Additive Presentation of ICP Results

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Introduction

- This presentation is a follow-up to my previous paper “The Gerschenkron Effect in International Comparisons, 2011 and 2017” (https://thedocs.worldbank.org/en/doc/29c71545f07864081057606e69491e05-0050022021/original/2-02-RA-Item-05-The-Gerschenkron-effect-in-ICP-2011-and-ICP-2017-Dikhanov-Presentation.pdf) presented at May 2021 TAG Meeting: https://www.worldbank.org/en/programs/icp/brief/tag-may-2021-doc;
- Additive indexes and Gerschenkron effect to be further discussed;
- World prices for neutral presentation to be introduced;
- Two versions of presenting ICP additive results to be shown.

Gerschenkron effect

- The *Gerschenkron effect* (*or bias*) in spatial comparisons can be defined as an overvaluation of a country's real GDP due to a deviation of the country's price structure from a base price structure.

- The base price structure can be another country’s price structure or, under an aggregation method that produces a set of international prices, the international price structure.

- Thus, the Gerschenkron effect translates into an overvaluation of the poorer country, while utilizing for the comparison the richer country's price structure, and, correspondingly, an overvaluation of the richer country, while using for the comparison the poorer country's price structure.

- In international comparisons, the Gerschenkron effect has been usually associated with poorer countries appearing richer versus the OECD countries. Normally, in order to determine the Gerschenkron effect of an index, we have to reference this “overvaluation” to some “unbiased” index. In this paper we will use the official 2011 and 2017 ICP index – the GEKS (Gini-Éltető-Köves-Szulc).

GK and IDB indexes

The traditional presentation of the GK (Geary-Khamis) system (in terms of international prices **π** and **PPP**) can be written down as follows:

$$\begin{cases} \pi_i = \sum_j p_j^i \nu_j \kappa_j^i \\ \nu_j = \sum_i \frac{\pi_i}{p_j^i} \omega_j^i \end{cases} \qquad (1)$$

where $\omega_j^i = \frac{p_j^i q_j^i}{\sum_k p_j^k q_j^k}$, $\kappa_j^i = \frac{q_j^i}{\sum_l q_l^i}$ and $\nu_j = 1 / PPP_j$.

The IDB (Ikle-Dikhanov-Balk) index can be written in the following way:

$$\begin{cases} \pi_i = \dfrac{\sum_j \omega_j^i}{\sum_j \dfrac{\omega_j^i}{p_j^i \nu_j}} \\ \nu_j = \sum_i \frac{\pi_i}{p_j^i} \omega_j^i \end{cases} \qquad (2)$$

Additivity is maintained through the second part of systems (1) and (2):

$$\nu_j = \sum_i \frac{\pi_i}{p_j^i} \omega_j^i = \sum_i \pi_i q_j^i \Big/ \sum_i p_j^i q_j^i = \frac{y_j}{e_j},$$

This is equivalent of saying that total real product of country *j* is a linear combination of quantities of that country $y_j = \sum_i \pi_i q_j^i$.

Thus, this property allows decomposition of the global product by country and commodity in an additive way.

THE GERSCHENKRON EFFECT IN ICP 2011 AND 2017: BY REGION

Average regional price levels, relative to GEKS (free), 2011

	AFR	ASI	OEC	LAC	WAS
US = 1.00					
GEKS (fixity)	1.03	1.03	1.02	1.02	1.01
GK	0.92	0.94	0.99	0.95	0.94
IDB	1.01	1.00	1.00	1.02	1.01
OECD = 1.00					
GEKS (fixity)	1.00	1.00	1.00	0.99	0.98
GK	0.93	0.95	1.00	0.96	0.95
IDB	1.01	1.00	1.00	1.02	1.01

THE GERSCHENKRON EFFECT IN ICP 2011 AND 2017: BY REGION

Average regional price levels, relative to GEKS (free), 2017

	AFR	ASI	OEC	LAC	WAS
US = 1.00					
GEKS (fixity)	1.00	0.99	0.99	0.98	1.00
GK	0.91	0.95	0.98	0.95	0.93
IDB	1.03	1.01	1.01	1.04	1.01
OECD = 1.00					
GEKS (fixity)	1.01	1.00	1.00	0.99	1.01
GK	0.92	0.96	1.00	0.97	0.94
IDB	1.02	1.00	1.00	1.03	1.00

Additivity in international comparisons

- Additivity is often considered a desired statistical property for international comparisons. Under additivity, the sum of expenditures of GDP components in PPP terms will equal the real expenditures obtained by dividing GDP totals in national currencies by the aggregated PPP for GDP, or, in other words, the real expenditures for components will add up to their totals in real terms.
- The GEKS method [the official ICP method] does not provide results that are additive, whereas both GK and IDB methods do. However, GEKS satisfies the economic approach to index number theory, it is one of the superlative multilateral methods (or, rather, it is based on a binary superlative method – the Fischer index. It can be based on any other binary superlative index as well).
- However, an additive method is not consistent with the economic approach to index number theory if the number of countries being compared is greater than two (if the number of countries is two, then one can always find an international price vector such that the resulting additive index employing this price vector will equal that superlative index). Thus, in the case with more than two countries, the countries whose price vectors diverge from the "international or world" prices used in additive methods should produce quantity shares that are biased upward. In the case of GK, this bias can be clearly seen.

Additivity in international comparisons

The inconsistencies due to the GEKS' non-additivity could be quite significant. Consider, for example, GROSS FIXED CAPITAL FORMATION. It consists of three sub-categories:

MACHINERY AND EQUIPMENT
CONSTRUCTION
OTHER PRODUCTS

For quite a few countries that participated in the ICP 2011 and 2017 (around 1/3 in fact), real expenditure on CONSTRUCTION was found to be greater than overall GROSS FIXED CAPITAL FORMATION. This situation was observed for most of the Asia region (for 12 countries out of 22 in 2011), and close to one half of non-OECD countries. For example, in Bangladesh CONSTRUCTION exceeded GROSS FIXED CAPITAL FORMATION by 57%, in Ethiopia – by 79% (estimated from the ICP 2017 results) .

Such inconsistencies could be observed for any GEKS aggregates. The GK and IDB results naturally would not have these inconsistencies.

Additivity in international comparisons

Importance of additivity and additive indexes has been widely discussed in the literature. Thus, Deaton and Heston (2009) advocate against GK for analyses of the world distribution of income or of world poverty, "and in favor of GEKS or even IDB type methods. For work on growth on or other macroeconomic questions, the additivity properties of GK or IDB may be more important, and the GK deficiencies may not be serious when the analysis is dominated by relatively similar rich countries. When such analyses involve poor countries in a substantial way, it should be borne in mind that the GK international prices that are used to value their goods and services are biased towards rich country prices... For these, IDB is worth serious consideration."

Diewert (2010) agrees with Deaton and Heston that "the ICP needs to provide users with at least two sets of results, one of which is an additive method (and it appears that IDB is "better" than GK for the reasons indicated by Deaton and Heston) and one of which is based on an economic approach to international comparisons such as GEKS."

It appears that publishing the two indices (GEKS and IBD) would indeed address multiple users' needs, and could bring in new users of ICP data working on problems where additive decomposition of real expenditures is required.

Having two indexes, however, may cause confusion for users. Yet, non-additivity itself is a source of great confusion for many a user who try to make sense out of GEKS expenditure shares. The solution, perhaps, would be publishing additive shares [but not additive real expenditures].

World Prices

Normally, a vector of international prices π is expressed in terms of a base country (it is the inverse of the base country's PPPs for each commodity or aggregate), which could be rather inconvenient for interpreting these international prices. However, it is possible to express the results in a country independent manner using World (or Region) as the reference.

In order to do so, first, we run aggregation (with any country as the base). Then, we express prices as PLIs w.r.t. World (or Region), i.e., as normalized prices:

$$ {}_N p_j^i = p_j^i / XR_j / \left(\sum_j y_j / \sum_j e_j / XR_j \right) \qquad \textbf{(3)} $$

Expressed in these terms World real output [$\boldsymbol{Y}$] will be scaled to World output nominal, converted by exchange rates [$\boldsymbol{E}$]:

$$ Y = \sum_j y_j \equiv \sum_j e_j / XR_j = E $$

World prices (i.e., the international prices π in this normalized system) will be independent of base country.

Note that the World prices are quite different from "world average prices", which was a way to reduce non-additivity in ICP 2005 and 2011 published results. However, non-additivity was still present (sometimes significantly), and users did not understand why those "world average prices" were not equal to US$.

A1. EXP nominal (US$)	NGA	IND	USA	BRA	World
DOMESTIC ABSORPTION	**335.5**	**2626.1**	**20094.8**	**2048.0**	**25104.4**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	258.5	1502.0	12898.3	1301.6	15960.4
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	1.5	0.0	413.8	28.5	443.9
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	4.3	86.8	1207.7	180.6	1479.5
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	13.1	188.2	1549.5	235.4	1986.2
GROSS CAPITAL FORMATION	58.1	849.1	4025.5	301.8	5234.5

A2. PLI (USA = 1)	NGA	IND	USA	BRA	World
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	0.378	0.294	1.000	0.744	0.781
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	0.252	0.309	1.000	0.446	0.917
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	0.278	0.396	1.000	0.500	0.820
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	0.229	0.458	1.000	0.773	0.856
GROSS CAPITAL FORMATION	0.798	0.451	1.000	0.763	0.821

A3. PLI (World = 1)					
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	0.484	0.377	1.281	0.953	1.000
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	0.275	0.337	1.090	0.486	1.000
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	0.339	0.483	1.219	0.610	1.000
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	0.268	0.536	1.169	0.904	1.000
GROSS CAPITAL FORMATION	0.973	0.549	1.218	0.930	1.000

A4. PLI normalized	NGA	IND	USA	BRA	World (1 / π)
DOMESTIC ABSORPTION	**0.518**	**0.423**	**1.254**	**0.918**	**1.000**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	0.501	0.389	1.324	0.985	1.034
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	0.412	0.505	1.635	0.728	1.499
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	0.424	0.604	1.527	0.764	1.253
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	0.293	0.587	1.280	0.990	1.095
GROSS CAPITAL FORMATION	0.799	0.451	1.000	0.763	0.821

A5. EXP real (US$), at World prices A5 = A1 / A4	NGA	IND	USA	BRA	World
DOMESTIC ABSORPTION	**647.6**	**6205.6**	**16020.6**	**2230.6**	**25104.4**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	516.4	3858.4	9741.6	1321.8	15438.2
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	3.7	0.0	253.1	39.2	296.0
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	10.2	143.7	790.8	236.5	1181.1
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	44.6	320.8	1210.5	237.8	1813.7
GROSS CAPITAL FORMATION	72.8	1882.7	4024.5	395.4	6375.4

One-stage (direct) aggregation

Valid both for additive and non-additive indexes. It was used, e.g., in published ICP GEKS results in 2005, 2011 (it was called "at world average prices"

This presentation relevant only for additive indexes.

Last column of A4 is the inverse of World price vector π in the normalized form, and it is *independent* of base country. World price vector π is a characteristic of input data (price and expenditure details).

Super-countries*

The ICP has hierarchical structure with regions being estimated independently and then linked together to form the global comparison. The ICP uses almost exclusively the GEKS index, but, certainly, the two-stage processing can be used for any index, including additive indexes.

There are two possible approaches to the two- (or multiple-) stage computation: (A) Super-countries and (B) Regional redistribution (called CAR in the ICP parlance). First, let us look at Super-countries. The Super-country approach involves (1) assembling groups of countries (regions) into Super-countries, and then (2) linking thus assembled super-countries in a separate aggregation. In the final thus globally linked comparison, the sums of real expenditures of individual countries comprising a region (a Super-country) should be equal to that region's real expenditures that went into stage (2) estimation. The two- (or multi-) stage linking is usually performed in order to retain regional fixity, but in our case, for an additive presentation, we are not worried about maintaining regional fixity, as the official regional results will use GEKS anyway. [Besides, an attempt to retain intra-regional fixity would break additivity]. Thus, the linking and staging for an additive presentation will be quite different from the regular ICP linking.

Then, correspondingly, the second approach would involve running an unrestricted comparison first, and the regions would be summations of individual countries, but without redistribution (and fixity).

For this discussion, we will consider four countries and combine them into two regions (Super-countries): REG1 combines NGA and IND, REG 2 – USA and BRA.

TWO-STAGE AGGREGATION

Regions (Super-countries) -> Countries

B1. EXP nominal (US$)	REG1	REG2	World
DOMESTIC ABSORPTION	**2961.6**	**22142.8**	**25104.4**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	1760.5	14199.9	15960.4
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	1.5	442.3	443.9
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	91.1	1388.3	1479.5
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	201.3	1784.9	1986.2
GROSS CAPITAL FORMATION	907.2	4327.3	5234.5

In Table B1 the regional sums are computed by mechanical addition of their constituent countries.

B2. PLI (USA = 1)	REG1	REG2	World
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	0.304	0.969	0.781
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	0.252	0.926	0.917
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	0.388	0.885	0.820
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	0.430	0.963	0.856
GROSS CAPITAL FORMATION	0.464	0.979	0.821

In Table B2, the regional price levels are obtained using formula (3). Note that World column is the same as in Table A2, thus the World and regional PLIs are consistent.

Aggregating the regions, with normalization to the World total, we obtain:

B4. PLI normalized	REG1	REG2	World
DOMESTIC ABSORPTION	**0.437**	**1.208**	**1.000**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	0.407	1.298	1.045
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	0.332	1.218	1.207
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	0.499	1.139	1.055
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	0.493	1.103	0.980
GROSS CAPITAL FORMATION	0.490	1.034	0.867

The last column of Table B4 is World prices obtained for aggregation with the super-countries - $p_w^i(2stage)$.

Which corresponds to the following real expenditures:

B5. EXP real (US$), at World prices	REG1	REG2	World
DOMESTIC ABSORPTION	**6774.6**	**18329.8**	**25104.4**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	4327.1	10942.7	15269.9
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	4.6	363.2	367.8
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	182.6	1219.4	1401.9
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	408.3	1618.5	2026.9
GROSS CAPITAL FORMATION	1852.0	4186.0	6038.0

Comparing A4 and B4 we note that the world prices are different, thus the regional and world results in Tables A5 and B5 are different too (except for the world total).

Table B4 can be understood as inter-regional linking factors. Thus, the PLI for individual countries linked via these super-countries are computed as Table A3 multiplied by inverse of World PLI $p_w^i(2stage)$ from Table B4 (last column, the inverse corresponds to the world price vector) and divided by $p_w^i(1stage)$ from Table A4 (last column). The results are shown in Table C4. (Alternatively, Table C4 can be obtained by multiplying Table A3 and by the last column of Table B4.).

Table C5 shows real expenditures for the linked countries.

C4. PLI normalized (linked)	NGA	IND	USA	BRA	World
DOMESTIC ABSORPTION	**0.519**	**0.429**	**1.252**	**0.899**	**1.000**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	0.506	0.394	1.339	0.996	1.045
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	0.332	0.407	1.316	0.586	1.207
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	0.358	0.509	1.287	0.644	1.055
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	0.262	0.525	1.145	0.886	0.980
GROSS CAPITAL FORMATION	0.843	0.476	1.056	0.806	0.867

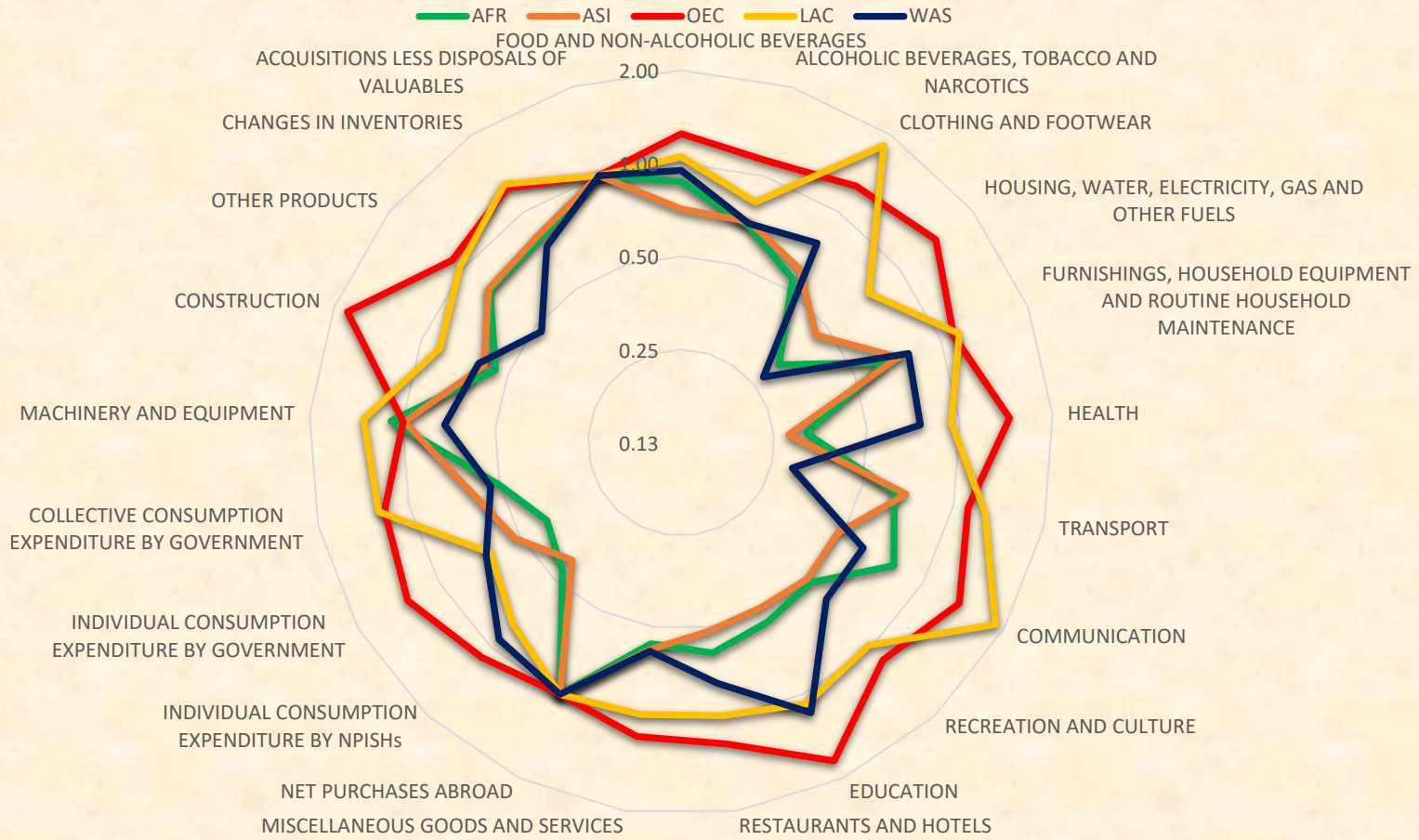
C5. EXP real (US$), at World prices, linked C5 = A1 / C4	NGA	IND	USA	BRA	World
DOMESTIC ABSORPTION	**646.2**	**6128.4**	**16052.9**	**2276.9**	**25104.4**
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	510.8	3816.4	9635.4	1307.4	15269.9
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	4.6	0.0	314.5	48.7	367.8
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	12.1	170.5	938.7	280.7	1401.9
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	49.8	358.5	1352.8	265.7	2026.9
GROSS CAPITAL FORMATION	69.0	1783.0	3811.5	374.5	6038.0

Thus, the two versions differ in how the vector of world prices is arrived at: (1) straight from individual countries (the inverse of last column of A4), or (2) via super-country assemblies (the inverse of last column of B4/C4).

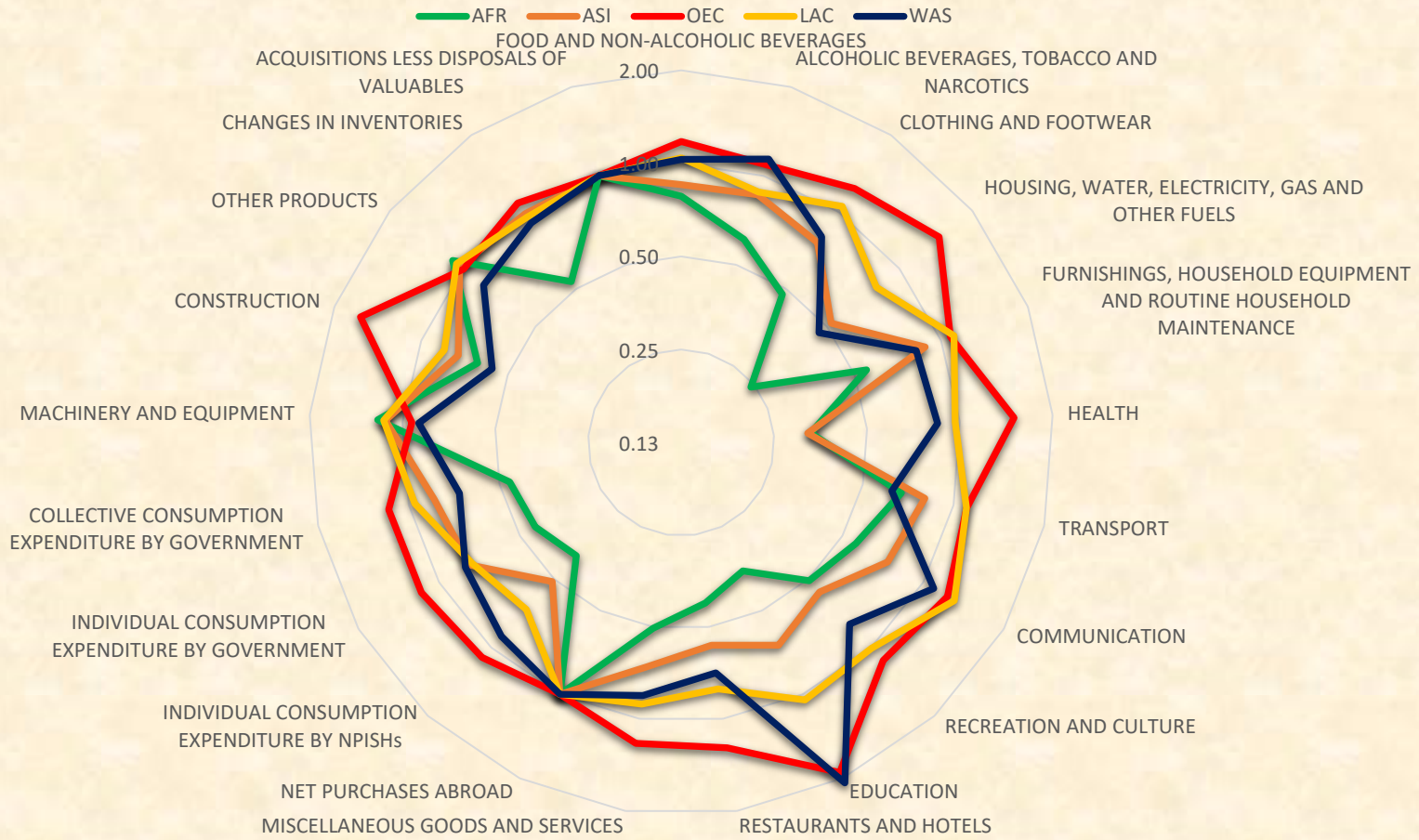
In both versions fixity is not maintained by design.

*) Note, that the Super-country concept shown in Tables B1 and B2, which is regional base country neutral, is quite different from the Super-country concept used in ICP 2005, which was not neutral to regional base's choice.

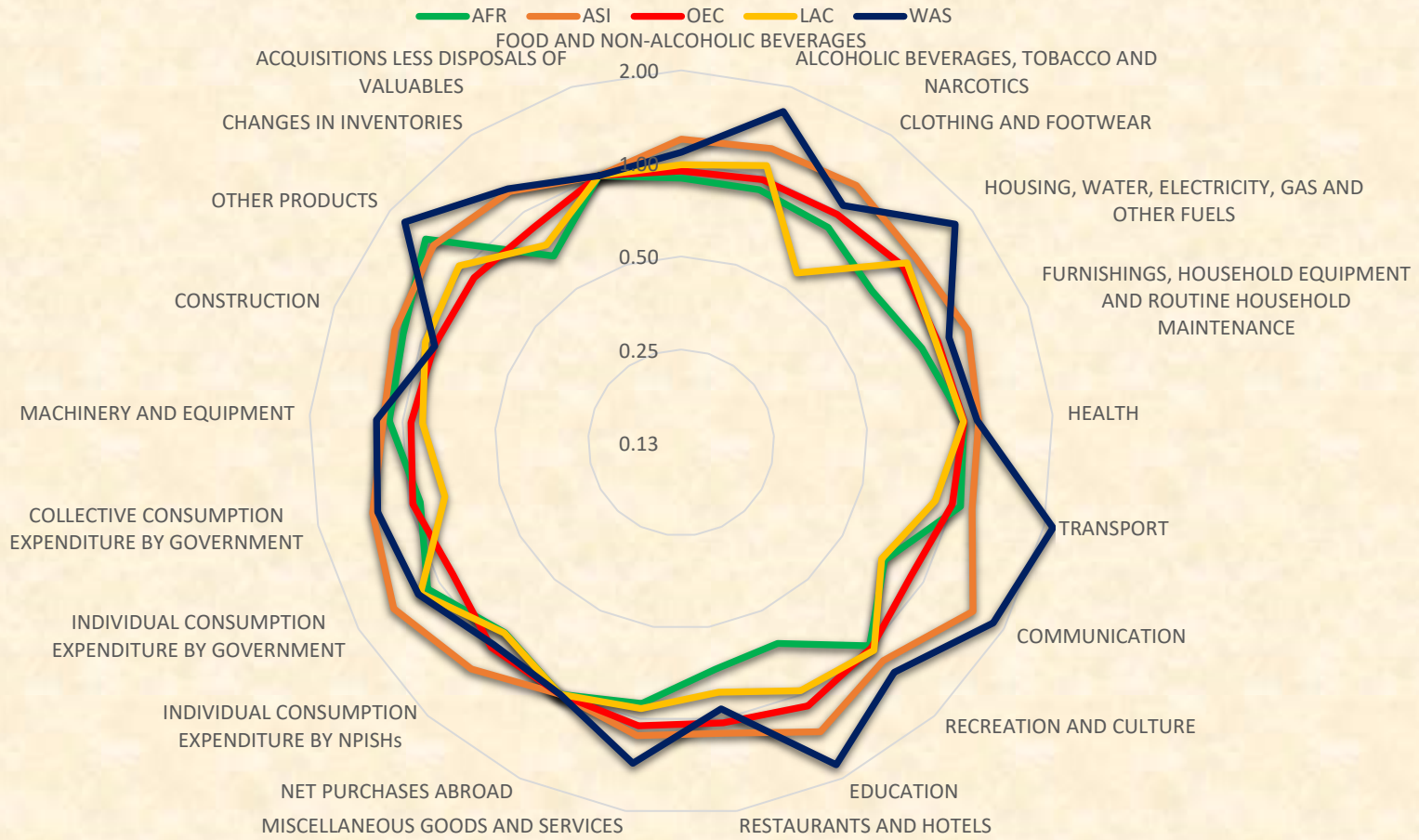
Relative regional price (world = 1.0), by category, 2011

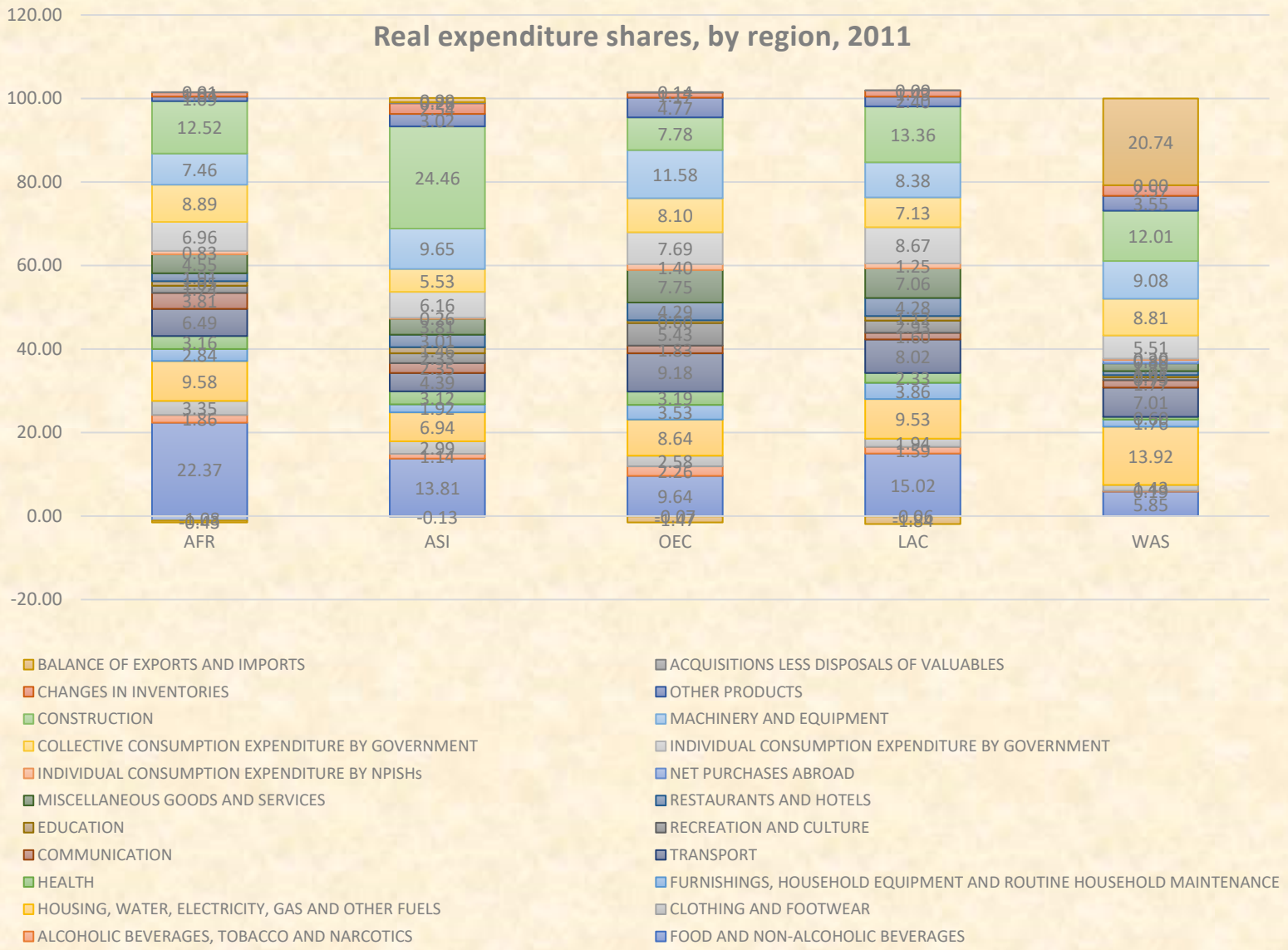


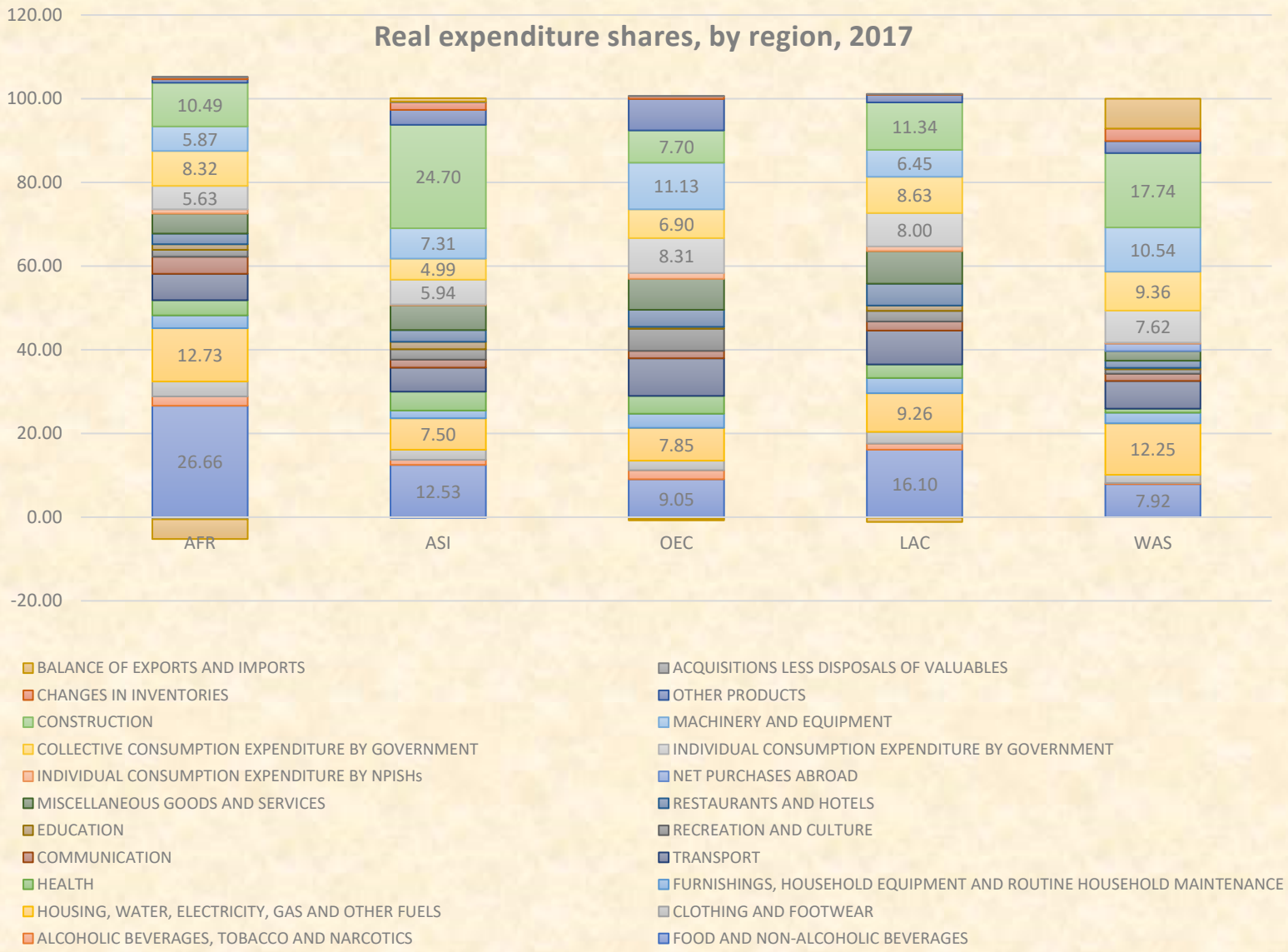
Relative regional price (world = 1.0), by category, 2017



Relative regional price change (world = 1.0), by category, 2011-17




Real expenditure shares, by region, 2011
120.00
100.00
80.00
60.00
40.00
20.00
0.00
-20.00
AFR
ASI
OEC
LAC
WAS
12.52
7.46
8.89
6.96
6.49
3.16
2.84
9.58
3.35
1.86
22.37
24.46
9.65
5.53
6.16
3.81
3.12
1.92
6.94
2.99
1.14
13.81
-0.13
7.78
11.58
8.10
7.69
1.40
7.75
4.29
5.43
9.18
3.19
3.53
8.64
2.58
2.26
9.64
13.36
8.38
7.13
8.67
1.25
7.06
4.28
8.02
3.86
9.53
1.84
15.02
20.74
3.55
12.01
9.08
8.81
5.51
7.01
13.92
5.85
BALANCE OF EXPORTS AND IMPORTS
CHANGES IN INVENTORIES
CONSTRUCTION
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs
MISCELLANEOUS GOODS AND SERVICES
EDUCATION
COMMUNICATION
HEALTH
HOUSING, WATER, ELECTRICITY, GAS AND OTHER FUELS
ALCOHOLIC BEVERAGES, TOBACCO AND NARCOTICS
ACQUISITIONS LESS DISPOSALS OF VALUABLES
OTHER PRODUCTS
MACHINERY AND EQUIPMENT
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT
NET PURCHASES ABROAD
RESTAURANTS AND HOTELS
RECREATION AND CULTURE
TRANSPORT
FURNISHINGS, HOUSEHOLD EQUIPMENT AND ROUTINE HOUSEHOLD MAINTENANCE
CLOTHING AND FOOTWEAR
FOOD AND NON-ALCOHOLIC BEVERAGES


Real expenditure shares, by region, 2017
120.00
100.00
80.00
60.00
40.00
20.00
0.00
-20.00
AFR
ASI
OEC
LAC
WAS
10.49
5.87
8.32
5.63
12.73
26.66
24.70
7.31
4.99
5.94
7.50
12.53
7.70
11.13
6.90
8.31
7.85
9.05
11.34
6.45
8.63
8.00
9.26
16.10
17.74
10.54
9.36
7.62
12.25
7.92
BALANCE OF EXPORTS AND IMPORTS
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Conclusions

The Gerschenkron effect (**GE**) for additive indexes [GK and IDB] was smaller in ICP 2011 and 2017 than in previous benchmarks, esp. when compared to GE from the 70s and 80s; though, GE for GK index was still significant;

IDB was found to be preferrable on that basis;

GE was smaller still at the regional level (for IDB); Thus, even publishing regional composites only, without individual countries, has its own benefits for structural analysis;

With this presentation, intertemporal comparison of shares is also facilitated, the changes are easier to decompose [additively] and interpret;

Two sets of world prices can be used in an additive presentation: (1) based on all individual countries, and (2) based on super-countries (i.e., regions assembled from respective countries); differences between the two are not significant;

Presentation 1 would be similar to the one published for IDB ICP 2005 results (though it was only for shares).

To avoid confusion, only shares and PLIs should be published, not real expenditures.