ICP: EXTRAPOLATION AND INTERPOLATION OF PPPS AND REAL EXPENDITURES FOR THE YEARS 2012 TO 2016

Robert Inklaar and D.S. Prasada Rao

4th Meeting of the International Comparison Program (ICP)
Technical Advisory Group (TAG)

October 28–29, 2019
World Bank, 1818 H St. NW, Washington, DC
International Comparison Program (ICP): Extrapolation and Interpolation of Purchasing Power Parities (PPPs) and Real Expenditures for the years 2012 to 2016

Robert Inklaar  
Groningen Growth and Development Centre (GGDC)  
University of Groningen

D.S. Prasada Rao (*)  
School of Economics  
The University of Queensland

6 October 2019

This version has benefited from the comments and suggestions made at various TAG meetings where content of the paper were presented.

(*) Corresponding author. Email contact: d.rao@uq.edu.au
1. Introduction

Since the completion of the ICP in 2011, the program has undergone a thorough evaluation conducted by the UN Statistical Commission. Based on an evaluation of the ICP, The Statistical Commission at its 47th meeting has endorsed the continuation of ICP and recommended that the ICP is conducted on a more regular basis using the rolling price survey approach. Recognizing the need for reliable and frequent international comparisons of PPPs and real expenditures, the DECDG is keen to establish a framework for compiling PPPs and real expenditure on an annual basis. The financial and human resource implications and the enormity of the task of getting all the participating countries to devote sufficient resources are major barriers to implementing ICP on an annual basis. A possible option is to follow the Eurostat practice of using rolling price surveys in compiling annual PPPs.

The main objective of this paper is to examine various options available for the purpose of extrapolating results from 2011 and to propose methodology that can be used in the compilation of PPPs for the years in between the two benchmark years, 2011 and 2017.

The paper is organized as follows. In section 2, we use the index number framework to identify the optimum level of disaggregation that minimizes inconsistencies between benchmarks and extrapolated PPPs. Section 3 explores the notion of a reduced information approach\(^1\) to international comparisons and then identifies the sources and methods to be used in assembling the basic data necessary for PPP extrapolation. In particular, we canvass the full use of all the information available from different sources in this process and then identify remaining gaps in data which may then need to be filled using targeted surveys or information from non-conventional sources. Section 4 describes various options available for interpolation of PPPs at the basic heading level (or higher levels) between benchmark years 2011 and 2017 and presents analytical results leading to the choice of the best option for interpolation.

\(^1\) The “reduced information approach” here differs from the approach canvassed in the work of Ahmad in the 1980s.
2. An analytical framework for identifying optimal level of disaggregation for extrapolation

We draw from a short but excellent exposition of the problem of updating by Deaton (2012). Though Deaton (2012) focuses on updating PPP exchange rates for consumption using consumer price indexes, it appears that his analysis is general enough to provide guidance in the choice of the level of disaggregation at which extrapolation is made using an appropriate price index number.

We consider the simple case of two countries where PPP is computed using Törnqvist index numbers. For simplicity, we assume that the same set of commodities enter PPP and national level index number computation. We also assume that the expenditure shares of commodities differ across countries but remain the same over time periods \( t \) and \( t + 1 \). Let \( p_{ij}^s \) represent the price of the \( i \)th commodity \((i = 1, 2, \ldots, N)\) in country \( j \) \((= 1, 2)\) in period \( s \) \((s = t \text{ or } t + 1)\). Let \( s_{ij} \) represent expenditure shares associated with commodity \( i \) in country \( j \) \((j = 1, 2)\).\(^2\) We further let \( \text{PPP}_2^s \) represent purchasing power parity of currency of country 2 with country 1 as the reference country in period \( s \).\(^3\) Let \( P_j \) represent the price index in country \( j \) \((1\text{ and } 2)\) over time \( t \) to \( t + 1 \). Then the logarithmic form of the three Törnqvist indices are given by:

\[
\ln \text{PPP}_2^s = \frac{1}{2} \sum_{i=1}^{N} (s_{i1} + s_{i2})(\ln p_{i2}^s - \ln p_{i1}^s) \text{ for } s = t \text{ or } t + 1
\] (1)

\[
\ln P_2 = \sum_{i=1}^{N} s_{i2}(\ln p_{i2}^{t+1} - \ln p_{i2}^t)
\] (2)

\[
\ln P_1 = \sum_{i=1}^{N} s_{i1}(\ln p_{i1}^{t+1} - \ln p_{i1}^t)
\] (3)

\(^2\) We do not have time superscript with expenditure share as we assume that expenditure shares remain the same over time. Expenditure shares tend to move slowly over time, so this is not a tenuous assumption.

\(^3\) We drop subscript 1 with PPP for ease of notation.
It is easy to see that $PP^S_2$ is a Törnqvist index that compares price levels across countries 1 and 2 whereas $P_1$ and $P_2$ represent Törnqvist indices for countries 1 and 2 measuring price changes from $t$ to $t + 1$.

Following Deaton (2012), we consider the change in PPP over time in logarithmic form. This is given by:

\[
\ln PPP^t_{2} - \ln PPP^t_{1} = \frac{1}{2} \sum_{i=1}^{N} (s_{i1} + s_{i2})[\ln(p_{i2}^{t+1} - p_{i1}^{t+1}) - (\ln p_{i2}^{t} - \ln p_{i1}^{t})]
\]

After simple rearrangement and definitions in (1), (2) and (3), we can show that equation (4) equals:

\[
\ln PPP^t_{2} - \ln PPP^t_{1} = \ln P_2 - \ln P_1 - \frac{1}{2} \sum_{i=1}^{N} (s_{i2} - s_{i1}) \left[ \ln \left( \frac{p_{i2}^{t+1}}{p_{i2}^{t}} \right) + \ln \left( \frac{p_{i1}^{t+1}}{p_{i1}^{t}} \right) \right]
\]

From equation (5), inconsistency between benchmark and updates is given by:

\[
\ln PPP^t_{2} - \ln PPP^t_{1} - (\ln P_2 - \ln P_1) = -\frac{1}{2} \sum_{i=1}^{N} (s_{i2} - s_{i1}) \left[ \ln \left( \frac{p_{i2}^{t+1}}{p_{i2}^{t}} \right) + \ln \left( \frac{p_{i1}^{t+1}}{p_{i1}^{t}} \right) \right]
\]

Deaton (2012) argues that this inconsistency depends on the covariance between differences in expenditure shares in the two countries and price movements in prices in the two countries under consideration.

However, we consider a different angle for equation (6). If the $N$ commodities considered here represent a commodity group, we ask the question as to when the inconsistency between updates and benchmarks is likely to zero or very small. The following result provides a useful direction.

**Result 1:** Under the set-up considered in equations (1) to (6) based on Törnqvist index for the measurement of price levels across countries and price change over time, inconsistency between benchmarks and updating using domestic measures of price changes vanishes if all the commodities considered in the computation show the same price change over time.
In order to verify this result, suppose prices of all the commodities in country 2 change by the same percentage $\alpha$ and price change is uniform across commodities in country 1 represented by a percentage change $\beta$, then equation (6) becomes:

$$\ln P_2^{t+1} - \ln P_2^t - (\ln P_2 - \ln P_1) = -\frac{1}{2} \sum_{i=1}^{N} (s_{i2} - s_{i1})[\alpha + \beta]$$

$$= -\frac{1}{2} (\alpha + \beta) \sum_{i=1}^{N} (s_{i2} - s_{i1}) = 0$$

(7)

The last equality in equation (7) follows from the fact that expenditure shares add up to 1.

We observe that the result reported here is based on the Törnqvist index. However it is easy to show that this result holds even when other index number formulae are used. Two further results are stated and proved below.

**Result 2:** Under the set-up considered in equations (1) to (6) and if the Fisher index is used for the purpose of price comparisons across countries and over time then the inconsistency between benchmarks and updating using domestic measures of price changes vanishes if all the commodities considered in the computation exhibit the same price change over time.

Given that all commodities exhibit the same level of price change over time, we can write the prices in period 2 for countries 1 and 2 respectively as:

$$p_{i2}^{t+1} = \beta \cdot p_{i2}^t \quad \text{and} \quad p_{i1}^{t+1} = \alpha \cdot p_{i1}^t$$

(8)

Consider the Fisher index which is the geometric mean of the Laspeyres and Paasche indices. Given (8) it follows that the price change from period $t$ to $t+1$ for counties 1 and 2 are respectively $\alpha$ and $\beta$, that is $p_2 = \beta$ and $p_1 = \alpha$.

Now we consider the change in the price level for country 2 with country 1 as the reference country. This is given by the ratio:
Substituting (8) into (9) and observing that the expenditure shares remain the same over time, we can show after simple algebraic manipulations that

\[
\frac{P_{t+1}^{12}}{P_t^{12}} = \left( \frac{\sum_{i=1}^{N} p_{t+1}^{12} q_{t+1}^{12} \sum_{i=1}^{N} p_{t}^{12} q_{t}^{12} \sum_{i=1}^{N} p_{t+1}^{12} q_{t+1}^{12} \sum_{i=1}^{N} p_{t}^{12} q_{t}^{12}}{\sum_{i=1}^{N} p_{t}^{12} q_{t}^{12} \sum_{i=1}^{N} p_{t+1}^{12} q_{t+1}^{12} \sum_{i=1}^{N} p_{t}^{12} q_{t}^{12} \sum_{i=1}^{N} p_{t+1}^{12} q_{t+1}^{12}} \right)^{0.5}
\]

Equation (10) implies: \( \ln P_{t+1}^{12} - \ln P_t^{12} = 0 \), which in turn implies that there is no inconsistency between the benchmark comparisons and temporal price changes observed in countries 1 and 2.

Now we turn to a more general result that does not depend upon the functional form for the price index. Here a binary index that compares prices in period or country 2 with the base period or reference country 1, denoted by \( P_{12} \), is a function of observed prices and quantities \( (p_2, p_1, q_2, q_1) \). We assume that the price index satisfies the following proportionality axioms\(^4\). The price index is given by a function of prices and quantities observed in the two periods/countries:

\[
P_{12} = P(p_2, p_1, q_2, q_1)
\]

Axiom of Proportionality in prices of current period: The price index \( P_{21} \) is said to satisfy this axiom if prices in period 2 are multiplied by a constant \( \lambda \) \(( > 0 \) then the index is itself multiplied by \( \lambda \). That is:

\(^4\) The axiomatic approach to index numbers is well researched. Comprehensive expositions of the axiomatic approach can be found in Balk (2008) and in ECE-ILO (2010) Manual on the Consumer Price Index.
\[ P(\lambda p_2, p_1, q_1, q_2) = \lambda P(p_2, p_1, q_1, q_2) \]  

Axiom of Proportionality in prices of base period: The price index \( P_{21} \) is said to satisfy this axiom if prices in period 1 are multiplied by a constant \( \lambda \) (>0) then the index is itself multiplied by \( 1/\lambda \). That is:

\[ P(p_2, \lambda p_1, q_1, q_2) = \frac{1}{\lambda} P(p_2, p_1, q_1, q_2) \]

The following result provides a sufficient condition for the consistency between benchmarks and temporal price movements.

Result 3: If the price index formula used for comparisons of prices across countries and over time are represented by a generic price index formula \( P_{12} = P(p_2, p_1, q_1, q_2) \) and if the index satisfies the axioms of proportionality in current and base period/country prices, then the cross-country price comparisons across two different benchmarks are consistent with relative price movements in the two periods.

The proof follows from the definitions that use notation in equations (1) to (6). We have:

\[ P_{2}^{'+1} = P(p_{2}^{'+1}, p_{1}^{'+1}, q_{2}^{'+1}, q_{1}^{'+1}) \]
\[ P_{2}^{'} = P(p_{2}^{'}, p_{1}^{'}, q_{2}^{'}, q_{1}^{'}) \]
\[ P_{2} = P(p_{2}^{'+1}, p_{1}^{'+1}, q_{2}^{'+1}, q_{1}^{'+1}) \]
\[ P_{1} = P(p_{1}^{'-1}, p_{1}^{'-1}, q_{1}^{'-1}, q_{1}^{'-1}) \]

Making use of the fact that \( p_{2}^{'+1} = \beta p_{2}^{'} \) and \( p_{1}^{'+1} = \alpha p_{1}^{'} \) and using the two axioms stated above, we can show that

\[ \frac{P_{2}^{'+1}}{P_{2}^{'}} = \frac{P_{2}}{P_{1}} = \frac{\beta}{\alpha} \]  

Therefore consistency between benchmarks and temporal price movements can be guaranteed in the case where price movements in the countries 1 and 2 are proportional and the index number formula used satisfies the two axioms of proportionality.

We make three observations:
1. The results stated here provide a sufficient condition but it is not a necessary condition. Further, the result is derived in a very special case.

2. We believe that this sufficient condition provides guidance as to the level of disaggregation at which we could extrapolate with minimum inconsistency. The answer according to the result is that the commodity group should be sufficiently homogeneous to exhibit similar price movements over time. In price index compilation, this concept is somewhat similar to commodity groups that underpin elementary indices.

3. This result suggests that it is best if extrapolation is undertaken at the basic heading level. It is generally expected that the products included in a basic heading are not only homogeneous but they also exhibit similar price level differences across countries and movements over time.

Adopting the framework considered in Deaton (2012) for our purpose of determining the optimum level of disaggregation, we find that extrapolation using national price deflators is best undertaken at the basic heading level. However, in actual implementation it may not be possible to obtain price deflators at a level of aggregation that matches the basic headings within the ICP. For example, the ICP has 110 consumption basic headings and most consumer price indices are available for 10 or 12 aggregate groups.

3. Reduced information approach to annual compilation of PPPs

The costs associated with the implementation of a fully-fledged ICP every year are prohibitively high and therefore is not an alternative that can be considered. Recognizing this reality, it is necessary to identify a reduced data set that is likely to approximate PPPs from detailed data set.

PPP compilation requires two sets of data: prices and weights. Prices are collected through price surveys conducted in each of the participating countries whereas weights are typically based on expenditure weights from national accounts. Price surveys are carefully designed to suit the characteristics of the aggregate under consideration. Different approaches are used for collecting prices for consumption, government expenditure, construction, and machinery and equipment.
3.1 National accounts weights

National accounts expenditure weights are not usually available in real time. Further, national accounts data are subject to significant revisions. For the purpose of PPP computation, use of an average of weights of the three years prior to a given year may provide a stable set of weights. While NA aggregates tend to be revised, it is rare the weights are revised.

3.2 Household Consumption

Use of rolling price survey approach can play a significant role in providing reliable price data for household consumption PPPs. However, we believe that some discretion needs to be used in the implementation of the rolling price survey data.

1. It is important to divide the basic headings into two classes, one class of basic headings where items included do not exhibit major changes from year to year. For example, basic headings like meat, milk, fresh vegetables, etc. are basic headings where the items remain the same over time. On the other hand, basic headings that cover communications, computers, sport and recreation exhibit considerable change from year to year. For basic headings where products change in quality and also specifications it may be necessary to price them on an annual basis instead of relying on rolling price surveys.

2. From the framework discussed in Section 3, it is clear that price deflators used in extrapolation must closely match the basic headings.

3. Given that both 2005 and 2011 benchmarks were conducted using structured product descriptions (SPDs), it may be feasible to compute a basic heading level inflation figure using prices of products that closely match between the two benchmarks. Closeness of the ICP based basic heading inflation figures with the CPI based deflators can be used in determining the suitability of CPI deflators under consideration. In fact, this type of analysis was conducted by the Asian Development Bank (ADB, 2015) as a data editing procedure to identify possible outliers.

4. There are non-conventional sources of price data available, such as scanner data and internet prices, for the purpose of PPP compilation. These sources could be used to supplement the extrapolation procedure using national price deflators. These sources are likely to provide a more accurate estimate of price level differences when products and services are rapidly changing from year to year.
5. There are some items like electricity and water charges, postal rates and rates fixed by the government where it would be feasible to collect these prices on an annual basis.

3.3 Government Consumption

*Compensation:* Data on government compensation would be straightforward to obtain from government sources where wage increases are formally recorded. Even if wages and salaries by occupational classification may not be available, percentage changes in government salaries can be obtained from annual report of government departments.

*Productivity adjustment:* In ICP 2011 productivity adjustments were made to government compensation. The adjustment was based on productivity factors estimated by Inklaar and Timmer (2013b). Given relative productivity changes are gradual and also small from one year to the other, these factors may either be kept constant or a three-year moving average of productivity measures are used so that it eliminates noise in the estimation of productivity levels. In practical terms, this suggests that extrapolating from productivity-adjusted PPPs until a subsequent round of wage surveys is conducted would be a sensible approach.

*Government consumption of goods and services:* These may be dealt similar to household consumption where rolling survey approach is used for items that tend to remain stable over time and use annually collected price data for items exhibiting significant change from one year to the other.

3.4 Gross Capital Formation

The two components of Construction and Machinery and Equipment need to be dealt with separately. The current approach to Construction is easy to deal with as only prices of basic materials used in construction are needed. In fact, most countries publish price of some of the basic items on an annual basis. For items not covered by such publications, suitable price deflators from the producer price index (PPI) can be considered. Changes in wages for construction labor are also frequently available from national sources which can be used in the construction of PPPs.

Machinery and equipment (M&E) is a more complex aggregate to deal with. Currently, PPPs are compiled using prices collected for a global list of products included in the M&E list. However, difficulties arise in the practical implementation of this approach. For purposes of
annual compilation, reliance could be made on the fact that PPPs for M&E are usually to the exchange rate as most of the items are traded and frequently imported. Blades (2013) examined this possibility and it offers a way of compiling adjustment factors to bring exchange rates in line with PPPs. This is an option that should be seriously explored.

3.5 Global Core List for Regional Linking

The global core list for household consumption has significant overlap with product lists used in different regions. So a blend of rolling price-survey and annual price-survey approach can be used in improving the quality of the linking factors. As the global core product lists and linking is done at the Global Office at the World Bank, it may be feasible to make use of alternative sources, such as internet and scanner data sources, of prices for products in the global core list.

3.6 Asymmetric data availability

Until the process of annual compilation of PPPs is set in place, it is quite possible that data available for PPP computation differs significantly and qualitatively across different countries in different regions. The current situation is that the level of disaggregation of CPI and other national price deflators differ. In the extreme case, only the GDP deflator may be available for a country where as deflators are available at a much higher levels of disaggregation.

In the case where availability and quality of data vary across countries, our approach is to use all the available information rather than to operate at the level of the lowest common denominator.

4. Interpolation of PPPs – 2011 to 2017

This part is essentially work in progress as we await the compilation of updated data for 2011 and for the release of final comparisons for the 2017 benchmark year. The problem of interpolation between two benchmarks is somewhat different from extrapolation exercise considered in Section 4 where we extrapolate from the 2011 benchmark to years 2012 and 2013. While waiting for the conclusion of the 2017 benchmark, we have explored the problem of how best to interpolate PPPs at the basic heading level given data for the two end-points. We report progress made in this direction.$^5$

$^5$ We acknowledge the contribution made by Alicia Rambaldi towards the contents of this section.
Interpolation of PPPs at the basic heading level - options

We use the following notation in this section. As we focus on the problem of extrapolation/interpolation of PPPs for a given basic heading, we do not use a separate identifier for the basic heading. Let \( PPP_t^c \) represent PPP for country \( c \) for the year \( t \) – all the PPPs are expressed relative to a reference country. We use USA as the reference country. We consider time series for \( t = 1, 2, ..., T \) and there are \( M \) countries in the comparison with \( c = 1, 2, ..., M \).

Data available for interpolation between the years 1 and T (2011 and 2017)

1. We have PPPs, for the basic heading under consideration, for all the countries for the two benchmark years 1 and T. For example, these two represent the benchmarks 2011 and 2017 respectively. These PPPs are denoted by \( PPP_1^c \) and \( PPP_T^c \), \( c = 1, 2, ..., M \)

2. National accounts deflators from each country for each year expressed relative to the year 1 as the base year. Let \( P_t^c \) represent the implicit national accounts deflator in country \( c \) for the year \( t \) with year 1 as the base year.\(^{6}\) These deflators may be computed using different formulae, e.g. fixed or chain base index numbers computed using Laspeyres or Fisher index numbers. At this stage we do not make any assumptions or restrict the use of the formulae for computation.

3. Expenditure, in national currency units, at current and constant prices. As constant price data are considered as volumes or implicit quantities (for the composite commodity group), we let \( E_t^c \) and \( Q_t^c \) represent, respectively, current and constant price expenditure in country \( c \) in period \( t \).\(^{7}\) Obviously these are linked through the national accounts deflator by the equation

\[
P_t^c = \frac{E_t^c}{Q_t^c} \quad \text{for } c = 1, 2, ..., M \text{ and } t = 1, 2, ..., M \tag{16}
\]

\(^6\) In practice, different countries have different base years in their respective national accounts. We assume, for our purpose, that all the deflators are suitably rebased to have year 1 as the base year.

\(^7\) We endeavor to maintain some similarity in notation with that used in Diewert and Fox (2015), but some differences remain.
4. We note here that $Q_t^c$, expenditure in country $c$ in period $t$ is expressed in the currency units of country $c$. Therefore, $Q_t^c$ can be compared over time to measure growth rates within each country but these cannot be compared across countries. To facilitate comparison across countries, we convert these into common currency units using purchasing power parities, thus we have

$$e_t^c = \frac{E_t^c}{PPP_t^c}; \quad p_t^c = \frac{P_t^c}{PPP_t^c}; \quad q_t^c = \frac{Q_t^c}{PPP_t^c} \quad \text{for } t = 1 \text{ and } T \quad (17)$$

5. Using PPP converted expenditure in (17), we can compute shares of each country in the total aggregate over all the countries in the two periods 1 and $T$. These shares are defined as:

$$s_t^c = \frac{e_t^c}{\sum_{c=1}^{M} e_t^c} \quad \text{for } c = 1, 2, \ldots, M \text{ and } t = 1, M \quad (18)$$

6. Using information provided here and following the framework suggested in Diewert and Fox (2015) and Balk, Rambaldi and Rao (2017), we can compute measures of volume growth at the country level using the ratio $Q_t^c / Q_1^c$ for each country $c$ and for the whole group of $M$ countries using either a Fisher index or Sato-Vartia index.$^{10}$

Now we have established notation and also the type of data available for the purpose of interpolation. We now consider options for interpolation between the two benchmarks years 1 and $T$ or 2011 and 2017 in the case of ICP.

**Option 1: Geometric version of PWT interpolation**

The Penn World Tables 8.0 onwards have adopted the following interpolation approach in generating PPPs for non-benchmark years in between benchmark years. PWT makes use of a weighted arithmetic average of extrapolated PPP for country $c$ for the year $t$ from the initial

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$^8$ We note here that we define these PPP converted aggregates and price deflators only for the years 1 and $T$ for which we have benchmark comparisons. This structure differs from Diewert and Fox (2015) where they have PPPs for all the years in between 1 and $T$. In a sense, our objective is different from theirs.

$^9$ The Diewert and Fox (2015) study uses chained-Fisher as they have data for all the years between 2000 and 2012. Here we have data only for the two endfi-points.

$^{10}$ For a discussion on the use of Fisher and Sato-Vartia indexes and the approach to measuring growth in the group of countries, see Balk, Rambaldi and Rao (2017) where world growth is computed. In contrast, Diewert and Fox (2015) focuses on OECD countries. As both studies use these for illustrating the approach proposed in their respective papers, the approaches can be used here.
benchmark year 1 and retrapolated PPP from the final benchmark year T, where the weights depend on the adjacency of t to the two benchmark years 1 and T. Using the notation in this paper, the PWT extrapolation is given by:

\[
PPP'_c = (1 - w') \cdot PPP^1_c \cdot \frac{P^t_c}{P^t_{USA}} + w' \cdot PPP^T_c \cdot \frac{P^t_c}{P^t_{USA}} = 1, 2, \ldots, M; t = 2, \ldots, T - 1
\]

where \( w' = (t - 1)/(T - 1) \).

Rationale for the use of (19) is intuitive in that extrapolations closer to benchmark are likely to be more reliable than extrapolations far from the benchmark. For example, in the case of interpolation between 2011 and 2017, it is intuitive that extrapolation from 2011 to 2012 would be more reliable than retrapolation from 2017 to 2012.\(^{11}\) Weights in (19) linearly decline as we move away from the benchmark, in either direction.

However, the use of arithmetic average has two problems. While the two components that make-up the extrapolation are essentially invariant to the choice of the base country, it is not clear if the arithmetic average is invariant to such a choice. Second, since PPPs are expected to satisfy transitivity in a multiplicative sense, it is important that the extrapolation in (19) makes use of a geometric average instead of arithmetic average. So we suggest the use of geometric version of PWT extrapolation given by:

\[
PPP'_c = \left[ PPP^1_c \cdot \frac{P^t_c}{P^t_{USA}} \right]^{(1-w')} \times \left[ PPP^T_c \cdot \frac{P^t_c}{P^t_{USA}} \right]^{w'} = 1, 2, \ldots, M; t = 2, \ldots, T - 1
\]

Diewert and Fox (2015) compare arithmetic average extrapolations from PWT with extrapolations from their method and suggest further work.

**Option 2: State-space approach from Rao, Rambaldi and Doran (2010)**

The Rao, Rambaldi and Doran (2010) approach adopted to the current problem of filling PPP data gaps in between two benchmarks in the years 1 and T, can be presented in the form of two equations. Here we assume that both benchmarks cover exactly the same list of

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\(^{11}\) This is consistent with the general econometric notion that predictions away from the sample are generally less reliable and have higher standard errors.
countries. In the case of Diewert and Fox (2015), the coverage includes all OECD countries through the years 2000 to 2012. However, in the case of ICP the 2011 benchmark covered 177 countries and ICP 2017 is likely to have greater coverage. We will revert to this problem toward the end of this paper.

The basic RRD approach postulates that the observed PPPs in the two benchmark years, 1 and T, are values of the true PPPs with measurement error. Thus

Observation equation: \( PPP^1_c = PPP^*1_c \cdot u^1_c \) and \( PPP^T_c = PPP^*T_c \cdot u^T_c \), \( c = 1, 2, ..., M \) (21)

where \( u^1_c \) and \( u^T_c \) are random disturbance terms with mean 1 and have variances reflecting the reliability of PPPs for each country in each of the two benchmark countries. PPPs for time periods in between 1 and T are obtained by updating PPPs using country-specific deflators over time.

Updating equation: \( PPP'_c = \left[ PPP^1_c \cdot \frac{P^t_c}{P^1_{USA}} \right] \cdot v'_c \), \( c = 1, 2, ..., M; t = 1, 2, ..., T \) (22)

The RRD paper outlines a state-space approach to construct extrapolated PPPs that are consistent with the stochastic framework governing (22) and (23). Estimation of parameters and construction of Kalman Filter and smoother are discussed in detail in Rao, Rambaldi and Rao (2010, 2013). The most pertinent part of the RRD work for the purpose of filling gaps in the years between 1 and T is the result which provides our Option 2.

The RRD approach in this case simplifies to the following option implemented in three stages: (i) First, extrapolate PPPs from benchmark year 1 to all the years using the updating equation in (22).

Forward extrapolation: \( \overline{PPP}^1_c = \left[ PPP^1_c \cdot \frac{P^t_c}{P^1_{USA}} \right] \cdot v'_c \), \( c = 1, 2, ..., M; t = 1, 2, ..., T \)

Second, retrace PPPs from benchmark year T backwards to year 1 using the following equation.
Backward extrapolation: $\text{PPP}_{c}^{t,T} = \left[ \text{PPP}_{c}^{T} \cdot \frac{P_{c}^{t}/P_{c}^{T}}{P_{\text{USA}}^{t}/P_{\text{USA}}^{T}} \right]^{v_{c}^{t}}$ for $c = 1, 2, ..., M$; $t = 1, 2, ..., T$

In the final step, the optimal predictor of PPP for all the years is given by the weighted average of the Forward and backward extrapolation.

When this procedure is implemented, PPPs at the benchmark years are also modified to a small degree. However, assuming that the observation equation in (21) is observed without errors, i.e. the random disturbance term equals 1, then the benchmark PPPs are preserved exactly (also one of the properties of the RRD Method). The extrapolated PPP's for the years 2, ..., $T-1$ are given by the weighted geometric average:

$$\text{PPP}'_{c}(\text{RRD}) = \left( \frac{\text{PPP}'_{c}^{1}}{\text{PPP}_{c}^{1}} \right)^{v_{c}^{1}} \cdot \left( \frac{\text{PPP}'_{c}^{T}}{\text{PPP}_{c}^{T}} \right)^{v_{c}^{T}} \text{ for } c = 1, 2, ..., M; t = 2, ..., T-1 \quad (23)$$

where $\gamma_{c}^{1} > 0$; and $\gamma_{c}^{T} > 0$ such that $\gamma_{c}^{1} + \gamma_{c}^{T} = 1$. Therefore the RRD approach gives a weighted average of the extrapolations from the two benchmarks with weights depending on the reliability of the benchmark PPPs and the updating equation. The point to note here is that the weights are the same over all the years. This is in contrast to PWT approach which provides weights for different years but not for the reliability of benchmark data.

**Option 3: Diewert and Fox (2015) approach**

The following steps are involved in their approach\(^{12}\).

**Step 1:** The Diewert and Fox (2015) approach involves the computation of growth rate in the quantity or volume of the whole region. In the current scenario where only two benchmarks are available, their approach requires the use of fixed-base approach to measure growth from period 1 to $T$. The basic data used are shares of different countries in periods 1 and $T$ defined in equation (19)

$$s_{c}^{t} = \frac{\varepsilon_{c}^{t}}{\sum_{c=1}^{M} \varepsilon_{c}^{t}} \text{ for } c = 1, 2, ..., M \text{ and } t = 1, M$$

\(^{12}\) We adapt the notation of Diewert and Fox (2015) to facilitate ease in reconciling our presentation with their results.
and quantity growth rates in each country over the period which are given by \( Q_t^c / Q_1^c \). The fixed-base Fisher index of overall growth in the world or group of countries, \( c=1,2,...,M \) is then given by:

\[
\Gamma_F = [\Gamma_L \cdot \Gamma_P]^{1/2} \text{ where } \Gamma_L = \sum_{c=1}^{M} s^L \cdot \left( \frac{Q_t^c}{Q_1^c} \right) ; \quad \Gamma_P = \left[ \sum_{c=1}^{M} s^P \cdot \left( \frac{Q_t^c}{Q_1^c} \right) \right]^{-1}
\]  

(24)

**Step 2:** Construction of interpolated quantities for each time period \( t \) for each country \( c \).

Interpolated quantities are denoted by \( q_{t,c}^l \), where \( l \) stands for the fact that these are interpolations. For the two end points

\[
q_{1,c}^l = s^L \quad \text{and} \quad q_{T,c}^l = s^P \cdot \Gamma_F
\]

(25)

The long term implied growth rate for country \( c \) from (26) is given by:

\[
g_c = \frac{q_T^c}{q_1^c}
\]

(26)

However, during the same period from 1 to \( T \), the observed growth rate is:

\[
G_c = \frac{Q_T^c}{Q_1^c}
\]

(27)

The discrepancy between the country-specific growth rates in (27) are compared with implied growth rates from the world growth in (27). Diewert and Fox (2015) define the country \( c \) proportional annualized discrepancy factor, \( \alpha_c \), is defined as

\[
\alpha_c = \left[ \frac{g_c}{G_c} \right]^{1/(T-1)}
\]

(28)

Using the discrepancy factor in (28), the interpolated quantity for period \( t \) is defined as:

\[
q_{t,c}^l = q_{1,c}^l \cdot \left[ \frac{Q_t^c}{Q_1^c} \right] \cdot \alpha_c \quad c = 1,2,3,...,M ; \quad t = 1,2,...,T-1
\]

(29)

**Step 3:** Computation of interpolated PPPs

Once implied quantities for each period are computed and given that these are already expressed in PPP terms, the interpolated PPPs are given by:

\[
PPP_{t,c}^l = \frac{E_t^l}{q_{t,c}^l} \quad \text{for } c = 1,2,...,M \text{ and } t = 2,...,T-1
\]
Diewert and Fox (2015) compare the interpolated PPPs from their approach with that of PWT and find significant differences between the two sets of results and they conclude by recommending further work.

Assessment of available options and recommendation
Following up on the recommendation from the TAG, we have undertaken an assessment of these three interpolation methods.

Equivalence of Geometric PWT and RRD methods:
We have been able to prove that under the conditions (i) benchmark PPPs are measured without error; and (ii) reliability of deflators in different countries remains the same over the interpolation period, the weights accorded in the state-space approach (option 2) to forward and backward interpolations are identical to the weights used in the geometric version of the interpolation used in PWT 8.0 onwards. However, the result does not hold if benchmarks are measured with error.

Equivalence of Diewert and Fox (2015) Extrapolation (Option 3) and Forward Extrapolation
Even though the Diewert and Fox (DF) (2015) extrapolation, described in Option 3 above, appeared to be complex after some simple algebraic manipulation we have been able to show that the extrapolated PPPs from DF method are simply based on forward extrapolation. Subsequently Diewert and Fox suggested a backward extrapolation and recommended an averaging process similar to the geometric extrapolation method used in the compilation of PWT.

In the light of the equivalence results discussed above and taking into consideration additional information required to implement the general RRD method, we have decided to make use of the geometric PWT method of interpolation.

As and when updated expenditure data for the 2011 benchmark and the finalized basic heading level PPPs, expenditure and related data (e.g., productivity adjustment factors) are available we will estimate interpolated PPPs for the years in between 2011 and 2017.
6. Conclusions

The main objectives of the paper are to: (i) examine the problem of extrapolation of PPPs from the 2011 benchmark to provided extrapolated data for the years 2012 to 2016; (ii) and to provide a framework for compilation of annual PPPs for the years in between 2011 and 2017. The paper provides an analytical framework that underpins the extrapolation approach. The main conclusion is that in order to minimize the discrepancies between benchmark comparisons and temporal movements in prices, extrapolations must be undertaken at the lowest aggregate possible where it is likely that movements of prices of products within the group are quite similar in magnitude over time. The paper adheres to the principle of making use of all the data available at any given point of time to construct PPPs instead of anchoring the price and real expenditure comparisons on the country or region with least amount of available data. The last section of the paper describes options available for interpolation between 2011 and 2017 at the basic heading level, and based on the analytical and computational exercises undertaken it has been determined that the use of geometric PWT method of interpolation is the best option available.

References


Deaton, Angus and Bettina Aten, 2016. “Trying to understand the PPPs in ICP 2011: why are the results so different?” forthcoming American Economic Journal: Macroeconomics.


