD is built on binary Tornqvists. And Fishers were found to be biased in the current setting. In one particular case (KWT-SOM), distortion due to Fisher index was found to be 94%. As the result, GEKS can be biased up to around plus 22% to minus 7%.

(see detailed description of this effect in Dikhanov (2024) and Dikhanov (2025)).

Multi-lateral indices (CCD and GEKS), one- and two-stage

Fig.4. GEKS(one-stage), CCD(two-stage) and GEKS(two-stage) (CCD(one-stage) = 1.00)



Multi-lateral indices (CCD and GEKS), one- and two-stage

Now, we compare the linking factors (PPPs of the base countries) using the CAR method (CCD(two-stage) and GEKS(two-stage)), and the PPPs of the base countries from the unrestricted global aggregations with 154 countries (CCD (one-stage) and EKS (one-stage)).

As we can see there isn't much difference among the four indices. This is saying that our choice of base countries is quite solid. [Ideally, we would like to have base countries that are most stable in their regions. It seems we found them: ZAF for Africa, HKG for Asia, USA for OECD, BRA for LAC, OMN for WAS, RUS for CIS].

PLIs of base countries	AFR	ASI	EUO	LAC	WAS	CIS
CCD(one-stage)	0.5227	0.8268	1.0000	0.4575	0.5388	0.3611
GEKS (one-stage)	0.5249	0.8365	1.0000	0.4684	0.5395	0.3652
CCD (two-stage)	0.5285	0.8379	1.0000	0.4623	0.5597	0.3631
EKS (two-stage)	0.5297	0.8406	1.0000	0.4682	0.5420	0.3636

Of course, if we change any base country, we may get much less consistent linking factors. Let's see what would happen with the linking factors for WAS region if we changed the base country for WAS from OMN to KWT (*changing base countries of one or all regions would not change the PPPs for any country, the linking and aggregation procedures are base-country invariant, here we discuss only the inter-regional linking factors which are changing*):

PLIs of base countries	AFR	ASI	EUO	LAC	WAS	CIS
CCD(one-stage)	0.5227	0.8268	1.0000	0.4575	0.7706	0.3611
GEKS (one-stage)	0.5249	0.8365	1.0000	0.4684	0.6288	0.3652
CCD (two-stage)	0.5285	0.8379	1.0000	0.4623	0.7595	0.3631
EKS (two-stage)	0.5297	0.8406	1.0000	0.4682	0.6720	0.3636

If we locate KWT in Figure 4 above, we will see those numbers for WAS shown in red, converted to relative terms.

MST-linked indices vs. CCD and GEKS

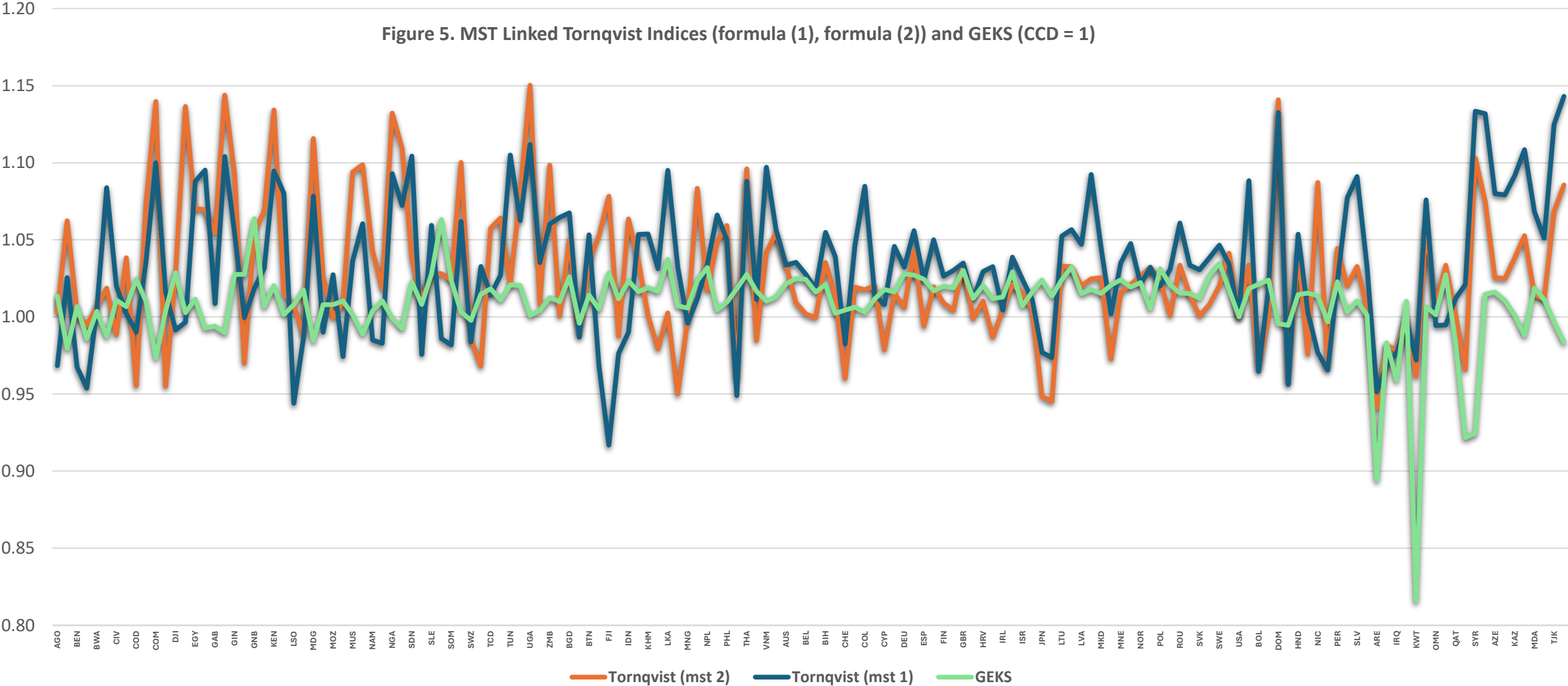
Finally, we can compare the MST-linked price indices to the multilateral price indices.

Figure (5) shows two MST indices (both Törnqvist-based), based on formulae (1) and (2), along with the GEKS compared to the CCD. One can see that the GEKS is much closer to the CCD than either of the MST-linked Törnqvist indices, except for the West Asia region where the GEKS deviates greatly from the CCD.

Thus, the MST-linked Törnqvist indices based on formulae (1) and (2) can differ systematically from each other while deviating further away from the CCD index on average more than the GEKS. (Note that red line in Fig.4 is the same as green line in Fig.5).

MST-linked indices vs. CCD and GEKS

Figure 5. MST Linked Tornqvist Indices (formula (1), formula (2)) and GEKS (CCD = 1)



Conclusions

First, the MSTs (Minimum Spanning Trees) are highly dependent on the similarity measure used. The second MST (formula (2) based) is better at discerning patterns embedded in the data.

Second, the results of the MST-linked PPPs differ significantly among themselves and from those of the multilateral indices. (Of course, we are already aware of the substantial differences among multilateral indices, such as CCD and GEKS.)

Third, the choice of base countries turned out to be quite good for the CAR regional linking (a bonus result).

Fourth, the second MST method reveals 'regionalization' patterns in global items. In other words, regional validation process appears to influence the 'qualities' of global items within a region, creating regional patterns that are 'discovered' by the second MST.

References

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Thank you!

