OPTIONS FOR PRODUCING "SMOOTHENED" PPP TIME-SERIES FOR THE YEARS BETWEEN REFERENCE YEAR COMPARISONS

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The Problem

Suppose we have data from two benchmarks – 2011 and 2017

- **PPPs at the Basic Heading level and at higher levels of aggregation**
- **Expenditure data in national currency units at BH level**

We have also data for the intermediate years:

- **year-on-year deflators**
- **expenditure in national currency units at the basic heading level**

The Problem: Construct smoothed series of PPPs for the intermediate years taking into account all the data available.

Question 1

What is the desired level of disaggregation at which extrapolation should be undertaken?

Our analytical results suggest that it is best if extrapolation is undertaken at the basic heading level. It is generally expected that the products included in a basic heading are not only homogeneous but they also exhibit similar price level differences across countries and movements over time.

Question 2

How do we construct smoothed PPP series at whatever level we choose to extrapolate?

- **Forward extrapolation – take benchmark PPPs in the initial benchmark (2011) and update with relative movements in prices**
- **Backward extrapolation – take benchmark PPPs in final year (2017) and take the series backward using relative price movements.**

We note that forward extrapolated series from 2011 will be different from 2017 benchmark and vice versa – inconsistency between price movements and benchmark comparisons

How do we combine the forward and backward extrapolated series to construct a predictor?

We discuss four options in the paper.

Option 1 – Penn World Table approach

From PWT 8.0 onwards, the approach for interpolation between benchmarks is to make use of a weighted arithmetic average of the forward and backward extrapolations. The following formula is used

$$PPP_c^t = (1-w^t)\cdot PPP_c^1 \cdot \frac{P_c^t / P_c^1}{P_{USA}^t / P_{USA}^1} + w^t \cdot PPP_c^T \cdot \frac{P_c^t / P_c^T}{P_{USA}^t / P_{USA}^T} \quad c = 1,2,\ldots,M; t = 2,\ldots,T-1$$

$$\text{where } w^t = (t-1)/(T-1)$$

Our preference is to make use of a multiplicative or geometric average which guarantees that the procedure is independent of the choice of the reference country and also transitive.

$$PPP_c^t = \left[PPP_c^1 \cdot \frac{P_c^t / P_c^1}{P_{USA}^t / P_{USA}^1}\right]^{(1-w^t)} \times \left[PPP_c^T \cdot \frac{P_c^t / P_c^T}{P_{USA}^t / P_{USA}^T}\right]^{w^t}$$

Weights used are intuitive but somewhat ad hoc.

Option 2 – State-space approach

Here we make use of the approach developed by Rao, Rambaldi and Doran (2010) and used in the series available on UQICD. We use a simplified version of the RRD approach.

- **We assume that benchmark data for 2011 and 2017 is given and taken to be without any errors. This means for each country we have**

$$PPP_c^1 \quad \text{and} \quad PPP_c^T$$

- **PPPs are updated from year *t* to *t+1* using**

$$PPP_c^t = \left[PPP_c^1 \cdot \frac{P_c^t / P_c^1}{P_{USA}^t / P_{USA}^1} \right] \cdot v_c^t$$

- **We note here that without some error term in the updating equation, we cannot reconcile the benchmarks.**

Option 2 – State-space approach

In this case, it is easy to show that the smoothed series for the years between 1 and T (2011 and 2017) are given by a weighted geometric average of the forward and backward extrapolations.

$$\text{Forward extrapolation: } \overleftrightarrow{PPP}_c^{t,1} = \left[PPP_c^1 \cdot \frac{P_c^t / P_c^1}{P_{USA}^t / P_{USA}^1} \right].$$

$$\text{Backward extrapolation: } \overleftrightarrow{PPP}_c^{t,T} = \left[PPP_c^T \cdot \frac{P_c^t / P_c^T}{P_{USA}^t / P_{USA}^T} \right].$$

The smoothed series for each *t* in the interval 1 to T is given by:

$$PPP_c^t(RRD) = \left(\overleftrightarrow{PPP}_c^{t,1} \right)^{\gamma_c^1} \cdot \left(\overleftrightarrow{PPP}_c^{t,\mathrm{T}} \right)^{\gamma_c^T} .$$

With the weights: $\gamma_c^1 > 0;\ and\ \gamma_c^T > 0\ such\ that\ \gamma_c^1 + \gamma_c^T = 1$

Option 3 – Mixed PWT and State-space approach

- **In this case of PWT weights depend on how far is the year from the initial and final periods and weights move linearly.**
- **In the case of RRD state-space approach, the weights are determined by the data and relative reliability of the deflators.**
- **We may consider hybrid approach between PWT and state-space approach where RRD and PWT weights are used.**

$$PPP_c^t(RRD\_PWT) = \left(\overleftrightarrow{PPP}_c^{t,1}\right)^{(\gamma_c^1+1-w_t)/2} \cdot \left(\overleftrightarrow{PPP}_c^{t,T}\right)^{(\gamma_c^T+w_t)/2}$$

Option 4 – Diewert and Fox (2015) approach

This approach is non-stochastic like PWT but the extrapolations are based on the time-space consistent approach developed by Diewert and Fox (2015). Their approach has several steps.

1. **In the first step, country specific growth rates are combined using the Fisher index to compute growth rate for the group of countries under consideration.**
2. **The Laspeyres, Paasche and Fisher indexes are computed using:**

$$\Gamma_L = \sum_{c=1}^{M} s_c^1 \cdot \left(\frac{Q_c^T}{Q_c^1} \right) \qquad \Gamma_P = \left[\sum_{c=1}^{M} s_c^T \cdot \left(\frac{Q_c^1}{Q_c^T} \right) \right]^{-1} \qquad \Gamma_F = \left[\Gamma_L \cdot \Gamma_P \right]^{1/2}$$

where the expenditure shares in the base and current (2011 and 2017) are given by

$$s_c^t = \frac{e_c^t}{\sum_{c=1}^{M} e_c^t}$$

where the expenditures are either exchange rate or PPP converted.

Option 4 – Diewert and Fox (2015) approach

Step 2: For each country, compute "interpolated" volume measures for the initial and end points 1 and T as:

$$q_{I,c}^{1} = s_c^1 \quad and \quad q_{I,c}^{T} = s_c^T \cdot \Gamma_F$$

For years in between 1 and T use the following approach:

Implied growth rate from interpolations: $g_c = \dfrac{q_c^T}{q_c^1}$

Actual or observed growth rate: $G_c = \dfrac{Q_c^T}{Q_c^1}$

Note that the interpolated and observed growth rates differ. For each country, the method suggests smoothing growth rates using:

$$q_{I,c}^{t} = q_{I,c}^{t-1} \cdot \left[\frac{Q_c^t}{Q_c^{t-1}}\right] \cdot \alpha_c \quad \text{where} \quad \alpha_c = \left[\frac{g_c}{G_c}\right]^{1/(T-1)}$$

Step 3: Using the interpolated quantity series, computed PPPs for the intermediate years using expenditure data and the quantity series.

Comments on Diewert-Fox approach

- This method is quite sensitive to the expenditure data at the basic heading level. So reliability of expenditure data at the level of extrapolation is important.
  - An implication is that this method would apply for extrapolation at an aggregated level.
- Since the method relies heavily on growth rate from year 1 to year T based on Fisher index, the method could be quite sensitive to the end-points. Further a fixed-base index is used which means that it would work well for interpolations in short intervals.
- The method relies on the growth rate of the group of countries under consideration. For example, growth rate could be influenced by two or three large countries with high growth rates (China and India). So the whole extrapolation relies on these two countries.
- The discrepancy between growth rate in interpolated series and observed series from 1 to T is applied at a fixed compound growth rate over all the years.

Illustration of PWT and RRD methods

Table 1:Tableau of PPPs (Reference country D)

	0 BM1	1	2	3 BM2
A	0.7	?	?	0.8
B	3.4	?	?	4.3
C	89	?	?	91

Table 2: Price deflators (expressed relative to reference country D)

	1	2	3 BM2
A	1.04	1.02	1.05
B	1.05	1.05	1.02
C	1	1.02	1.01

$$Q_1 = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}; \; Q_1 = \begin{bmatrix} 0.03 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.02 \end{bmatrix}; \; Q_3 = \begin{bmatrix} 0.021 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.02 \end{bmatrix}$$

Illustration of PWT and RRD methods

					State-space						
Country	t=0	t=1					t=2				
	Bench1	F	weight	B	weight	S	F	weight	B	weight	S	Bench 2
A	0.700	0.728	0.718	0.747	0.282	0.732	0.743	0.296	0.762	0.704	0.756	0.8
B	3.400	3.570	0.640	4.015	0.360	3.669	3.749	0.353	4.216	0.647	4.045	4.3
C	89.000	89.000	0.750	88.332	0.250	88.874	90.780	0.250	90.099	0.750	90.269	91

F =	Forward Filter	B=	Backward	S =	Smoothe

					Penn World Table							
Country	t=0	t=1					t=2					
	Bench1	F	weight	B	weight	PWT	F	weight	B	weight	PWT	Bench 2
A	0.700	0.728	0.667	0.747	0.333	0.734	0.743	0.333	0.762	0.667	0.755	0.8
B	3.400	3.570	0.667	4.015	0.333	3.718	3.749	0.333	4.216	0.667	4.060	4.3
C	89.000	89.000	0.667	88.332	0.333	88.778	90.780	0.333	90.099	0.667	90.326	91

PWT and RRD methods - equivalence

We have been able to prove that under the conditions:

- **Benchmark PPPs are measured without error**
- **Reliability of deflators in different countries remains the same over the interpolation period**

the weights accorded in the state-space approach to forward and backward interpolation are identical to the weights used in PWT 8.0 onwards.

However, the result does not hold if benchmarks are measured with error.