Outlier detection and treatment

LECTURE 12

Today is mainly about outliers

1) Definitions
   What do we mean by an outlier, exactly?

2) Motivation
   Do outliers really matter?

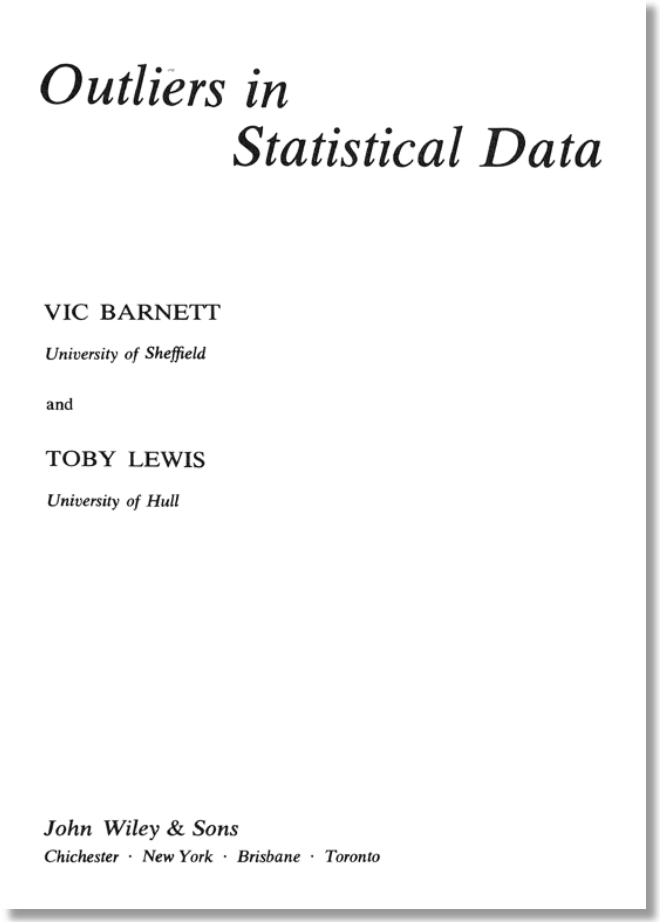
3) Detection
   How to detect outliers?

4) Treatment
   How to deal with outliers?

Definitions

What is an outlier?

- An outlier is an observation "that appears to deviate markedly from other members of the sample in which it occurs" (Grubbs, 1969)
- Note: we focus on univariate outliers, those found when looking at a distribution of values in a single dimension (e.g. income).


Outliers in Statistical Data

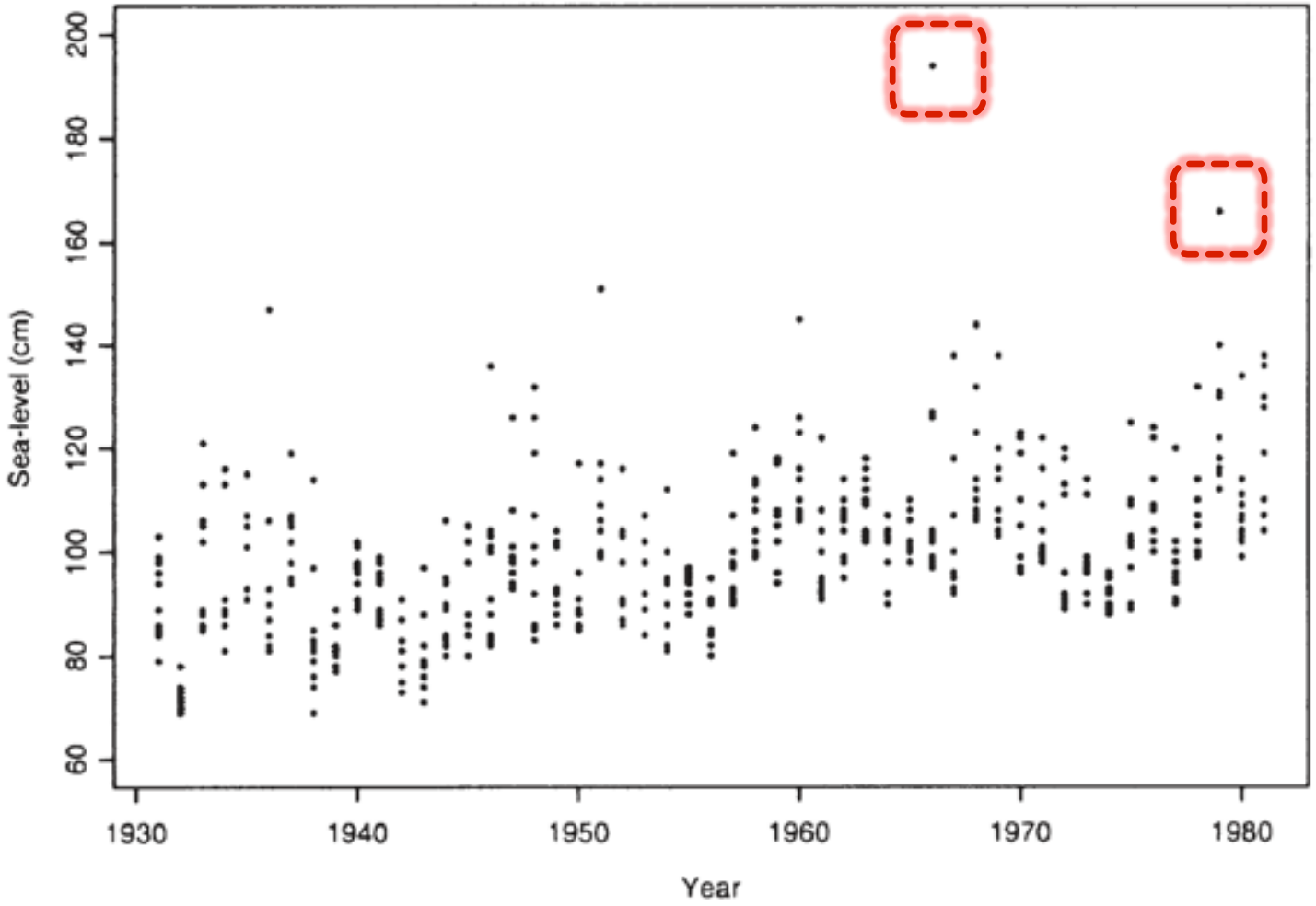
VIC BARNETT
University of Sheffield

and

TOBY LEWIS
University of Hull

John Wiley & Sons
Chichester · New York · Brisbane · Toronto

Highest sea-levels in Venice



Other classical definitions

- An outlier is “an observation that deviates so much from other observations as to arouse suspicion that it was generated by a different mechanism” (Hawkins 1980)
- Aguinis et al (2013) provide 14 definitions of outliers based on a litterature review of 28 papers.

What causes outliers?

- Human errors, e.g. data entry errors
- Instrument errors, e.g. measurement errors
- Data processing errors, e.g. data manipulation
- Sampling errors, e.g. extracting data from wrong sources
- Not an error, the value is extreme, just a ‘novelty’ in the data

A dilemma

- Outliers can be genuine values
- The trade-off is between the loss of accuracy if we throw away “good” observations, and the bias of our estimates if we keep “bad” ones
- The challenge is twofold:
    1. to figure out whether an extreme value is good (genuine) or bad (error)
    2. to assess its impact on the statistics of interest

Do outliers matter?

Theory first

- Three papers:

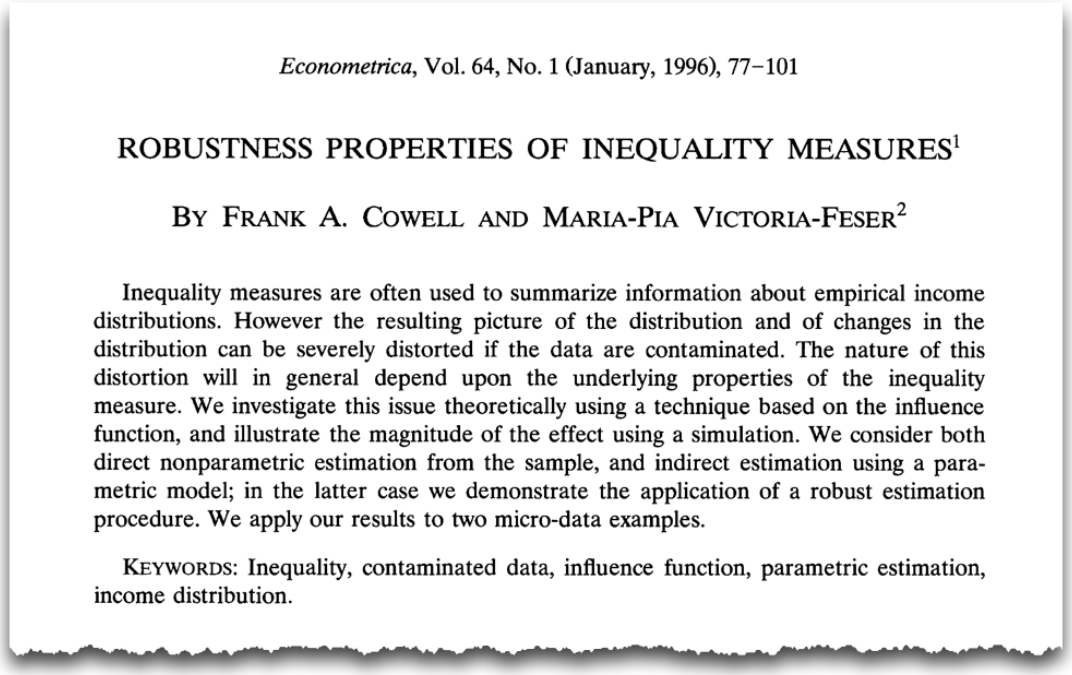
I. 1996a
   Frank Cowell and Maria-Pia Victoria-Feser

II. 2007
   Frank Cowell and Emmanuel Flachaire (*)

III. 1996b
   Frank Cowell and Maria-Pia Victoria-Feser

Outliers and inequality measures – I

Cowell and Victoria-Feser (1996a)



Econometrica, Vol. 64, No. 1 (January, 1996), 77–101

ROBUSTNESS PROPERTIES OF INEQUALITY MEASURES[1]

BY FRANK A. COWELL AND MARIA-PIA VICTORIA-FESER[2]

Inequality measures are often used to summarize information about empirical income distributions. However the resulting picture of the distribution and of changes in the distribution can be severely distorted if the data are contaminated. The nature of this distortion will in general depend upon the underlying properties of the inequality measure. We investigate this issue theoretically using a technique based on the influence function, and illustrate the magnitude of the effect using a simulation. We consider both direct nonparametric estimation from the sample, and indirect estimation using a parametric model; in the latter case we demonstrate the application of a robust estimation procedure. We apply our results to two micro-data examples.

KEYWORDS: Inequality, contaminated data, influence function, parametric estimation, income distribution.

- This is a beautiful paper
- Explains why outliers (contaminants) are a serious threat to most inequality measures.
- "if the mean has to be estimated from the sample then all scale independent or translation independent and decomposable measures have an unbounded influence function" (p. 89)
- An unbounded IF is a catastrophe.

The catastrophe

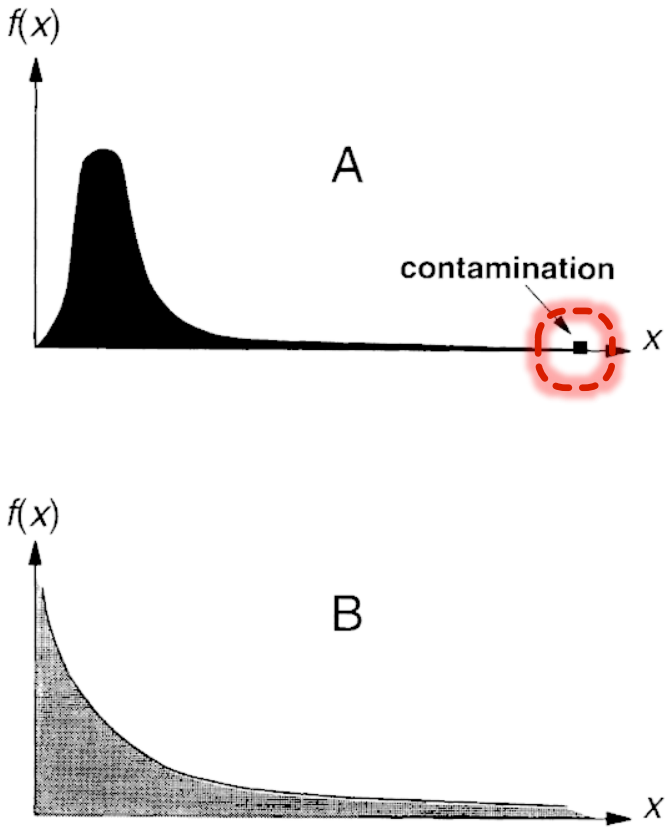


FIGURE 1

- Suppose the shape of the income distribution is represented by the continuous frequency distribution in part A
- Suppose that in the sample there are some rogue observations represented by the point mass labelled “contamination”.
- Then, according to inequality statistics that are sensitive to the top end of the distribution, the income distribution in A will be indistinguishable from that represented in B (that is, IF is unbounded).

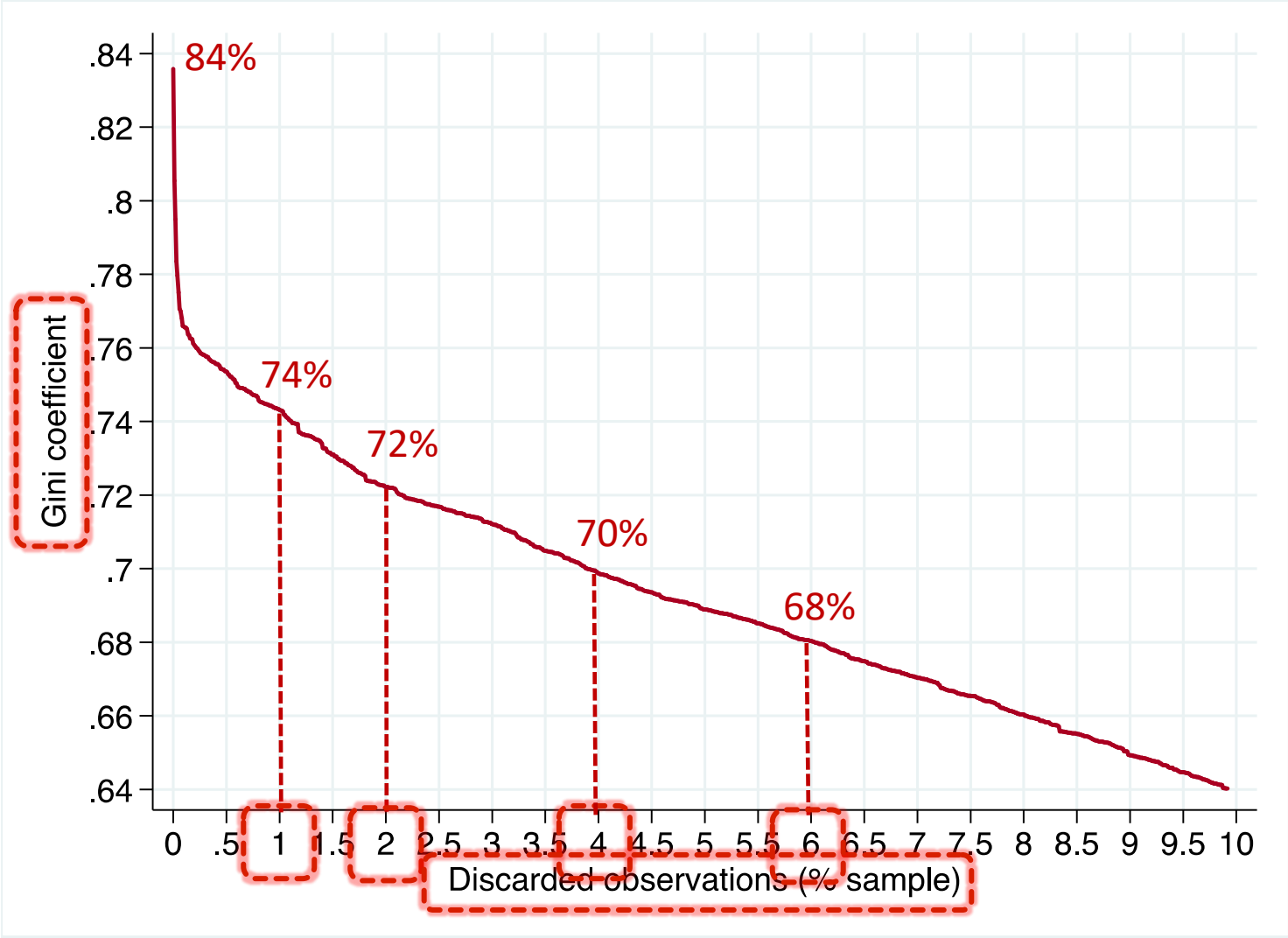
In practice

Hlasny and Verme (2018: 191)

- Many researchers routinely trim outliers or problematic observations or apply top coding with little consideration of the implications for the measurement of inequality
- One example to illustrate

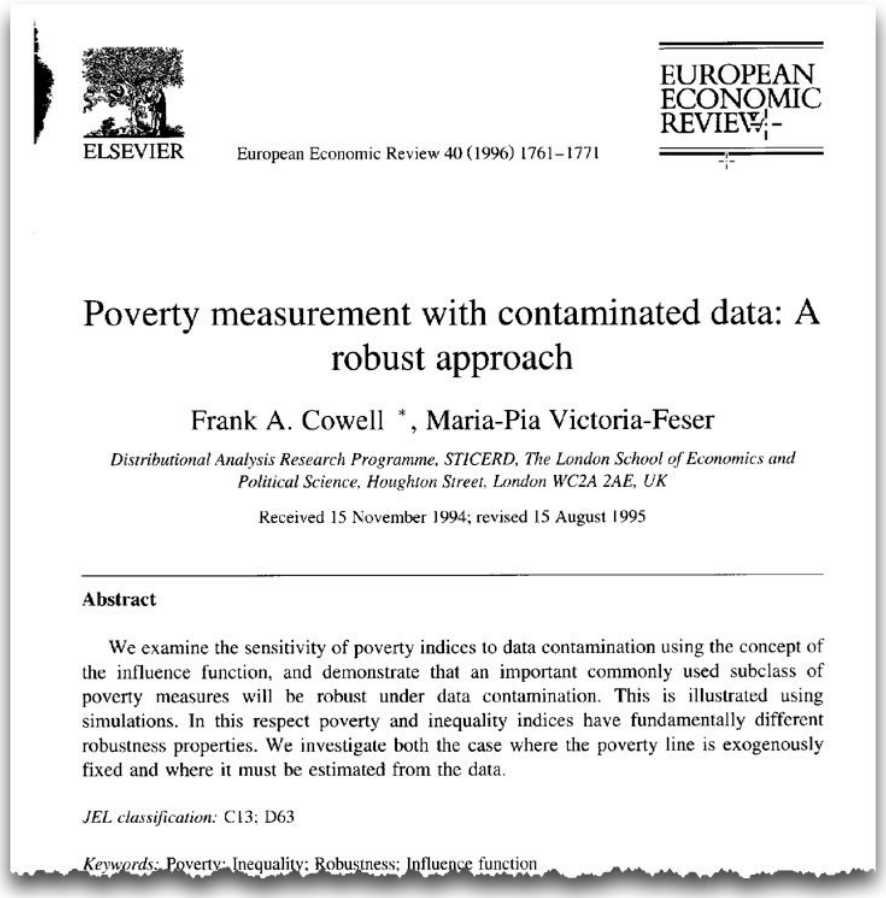
Sensitivity of the Gini index to extreme values

iterative trimming



Outliers and poverty measures

Cowell and Victoria-Feser (1996b)



ELSEVIER

European Economic Review 40 (1996) 1761–1771

EUROPEAN ECONOMIC REVIEW

Poverty measurement with contaminated data: A robust approach

Frank A. Cowell *, Maria-Pia Victoria-Feser

Distributional Analysis Research Programme, STICERD, The London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK

Received 15 November 1994; revised 15 August 1995

Abstract

We examine the sensitivity of poverty indices to data contamination using the concept of the influence function, and demonstrate that an important commonly used subclass of poverty measures will be robust under data contamination. This is illustrated using simulations. In this respect poverty and inequality indices have fundamentally different robustness properties. We investigate both the case where the poverty line is exogenously fixed and where it must be estimated from the data.

JEL classification: C13; D63

Keywords: Poverty; Inequality; Robustness; Influence function

- Explains why outliers only rarely are a serious threat to most poverty measures.
- Poverty measures are not sensitive to the values (real or contaminated) of the incomes of the rich

Recap

- The answer to the question on whether outliers matter depends on the statistic of interest
- Inequality: both theory (unbounded IF) and practice (incremental truncation) suggest that they matter (tremendously). Not taking this issue into proper account puts inequality comparisons at risk.
- Poverty: not so much

How to detect outliers?

Visual inspection

- Our procedures are part graphical, and part automatic. For each commodity, we draw histograms and one-way plots of the logarithms of the unit values, using each to detect the presence of gross outliers for further investigations. […] [Automatic method] does not remove the need for the graphical inspection
(Deaton and Tarozzi 2005)

Visual inspection

Malawi IHS3, Cassava tuber expenditure

Malawi - Integrated Household Panel Survey 2010-2013 (Short-Term Panel, 204 EAs)

Reference ID	MWI_2010-2013_IHPS_v01_M
Year	2010 - 2013
Country	Malawi
Producer(s)	National Statistical Office - Government of Malawi
Sponsor(s)	Government of Malawi - GovMWI - Funded the study The World Bank - WB - Funded the study Millennium Challenge Corporation - MCC - Funded the study Irish Aid - IA - Funded the study German Development Corporation - GTZ - Funded the stud
Collection(s)	Living Standards Measurement Study (LSMS) Central
Metadata	Documentation in PDF

Created on	Apr 21, 2015
Last modified	Dec 13, 2017

Related Materials | Study Description | Data Dictionary | Get Microdata

Visual inspection

Malawi IHS3, Cassava tuber expenditure

- Example 1: look at descriptive statistics

MODULE G: FOOD CONSUMPTION OVER PAST ONE WEEK

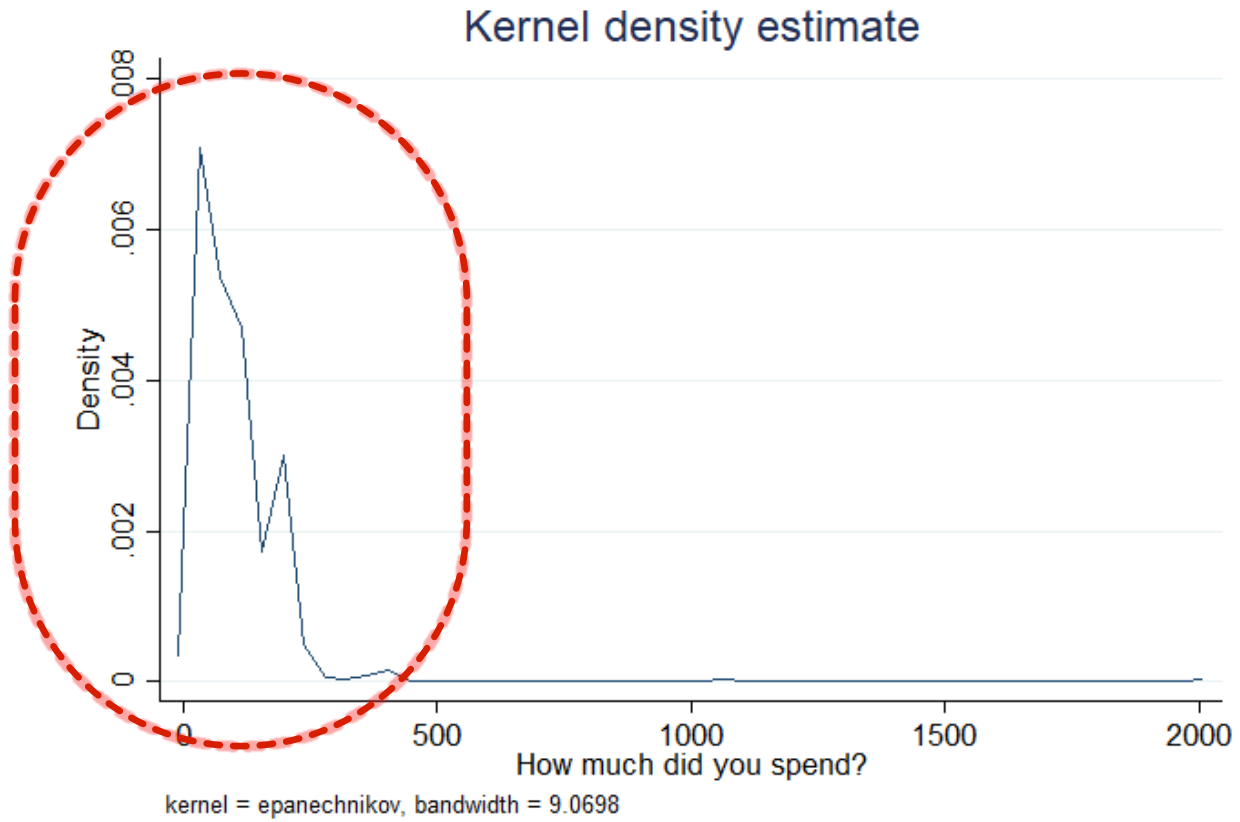
DATA ENTRY LINE NUMBER		G01	G02	G05
	Over the past one week (7 days), did you or others in your household consume any [. . .]? INCLUDE FOOD BOTH EATEN COMMUNALLY IN THE HOUSEHOLD AND THAT EATEN SEPARATELY BY INDIVIDUAL HOUSEHOLD MEMBERS.	YES..1 NO...2>> **NEXT ITEM**	ITEM CODE	How much did you spend? MK
19	**Roots, Tubers, and Plantains**			
20	Cassava tubers		201	

```
. sum hh_g05 if hh_g02==201,d

                    How much did you spend?
-------------------------------------------------------------
      Percentiles      Smallest
 1%           5              0
 5%          20              0
10%          20              0       Obs                 673
25%          50              0       Sum of Wgt.         673

50%          75                      Mean           94.95097
                        Largest      Std. Dev.      106.2379
75%         100            400
90%         200            400      Variance       11286.5
95%         220           1050      Skewness       10.0151
99%         350           2000      Kurtosis       164.7054
```

Visual inspection

Malawi IHS3, Cassava tuber expenditure

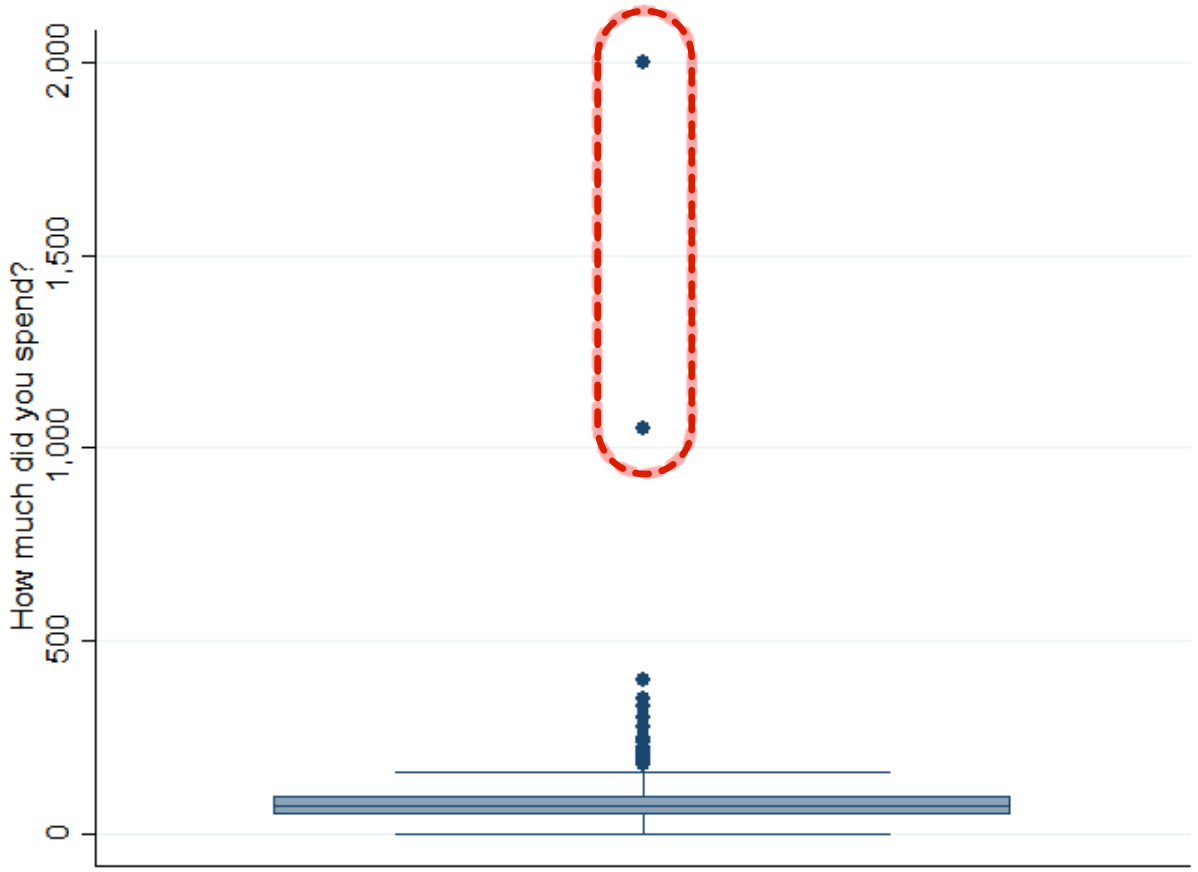
- Example 2: graph the distribution of the data



Visual inspection

Malawi IHS3, Cassava tuber expenditure

- Example 3: use graphical diagnostic tools, e.g. the boxplot graph



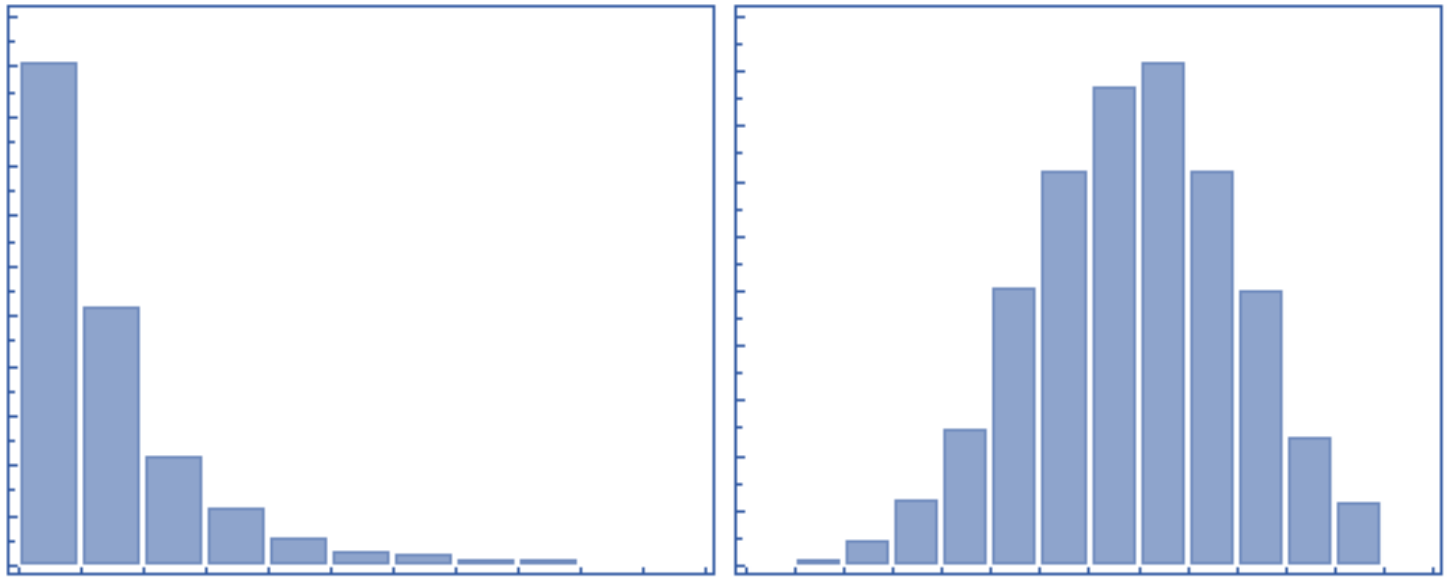
Statistical methods

- The literature is rich with methods to identify outliers; in practice, most methods used in empirical work hinge on the underlying distribution of the data.

- The idea is simple:
  - transform the variable to induce normality
  - set thresholds to identify extreme values

Transform the variable to induce normality

- The easiest transformation relies on taking the logarithm of the variable of interest
- The log “squeezes” large values more, so that skewed distributions become more symmetrical and closer to a Normal distribution.



Set a threshold

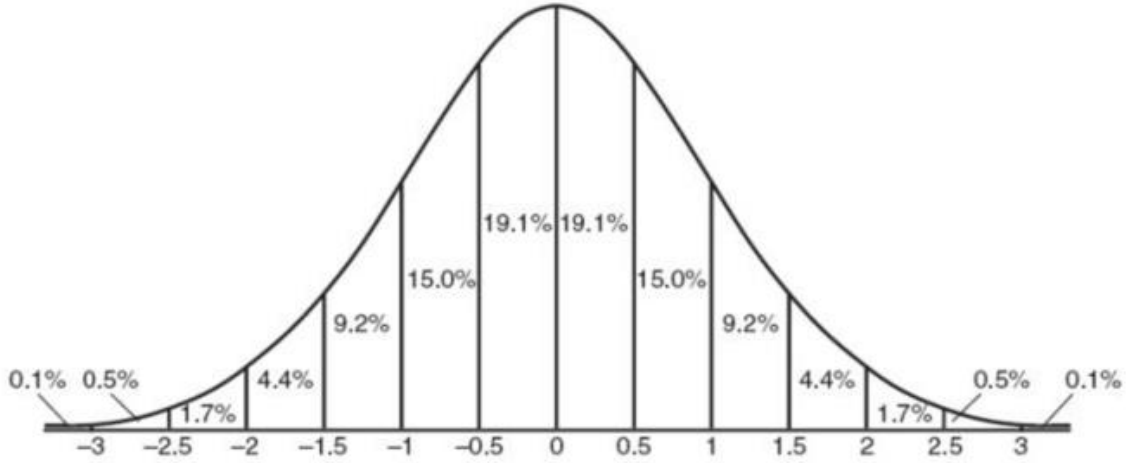
- We must specify a threshold for deciding whether each observation is 'too extreme' (outlier or not?)
- Common 'thumb-rule' thresholds : an observation is considered an outlier if it is more than 2.5, 3, 3.5 standard deviations far from the mean of the distribution
- In formulas: x is an outlier if $x > \bar{x} + z_\alpha \, s$

  where z_α equals, say, 2.5.
- We can express the same criterion as $\dfrac{x - \bar{x}}{s} > z_\alpha$

  where the left-hand side is called a z-score (a variable with mean = 0 and var = 1)

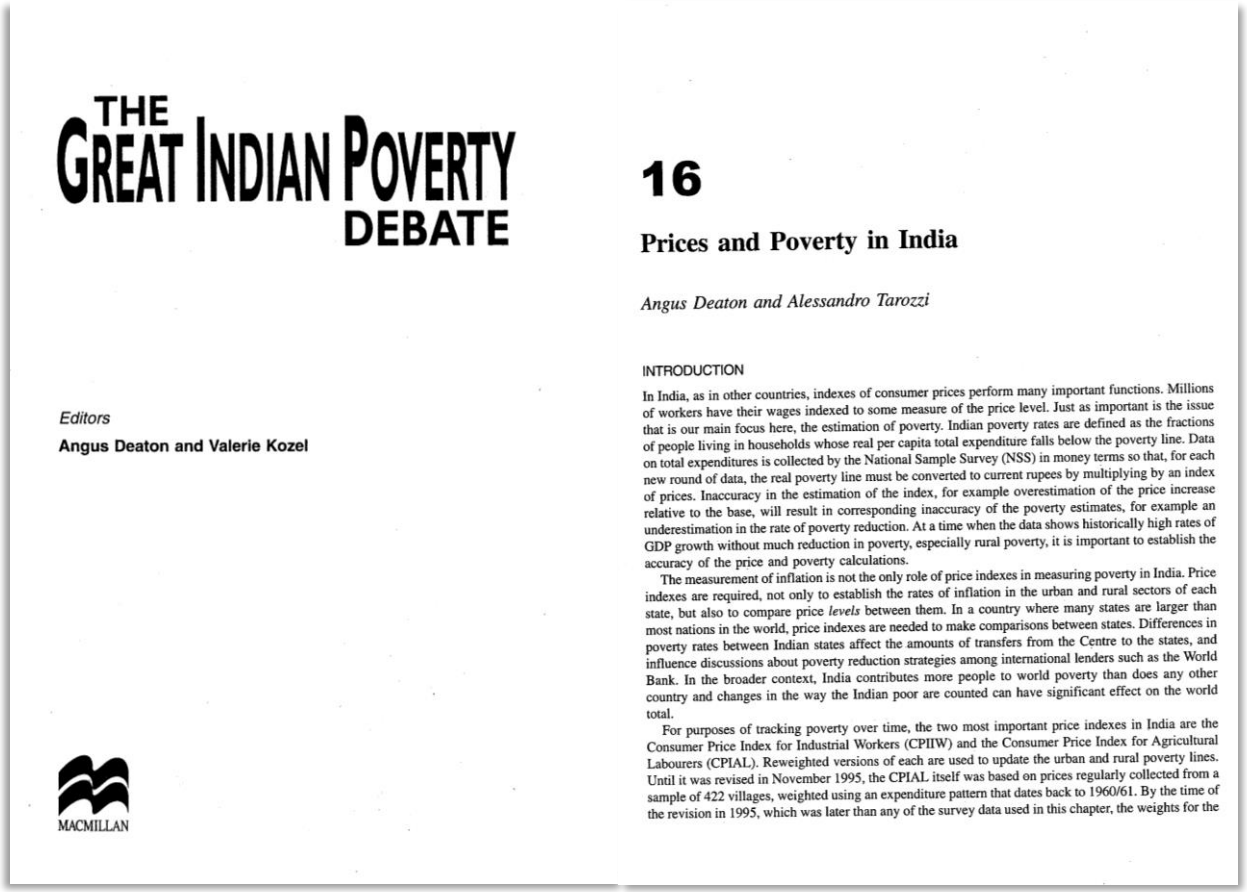
Why 2.5, 3, or any other number?

- Under the assumption of normality:



- $z_\alpha = 2.5$ implies that outliers are in the region where $\alpha = 0.5$ percent of other observations normally are.

Deaton and Tarozzi (2005)



THE GREAT INDIAN POVERTY DEBATE

Editors
Angus Deaton and Valerie Kozel

MACMILLAN

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Prices and Poverty in India

Angus Deaton and Alessandro Tarozzi

INTRODUCTION

In India, as in other countries, indexes of consumer prices perform many important functions. Millions of workers have their wages indexed to some measure of the price level. Just as important is the issue that is our main focus here, the estimation of poverty. Indian poverty rates are defined as the fractions of people living in households whose real per capita total expenditure falls below the poverty line. Data on total expenditures is collected by the National Sample Survey (NSS) in money terms so that, for each new round of data, the real poverty line must be converted to current rupees by multiplying by an index of prices. Inaccuracy in the estimation of the index, for example overestimation of the price increase relative to the base, will result in corresponding inaccuracy of the poverty estimates, for example an underestimation in the rate of poverty reduction. At a time when the data shows historically high rates of GDP growth without much reduction in poverty, especially rural poverty, it is important to establish the accuracy of the price and poverty calculations.

The measurement of inflation is not the only role of price indexes in measuring poverty in India. Price indexes are required, not only to establish the rates of inflation in the urban and rural sectors of each state, but also to compare price *levels* between them. In a country where many states are larger than most nations in the world, price indexes are needed to make comparisons between states. Differences in poverty rates between Indian states affect the amounts of transfers from the Centre to the states, and influence discussions about poverty reduction strategies among international lenders such as the World Bank. In the broader context, India contributes more people to world poverty than does any other country and changes in the way the Indian poor are counted can have significant effect on the world total.

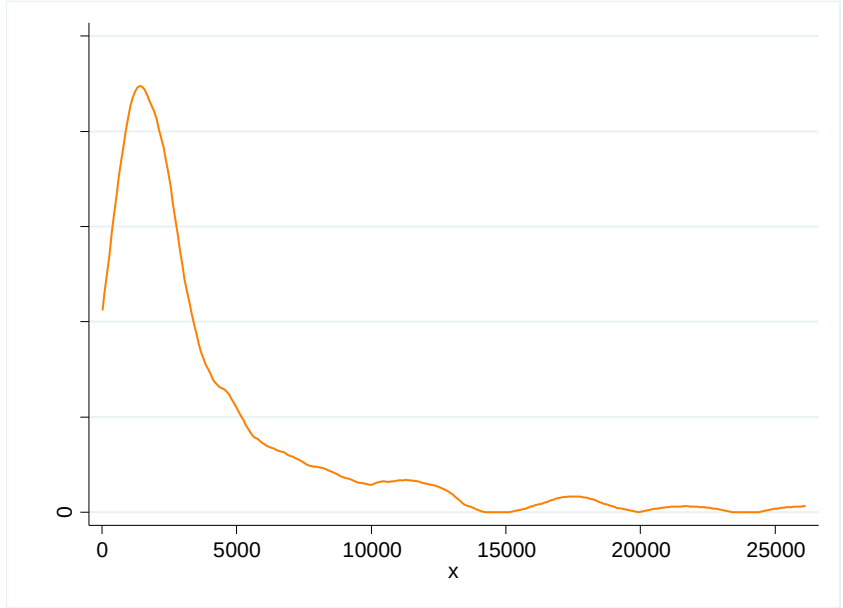
For purposes of tracking poverty over time, the two most important price indexes in India are the Consumer Price Index for Industrial Workers (CPIIW) and the Consumer Price Index for Agricultural Labourers (CPIAL). Reweighted versions of each are used to update the urban and rural poverty lines. Until it was revised in November 1995, the CPIAL itself was based on prices regularly collected from a sample of 422 villages, weighted using an expenditure pattern that dates back to 1960/61. By the time of the revision in 1995, which was later than any of the survey data used in this chapter, the weights for the

In the case of India, D&T (2005) flagged as outliers prices whose logarithms exceeded the mean of logarithms by more than 2.5 standard deviations:

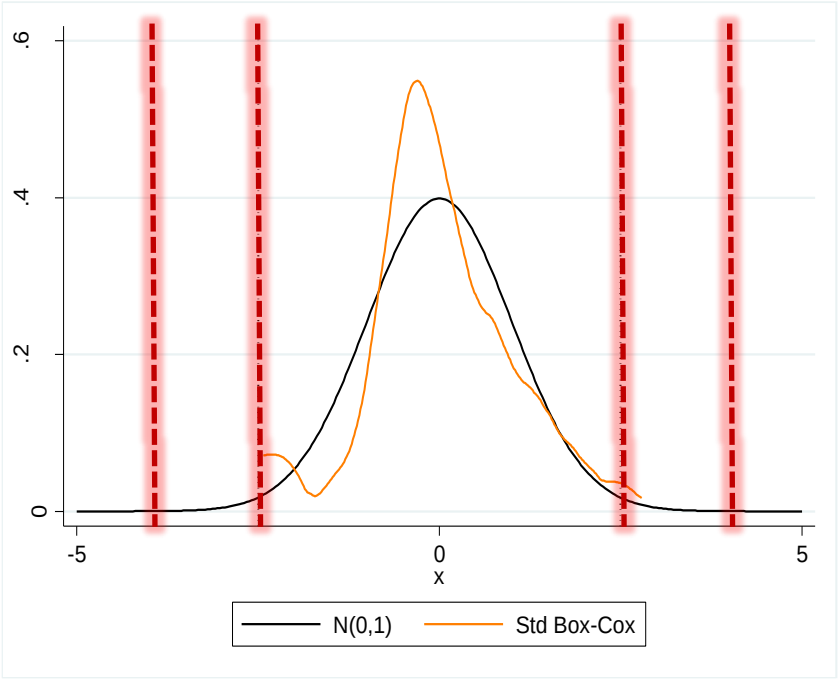
$$\frac{ln(x) - E[ln(x)]}{sd[ln(x)]} > 2.5$$

Transformation and thresholds

Raw untransformed data



Transformed data



Two questions

1) How good is such an approach?

2) What to do after flagging outliers?

How good is such an approach?

- Log-transformation is very basic – how to deal with negative values?
- Not recommended when the log-distribution can not be assumed to be a Normal distribution
- Why should we set the threshold using the mean and standard deviation, which are sensitive to extreme values, if this is exactly what we are worried about?

$$\frac{ln(x) - E[ln(x)]}{sd[ln(x)]} > 2.5$$

- We can do better

A popular strategy

robustification

- While there is no agreement on the best method, a common solution is to use robust measures of scale and location to set the threshold for flagging outliers

- the idea is to replace the sample average $\bar{x}$ with a robust estimator (e.g. the median), and the standard deviation s with a robust estimator. A popular option is the median absolute deviation (MAD).

The median absolute deviation (MAD)

$$z_h = \frac{x_h - \bar{x}}{s}$$

$$z_h = \left| \frac{x_h - med[x_h]}{MAD} \right|$$

$$MAD = b \times med|x - med[x]|$$

b = 1.4826

if the distribution is Gaussian

We can do better

Rousseeuw and Croux (1993, JASA)

Alternatives to the Median Absolute Deviation

Peter J. Rousseeuw and Christophe Croux*

In robust estimation one frequently needs an initial or auxiliary estimate of scale. For this one usually takes the median absolute deviation $\mathrm{MAD}_n = 1.4826\ \mathrm{med}_i\{|x_i - \mathrm{med}_j x_j|\}$, because it has a simple explicit formula, needs little computation time, and is very robust as witnessed by its bounded influence function and its 50% breakdown point. But there is still room for improvement in two areas: the fact that MAD_n is aimed at symmetric distributions and its low (37%) Gaussian efficiency. In this article we set out to construct explicit and 50% breakdown scale estimators that are more efficient. We consider the estimator $S_n = 1.1926\ \mathrm{med}_i\{\mathrm{med}_j|x_i - x_j|\}$ and the estimator Q_n given by the .25 quantile of the distances $\{|x_i - x_j|; i < j\}$. Note that S_n and Q_n do not need any location estimate. Both S_n and Q_n can be computed using $O(n \log n)$ time and $O(n)$ storage. The Gaussian efficiency of S_n is 58%, whereas Q_n attains 82%. We study S_n and Q_n by means of their influence functions, their bias curves (for implosion as well as explosion), and their finite-sample performance. Their behavior is also compared at non-Gaussian models, including the negative exponential model where S_n has a lower gross-error sensitivity than the MAD.

KEY WORDS: Bias curve; Breakdown point; Influence function; Robustness; Scale estimation.

Rousseeuw and Croux (1993)

- Rousseeuw and Croux (1993) propose to substitute the MAD with a different estimator:

- $S = c \times \mathrm{med}_i\{\mathrm{med}_j|x_j - x_i|\}$

- For each i we compute the median of $|x_i - x_j|$ (j = 1, …, n). This yields n numbers, the median of which gives our final estimate S.

$$z_h = \left|\frac{x_h - med[x_h]}{S}\right|$$

c = 1.1926 at the Gaussian model.

Recap

- “take the log and run” is not a recommended practice
- taking the log and robustifying the z-score is a better practice
- Belotti and Vecchi (2019) provide `outdetect.ado`

Malawi, 2013

Method	Overall	Left	Right
Z-score	2.08	0.20	1.88
MAD-score	3.05	0.35	2.70
S-score	3.02	0.35	2.67
Q-score	3.00	0.35	2.65

- ‘take the log and run’: 2.08% of outliers (most of which in the right tail)
- ‘take the log, robustify the z-score, and run’: 3.00% (most of which in the right tail)

How to deal with outliers?

(in one slide)

Treatment of outliers

Three main methods of dealing with outliers, apart from removing them from the dataset:

1) reducing the weights of outliers (trimming weight)
2) changing the values of outliers (Winsorisation, trimming, imputation)
3) using robust estimation techniques (M-estimation).

- Documentation, transparency & reproducibility

Lessons learned

- Outliers can be genuine observations… be gentle to the data and document each and every step of the data processing
- As far as inequality is concerned, outliers are the worst enemy (unbounded IF)
- Outlier detection:
  - go beyond the "take the log and run" strategy. It works well only if you can describe the data with a Gaussian distribution. Typically, however, distributions are skewed.
  - Use a "take the log, robustify the z-score and run", strategy.
- Outlier treatment: it depends. Quantile regression is a good candidate.

References

Required readings

Barnett, V., & Lewis, T. (1994). Outliers in Statistical Data. 3rd edition. J. Wiley & Sons (Chapter 1 & 2)

Suggested readings

Alvarez, E., Garcıa-Fernández, R. M., Blanco-Encomienda, F. J., & Munoz, J. F. (2014). The effect of outliers on the economic and social survey on income and living conditions. World Acad. Sci., Eng. Technol., Int. J. Soc., Behav., Educ., Econ., Bus. Ind. Eng, 8, 3276-3280.

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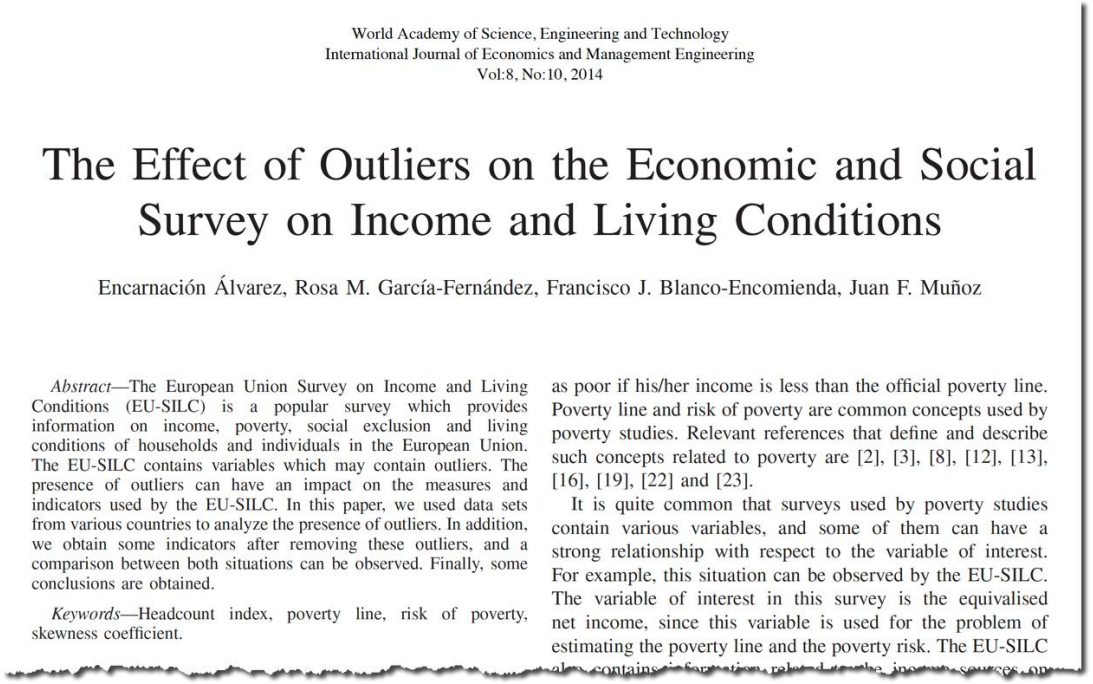
Thank you for your attention

Homework

Exercise 1 - Engaging with the literature



World Academy of Science, Engineering and Technology
International Journal of Economics and Management Engineering
Vol:8, No:10, 2014

The Effect of Outliers on the Economic and Social Survey on Income and Living Conditions

Encarnación Álvarez, Rosa M. García-Fernández, Francisco J. Blanco-Encomienda, Juan F. Muñoz

Abstract—The European Union Survey on Income and Living Conditions (EU-SILC) is a popular survey which provides information on income, poverty, social exclusion and living conditions of households and individuals in the European Union. The EU-SILC contains variables which may contain outliers. The presence of outliers can have an impact on the measures and indicators used by the EU-SILC. In this paper, we used data sets from various countries to analyze the presence of outliers. In addition, we obtain some indicators after removing these outliers, and a comparison between both situations can be observed. Finally, some conclusions are obtained.

Keywords—Headcount index, poverty line, risk of poverty, skewness coefficient.

as poor if his/her income is less than the official poverty line. Poverty line and risk of poverty are common concepts used by poverty studies. Relevant references that define and describe such concepts related to poverty are [2], [3], [8], [12], [13], [16], [19], [22] and [23].

It is quite common that surveys used by poverty studies contain various variables, and some of them can have a strong relationship with respect to the variable of interest. For example, this situation can be observed by the EU-SILC. The variable of interest in this survey is the equivalised net income, since this variable is used for the problem of estimating the poverty line and the poverty risk. The EU-SILC

Summarize the main conclusions of the paper: do outliers matter? Why or why not?

Exercise 2 - Do-it-yourself….

English

1) Generate a log-normal looking wealth distribution

2) Estimate the Gini index

3) Contaminate the distribution with a few extreme values

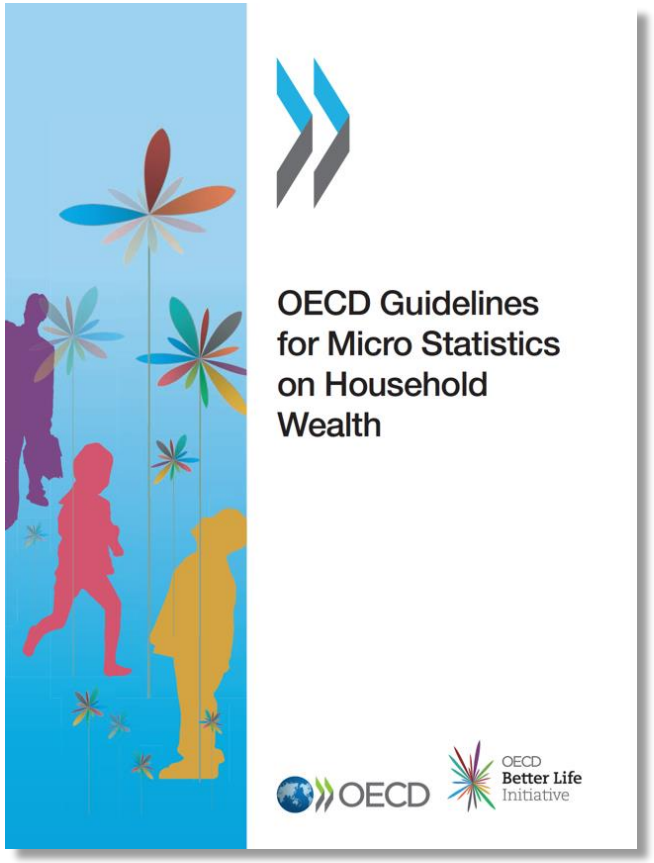
4) Re-estimate the Gini index

Stata/R/SPSS/Excel/…

```
clear

set obs 5000
set seed 198607
gen n  = rnormal(0,1)
gen ln = exp(n)

* simulate order of magnitude mistake:
* take 100 obs around the median
* of the distribution and multiply
* them by 100

sort ln

gen cont100 = 1
replace cont100 = 100 in 2480/2520
gen ln_cont100 = ln*cont100
```

Exercise 3 – Inequality measures

- Comment on table 7.3 from OECD (2013) p.172 (see next slide).
- What can you say about the sensitivity of estimates to the treatment of outliers?



Exercise 3 – Inequality measures

OECD (2013)

Table 7.3. **Effect of the treatment of outliers on summary measures of wealth inequality in the United States, 2007**

	Raw	Shave top and bottom 1%	Shave top 1% and bottom 0.5%
Mean	556 846	378 215	559 361
Median	120 780	120 780	123 800
Gini	0.82	0.74	0.81
$\frac{1}{2}CV^2$	18.1	2.4	14.6
P90/P10	30 000	3 369	3 061
P75/P25	26.3	24.5	24.3
P90/P50	7.6	7.0	7.4
n	4 418	3 698	4 359

Source: 2007 Survey of Consumer Finances.