

Linking and Calculation of Global Results

This chapter describes the steps and procedures used to calculate global purchasing power parities (PPPs) in the 2011 round of the International Comparison Program (ICP). The steps and procedures were recommended by the ICP's Technical Advisory Group (TAG).

To calculate the global PPPs, the ICP regions and the Eurostat–Organisation for Economic Co-operation and Development (OECD) PPP Programme provided for the economies in their regions the following inputs:

- Global core list (GCL) item prices in national currency
- Within-region PPPs at the basic heading (BH), aggregate, and analytical category levels (hereafter referred to as regional PPPs) following the ICP Classification
- National accounts expenditures at the basic heading level in national currency
- The 2011 annual average exchange rates to the U.S. dollar and population data.

The first two sections that follow describe the linking steps for the standard linking approach. The next section deals with the nonstandard linking approaches and special participation cases. The standard ICP regions in ICP 2011 were Africa, Asia and the Pacific, Latin America, and Western Asia, as well as the Eurostat-OECD region. Special participation cases were the two singleton economies, the Islamic Republic of Iran and Georgia; the Commonwealth of

Independent States (CIS); the Caribbean; Cuba; the Pacific Islands; and the dual-participation economies: the Russian Federation, the Arab Republic of Egypt, and Sudan.

LINKING AT THE BASIC HEADING LEVEL

Regional BH PPPs were calculated using either the Éltető-Köves-Szulc* (EKS*) method (Eurostat-OECD and CIS regions) or the weighted country product dummy (CPD-W)¹ method (other ICP regions except Asia and the Pacific, which used the unweighted country product dummy [CPD] method). Interregional linking factors were calculated using the CPD-W method. In this method, explicit weights of 3:1 were used to weight the important items and the less important ones.

Calculation of the BH PPPs in the global comparison involves four steps:

- *Step A1.* Calculate the regional BH PPPs based on both the regional and GCL items, using either the EKS* method or the CPD-W method.² BH PPPs must follow the ICP BH classification. When a region is not following the ICP BH classification,³ calculate the BH PPPs according to this classification by aggregating the BH PPPs using the respective national accounts expenditures.
- *Step A2.* Convert all GCL item prices in national currency for the economies in a region into a

- common regional numéraire using the economy's regional BH PPPs from step A1.
- *Step A3.* Using the CPD-W method, process the converted GCL item prices for all regions resulting from step A2 to generate the interregional linking factors for the basic headings. These interregional linking factors are interregional BH PPPs, expressed in the regional numéraires to the world numéraire.
 - *Step A4.* To calculate the global economy's PPPs to the world numéraire and ensure fixity of an economy's regional BH PPPs in the global comparison, multiply each economy's regional BH PPP from step A1 by the interregional linking factor resulting from step A3. The PPPs derived from this step are the global BH PPPs with regional fixity.

LINKING AT THE AGGREGATE LEVEL

The Gini-Éltető-Köves-Szulc (GEKS)⁴ aggregation method, with further redistribution of regional volumes in accordance with an economy's regional volume shares (known as the country aggregation with volume redistribution procedure, CAR-volume), was used to obtain real expenditures (hereafter referred to as volumes) and indirect aggregated PPPs with regional fixity. All economies in the standard ICP regions participated simultaneously and equally in the global aggregation using the GEKS method.

The steps for linking at the aggregate levels are the following:

- *Step B1.* Calculate the regional PPPs by applying the GEKS⁵ aggregation to the regional BH PPPs from step A1 and the BH national accounts data derived in accordance with the regional fixity schema.⁶ Economy regional volumes are derived as nominal values in national currency divided by the regional aggregated PPPs.⁷
- *Step B2.* Obtain an economy's volume shares in the regional results for each level of aggregation up to the gross domestic product (GDP) using data from step B1. These data for aggregated headings and ICP analytical categories were supplied by the regions.

- *Step B3.* Calculate an economy's aggregated PPPs in the global comparison by applying the unrestricted GEKS aggregation to the global BH PPPs derived from step A4 and the national accounts BH expenditures in national currency for each level of aggregation up to GDP. The starting point is a matrix of 150-plus global BH PPPs for 140-plus economies from the standard ICP regions and another matrix of the same size containing BH national accounts expenditures in national currency. Economy volumes in the global unrestricted comparison in a global numéraire are derived as nominal values in national currency divided by the aggregated PPPs from the unrestricted global comparison.
- *Step B4.* Obtain the regional volume totals in the global comparison by summing up the total volumes for individual economies for each region derived from step B3 for each level of aggregation up to GDP.
- *Step B5.* Distribute the regional volume totals from step B4 among the economies in the regions according to the economy shares in the regional results derived from step B2 in order to uphold regional fixity for each level of aggregation up to GDP.
- *Step B6.* Calculate the aggregated global PPPs' national currency/world numéraire *indirectly* by dividing economies' nominal expenditures by the volumes derived from step B5 for each level of aggregation up to GDP.

SPECIAL CASES

In ICP 2011, in two categories of special linking cases the linking approach differed from the one just described. The first category covered nonstandard survey methodologies; the second covered nonstandard regional or economy participation.

Nonstandard Linking Approaches

Housing

All economies participating in ICP 2011 were asked to collect annual average rents for a global list of dwelling types and dwelling stock data: number of dwellings, usable surface area in

square meters, and information on three quality indicators—availability of electricity, water supply, and in-house toilet. National accounts expenditure data on actual and imputed rentals were collected by means of expenditure questionnaires. However, not all economies were able to report rents and dwelling stock data, and some economies were only able to provide rents for a limited subset of dwelling types or limited dwelling stock data. Each regional coordinating agency then decided on the best way to use the collected data for its region:

- The Africa, Latin America, Caribbean, and Western Asia regions calculated their regional PPPs on the basis of the rents collected for the global list of dwelling types, relying on the same CPD method used for the rest of household consumption but without the importance indicators.
- The Asia and the Pacific region, after in-depth analysis of the available data, resorted to using a reference volume approach and the respective indirect PPPs. This implies that the relative volumes of housing services between economies were equal to the relative volumes of household expenditure, excluding rents.
- The Eurostat-OECD region used a mix of rents and dwelling stock data. Generally, for economies that have a well-developed rental market—that is, those economies that use the stratification method for rents in national accounts—PPP were determined on the basis of the rental data, whereas for other economies dwelling stock data were used to obtain estimates of PPPs indirectly—that is, for those economies that were using the cost approach for rents in national accounts. Indirect PPPs are based on the relationship $\text{price} \times \text{quantity} = \text{expenditure}$. An indirect PPP can be derived by dividing the expenditure on rents from an economy's national accounts by the real expenditure on rents estimated using dwelling stock data adjusted for quality. This is known as the quantity approach of estimating real expenditures directly and the PPPs indirectly.
- The quantity approach was used in the CIS region, which was then linked to other regions using Russia, a member of the OECD comparison, as the bridge economy.

The rental data used to link the Africa, Latin America, and Western Asia regions were the same as those that entered the calculation of their regional PPPs. The linking factors for these three regions were calculated by means of the same CPD method used for the link of the rest of household expenditures.

The Asia and the Pacific and the Eurostat-OECD regions were linked to each other and to the rest of the world by the quantity approach through use of the dwelling stock data.

The plausibility of each economy's estimate of number of dwellings was evaluated by calculating the ratio of the number of dwellings to the total population. Economies with very high or very low ratios were not included in the linking process. For each economy with a plausible estimate of number of dwellings, the data on housing quality were reviewed. Three quality indicators were available: share of dwellings with electricity, share of dwellings with inside water, and share of dwellings with a private toilet. Only economies for which a plausible estimate for all three indicators was available or could be consciously imputed were included in the linking process.

Government Compensation

Adjustments for productivity differences were taken into account in some regional comparisons as well as in the global linking. Adjustments were made to the PPPs and respectively to the real expenditure estimates for government in the Africa, Asia and the Pacific, Latin America, and Caribbean regions. No productivity adjustments were applied to the Eurostat-OECD, CIS, and Western Asia regions because differences in labor productivity within each of those regions were considered not very high. However, productivity adjustments were made to all regions when the interregional linking factors were estimated to maintain consistency in the global comparison.

For education, the Eurostat-OECD region applied an output approach in which volumes are based on numbers of students and average student scores from the Programme for International Student Assessment (PISA). Within-region PPPs were linked to the rest of the world using five Latin American economies—Brazil, Colombia, Panama, Peru,

and Uruguay—that had data for both the input approach used by ICP regions and the Eurostat-OECD output approach. The specific linking steps were as follows:

- Global education basic heading PPPs⁸ for the standard ICP regions, excluding Eurostat-OECD, were obtained by multiplying regional PPPs by the productivity-adjusted linking factors.
- The global basic heading PPPs for the standard ICP regions, excluding Eurostat-OECD, were aggregated using the GEKS method to obtain the total education PPPs.
- The total education linking factor for the Eurostat-OECD region was calculated via five Latin American economies for which the output approach was feasible as the ratio of the geometric mean of PPPs from the output approach to the geometric mean of PPPs from the input approach. The result was the total education linking factor for each region.
- For global aggregation covering all five standard regions, the six basic headings under the individual government consumption expenditure for education were collapsed into one. The result was one basic heading under household consumption expenditure and one heading under individual government consumption expenditure.² The global aggregation resulted in linked global PPPs, maintaining regional fixity for all regions.

For health, the Eurostat-OECD comparison applied an output approach for government health providers (hospitals) to its regional comparison, which was based on quasi-prices for a set of hospital services. Basic heading weights from the health accounts were used for the health PPP aggregation. For linking, Eurostat-OECD provided input data for government health providers based on the former input cost approach. Input data for the other health basic headings were identical to those produced by the standard ICP method.¹⁰ However, when fixity of regional results was applied in the country aggregation with redistribution (CAR) procedure, the fixity preserved was that of the aggregate analytical categories resulting from the current health approach used by the Eurostat-OECD comparison.

Construction and Civil Engineering

The standard ICP 2011 approach for estimating construction and civil engineering PPPs covered four separate but consecutive steps:

- Input prices collected for materials, labor, and hire of equipment were allocated to the three construction category basic headings (residential buildings, nonresidential buildings, and civil engineering works) using product relevancy information.
- PPPs for the input groups (materials, labor, and equipment), or subheadings under the three basic headings, were calculated using the CPD, resulting in nine sets of subheading PPPs.
- The subheading PPPs were aggregated using resource mixes as weights, resulting in three sets of basic heading PPPs.
- PPPs for the three basic headings were aggregated using national accounts expenditure data as weights, resulting in PPPs for the construction category.

The Eurostat-OECD approach to estimating construction and civil engineering PPPs differs from the ICP approach,¹¹ and thus several economies in the Eurostat-OECD comparison conducted the ICP survey, which provided a link for construction between the Eurostat-OECD economies and the rest of the world. Because of the peculiarities of the available input data, a modified approach was used for linking:

- Prices collected for the machinery and equipment items were used to calculate reference PPPs for the equipment hire subheading at the global level.
- Regional GCL prices in local currency were used to calculate linking factors instead of prices converted to a common regional numéraire with the regional PPPs, as is done in the standard linking approach. More concretely, economies' PPPs were calculated by applying the unrestricted, unweighted CPD method.
- Linking factors for the basic headings were calculated as the geometric mean of the subheading PPPs for the economies in a region.¹²

Nonstandard Participation

Following TAG's recommendation, the two singleton economies (the Islamic Republic of Iran and Georgia), the CIS economies, the Caribbean

economies, Cuba, and the Pacific Islands were included in the global linking process in a way that had no effect on the multilateral PPPs for all other economies in the global comparison. This was achieved by the special linking approaches, as explained in the sections that follow.

Singleton Economies: The Islamic Republic of Iran and Georgia

PPPs for the Islamic Republic of Iran (IRN) and Georgia (GEO) were calculated by means of a bilateral comparison with Turkey (TUR), a participant of the Eurostat-OECD comparison, and Armenia (ARM), a CIS economy, respectively. The Islamic Republic of Iran is thus linked in the global comparison through Turkey's results and Georgia through Armenia's results.

The Eurostat-OECD item list was used as the basis for the IRN-TUR comparison,¹³ and the CIS 2011 list was used as the basis for the GEO-ARM comparison.¹⁴ National annual average prices for the Islamic Republic of Iran and Georgia were computed based on the price surveys. These prices were used for the calculation of bilateral IRN-TUR and GEO-ARM BH PPPs. Bilateral PPPs for the Islamic Republic of Iran and Georgia were bridged to the global comparison using Turkey's and Armenia's results in the global comparison, which were derived using the "standard" approach described earlier.

Calculation of the bilateral BH and aggregated PPPs requires the following steps:

- *Step C1.* Use the Eurostat-OECD item list as the basis for the IRN-TUR comparison. Reclassify the items from the Eurostat list priced by the Islamic Republic of Iran and Turkey to follow the ICP BH classification.¹⁵ Carry out a similar procedure for the GEO-ARM comparison.
- *Step C2.* Calculate bilateral PPPs for the IRN-TUR and GEO-ARM comparisons on the basis of the national annual average prices in national currency from step C1 using the EKS* method.¹⁶ Turkey is the numéraire in the IRN-TUR bilateral comparison (i.e., Turkey's PPPs equal 1). Similarly, Armenia is the numéraire in the GEO-ARM bilateral (i.e., Armenia's PPPs equal 1).
- *Step C3.* Aggregate the bilateral BH PPPs for the IRN-TUR and GEO-ARM comparisons

using the GEKS procedure for each level of aggregation up to GDP and for ICP analytical categories. Turkey is the numéraire in the IRN-TUR comparison, and Armenia is the numéraire in the GEO-ARM comparison.

The following steps are needed to link the bilateral aggregated PPPs (from step C3) for the Islamic Republic of Iran and Georgia to the global comparison:

- *Step D1.* Obtain the BH and aggregated PPPs for the Islamic Republic of Iran and Georgia in the global comparison by using Turkey's and Armenia's final PPPs from step B6 in the global comparison (after the CAR-volume redistribution) as a bridge to the Islamic Republic of Iran's and Georgia's PPPs from step C3. The Islamic Republic of Iran's global PPP is thus the bilateral PPP with Turkey multiplied by Turkey's global PPP in the global comparison (world = 1). Georgia's global PPP is the bilateral PPP with Armenia multiplied by Armenia's global PPP in the global comparison (world = 1).
- *Step D2.* Calculate volumes for the Islamic Republic of Iran and Georgia by dividing the nominal expenditures in national currency by the PPPs obtained from step D1.

For the Islamic Republic of Iran, the linking method just described was possible for the GDP components for which the bridge economy, Turkey, followed the same comparison approach. An alternative linking method was required for housing and construction and civil engineering because Turkey followed the Eurostat approach, whereas the Islamic Republic of Iran followed the ICP approach. For these components, the Islamic Republic of Iran was linked through economies using the ICP housing and construction and civil engineering surveys.

Commonwealth of Independent States

The CIS economies were linked to the global comparison via Russia. Russia participated in both the CIS and Eurostat-OECD comparisons. Because of the need for regional fixity, which is impossible for dual participants, in accordance with an agreement the results for Russia from the Eurostat-OECD comparison were used as official results. The CIS economies, except

Russia, were excluded from the calculation of the interregional linking factors.

The following steps are required to calculate the regional and aggregate level PPPs:¹⁷

- *Step E1.* Calculate the PPPs for the CIS on the basis of the national annual average prices in national currency using the EKS* method. For nonstandard ICP BHs, aggregate the specific CIS BH PPPs using the EKS procedure, with the respective national accounts expenditures as weights, to follow the ICP BH classification. Russia is the numéraire (i.e., Russia = 1).
- *Step E2.* Calculate the aggregated PPPs for the CIS by applying an unrestricted GEKS aggregation to the BH PPPs obtained from step E1 for each level of aggregation up to GDP.

The following steps are required to link at the aggregate level:

- *Step F1.* Obtain the BH and aggregated PPPs for the CIS economies in the global comparison by using as a bridge Russia's PPPs from step B6 in the final global comparison, after the CAR-volume redistribution is used as a bridge. Global PPPs for the CIS economies are thus their PPPs from the CIS comparison (Russia = 1) multiplied by Russia's global PPPs in the global comparison (world = 1).
- *Step F2.* Calculate the volumes for the CIS economies by dividing the nominal expenditures in national currency by the PPPs obtained from step F2.

The Caribbean

The Caribbean was linked to the global comparison via the Latin American economies. As an initial step, the 22 Caribbean economies were linked to the 16 Latin American economies at the basic heading level. Linking at the basic heading level was carried out by calculating separate sets of CPD-W PPPs for Latin America and the Caribbean, by subsequently calculating a combined set of respective PPPs, and finally by re-indexing the combined set of PPPs in accordance with the intraregional results (the CAR-PPP approach¹⁸) in order to maintain fixity of both the Latin America and Caribbean basic heading PPPs.

Linking at the aggregate level was carried out using the CAR-volume approach. The GEKS

aggregation was carried out first for Latin America and the Caribbean separately and then for the combined set of data. Finally, subregional totals of real expenditures were redistributed in accordance with the economies' real expenditure shares from separate Latin America and the Caribbean aggregations in order to maintain fixity of both the Latin America and the Caribbean results at all aggregate levels. As for the standard ICP regions, the aggregated PPPs were calculated indirectly by dividing the nominal expenditures by the real expenditures. This approach enabled regional linking of the Caribbean economies, using the Latin American economies as a base, while maintaining base economy invariance and fixity of results for both subregions. As a second step, the Caribbean results were linked to the global comparison using Latin America's global results as a bridge.

Cuba

Cuba was linked to the Latin America comparison via Peru for household consumption, government compensation, machinery and equipment, and construction. For housing, Cuba was linked via República Bolivariana de Venezuela, which had a typical housing volume index per capita for the Latin America comparison as well as the dwelling stock quantity and quality data needed for the bilateral comparison. The price and expenditure data used for Cuba in the calculations were expressed in convertible pesos. The linking steps for the PPPs were similar to those described for the Islamic Republic of Iran and Georgia.¹⁹

The Pacific Islands

The Pacific Islands participated in ICP 2011 on a limited basis. Twenty islands provided price data on some 87 household consumption items. No specific national accounts activities were conducted, but there was an opportunity to use existing household income and expenditure survey (HIES) data for the purpose of compiling consumption data. The Pacific Islands were bridged to the ICP via economies participating in other regions:

- Fiji participated in both the Asia and the Pacific and the Pacific Islands comparisons.
- New Zealand and Australia participated in both the OECD and ICP Pacific Islands comparisons.

The weighted geometric mean of PPPs to the bridge economies was used in the linking (the weights were Fiji, 0.5; Australia and New Zealand, 0.25) in order to have symmetry between Asia and the Pacific and the OECD in regional influence.

Dual Participation

In ICP 2011, Russia participated in both the CIS and Eurostat-OECD comparisons. To preserve the fixity of Russia's results within the Eurostat-OECD region, it was agreed that for the global comparison Russia would be included only in the Eurostat-OECD region. Thus Russia's dual participation did not pose any computational issues.

Egypt and Sudan required special treatment because both the Africa (AF) and Western Asia (WA) regions included these economies in their regional results. Because it was not possible to maintain fixity for these economies in both regional comparisons, a suggested approach was to use geometric averages from the regional results as was done for Egypt in the 2005 round of the ICP.

Two sets of price data for Egypt and Sudan for each region were included in the computations of the interregional BH PPPs (linking factors) using the CPD-W method. One set included their GCL prices expressed in the AF numéraire on the basis of the BH PPPs from the AF comparison. Another set included their GCL prices expressed in the WA numéraire on the basis of the BH PPPs from the WA comparison. The use of these two sets was logical in the regional approach because both economies participated in both comparisons, and all interregional PPPs should be based on input data from both regional comparisons for Egypt and Sudan.

The global PPPs from regions were calculated for Egypt and Sudan using the standard approach as follows:

$$\begin{aligned}
 \text{PPP}_1 \text{ "Egypt/ world"} &= \frac{\text{PPP "Egypt/ AF"}}{[\textit{regional comparison}]} \times \frac{\text{PPP "AF/ world"}}{[\textit{interregional linking factor}]} \\
 \text{PPP}_2 \text{ "Egypt/ world"} &= \frac{\text{PPP "Egypt/ WA"}}{[\textit{regional comparison}]} \times \frac{\text{PPP "WA/ world"}}{[\textit{interregional linking factor}]}
 \end{aligned}$$

The geometric means of PPP₁ and PPP₂ were considered final global BH PPPs for Egypt:

$$\text{PPP "Egypt/ world"} = \left(\frac{\text{PPP}_1 \text{ "Egypt/ world"} \times \text{PPP}_2 \text{ "Egypt/ world"}}{2} \right)^{1/2}$$

The same procedure was carried out for Sudan:

$$\begin{aligned}
 \text{PPP}_1 \text{ "Sudan/ world"} &= \frac{\text{PPP "Sudan/ AF"}}{[\textit{regional comparison}]} \times \frac{\text{PPP "AF/ world"}}{[\textit{interregional linking factor}]} \\
 \text{PPP}_2 \text{ "Sudan/ world"} &= \frac{\text{PPP "Sudan/ WA"}}{[\textit{regional comparison}]} \times \frac{\text{PPP "WA/ world"}}{[\textit{interregional linking factor}]}
 \end{aligned}$$

The geometric means of PPP₁ and PPP₂ were considered final global BH PPPs for Sudan:

$$\text{PPP "Sudan/ world"} = \left(\frac{\text{PPP}_1 \text{ "Sudan/ world"} \times \text{PPP}_2 \text{ "Sudan/ world"}}{2} \right)^{1/2}$$

This was a symmetrical way to include Egypt and Sudan in the global comparison at the basic heading level on the basis of input data from both regions.

The CAR-volume approach was used for the global linking at the aggregate level. Therefore one set of input data for each economy in question was used in the unrestricted GEKS method to avoid these economies having a double impact on the global linking through use of the CAR-volume approach.

Egypt and Sudan had the same national accounts expenditure data in both regions. BH-PPPs were derived as described earlier. Aggregated PPPs for Egypt and Sudan from the unrestricted global GEKS were used in the further computations. Regional volumes for both regions, volume AF and volume WA, were calculated with the inclusion of Egypt and Sudan, and two volumes from both regions were calculated for Egypt and Sudan using the standard procedure. Aggregated volumes for Egypt and Sudan were then obtained as follows:

$$\text{volume}_1 \text{ "Egypt"} = \frac{\text{volume "AF"} \times \text{volume share "Egypt/AF"}}{[\textit{global comparison}] \times [\textit{regional comparison}]}$$

$$\text{volume}_2 \text{ "Egypt"} = \frac{\text{volume "WA" [global comparison]}}{\text{volume share "Egypt/WA" [regional comparison]}}$$

$$\text{volume}_1 \text{ "Sudan"} = \frac{\text{volume "AF" [global comparison]}}{\text{volume share "Sudan/AF" [regional comparison]}}$$

$$\text{volume}_2 \text{ "Sudan"} = \frac{\text{volume "WA" [global comparison]}}{\text{volume share "Sudan/WA" [regional comparison]}}$$

Geometric means from volume₁ and volume₂ were considered volumes in the global comparison for Egypt and Sudan:

$$\text{volume "Egypt"} = (\text{volume}_1 \text{ "Egypt"} \times \text{volume}_2 \text{ "Egypt"})^{1/2}$$

$$\text{volume "Sudan"} = (\text{volume}_1 \text{ "Sudan"} \times \text{volume}_2 \text{ "Sudan"})^{1/2}$$

The indirect global PPPs for Egypt and Sudan were derived after this in the standard way as PPPs = expenditures in national accounts/volumes.

Annex A

Weighted Country Product Dummy Method

The weighted country product dummy (CPD-W) method uses explicit weights.²⁰

Representative/important items receive higher weights, such as 2 or 3, than the weights, such as 1, assigned to nonrepresentative/less important items.²¹

The unweighted original country product dummy (CPD) derives estimators of regression parameters (26A.1) through minimization of the squares in logarithmic terms using a standard least squares procedure:

$$\sum_{i=1}^n \sum_{j=1}^r \sum_{l=1}^{c_j} [\ln(p_{ijl}) - \alpha'_j - \beta'_n - k']^2; \quad (26A.1)$$

$$\alpha'_{j1} = 0; \beta'_{11} = 0.$$

Representative/important items and nonrepresentative/less important items receive the same weight, 1, in the unweighted CPD.

The CPD-W approach suggests that each price corresponding to a product in a given economy in a region be given a prespecified weight in the least squares estimation depending on the representativity/importance of the product. Suppose w is a set of weights for representative/important (w^{repr}) and nonrepresentative/less important ($w^{nonrepr}$) items to be applied to the price p_{ij} (Rao 2004). The weighted least squares (WLS) approach applied to (26A.1) can then be presented as

$$\sum_{i=1}^n \sum_{j=1}^r \sum_{l=1}^{c_j} w [\ln(p_{ijl}) - \alpha'_j - \beta'_n - k']^2; \quad (26A.2)$$

$$\alpha'_{j1} = 0; \beta'_{11} = 0.$$

It is often declared that the advantage of the stochastic CPD approach is that it allows one to derive standard errors for the CPD estimates of PPPs, common prices, and so forth.²² However, it is not easy to implement these standard errors in the analysis because there are numerous problematic points.²³ In effect, standard errors of the CPD parameters are not used in the ICP.

Therefore, to make the CPD method more transparent and understandable for a broad array of users, the CPD-W method is presented in the equations that follow in a more traditional index form specific to the Geary-Khamis

(GK) method in geometric (logarithmic) terms (Diewert 2002; Sergeev 2004):

$$\pi_i = \left\{ \prod_{r=1}^R \prod_{j=1}^{N(r)} [P_{ijr}/PPP_r]^{q_{ijr}} \right\}^{1/r \sum q_{ijr}}; \quad (26A.3)$$

$$i = 1, 2, \dots, M$$

and

$$PPP_r = \left\{ \prod_{i=1}^M \prod_{j=1}^{N(r)} [P_{ijr}/\pi_i]^{q_{ijr}} \right\}^{1/i \sum q_{ijr}}; \quad (26A.4)$$

$$r = 1, 2, \dots, R-1 \quad (PPP_R = 1)$$

where

π_i is the international average price of item i in the currency of the numéraire region (here region R);

π_i is an analogue of α_i from the CPD regression;

PPP_r is the PPP of region r relative to the base region R ($PPP_R = 1$);

PPP_r is an analogue of β_r from the CPD regression;

R is the number of regions;

$N(r)$ is the number of economies in region r ;

M is the number of products within a basic heading;

P_{ijr} is the price for product i in economy j from region r in the regional numéraire (by economy's regional PPPs);

q_{ijr} is the weight for product i in economy j from region r —the appropriate values can be 3 (for important products) and 1 (for less important products);

$r \sum q_{ijr}$ is the cumulative value of the importance of item i among all economies in all regions; and

$i \sum q_{ijr}$ is the cumulative value of the importance of items priced in all economies of region r .

The average "international price" of the i -th item (π_i) is presented as an implicit quantity-weighted geometric average of the PPP-adjusted national prices (indicators of importance are used as weights).²⁴

The PPP for the j -th region (PPP_j) is presented as the geometric average (implicit expenditure weighted) deviation of its regional prices from the international prices.

The system shown in (26A.3) and (26A.4) can be efficiently solved by an iterative method.²⁵ The geometric mean of the price ratios or simply the exchange rates of the economy selected as the base can be used as an initial set of unknown PPPs.

Annex B Using the GEKS Method for Aggregation

The Gini-Éltető-Köves-Szulc (GEKS) method was recommended by the ICP's Technical Advisory Group for calculating aggregations within the regions as well as for the global comparison.²⁶ All calculations by the GEKS method are carried out for each aggregated heading separately.

First, calculate the bilateral Fisher PPPs (F-PPPs)²⁷ for all pairs of economies. For any two economies j and k , the binary F-PPP (price index) is computed as

$$P_{F,j,k} = \left(\left(\frac{\sum_i p_j^i q_k^i}{\sum_i p_k^i q_k^i} \right) \left(\frac{\sum_i p_j^i q_j^i}{\sum_i p_k^i q_j^i} \right) \right)^{1/2}. \quad (26B.1)$$

Second, average geometrically all direct and indirect F-PPPs (equally, all F-PPPs of economy j to all other economies) to obtain the transitive GEKS-PPPs. The GEKS-PPP is computed as

$$P_{GEKS,j,k} = \left(\prod_l P_{F,j,l} / P_{F,k,l} \right)^{1/K}, \quad (26B.2)$$

where K is the total number of economies. The respective volume is calculated as

$$Q^j \equiv p^j \cdot q^j / P^j. \quad (26B.3)$$

Economies within and across regions can be very different. The bilateral indexes for the economies with different price and quantity structures can be unreliable.²⁸ In this aspect, analysis of the Paasche-Laspeyres spread (PLS) is very important—for details, see Vogel (2013).

Annex C The Country Aggregation with Redistribution (CAR) Method of Linking above the Basic Heading Level

The Technical Advisory Group of the International Comparison Program (ICP) recommended that the country aggregation with volume redistribution

(CAR-volume) approach²⁹ serve as the official ICP 2011 method (Sergeev 2011).

In this method, the unrestricted global Gini-Éltető-Köves-Szulc (GEKS) purchasing power parities (PPPs) for the aggregates are used to recalculate the economy aggregates in national currencies (NCs) into volumes (real expenditure) measured in a world numéraire. Economy volumes are summed up by the region, and these regional volumes are redistributed in accordance with the economy shares of volumes in the regional comparisons to preserve the regional fixity:

$$\begin{array}{rcc} & \text{volume} & \text{share} \\ \text{volume} & \text{"region in"} & \text{"economy/"} \\ \text{"economy in"} & \text{the world"} & \text{region"} \\ \text{the world"} & = & \\ & \text{[global} & \text{[regional} \\ & \text{comparison]} & \text{comparison]} \end{array} \times$$

Because the indicator volume share of an economy within the region can be also considered the volume index "economy/region" and the volume share of a region within the world total can be considered the volume index "region/world," this procedure can also be presented in the form of volume indexes (VIs):

$$\begin{array}{rcc} \text{VI} & \text{VI "region/"} & \text{VI "economy/"} \\ \text{"economy/"} & \text{world"} & \text{region"} \\ \text{world"} & = & \\ & \text{[global} & \text{[regional} \\ & \text{comparison]} & \text{comparison]} \end{array} \times$$

The respective PPPs (with fixity) are calculated in an indirect way as

$$\text{PPPs "national currency/"} = \text{expenditures in} \\ \text{world numéraire"} = \text{NC/volumes.}$$

NOTES

1. Annex A describes the CPD-W method.
2. These PPPs were supplied by the regions.
3. When different, the region-specific and ICP BH classifications have hierarchical relations. Therefore region-specific basic headings can be aggregated to the ICP basic heading using the Gini-Éltető-Köves-Szulc (GEKS) procedure (with national accounts expenditure weights). For example, if the regional

- classification has three separate basic headings for coffee, tea, and cocoa, but the ICP Classification has only one combined basic heading, it is possible to run the GEKS procedure for these three basic headings in order to obtain the region-specific PPPs for the ICP basic heading coffee, tea, and cocoa. In addition, in ICP 2011 treatment of nonprofit institutions serving households (NPISHs) had some specificities. NPISH expenditures were distributed among the respective household consumption expenditure categories in the global comparison. However, to maintain regional fixity, the final linking steps were carried out using expenditures as per the regional practice for distribution.
4. Annex B describes the GEKS method.
 5. The GEKS method was recommended by TAG for the aggregation within the regions as well as for the global comparison. However, it is likely that the differences in the regional methodologies and the classifications will have some marginal impact on the (in)comparability of the global results with fixity.
 6. For example, the Eurostat-OECD program used fixity for the 37 economies in its comparison.
 7. These PPPs were supplied by the regions.
 8. In total seven basic headings, of which one was under the household consumption expenditure, and the remaining six were under the individual government consumption expenditure.
 9. An equal way would be use of the same PPPs for the education total of all education BHs.
 10. Much of the former data were used in the Eurostat-OECD approach—for example, all prices of pharmaceuticals, medical products, and outpatient medical services by private providers were used for the respective basic headings. The impact of the new approach on GDP PPPs was generally not very high.
 11. The CIS applies a different regional approach, however. The CIS economies were linked via Russia in the Eurostat-OECD exercise.
 12. This approach is similar to the country approach method with the reindexation of PPPs (CAR-PPP). See note 33 in annex C for further information.
 13. The BH classification used in the Eurostat-OECD comparison differs from the ICP BH classification.
 14. The CIS BH classification is based on the Eurostat-OECD classification, but it has a few differences.
 15. Because national accounts expenditure data for the Islamic Republic of Iran are available only for the ICP BH classification, the ICP BH PPPs are calculated by reclassifying the items rather than by aggregating the BH PPPs.
 16. The EKS* method was selected because it is the official PPP calculation method of the Eurostat-OECD and CIS programs.
 17. These PPPs were provided by the region.
 18. See note 33 in annex C for further information on the CAR-PPP approach.
 19. Volumes were not calculated for Cuba because of peculiarities in the national accounts expenditure data.
 20. The version used in regional linking—an economy's prices in regional numéraires as an input basis—is presented here. This annex is based on Sergeev (2011), with inputs from Yuri Dikhanov.
 21. The weights 3 and 1 were recommended by the ICP's TAG. The weights 1 and 0 are applicable to the EKS* method—1 for representative items with an asterisk (*) and 0 for nonrepresentative items without an asterisk. The weights are not applicable, however, to the CPD method because the items with 0 weights are items not priced, and they are eliminated from the calculations.
 22. It is unclear how the stochastic estimations should be used in practice. In addition, the estimations of errors depend on the regression specification.
 24. Theoretically, quantity weights should be used here, but this information is not available.
 25. The numerical results by (26A.3) and (26A.4) are identical to the results by (26A.1) and (26A.2).
 26. This annex is based on Sergeev (2011) with inputs from Yuri Dikhanov.
 27. The same can be applied to the volume/quantity indexes because the Fisher index is

- relatively symmetrical to variables (prices and quantities). The product of $PPP(F)$ and $Q(F)$ is the expenditure ratio. In principle, each bilateral index (e.g., Tornqvist) can be used in the GEKS procedure. The Fisher indexes, but not the Törnqvist indexes, are used traditionally because the product of $PPP(T)$ and $Q(T)$ is not equal to the expenditure ratio.
28. The so-called selective GEKS procedure can be used to eliminate the impact of "weak" bilateral links. In accordance with this, bilateral F-PPPs with a high PLS and a PLS of less than 1 are eliminated and replaced by indirect PPPs calculated via a third economy. Such an approach was used during the linking of Group I and II in the 1993 round of the European Comparison Program. For details, see United Nations Economic Commission on Europe (1997).
 29. This method is described in chapter 15 of the *ICP 2005 Methodological Handbook* as calculation of the regional linking factors (between-region PPPs) by the weighted harmonic mean method (World Bank 2007). It is also possible to use the country approach method with the reindexation of PPPs (CAR-PPP method), in which regional PPPs from the global comparison are combined with the economy's regional PPPs:

$$\begin{array}{rcccl}
 & & \text{PPP "economy/} & & \text{PPP "region/} \\
 & & \text{region"} & & \text{world"} \\
 \text{PPP "econo-} & = & & \times & \\
 \text{my/world"} & & \text{[regional} & & \text{[global} \\
 & & \text{comparison]} & & \text{comparison]}
 \end{array}$$

PPP "region/world" is calculated as the unweighted geometric mean from the economy's PPPs belonging to the region. This method was described in chapter 15 of the *ICP 2005 Methodological Handbook* as calculation of the regional linking factors (between-region PPPs) by the unweighted geometric mean method (World Bank 2007). Also, a modification of the CAR-PPP

method is used in the Eurostat–Organisation for Economic Co-operation and Development (OECD) comparison in order to preserve the fixity of the Eurostat comparison. Both CAR methods (CAR-volume and CAR-PPP) maintain the fixity of the regional results, but the between-region results obtained by these methods can be slightly different.

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