

# Long-Term Growth Model (LTGM)

## MTI Forum Learning Module

Presenters: Steven Pennings (DECMG, [spennings@worldbank.org](mailto:spennings@worldbank.org) )

and Jorge Guzmán (DECMG, [jguzmancorrea@worldbank.org](mailto:jguzmancorrea@worldbank.org))

13 November 2019 (updated 25 September 2020)

[www.worldbank.org/LTGM](http://www.worldbank.org/LTGM) (internet)

<http://LTGM> (intranet FURL)



# Model Overview

- Countries want to grow at high rates
  - What growth rates are feasible? What would it take to achieve these goals?
- A simple model to analyze long-term growth
  - Based on celebrated Solow-Swan Model: savings and investment key
  - Also TFP, human capital, demographics, labor participation, FDI & external debt
  - Implications of growth for poverty
- Toolkit for use by country economists/policymakers in many countries
  - Spreadsheet-based for simplicity.
  - No macros; transparent, flexible & easy-to-learn
- Many extensions: public investment, WB HCI, TFP, Natural Resources....

# Objectives of the Main LTGM

- Help policy makers in finding answers to 3 important policy questions:
  - Submodel 1: How much growth from a given investment profile?
  - Submodel 2: How much investment is needed to achieve given growth profile?
  - Submodel 3: How much growth from a given savings profile?
    - Requires assumptions on debt or current account balance
- Allow policy makers ample flexibility
  - Scenario analysis using many other variables: Productivity, Human Capital, Demographics, External sector
  - Growth → Poverty
- For long-run scenario analysis -- not short-run analysis or forecasting

# Some examples of work using the LTGM

Used in 40+ countries for growth analysis and country reports (CEMs and SCDs):

- **Sub-Saharan Africa:** Cameroon (CEM), Cape Verde (SCD), Eswatini, Gabon, Guinea (SCD), Seychelles (SCD), Ghana (SCD), Malawi, South Africa, Ivory Coast, Mauritania Zambia (SCD), Zimbabwe
- **South Asia:** Bangladesh, Nepal (CEM), Sri Lanka (CEM)
- **Latin America & Caribbean :** Brazil, Costa Rica, El Salvador, Guatemala, Honduras, Nicaragua, Panama, Peru
- **East Asia & Pacific:** Cambodia, Korea, Laos, Malaysia, Philippines, Vietnam, Thailand
- **Europe and Central Asia:** Armenia (SCD), Bosnia, Georgia (SCD), Kyrgyz Republic (SCD), Tajikistan (CEM)
- **MENA:** Egypt (CEM), Syria
- **Eg Cameroon CEM 2016** – goal to boost growth to 8% become an UMI country by 2035.
  - Planned ↑Investment insufficient higher TFP growth → ↑competition to boost TFP
- **Honduras, Panama, Peru, Zambia, Bangladesh, Malaysia, Cambodia** – LTGM Training for govt officials

# Outline of the Rest of the Talk

## **Part A: Main LTGM**

1. Explanation of how the growth model works
  - Equations, parameters, assumptions and drivers of growth
2. Hands-on demonstration and tutorial
  - Examples: investment path  $\rightarrow$  growth, growth+inequality  $\rightarrow$  poverty, growth target  $\rightarrow$  required investment, savings+ CAB  $\rightarrow$  growth

## **Part B: Public Capital Extension (and other extensions)**

- List of extensions
- Overview of LTGM-Public Capital extension
- Hands-on demonstration using LTGM-PC

Comments/Questions/Suggestions

# A1. The Growth Model

# Three Building Blocks of the Model

## 1. Production Function

$$Y_t(GDP) = A_t K_t^{1-\beta} (h_t L_t)^\beta$$

## 2. Capital Accumulation

$$K_{t+1} = (1 - \delta)K_t + I_t$$

## 3. Demographics and Labor Market:

$$y_t^{PC}(GDP \text{ per capita}) = \frac{Y_t}{N_t} = \frac{Y_t}{L_t} \frac{L_t}{W_t} \frac{W_t}{N_t} = A_t k_t^{1-\beta} h_t^\beta \rho_t \omega_t$$

( $W_t$ : working-age pop;  $N_t$ : total population;  $\rho_t$ : participation rate;  $\omega_t$ : working-age-pop. to pop. ratio  $A_t$ : TFP;  $K_t$ : capital;  $h_t$ : human capital per worker;  $L_t$ : workers)

# Growth Drivers

$$g_{y,t+1} \approx g_{A,t+1} + \beta(g_{h,t+1} + g_{\omega,t+1} + g_{N,t+1} + g_{\rho,t+1}) + \left[ \frac{1-\beta}{K_t/Y_t} \right] \frac{I_t}{Y_t} - (1-\beta)\delta$$

[GDP Growth]    [TFP]    [Human Cap]    [Demographics]    [Participation]    MPK=1/mICOR    [Investment]

- Common policy message: investment-led growth [by itself] is not sustainable in long run
  - $\uparrow$  K/Y reduces the effectiveness of investment over time ( $\downarrow$ MPK)
  - Leads to an increase in the  $mICOR_t = \frac{1}{1-\beta} \frac{K_t}{Y_t}$  (ppt increase I/Y needed for extra 1% growth)
  - Needs to be accompanied by other sources (e.g., human capital, TFP, participation)



# External Sector (how to fund investment?)

1. Current Account Balance (CAB):  $I_t/Y_t = S_t/Y_t - CAB_t/Y_t$

2. External Debt (we assume  $\Delta NFA_t \approx 0$ )

$$CAB_t = \Delta NFA_t - \Delta NFL_t = -(FDI_t + (D_t - D_{t-1}))$$

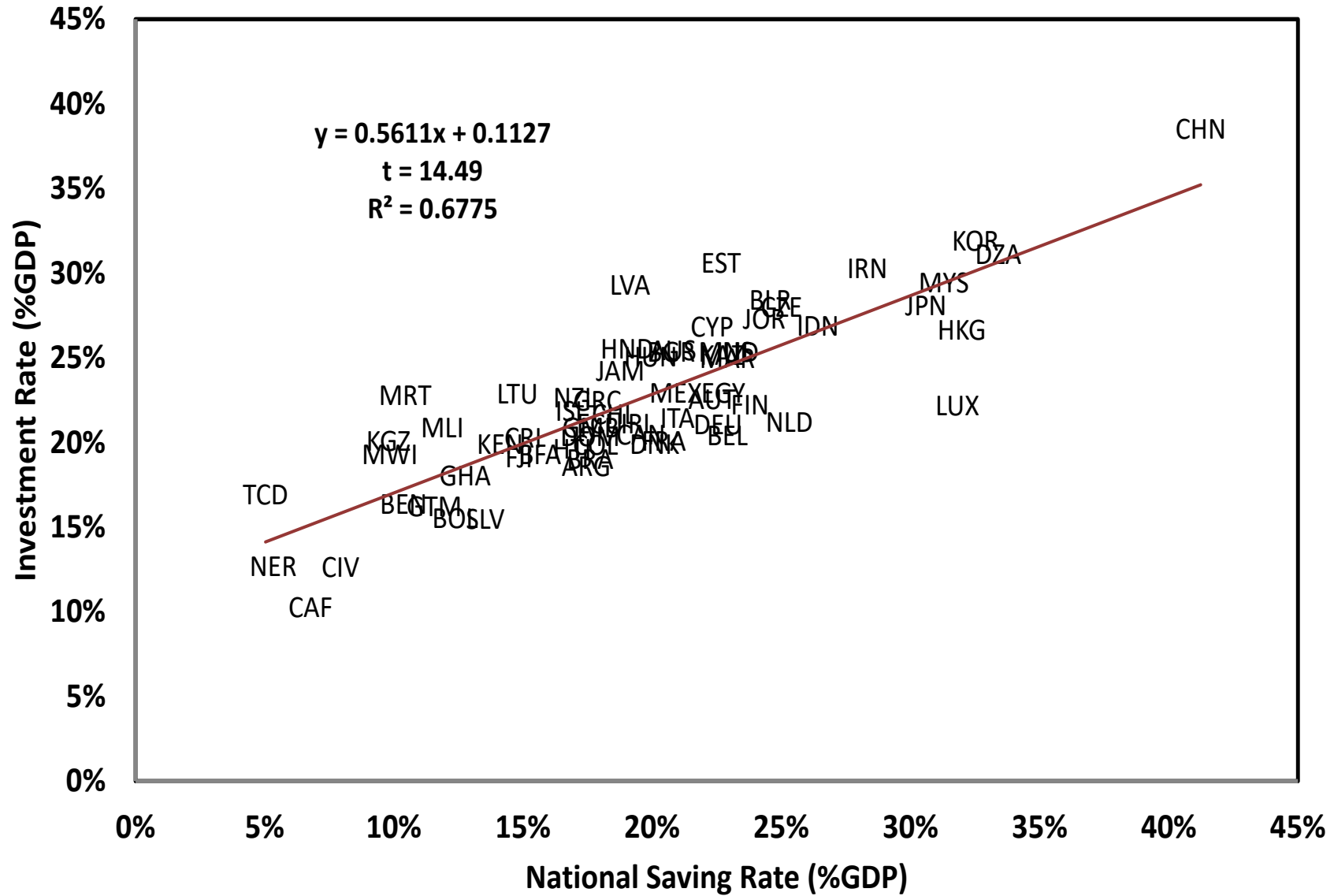
Change Net Foreign Liabilities; Foreign Direct Inv.; Change Total External Debt

$$\Rightarrow \frac{I_t}{Y_t} = \frac{S_t}{Y_t} + \frac{FDI_t}{Y_t} + \frac{D_t}{Y_t} - \frac{D_{t-1}/Y_{t-1}}{(1 + g_{y,t}^{pc})(1 + g_{N,t})}$$

- Common policy message: need to increase savings or attract FDI to fund investment plans

# Saving and Investment

Average, 1980-2008



Source: Hevia & Loayza (2012)

# Solving the Model - Parameters

- Can solve the model in simple spreadsheet *without macros*
- Minimal Data requirements - only need data on **three** parameters
  - Labor share ( $\beta$ )
  - Depreciation rate ( $\delta$ )
  - Initial Capital-to-Output Ratio ( $K_0/Y_0$ )
- $\uparrow \beta$ ,  $\uparrow \delta$  and  $\uparrow K_0/Y_0$  all make growth harder via capital accumulation
- Users can choose preloaded data source & time horizon via dropdown menu – and compare in “data summary” tab

# Solving the Model - required assumptions (future)

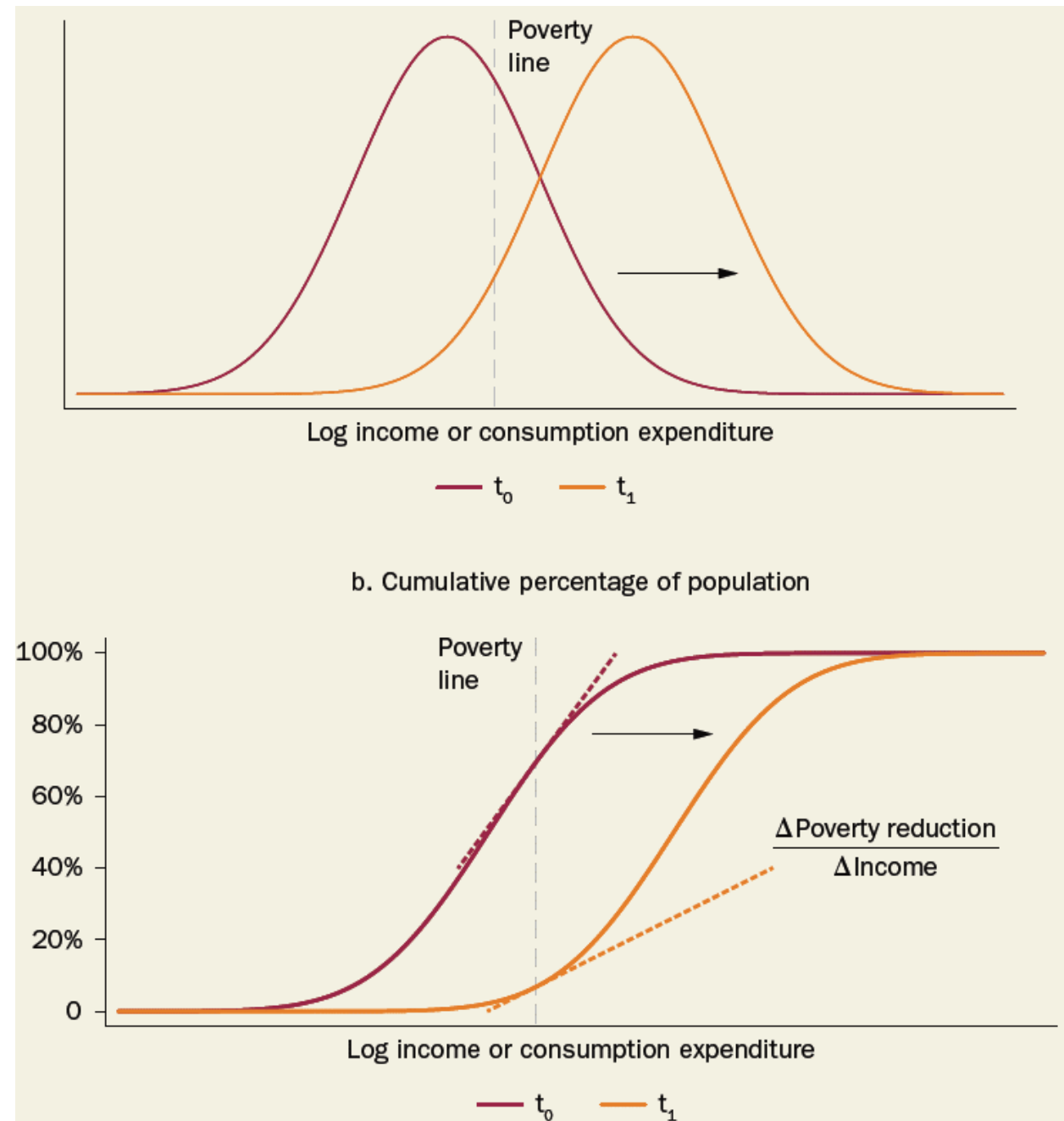
- Needed for all submodels :
  - Growth rate of TFP ( $g_{A,t}$ ); Human Capital per worker ( $g_{h,t}$ );
  - Demographics: Population ( $g_N$ ) & Working-age-pop ratio ( $g_\omega$ )
  - Participation rates ( $g_{\rho,t}$ )
- Submodel 1: Choose Investment share of GDP ( $I_t/Y_t$ )
  - Model calculates returns the growth rate of GDP per capita (or GDP per worker)
- Submodel 2: Choose Growth rate of GDP per capita ( $g_{y^{PC},t+1}$ )
  - Model calculates the investment share of GDP
- Submodel 3: Choose Savings share of GDP ( $S_t/Y_t$ ) and CAB/Y or Ext. Debt/Y & FDI/Y
  - Model calculates the growth rate of GDP per capita (or GDP per worker)

# Poverty and Growth

- 2030 Goals to eliminate extreme poverty & halve poverty (at national lines)
  - But what growth rates are required? How do current growth paths affect poverty?
- Based on Log-Normal approx. of the income distribution
  - Can analyze in Excel simply using preloaded data (no microdata required)
  - Automatically produces a Growth Elasticity of Poverty (GEP) (or users can add their own)
- Can assume constant inequality or reduced inequality (income Gini)
  - Lower inequality: (i) reduces poverty directly & (ii) increases effect of growth on poverty
- “Shared prosperity premium” where income of B40 grows faster
  - Translate this into path for Gini coefficient and examines effect on poverty rates
- Caveat: the “type” of growth doesn’t impact poverty (eg which sector grows)

# How poverty model works

- Assume a constant Gini coefficient over time.
- Growth increases everyone's income or consumption by the same percentage
  - Shifts the log distribution to the right
- Effect on poverty varies by how many people are near the poverty line
  - Larger ppt fall in poverty when poverty rate is close to 50%
  - Varies by country, poverty line & time
- In more equal countries (lower Gini coeff)
  - more compressed distribution
  - more ppl near poverty line
  - larger effect of growth on poverty



# A2: Spreadsheet Tutorial

(Hands-on demonstration)

Download LTGM spreadsheet from [www.worldbank.org/LTGM](http://www.worldbank.org/LTGM)

# LTGM Spreadsheet Structure

- **Yellow** - user can change/edit (dropdown menu or text box)
- *InputDataA\_GeneralAssumptions* – Assumptions/parameters that affect all simulations (country, start year, TFP growth, K/Y, poverty etc)
- *GraphsA* plots all general assumptions in *InputDataA*.
- *InputDataB\_ModelSpecAssumptions* – Assumptions for specific models
  - Model 1: Investment share of GDP → GDP Growth
  - Model 2: Growth rate → Investment share of GDP
  - Model 3: Savings share of GDP → GDP Growth
- *GraphsB* plots **results** of each model (and assumptions from *InputDataB*).
- Submodel 1/1s/2/2s/3/3s -- see the formulas here (no macros)
- *DataSummary* – overview of historical data and parameters from different sources



# Baseline

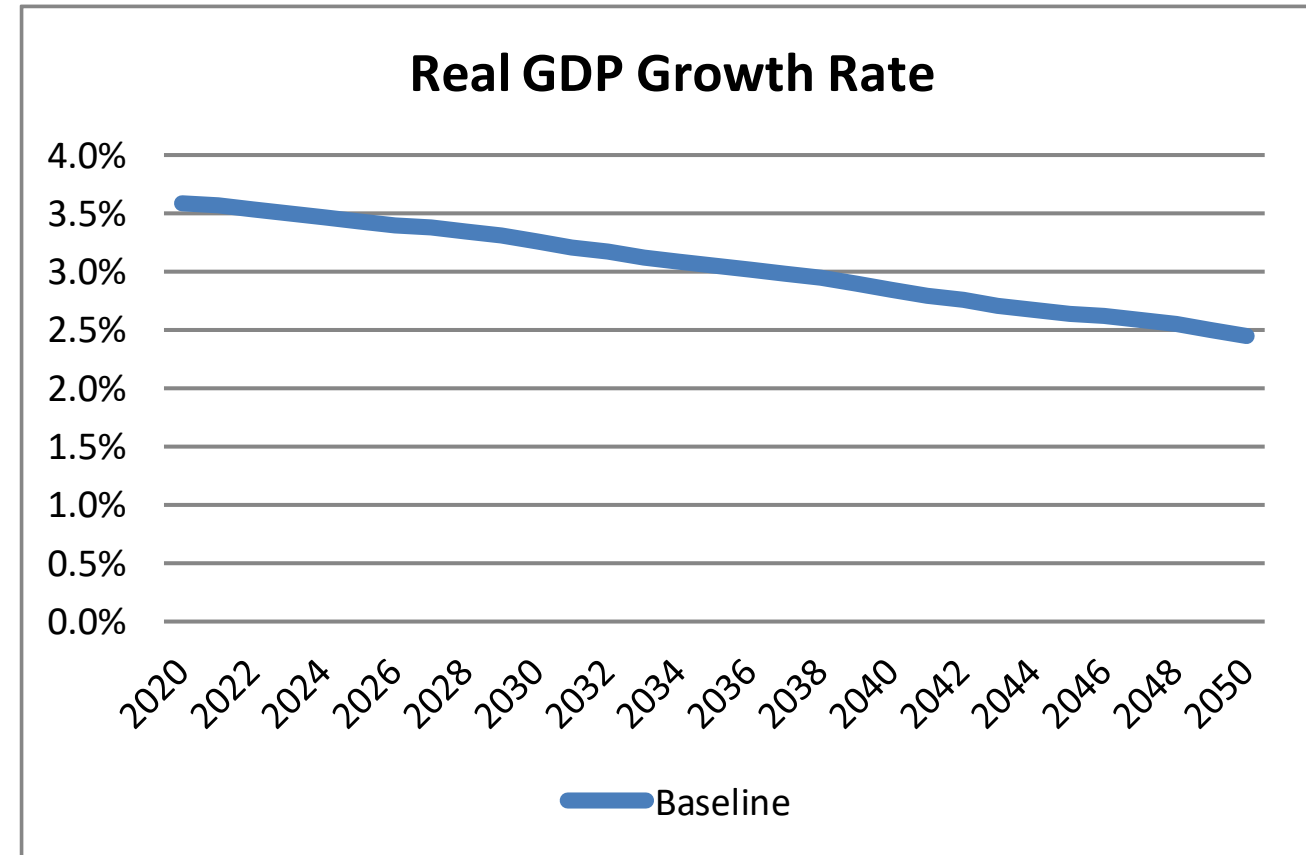
Tab *InputdataA*: Choose general parameters

- Country=Peru [Cell B3; the Default]
- Initial year=2019 [Cell D/E7; the Default]
- Labor share ( $\beta$ ) = 53.8%

[in cell C15 select PWT 9 LS. 3 Avg Wage]

- Depreciation rate ( $\delta$ ) = 4.2% [the default]
- Initial ( $K_0/Y_0$ ) = 2.62 [Cells I/J13; default]
- 0.004 (0.4%) Human capital growth initial & target [Cell D/E22; the default]
- 0.008 (0.8%) TFP growth – initial & target [Cell D/E31; the default]

Tab *InputDataB*: Choose the investment-to-GDP Ratio = 21% [Cell D/E6; the default]

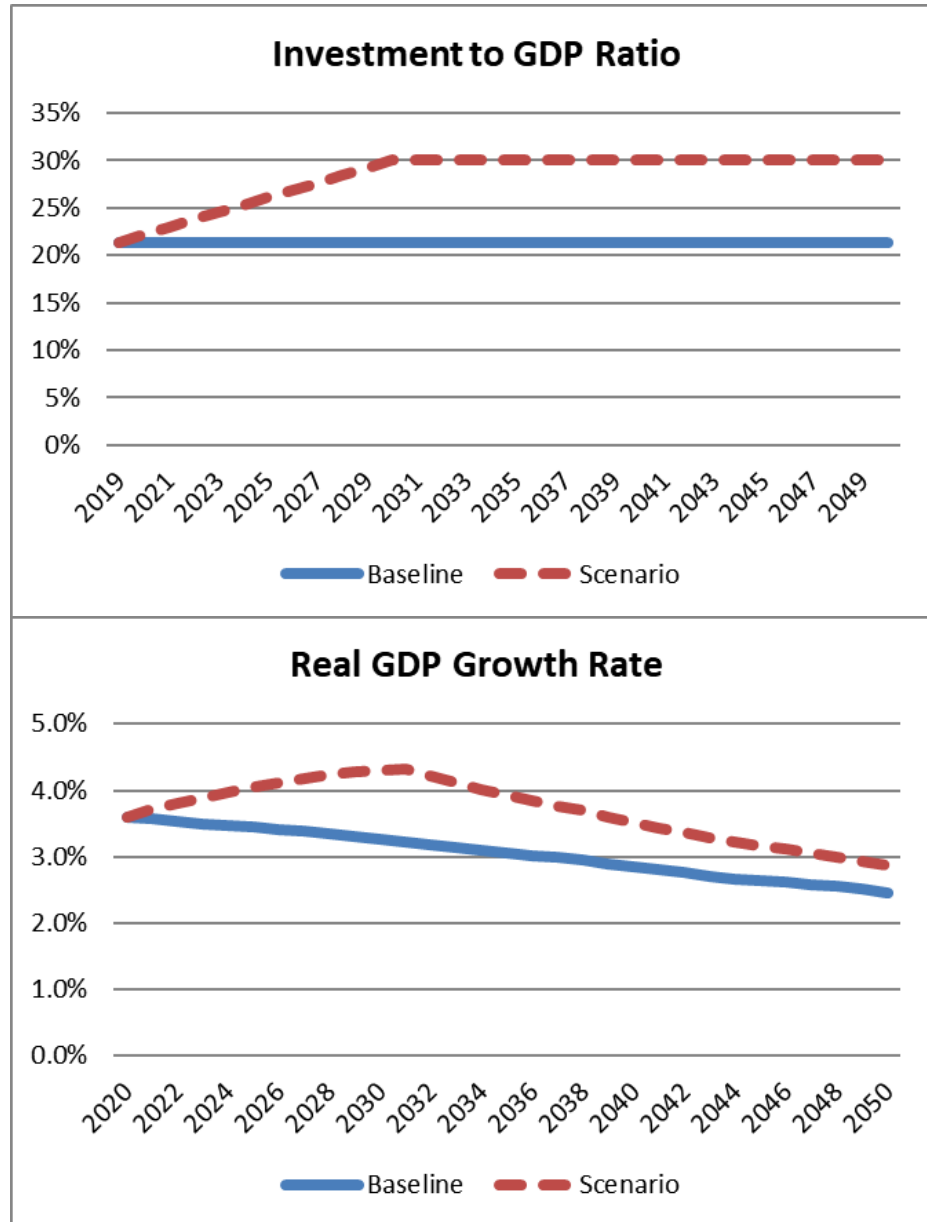


Should get this graph:

- 2020 growth of 3.6%
- 2050 growth of 2.5%

# (i) Submodel 1: Investment → Growth

- Tab *InputdataB*: Submodel 1 specific assumptions
- Baseline – no change (target=initial)
- Scenario – 0.30 I/Y [Cell E9] by 2030 (set as target) [Cell E10]
- Tab *GraphsB* - check get these graph for investment and growth→
- Growth should peak at 4.3% in 2031 in scenario



## (ii) Submodel 1: effect of growth on poverty

Start with Model 1 simulation in (i)

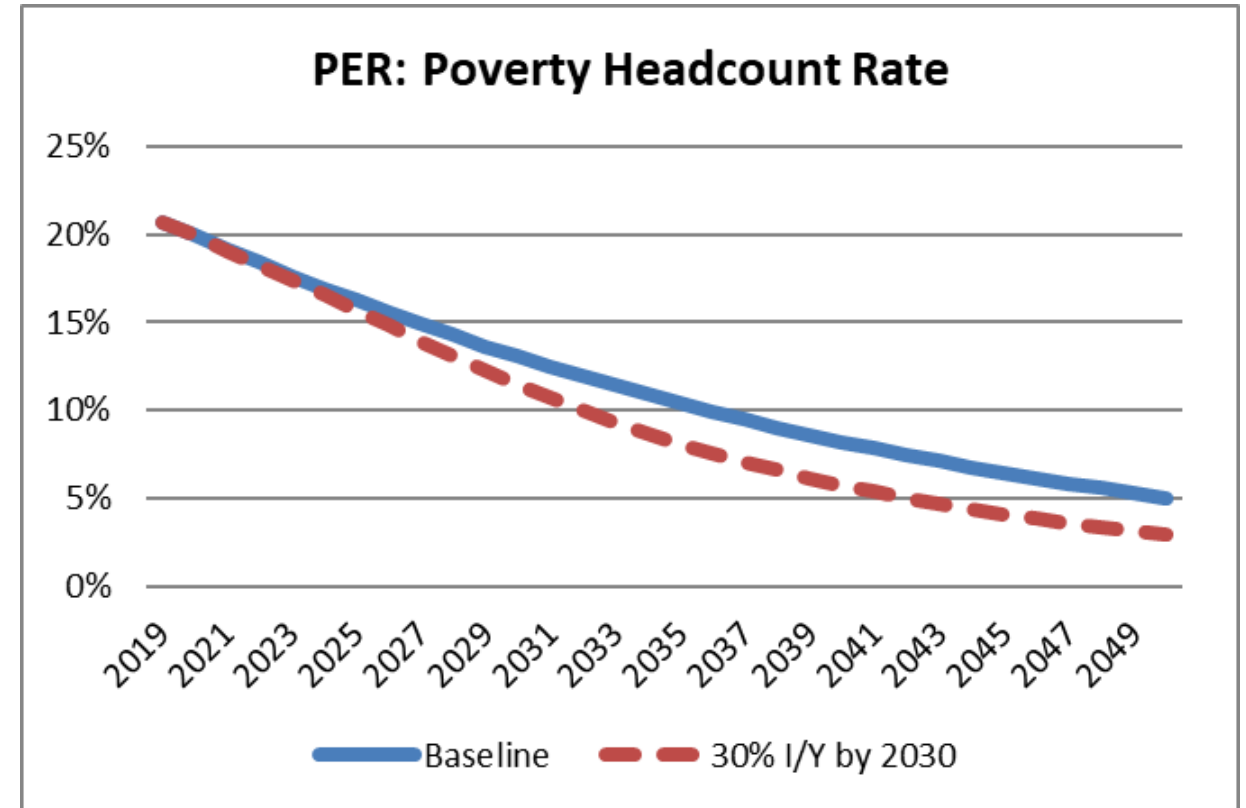
InputdataA: Choose general parameters

- National Poverty line (default) [Cell C130]
- Initial poverty rate at 0.207 (default) [Cell D/E125]
- Set the Growth Elasticity of Poverty (GEP) on Automatic (the default) [Cell D/E133]
- Make sure the Gini coefficient option is chosen [Cell D/E140, the default]
- Initial Gini coefficient of income 0.438
  - Constant (initial= target) [Cell D/E137, default]

*GraphsB - make sure you get this poverty →*

*By 2050 Results (Submodel 1/1s sheet bottom)*

- *Poverty: 4.8% (baseline) [Cell AJ 65 Submodel1]  
vs 2.9% (scenario) [Cell AJ 65 Submodel1s]*



# (iii) Submodel 1: Effect of ↓ inequality on poverty

**Goal:** Lower inequality 43.8% → 40% by 2030

(in addition to effects of higher investment)

*InputDataA:* Make sure the Gini coefficient option is chosen [Cell D/E140, the default]

Start with Model 1 simulation in (ii)

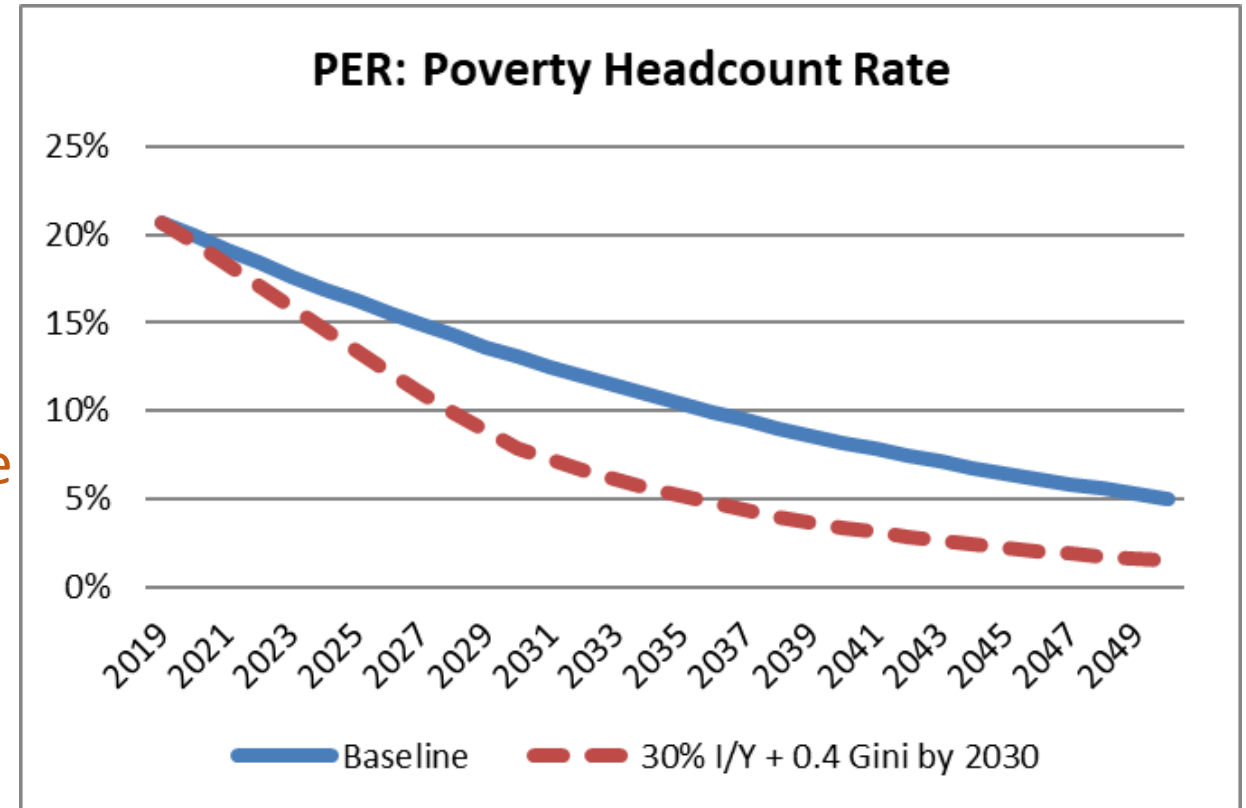
*InputdataA:* parameters for poverty

- Scenario target Gini of 0.40 by 2030 [change Cell E144 in InputdataA]

*GraphsB* - make sure you get this poverty →

*By 2050 Results (Submodel 1/1s sheet bottom)*

- Poverty: 4.8% (baseline) [Cell AJ 65 Submodel1]  
vs 1.4% (scenario) [Cell AJ 65 Submodel1s]



# (iv) Submodel 2: Growth → Investment (w/ TFP)

Tab *InputDataB*: target GDP growth rate is 3.5% in baseline and scenario [Cell D/E13 type 0.035]

InputdataA parameters for TFP growth

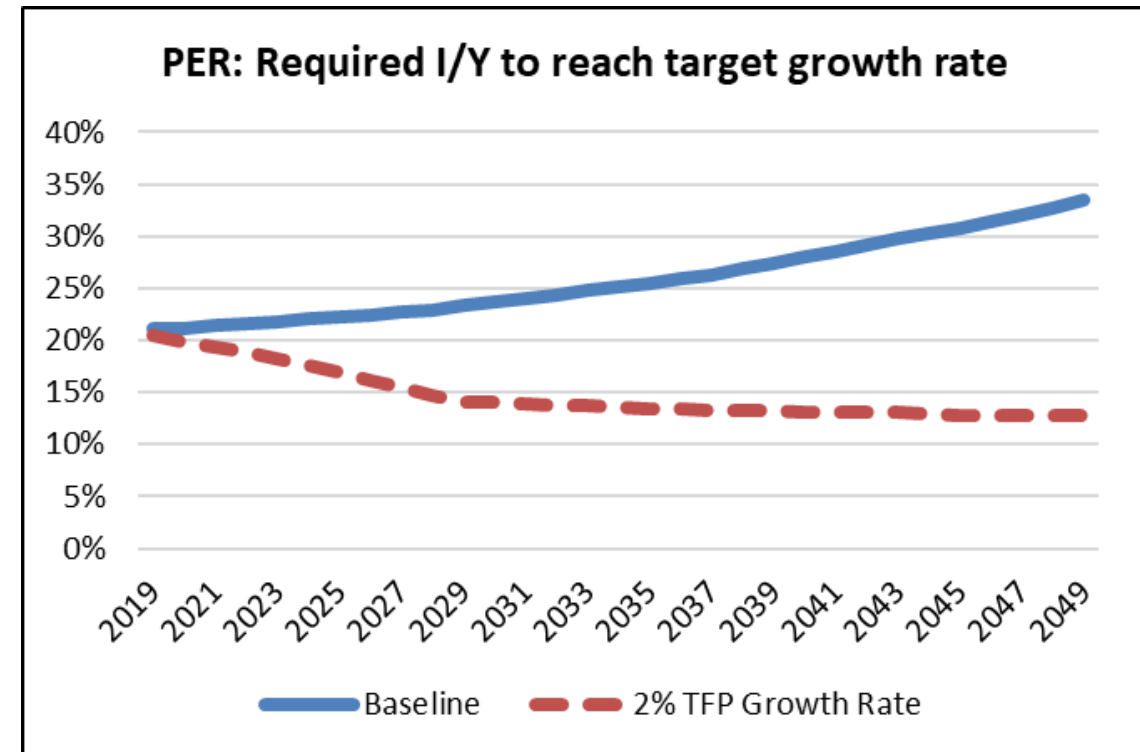
- Initial TFP growth of 0.008 (0.8% TFP growth) [Default Cell D/E29]
- Scenario: 0.02 (2%) [Cell E34] TFP growth target by 2030

*GraphsB* - check get this graph for investment →

By 2049: Required investment (Submodel 2/2s sheet)

- 0.327 (baseline) [Cell AI18] vs 0.124 (scenario) [Cell AI18]

Explanation: Declining marginal product of capital

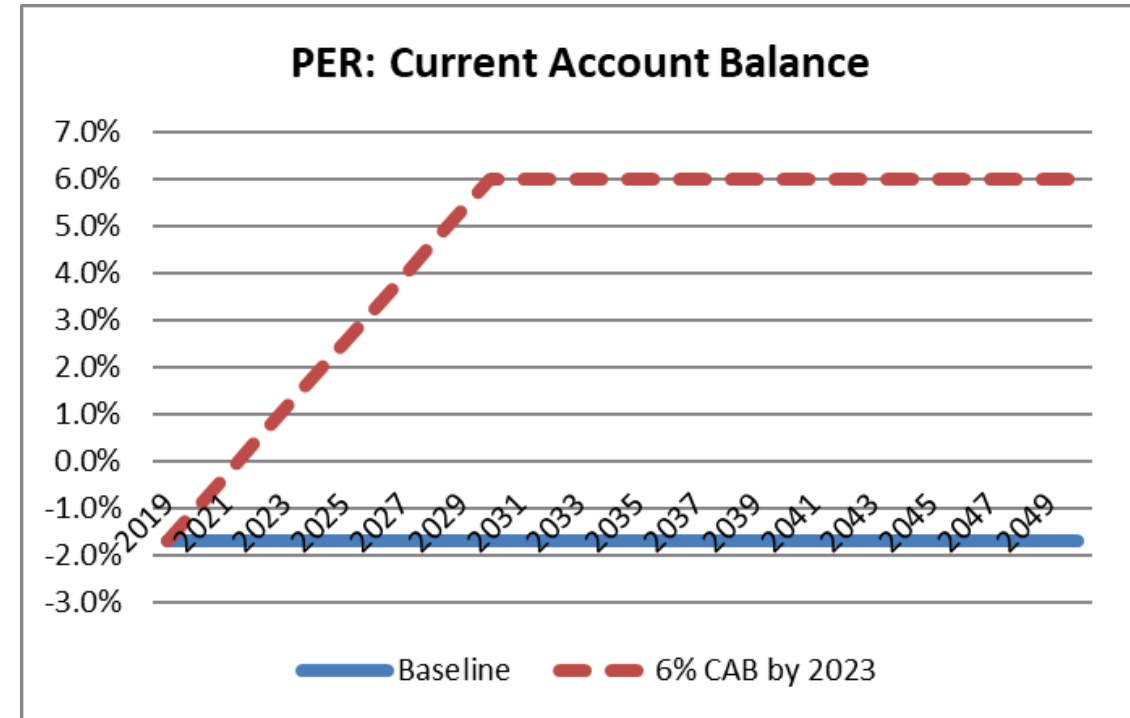


# (v) Submodel 3: Savings + CAB → Growth (ASSUMPTIONS)

Reset TFP growth rate in scenario: type  
“=E31” in Cell E34 tab InputdataA

*InputdataA*: External Balance (previously didn't matter)

- Current Account Balance Constraint (dropdown menu) [Cell D/E60; the default]
- CAB target in Scenario: 6% GDP (0.06 CAB/GDP [Cell E69] ) by 2030
- *GraphsA* - check get this graph (CAB red/blue only) →



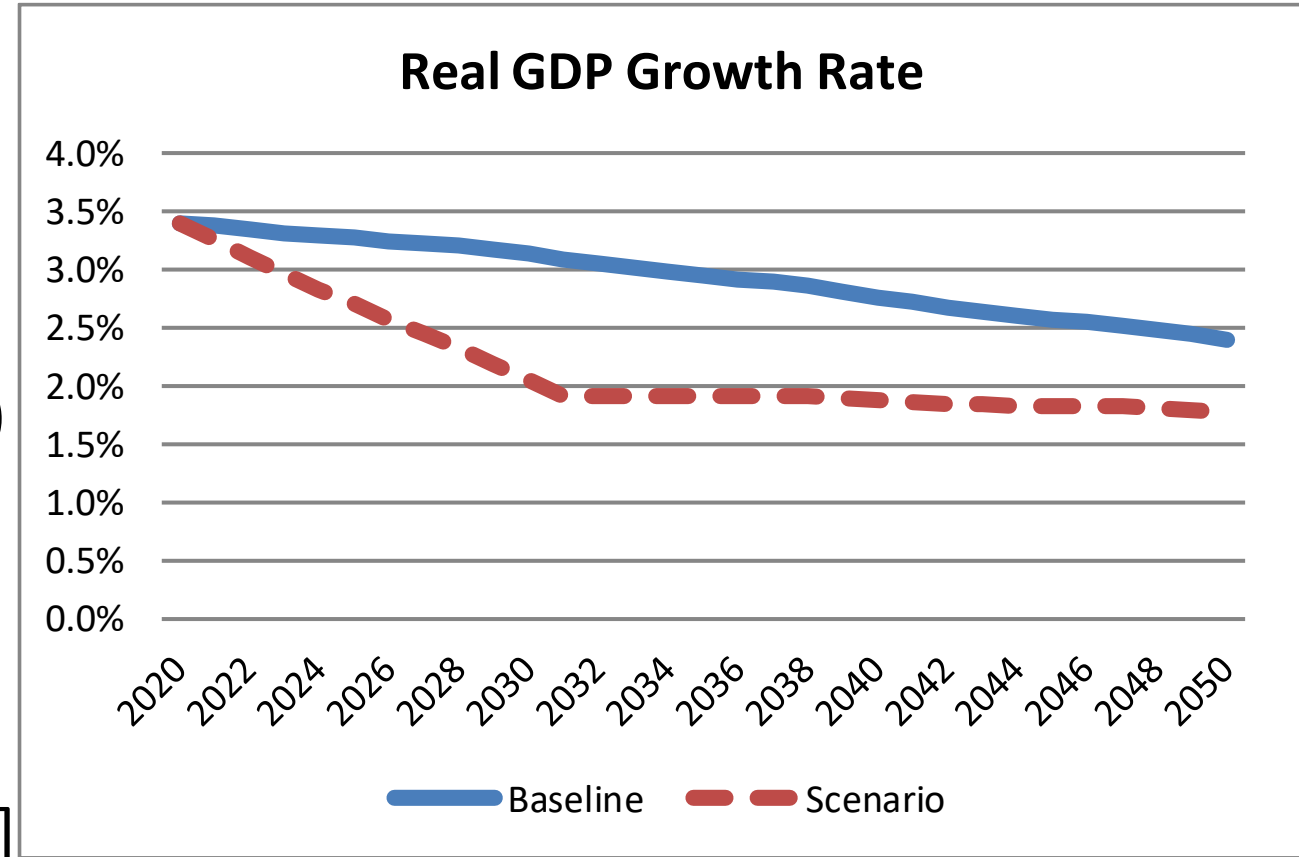
# (v) Submodel 3: Savings + CAB → Growth (RESULTS)

*InputDataB* - check savings is constant  
19% of GDP [Cell D26 & E26]

*GraphsB* - check get GDP Growth →  
(Headline GDP Growth, not per capita)

By 2030: GDP Growth result:

- 3.1% (baseline) [Cell P32 Submodel 3]
- 2.1% (scenario) [Cell P32 Submodel 3s]



# Part B. Public Capital Extension (LTGM-PC) and other Extensions

Download LTGM-PC spreadsheet from [www.worldbank.org/LTGM](http://www.worldbank.org/LTGM)



# LTGM Extensions

## **1. Public Capital extension** (Devadas & Pennings 2018)

- TODAY Private & Public investment (quantity/quality) → growth

## **2. TFP Extension** (Kim & Loayza 2018) - download at [www.worldbank.org/LTGM](http://www.worldbank.org/LTGM)

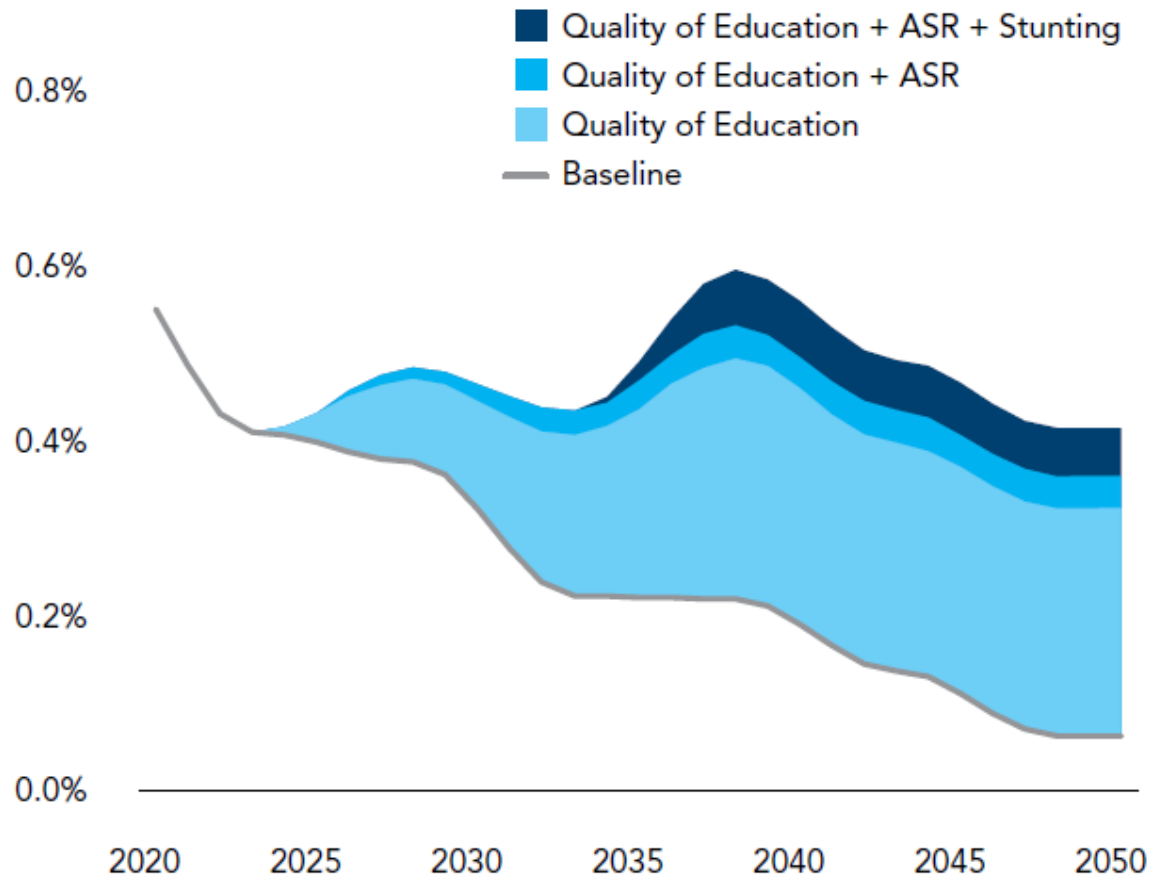
- Uses cross country regression to calculate path for TFP growth based on country's scores for:
  - innovation, education, market efficiency, infrastructure, and institutions

## **3. Human Capital Extension** (beta)

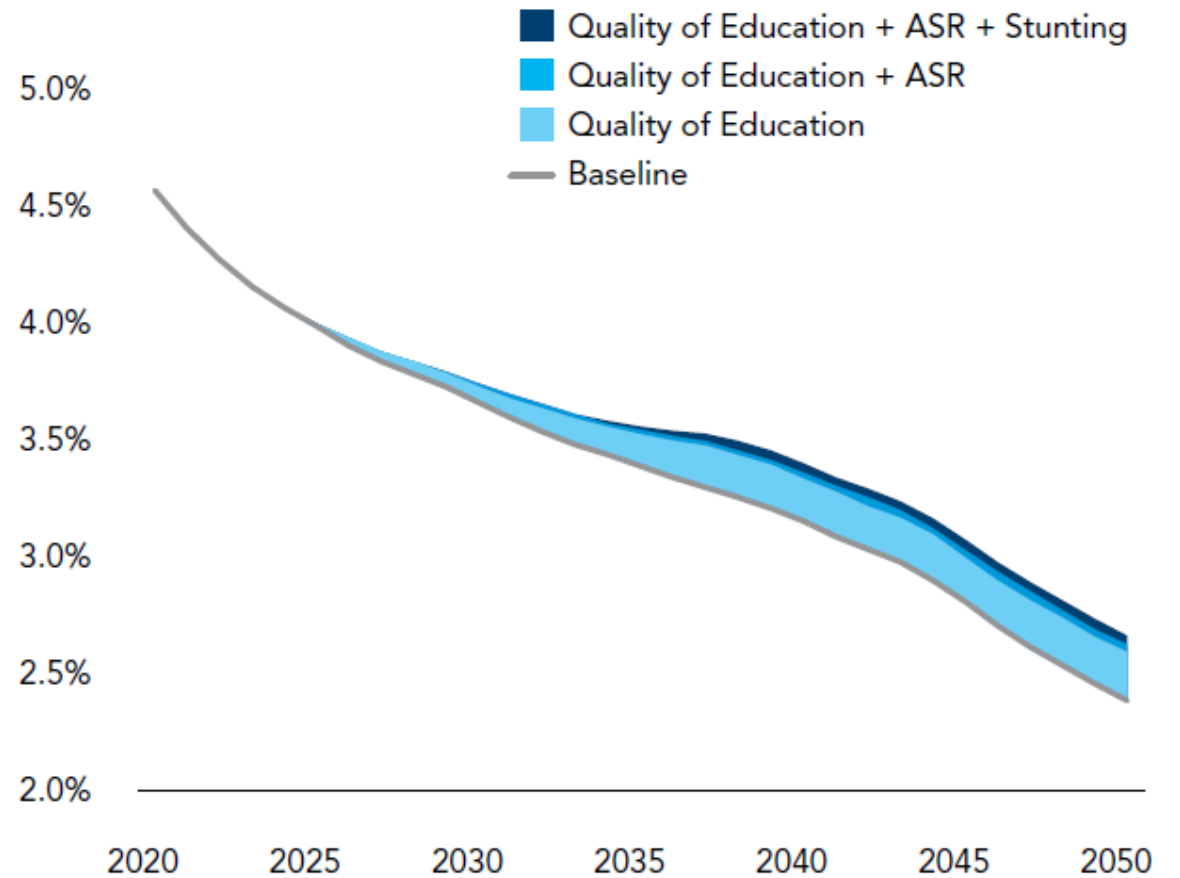
- Adapts the World Bank HCI for growth analysis (schooling quantity/quality, health)
- Based on population cohorts – reforms affecting today's children affect growth in the future.
- Example from Malaysia box: increase HCI components to median of high income country

**FIGURE 21****Simulations for Malaysia's rate of human capital growth**

Human Capital, y/y, Selected Simulations

**FIGURE 22****Simulations for Malaysia's long-term rate of GDP growth**

GDP, y/y, Selected Simulations



Source: Malaysia Economic Monitor (June 2019)

# LTGM Extensions (cont.)

## **4. Natural Resource extension (beta)**

- Commodity export sector calibrated to commodity exporters all over the world
- Analyze the effect of long-run growth of
  - Commodity price changes
  - Commodity discoveries
- Analyze different fiscal rules for commodity revenues:
  - Balance Budget Rule, Structural Surplus Rule, Hartwick's Rule

## **5. Advanced Sectoral Growth Tool (in progress)**

- 3 sectors - agriculture, manufacturing and services
- Structural transformation
- Distortions in the allocation of factors of production across sectors

# LTGM-Public Capital Extension - Overview

1. Disaggregate total investment into public  $I_t^G$  (infrastructure) and private  $I_t^P$ 
  - What is effect on growth of 1ppt GDP  $\uparrow$  **public** investment? And when?
  - Is that larger or smaller than the same size effect of private investment?
2. Introduce new quality/efficiency of public capital  $\theta_t$  (between 0 and 1)
  - Measured as new Infrastructure Efficiency Index (IEI) based on:
    - Water leaks, Power transmission losses, Unpaved roads
  - What is the effect of **efficiency/quality** of public investment on growth?

# The LTGM-PC production function

- New/changed parts in the LTGM-PC in red:

$$Y_t(\text{GDP}) = A_t (\theta_t K_t^G)^\phi (K_t^P)^{1-\beta-\phi} (h_t L_t)^\beta$$

GDP →  $Y_t$   
 Total factor productivity →  $A_t$   
 Efficiency of public capital →  $\theta_t$   
 Measured public capital stock →  $K_t^G$   
 Private capital →  $K_t^P$   
 Human capital (schooling) →  $h_t$   
 Labor →  $L_t$

- Log linear approximation:

$$g_{Y,t+1} \approx \tilde{g}_{t+1} + \phi \left( \frac{\theta_t^N - \theta_t}{\theta_t} \right) \frac{I_t^G / Y_t}{K_t^G / Y_t} + \phi \left[ \frac{I_t^G / Y_t}{K_t^G / Y_t} - \delta^G \right] + (1 - \beta - \phi) \left( \frac{I_t^P / Y_t}{K_t^P / Y_t} - \delta^P \right)$$

Growth due to quality of public K (more  $\theta$ ) →  $\left( \frac{\theta_t^N - \theta_t}{\theta_t} \right) \frac{I_t^G / Y_t}{K_t^G / Y_t}$   
 Growth due to quantity of public K (more  $K_t^G$ ) →  $\left[ \frac{I_t^G / Y_t}{K_t^G / Y_t} - \delta^G \right]$   
 Growth due to private Investment →  $\left( \frac{I_t^P / Y_t}{K_t^P / Y_t} - \delta^P \right)$

Growth due to quality of public K (more  $\theta$ )

# Calibration of LTGM-PC to Malaysia

Variable	Baseline Value	Source/Comments
Labor share income	$\beta = 0.5$	Penn World Tables v9 (2014)
Deprecation rate	$\delta = 5.8\%$	Penn World Tables v9 (2014)
Total factor productivity (TFP) growth	$g_A = 0.9\%$ initially 0.6% by 2050	Similar to PWT v9: 30 years median; 15 year average
Human capital (HC) growth	$g_h = 0.6\%$ initially 0.1% by 2040	LTGM-Human Capital Extension Similar to 2011-14 average from PWT 9
Total investment rates	$I/Y = 24\%$	IMF Article IV (2019) -2020-23 average
Public Investment rate	$I^G/Y = 6\%$	
Private Investment rate	$I^P/Y = 18\%$	
Total Capital-to-output ratio	$K/Y = 2.25$	Steady state $K/Y=(I/Y)/(g_Y + \delta)$
Public capital-to-output ratio	$K_G/Y = 1.14$	IMF FAD Investment and Capital Stock Dataset. $K_G/K=51\%$
Private capital-to-output ratio	$K_G/Y = 1.11$	
Population growth (2019-2050)	$g_N=1.3\% \rightarrow 0.4\%$	UN Population projections (via WB HDN)
Headline GDP growth in 2020	$g_Y=4.5\%$	World Bank MTI forecasts (2019-2021)

Note: also assume that quality  $\theta$  is constant

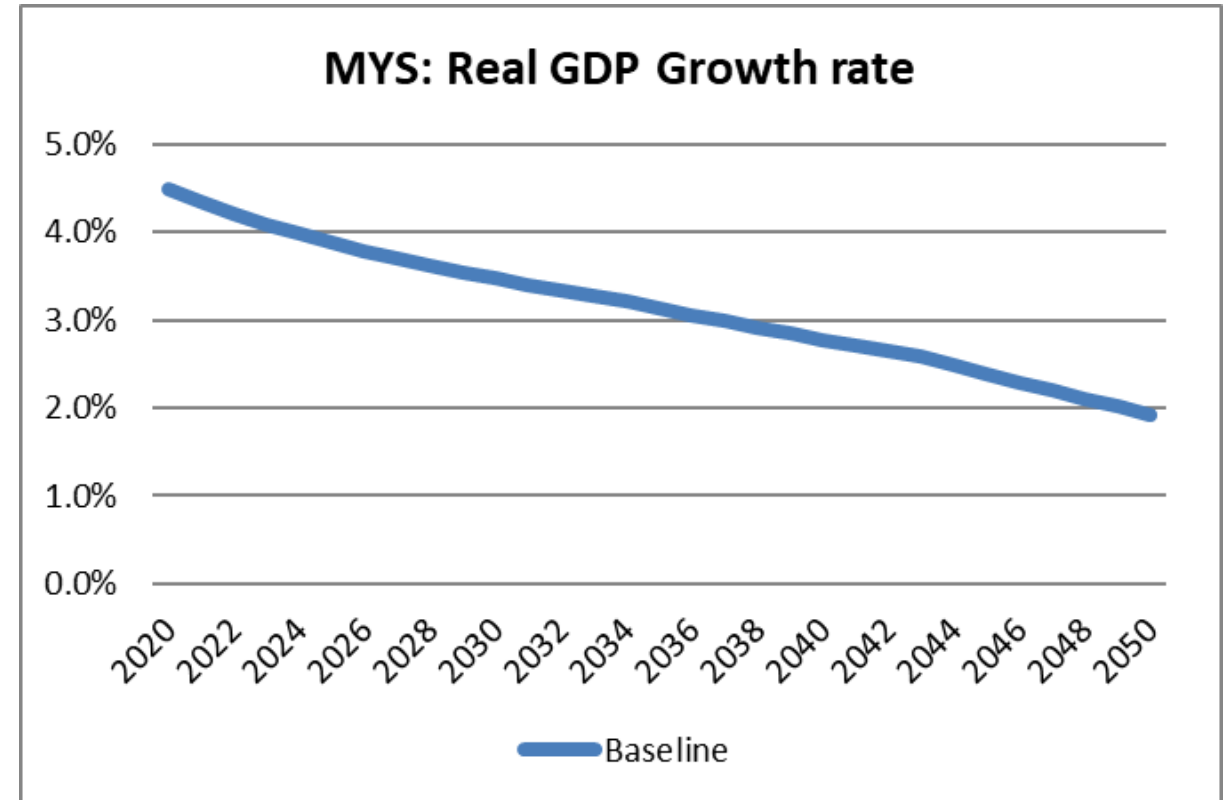
# Baseline for the LTGM-PC

Tab *InputdataA*: Choose general parameters

- Country=Malaysia [Cell B3]
- Initial year=2019 [Cell D/E7; default]
- Labor share ( $\beta$ ) = 50% [Cell D16 & E16]
- Deprec. rate ( $\delta$ ) = 5.8% [Cell D/E13; default]
- Initial ( $K_0/Y_0$ ) = 2.25 [Cell I13 & J13]
- Human capital growth
  - Initial 0.006 (0.6%) [Cell D32 & E32]
  - Target: 0.001 (0.1%) [Cell D37 & E37] by 2040
- TFP growth
  - Initial 0.009 (0.9%) [Cell D41 & E41]
  - Target 0.006 (0.6%) [Cell D46 & E46]
    - by 2050 [Cell D47 & E47]

Tab *InputDataB*:

- Choose the public investment-to-GDP Ratio = 6% [Cell D8 & E8]
- Choose the private investment-to-GDP Ratio = 18% [Cell D18 & /E18]



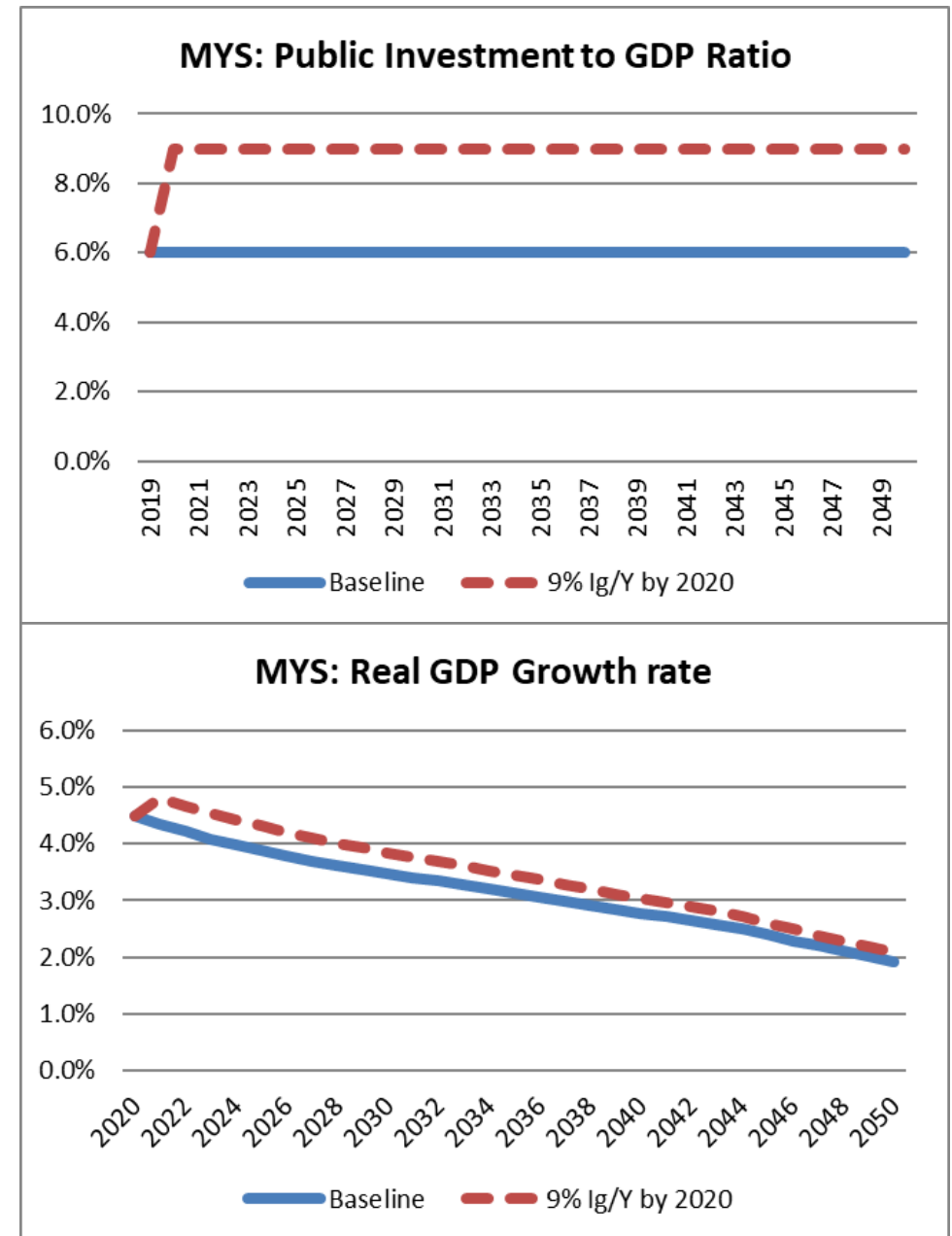
Should get this graph:

- 2020 GDP growth of 4.5%
- 2050 GDP growth of 1.9%

# (vi) Submodel 1: Public Investment → Growth

- Public Investment Shock (InputDataB):
- Permanent 9% (0.09) of GDP [Cell E13] by 2020 [Cell E14]
- *GDP Growth results in 2030:*
  - 3.5% (baseline) [Cell P38 in Submodel 1]
  - 3.8.% (scenario) [Cell P38 in Submodel 1s]

*GraphsB - make sure you get these graphs for Public Investment & GDP Growth →*





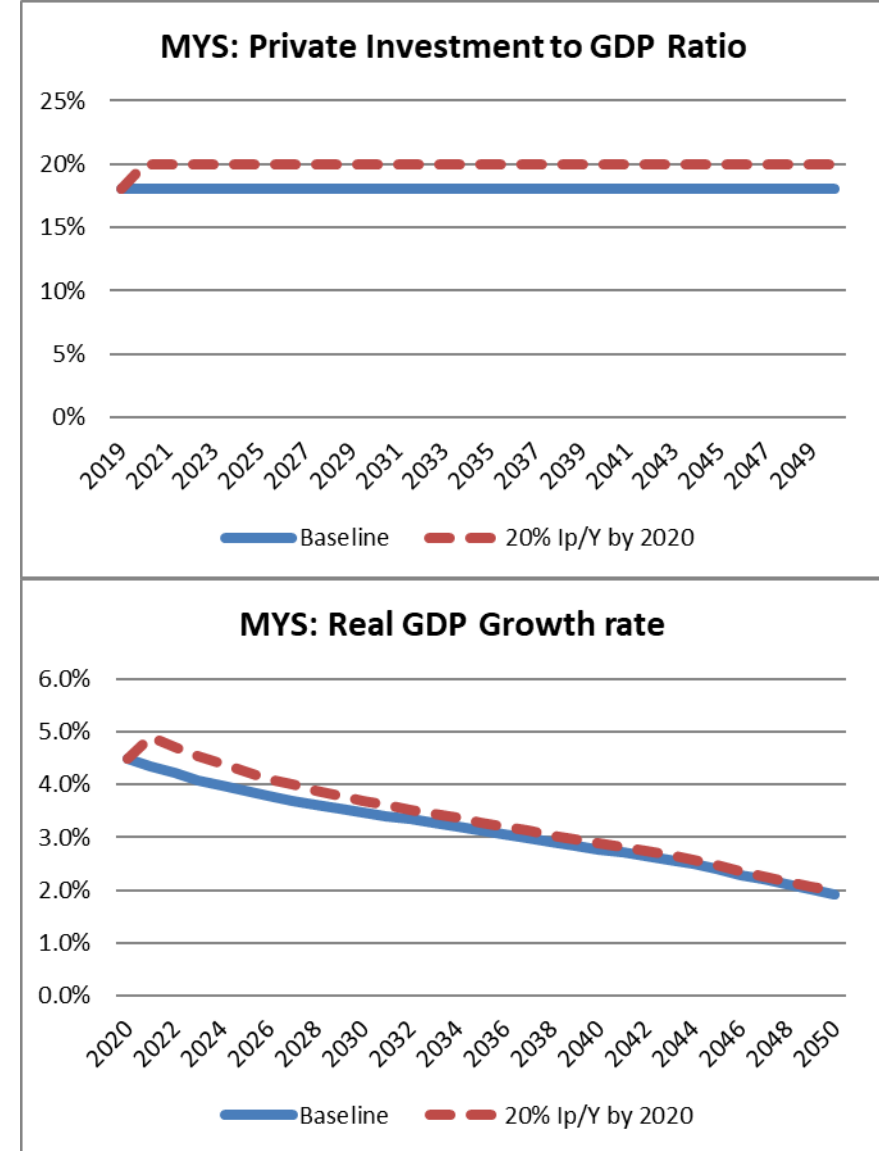
# (vii) Submodel 1: Private Investment → Growth

- **Reset Public Investment: 6% (0.06) of GDP target [Cell E13 in InputDataB]**
- **Private Investment Shock:**
  - permanent 20% (0.20) of GDP [Cell E23]
  - by 2020 [Cell E24] in InputDataB

## *GDP Growth results in 2030:*

- 3.5% (baseline) [Cell P38 in Submodel 1]
- 3.7% (scenario) [Cell P38 in Submodel 1s]
- .

*GraphsB - check get this graph for Public Investment & GDP Growth →*



Thank you!

# Questions/comments/suggestions

- Latest version available at [www.worldbank.org/LTGM](http://www.worldbank.org/LTGM) (or <http://LTGM> on intranet)
- Always trying to improve the LTGM -- your comments and suggestions are welcome
- Please contact us if you would like to use the model in your country:
  - Steven Pennings ([spennings@worldbank.org](mailto:spennings@worldbank.org)),
  - Norman Loayza ([nloayza@worldbank.org](mailto:nloayza@worldbank.org)), or
  - Jorge Guzmán ([jguzmancorrea@worldbank.org](mailto:jguzmancorrea@worldbank.org))
- We can also provide help with analysis, presentations, training etc

# Input/Output: Three Versions of the Model

	Submodel 1	Submodel 2	Submodel 3
<b>Purpose</b>	<b>Growth</b> given <b>Investment</b>	<b>Investment</b> given <b>Output/Growth Target</b>	<b>Investment/Growth</b> given <b>Savings</b>
<b>Inputs:</b>	Investment rate $\left(\frac{I_t}{Y_t}\right)$	Growth rate of GDP ( $g_Y$ ) <b>OR</b> Growth rate of GDP per capita ( $g_Y^{pc}$ ) <b>OR</b> Time path of GDP ( $GDP_t$ ) <b>OR</b> Poverty	Savings rate $\left(\frac{S_t}{Y_t}\right)$
<b>Outputs:</b>			
<b>GDP</b>	Growth rate of GDP ( $g_Y$ ), Growth rate of GDP per capita ( $g_Y^{pc}$ ), Level of GDP ( $GDP_t$ ), Poverty rate	Other three of the four measures	Growth rate of GDP ( $g_Y$ ), Growth rate of GDP per capita ( $g_Y^{pc}$ ), Level of GDP ( $GDP_t$ ), Poverty rate
<b>Savings/ Investment</b>	Savings rate $\left(\frac{S_t}{Y_t}\right)$	Investment rate $\left(\frac{I_t}{Y_t}\right)$ , Savings rate $\left(\frac{S_t}{Y_t}\right)$	Investment rate $\left(\frac{I_t}{Y_t}\right)$
<b>External Sector</b>	CAB to GDP $\left(\frac{CAB_t}{Y_t}\right)$ <b>OR</b> External Debt to GDP $\left(\frac{D_t}{Y_t}\right)$	CAB to GDP $\left(\frac{CAB_t}{Y_t}\right)$ <b>OR</b> External Debt to GDP $\left(\frac{D_t}{Y_t}\right)$	CAB to GDP $\left(\frac{CAB_t}{Y_t}\right)$ <b>OR</b> External Debt to GDP $\left(\frac{D_t}{Y_t}\right)$