International Trade and Macroeconomic Dynamics with Sanctions

Fabio Ghironi*
University of Washington
CEPR, EABCN, and NBER

Daisoon Kim†
North Carolina State University

Galip Kemal Ozhan‡
Bank of Canada

December 15, 2022
(click here for the latest version)

Abstract

We study international trade and macroeconomic dynamics triggered by the imposition of sanctions. We begin with a tractable two-country model where Home and Foreign countries have comparative advantages in production of differentiated consumption goods and commodity (e.g., gas), respectively. Home imposes sanctions on Foreign. Financial sanctions imply exclusion of a fraction of Foreign agents from the international bond market. When all Foreign agents are excluded, financial sanctions imply financial autarky. We introduce trade sanctions through (i) a ban on international gas trade, or (ii) an exclusion of a fraction of Foreign/Home exporters from international trade. Sanctions generate reallocation of resources in both economies. Exchange rate reflects the type of actions taken instead of the success of sanctions. Welfare analysis shows that gas sanctions are more costly for Home, while consumption good trade sanctions are more costly for Foreign. A third country that does not join sanctions dampens the welfare losses in Foreign. Third country also benefits from not joining sanctions. Our results highlight the importance of international coordination when imposing sanctions.

JEL Classification: F31, F41, F42, F51.
Keywords: Exchange Rate; Macroeconomic Dynamics; Sanctions; Trade.

*Department of Economics, University of Washington, Savery Hall, Box 353330, Seattle, WA 98195, US. Email: fabio.ghironi.1@gmail.com URL: http://faculty.washington.edu/ghiro.
†North Carolina State University, Nelson Hall 4114, Raleigh, NC 27695, US. Email: dkim29@ncsu.edu URL: https://sites.google.com/site/fatherofseoyoon/.
‡Bank of Canada, 234 Wellington Street, Ottawa, ON K1A 0G9, Canada. Email: gozhan@gmail.com URL: http://galipkemalozhan.com.

We would like to thank Linda Tesar, Andrei Levchenko, and participants at the Bank of Canada's seminar series and the IMF's Jacques Polak Annual Research Conference. All errors are ours.

The views expressed in this paper are solely those of the authors and may differ from official Bank of Canada views. No responsibility for them should be attributed to the Bank. On behalf of all authors, the corresponding author states that there is no conflict of interest.

1. Introduction

Understanding the mechanisms of international economic interdependence is at the forefront of policy debates in times of geopolitical tensions. Russia launched its invasion of Ukraine on February 24, 2022. This prompted the governments of 38 countries to impose various types of sanctions on the Russian economy in an effort to punish a blatant act of aggression, reduce the resources that support it, and, in so doing, help Ukraine by means of economic warfare.[1] In contrast to other countries that were sanctioned since the end of World War II, Russia is a large economy—the 11th largest economy by nominal GDP when it attacked Ukraine. Not since the 1930s had a similar size economy been placed under such restrictions. Moreover, in contrast to sanctioned economies in the 1930s, Russia is a major exporter of gas and oil, and the global economy is far more integrated than it was in the 1930s. Imposing sanctions on Russia has consequences for the global economy and, therefore, also for the economies that impose those sanctions. When a country (or group of countries) imposes sanctions on another country (or group of countries), and the countries involved are large enough that the use of sanctions does not affect only the targeted economy, it is challenging to understand the mechanisms through which sanctions generate intended and unintended outcomes. Our goal of this study is to contribute to understanding the effects of sanctions by using a micro-founded model of international trade and macroeconomic dynamics.

Our starting point is a suitable extension of the two-country model in Ghironi and Melitz (2005)—henceforth, GM. We modify the GM model by assuming that both countries, Home and Foreign, are endowed with a raw source of energy—for convenience, natural gas. In each country, an upstream, perfectly competitive production sector combines sector-specific labor and natural gas to produce usable gas. A downstream, monopolistically competitive sector uses gas and sector-specific labor to produce differentiated consumption goods. There is endogenous firm entry into this sector subject to an initial sunk cost, and firms produce with heterogeneous productivities that are drawn upon entry. Fixed trade costs imply that only the relatively more productive firms export. In the absence of sanctions, Home and Foreign gas are perfect substitutes, with price determined by equalization of world demand and supply. We assume that Foreign has a larger endowment of

[1] See Blackwill and Harris (2016) and Mulder (2022) on the history of sanctions as an instrument of geopolitical power.

natural gas, but it is characterized by higher sunk costs of firm entry. These assumptions imply that, in the absence of sanctions, Home imports gas from Foreign, and there is a larger mass of producers of differentiated goods in Home than in Foreign. Households in the two countries hold non-contingent bonds and shares. As in GM, we assume that only bonds are traded internationally. Each household consists of gas-sector and consumption-sector workers. Household members pool their incomes so that, in the absence of sanctions, there is a representative household in each country.[2]

We assume that sanctions are imposed by Home. There are two types of sanctions: financial and trade sanctions, and they take the form of exclusion from the international market. Financial market sanctions exclude a fraction of Foreign households from international bond trading (in the limit, all Foreign households are excluded). This implies that there are now two types of Foreign households: the representative sanctioned household and the representative non-sanctioned one. The latter can still trade bonds with the Home household, but the former is restricted to trading bonds only with the non-sanctioned Foreign household.

Trade sanctions can apply to gas trade and/or differentiated consumption-good trade. In the gas market, we consider the scenario of a constraint on the quantity of traded gas. In the market for consumption goods, we assume that the Home country prohibits export of the outputs of Home firms with productivity above a certain threshold and/or imports of the outputs of Foreign firms with productivity above a (possibly different) threshold. The idea is that sanctions are imposed on the relatively larger firms. In the absence of sanctions, all Home (Foreign) firms with productivity above the cutoff implied by the fixed cost of trade export to Foreign (Home). When sanctions are introduced, there is a second, higher productivity cutoff, such that only the firms with productivity between the two cutoffs export. In the limit, the sanction-determined cutoff coincides with the trade-cost determined one, and there is no international trade in differentiated goods.[3]

[2]We intentionally keep our setup simple relative to quantitative extensions of the GM framework that have appeared in the subsequent literature and relative to analyses of sanctions, such as Bachmann et al (2022). Our goal is to provide a set of benchmark results that future literature can build on.

[3]Note that it will not matter for our results whether firms stop exporting because of a government-imposed sanction or because they choose by themselves to stop exporting. Several American companies stopped engaging in trade with Russia after its 2022 invasion of Ukraine without involvement by the U.S. government, but simply because of their own concerns about trade with an aggressor (concerns that can range from moral principles to bad publicity). We do not model these concerns and, therefore, do not differentiate between government-imposed and self-imposed termination of trade.

We study the effects of sanctions in the short, medium, and long term. We are interested in the effects on international relative prices, balances, standard macroeconomic aggregates, and ultimately how the Home and Foreign economies respond in terms of welfare. Like most conventional open macroeconomic models, our model cannot be fully solved analytically. However, like in GM, it is possible to obtain analytical results that are important for understanding the effects of sanctions. We do this for two variables of special interest: the price of gas and the real exchange rate.

We show that, as long as there is some gas trade, the price of gas depends on two key variables: Home consumption and the extent to which relative consumption dynamics deviate from the outcome under internationally complete asset markets. If Home demand for consumption goods rise, consumption good producers demand more inputs to satisfy the demand and input prices increase, including Home gas price. In response to a rise in Foreign demand for consumption goods, supply of Foreign gas to Home firms decrease because it has more domestic use. Therefore, Home consumption good producers substitute for Home gas and Home gas price increases.

Concerning the real exchange rate, the relative cost of effective labor across countries (the terms of labor, using GM's terminology) is a key driver of its fluctuations: As in GM, appreciation of the terms of labor causes real exchange rate appreciation. Moreover, the real exchange rate changes in response to changes in average exporter productivity and/or changes in the composition of consumption baskets. For instance, a decrease in the average productivity of Foreign exporters causes Home real exchange rate appreciation because less productive Foreign exporters charge higher prices. If the share of imported goods in total product variety available to Home consumers rises relative to Foreign, the real exchange rate depreciates because, on average, exporters charge lower prices than non-exporters.

These and other analytical results help us understand the effects of sanctions that cause changes in gas price and real exchange rate determinants. Numerical exercises illustrate the analytical results and explore the implications of our model for how other variables respond to sanctions.

We find that financial sanctions have a sizable effect on Foreign consumption and welfare only if a very large share of Foreign households is sanctioned. Intuitively, if the share of Foreign households excluded from international asset trade is not sufficiently large, international borrowing and lending by non-sanctioned households on behalf of sanctioned ones significantly dampens the consumption and welfare effects of sanctions. When a large share of Foreign households is sanctioned, less resources are available to finance domestic producer entry into the Foreign economy, with a negative effect on entry and the number of Foreign firms.[4] Notwithstanding the decrease in the overall number of Foreign producers, the number of Foreign exporters must increase for Home and Foreign to reach a steady state in which trade is balanced. The increase in the number of exporters translates into lower average Foreign exporter productivity and, therefore, higher average price of Home imports of goods, leading to Home real exchange rate appreciation.

Sanctions on Foreign gas force the Foreign economy to shift resources toward production of differentiated goods to replace lost labor income and export revenue. As a result, there is an increase in the number of entrants and producers in Foreign. Whereas in Home economy, resources are reallocated toward gas production to compensate for the forgone gas imports. Sanctions that reduce exports of Home consumption goods cause Foreign consumption demand to shift toward domestic goods. Similarly to sanctions on Foreign gas, this sanction also generates a higher number of entrants and producers in the consumption good sector of the Foreign economy. Reallocation in Foreign reduces the gas production in Foreign and subsequently decreases Home imports of gas from Foreign. Home rebalances its economy toward the gas sector.

The real exchange rate, however, behaves differently under in response to these different types of sanctions. The Home real exchange rate appreciates in response to gas sanctions, but it depreciates in response to consumption good sanctions. Different responses of the number of Foreign exporters are responsible for the difference in real exchange rate responses. After a gas sanction, Foreign consumption good exports expand—and the number of Foreign exporters rises—to compensate for the loss of gas exports. After a consumption good sanction, domestic demand in Foreign rises, and the number of exporters decreases. The larger number of Foreign exporters causes Home real exchange rate appreciation after gas sanctions; in contrast, the real exchange rate depreciates after consumption good sanctions because the number of Foreign exporters falls.

The welfare effects of sanctions show that imposing them is costly for Home households. However, it is less costly than for Foreign. Economic warfare can hurt one's adversary, but no war can

[4] As in GM, we assume a one-to-one identification between a producer, a product, and a firm for convenience. However, continuity makes it possible to interpret our model as one of multi-product firms whose boundaries are left unspecified, and where product-line managers within each firm act independently of each other. See Ghironi and Melitz (2005); Bilbiie, Ghironi, and Melitz (2012)for more discussion.

be fought without costs.

After completing the analysis of our two-country model, we turn to a three-country version to address questions that cannot be tackled in a two-country model: Are the effects of sanctions dampened when the sanctioned economy can substitute toward a third country? What are the impacts of sanctions to a third country when two other large countries are sanctioning each other?. We show that while the mechanisms from our analytical results and from the simulations of the two-country model are preserved, introducing a third country dampens the impact of sanctions if the third country does not join sanctioning. Coordinating sanctions with the third country generates the most pronounced welfare losses in the sanctioned economy, while the third country shares the burden with the first country.

Our paper is related to at least two literatures. Russia's invasion of Ukraine triggered a wave of papers on the effects of sanctions.[5] These papers present quantitative, multi-country, static analyses of trade effects (for instance, Bachmann et al 2022), analyses that abstract from extensive margin effects (for instance, Lorenzoni and Werning 2022), or small open economy, New Keynesian models that cannot address the full range of consequences of sanctioning a large economy (for instance, Itskhoki and Mukhin 2022). Ours is not a multi-country, quantitative model, but it allows us to analyze dynamic effects of sanctions in a canonical, easy-to-understand trade and macro framework; the model incorporates extensive margin effects that we believe are central to the working of sanctions, and it makes it possible to study their repercussions for the country that imposes them and the global economy. Our results confirm the conclusion that Eichengreen et al. (2022) reach in their empirical analysis of sanctions in 1914-1945: "Exchange rate movements are [...] not an adequate metric of the success or failure of sanctions but a reflection of the type and scale of the measures taken."

Our paper also contributes to the literature on international trade and macroeconomics that developed following GM.[6] These papers—and many others—extend the GM framework to analyze a

[5] The list of those we are aware of includes Albrizio et al. (2022); Bachmann et al (2022); Bianchi and Sosa-Padilla (2022); Eichengreen et al. (2022); Itskhoki and Mukhin (2022); Lorenzoni and Werning (2022); Kim (2021); Strum (2022). Work that pre-dates Russia's attack on Ukraine includes Korhonen (2019); van Bergeijk (2021), and references therein.

[6] Among others, see Auray and Eyquem (2011); Bergin and Corsetti (2019); Bergin, Feng, and Lin (2021); Cacciatore (2014); Cacciatore and Ghironi (2021); Corsetti, Martin, and Pesenti (2007, 2013); Dekle, Jeong, and Kiyotaki (2015); Hamano and Zanetti (2017); Imura and Shukayev (2019); Kim (2021); Kim, Ozhan, and Schembri (2021); Zlate (2016).

host of questions in international macroeconomics. We contribute to this literature by incorporating a simple model of energy production in GM and by using the extended model (and a three-country version) to study the effects of sanctions.

The rest of the paper is organized as follows: Section 2 builds the two-country model. Section 3 presents analytical results on gas price and real exchange rate determination. Section 4 discusses calibration and model dynamics. Section 5 studies the impact of various sanctions in the two-country model. Section 6 discusses the role of international coordination in a three-country model. The last section concludes. The Appendix provides details of analytical derivations.

2. The Model

The world is composed of two asymmetric regions, Home and Foreign. Both Home and Foreign are populated by a unit mass of atomistic households. The representative household in each country consists of two groups of workers who supply labor to the two sectors of the economy, consumption goods producers and gas producers. Labor is assumed immobile across the two sectors in each country and across countries. Home is an importer of gas, whereas Foreign is an exporter of gas. We use Melitz (2003)'s monopolistic competition and heterogenous producers framework for the microeconomic underpinning of the consumption good producing sector as in GM. Prices are flexible. Figure 1 exhibits the model architecture.

2.1. Household Preferences

The representative household obtains utility from consumption of a basket of goods, C_t, and disutility from supplying labor, L_t, to the sector that produces consumption goods and $L_{G,t}$ to the sector that produces gas. The expected intertemporal utility function that the household maximizes is:

$$\mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left(\ln C_s - \frac{\kappa}{2}L_s^2 - \frac{\kappa_G}{2}L_{G,s}^2\right)\right]$$

with $\beta \in (0,1)$ and $\kappa, \kappa_G > 0$. The consumption basket is defined over a continuum of goods Ω: $C_t = \left(\int_{\omega\in\Omega} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega\right)^{\frac{\theta}{\theta-1}}$ where $\theta > 1$ is the symmetric elasticity of substitution across goods. At any time t, only a subset of goods $\Omega_t \subset \Omega$ is available. Demand for individual goods

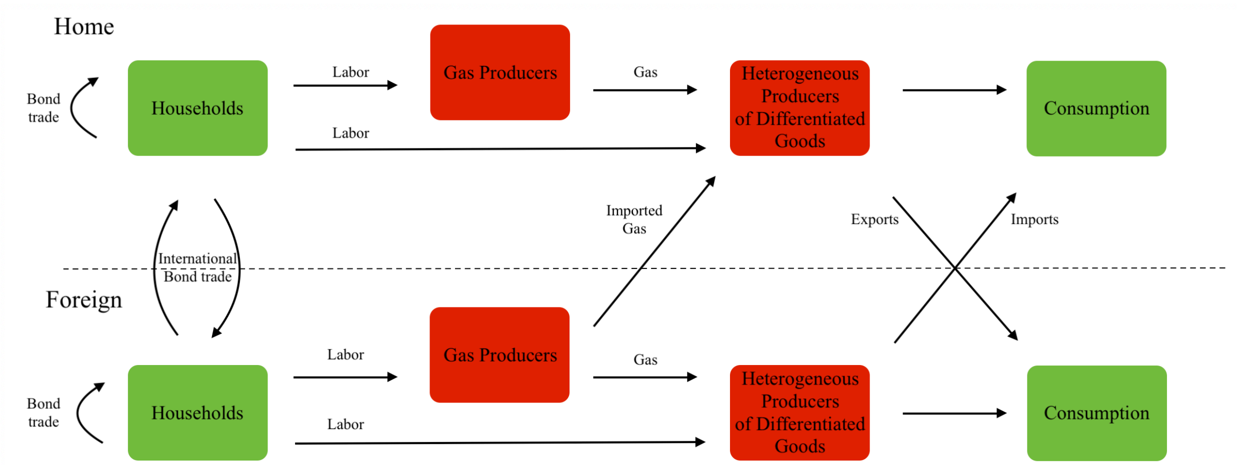

Figure 1: Model Architecture

is $c_t(\omega) = (p_t(\omega)/P_t)^{-\theta} C_t$ where $p_t(\omega)$ is the home currency price of a good $\omega \in \Omega_t$ and $P_t = \left(\int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} d\omega\right)^{\frac{1}{1-\theta}}$. Letting $\rho_t(\omega)$ be the price of good ω relative to the price of the basket, demand for good ω is $c_t(\omega) = (\rho_t(\omega))^{-\theta} C_t$. Everything is similar in Foreign unless otherwise noted. Foreign variables are denoted with a star, and the location of gas use or good consumption below is denoted with a subscript H or F.

2.2. Gas Production

Home and Foreign are endowed with amounts of natural gas G_N and G_N^*, respectively, and we assume that Foreign has a larger endowment, i.e., $G_N^* > G_N$. A perfectly competitive, upstream sector in each country produces usable gas by combining labor and natural gas. Production of usable gas by Home is

$$G_t = G_N L_{G,t}. \tag{1}$$

This gas can be used domestically ($G_{H,t}$) or exported ($G_{F,t}$). Hence, in equilibrium, it will be $G_N L_{G,t} = G_{H,t} + G_{F,t}$. Similarly, Foreign production of usable gas is

$$G_t^* = G_N^* L_{G,t}^*, \tag{2}$$

and equilibrium will imply $G_N^* L_{G,t}^* = G_{H,t}^* + G_{F,t}^*$. First-order conditions for optimal labor demand in gas production in Home and Foreign imply, respectively, $w_{G,t} = \rho_{G,t} G_N$ and $w_{G,t}^* = \rho_{G,t}^* G_N^*$, where $w_{G,t}$ and $w_{G,t}^*$ are the real wages paid to workers in this sector in Home and Foreign, and $\rho_{G,t}$ and $\rho_{G,t}^*$ are the real prices of usable gas in the two countries (both wages and prices are in units of the relevant country's consumption basket). Foreign exports gas to Home. Home and foreign produced gas is perfectly substitutable, and thus home gas market price determination ensures $\rho_{G,t} = \tau_{G,t} Q_t \rho_{G,t}^*$, where $\tau_{G,t}$ is iceberg gas trade costs, and Q_t is the consumption-based real exchange rate (units of Home consumption per unit of Foreign).

2.3. Consumption Good Production

Consumption Goods Producer. Differentiated consumption goods are produced by monopolistically competitive firms using gas and labor as inputs. Home and Foreign gas are perfect substitutes in production of consumption goods. Home firm ω produces output $y_t(\omega)$ of good ω with production function:

$$y_t(\omega) = z Z_t \left(G_{H,t}(\omega) + \frac{G_{H,t}^*(\omega)}{\tau_{G,t}} \right)^{\alpha} L_t(\omega)^{1-\alpha}, \tag{3}$$

where z is exogenous, heterogeneous productivity determined upon firm entry, Z_t is an exogenous sector-wide productivity shock, $G_{H,t}(\omega) + G_{H,t}^*(\omega)/\tau_{G,t}$ is the firm's total use of gas (domestic and imported, with gas import subject to an iceberg trade cost $\tau_{G,t} \geq 1$), $L_t(\omega)$ is the firm's use of labor, and $0 \leq \alpha < 1$. We set Foreign not to import gas from Home. Foreign firms use only domestic gas, $G_{F,t} = 0$.

Using w_t to denote the real wage paid to consumption-sector workers (in units of consumption), the firm's marginal cost is $\rho_{G,t}^{\alpha} w_t^{1-\alpha}/(zZ_t)$. Given Dixit-Siglitz preferences, the real price charged by the firm for sales in the Home market is

$$\rho_{H,t}(z) = \left(\frac{\theta}{\theta-1}\right)\frac{\rho_{G,t}^{\alpha} w_t^{1-\alpha}}{zZ_t}, \tag{4}$$

where we dropped the identifier ω and replaced it with the heterogeneous productivity z. Exporting is costly, and producers are subject to an iceberg trade cost, $\tau_t \geq 1$, and a per-period fixed export cost, $f_{X,t}$. The fixed export cost requires use of consumption-sector labor with effectiveness determined by the aggregate shock Z_t. We assume that $f_{X,t}$ is in units of effective labor. Hence, the fixed export cost in units of consumption is $w_t f_{X,t}/Z_t$. The fixed export cost implies that only firms with sufficiently high productivity z will export. The iceberg cost implies that, if a firm exports, the price it charges in the Foreign market (in units of the Foreign consumption basket) is

$$\rho_{F,t}(z) = \left(\frac{\theta}{\theta-1}\right)\frac{\tau_t \rho_{G,t}^{\alpha} w_t^{1-\alpha}}{Q_t z Z_t}. \tag{5}$$

Number of Firms, Exporters, and Their Averages. Following Melitz (2003), define the market-share weighted productivity average $\tilde{z}_D$ for all producing firms in each country as:

$$\tilde{z}_D \equiv \left(\int_{zmin}^{\infty} z^{\theta-1} dF(z)\right)^{\frac{1}{\theta-1}}, \tag{6}$$

and the market-share weighted productivity averages for Home and Foreign exporters as, respectively:

$$\tilde{z}_{X,t} \equiv \left(\frac{1}{1-F(z_{X,t})}\int_{z_{X,t}}^{\infty} z^{\theta-1} dF(z)\right)^{\frac{1}{\theta-1}}, \tag{7}$$

and

$$\tilde{z}_{X,t}^{*} \equiv \left(\frac{1}{1-F(z_{X,t}^{*})}\int_{z_{X,t}^{*}}^{\infty} z^{\theta-1} dF(z)\right)^{\frac{1}{\theta-1}}. \tag{8}$$

As shown by Melitz (2003), the model is isomorphic to one in which $N_{D,t}$ ($N_{D,t}^{*}$) firms with productivity $\tilde{z}_D$ produce in the Home (Foreign) country and $N_{X,t}$ ($N_{X,t}^{*}$) firms with productivity $\tilde{z}_{X,t}$ ($\tilde{z}_{X,t}^{*}$) export to Foreign (Home). The expression of the Home price index P_t then implies $N_{D,t}(\tilde{\rho}_{D,t})^{1-\theta} + N_{X,t}^{*}\left(\tilde{\rho}_{X,t}^{*}\right)^{1-\theta} = 1$, where $\tilde{\rho}_{D,t} \equiv \rho_{D,t}(\tilde{z}_D)$ and $\tilde{\rho}_{X,t}^{*} \equiv \rho_{X,t}^{*}\left(\tilde{z}_{X,t}^{*}\right)$ are the average relative prices of Home producers and Foreign exporters in the Home market. Moreover, given average profits from domestic and export sales$\tilde{d}_{D,t} \equiv d_{D,t}(\tilde{z}_D)$ and $\tilde{d}_{X,t} \equiv d_{X,t}(\tilde{z}_{X,t})$, average total profits of Home firms are $\tilde{d}_t \equiv \tilde{d}_{D,t} + [1 - F(z_{X,t})]\,\tilde{d}_{X,t}$, where $1 - F(z_{X,t})$ is the proportion of Home firms that export, i.e., $1 - F(z_{X,t}) = N_{X,t}/N_{D,t}$.

Firm Entry and Exit. There is an unbounded mass of potential entrants in each country. Entry requires use of consumption-sector labor with effectiveness determined by the aggregate shock Z_t. Prior to entry, all firms are identical and face a sunk entry cost $f_{E,t}$ in units of effective labor. Hence, the sunk entry cost in units of consumption is $w_t f_{E,t}/Z_t$. Upon entry, firms draw the firm-specific productivity level z from a cumulative distribution function $F(z)$ with support $[z_{min}, \infty)$. This productivity level remains fixed thereafter. We assume that $f^*_{E,t} \geq f_{E,t}$, allowing for the possibility that the gas-rich country features less consumption-sector firms as a consequence of inefficiencies of various type that can characterize the firm creation process.

We also assume a one-period time-to-build requirement: It takes one period between the time of entry and the time when firms start producing and generating profits. All firms in the economy, incumbent and new entrants, are subject to an exogenous shock that causes them to exit with probability $\delta \in (0, 1)$ at the end of each period. Therefore, the mass $N_{D,t}$ of producing Home firms in period t is determined by $N_{D,t} = (1 - \delta)(N_{D,t-1} + N_{E,t-1})$, where $N_{E,t-1}$is the number of firms that entered in period t-1.

Given these definition, firm entry decisions are determined as follows. Prospective entrants are forward looking and compute the rational expectation of the stream of average total profits that they will generate post entry. This determines the average value of an entrant, $\tilde{v}_t$, as:

$$\tilde{v}_t \equiv \mathbb{E}_t \left\{ \sum_{s=t+1}^{\infty} [\beta(1 - \delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-1} \tilde{d}_s \right\}. \tag{9}$$

Entry occurs until this value is equated to the sunk entry cost, implying the free-entry condition $\tilde{v}_t = w_t f_{E,t}/Z_t$. We assume that macroeconomic shocks are never large enough to cause zero entry in any period (or $\tilde{v}_t < w_t f_{E,t}/Z_t$) so that the entry condition always holds with equality (in other words, there is always a positive number of entrants). Since both new entrants and incumbent firms face the same probability δ of exit at the end of each period regardless of their firm-specific productivity, $\tilde{v}_t$ is also the average value of incumbent firms after production has occurred.

2.4. Household Budget Constraint, Asset Holding, and Labor Supply Decisions

International financial markets are incomplete as only non-contingent, riskless real bonds are traded internationally. The representative Home household's holdings of Home bonds entering period t are denoted with $B_{H,t}$. The household receives the risk-free real interest rate r_t on these bonds during period t. The household's holdings of Foreign real bonds entering period t are denoted with $B^*_{H,t}$, and they pay the risk-free real interest rate r^*_t (Foreign bonds and interest rate are in units of Foreign consumption). We assume that firms are fully owned domestically. Specifically, the representative household enters the period with share holdings x_t in a mutual fund of $N_{D,t}$ Home producing firms. During period t, the household receives dividends from its share holdings, $\tilde{d}_t$ per share, and the value of selling its share portfolio at the price $\tilde{v}_t$ per share. Besides its financial assets and the income they generate, the representative household's resources in period t also include the income from labor supplied in the gas production sector ($w_{G,t}L_{G,t}$) and in the consumption sector ($w_t L_t$). Finally, the household also receives a lump-sum rebate of fees that it pays to financial intermediaries in order to enter period $t+1$ (these fees serve the purpose of pinning down holdings of Home and Foreign bonds at their steady state values in the deterministic steady state of the model). During period t, the household uses its resources to buy consumption, to buy bonds with which it will enter period $t+1$ ($B_{H,t+1}$ and $B^*_{H,t+1}$), to pay fees $\frac{\eta}{2}(B_{H,t+1} - B_H)^2$ and $\frac{\eta}{2}Q_t(B^*_{H,t+1} - B^*_H)^2$, with $\eta > 0$, and to buy share holding x_{t+1} in a mutual fund of $N_t \equiv N_{D,t} + N_{E,t}$ firms. Only $1-\delta$ of these N_t firms will be around to produce and generate profits in period $t+1$. The household does not know which firms will be hit by the exit-inducing shock and, therefore, it finances continued operations by all currently producing firms and entry by all producers who choose to enter the market, with the risk of firm exit at the end of period t reflected in the share price that will be determined by the Euler equation for optimal share holdings. The budget constraint of the representative Home household is thus:

$$\begin{aligned} C_t + B_{H,t+1} + Q_t B^*_{H,t+1} + \tfrac{\eta}{2}(B_{H,t+1} - B_H)^2 + \tfrac{\eta}{2}Q_t(B^*_{H,t+1} - B^*_H)^2 + \tilde{v}_t N_t x_{t+1} + \tfrac{\eta}{2}(x_{t+1} - 1)^2 \\ = (1 + r_t) B_{H,t} + Q_t (1 + r^*_t) B^*_{H,t} + w_{G,t}L_{G,t} + w_t L_t + \left(\tilde{d}_t + \tilde{v}_t\right) N_{D,t} x_t + T^f_t . \end{aligned} \tag{10}$$

where $T_t^f = 0.5\eta(B_{H,t+1} - B_H)^2 + 0.5\eta Q_t(B^*_{H,t+1} - B^*_H)^2 + 0.5\eta(x_{t+1} - 1)^2$.

The Euler equations for optimal holdings of Home and Foreign bonds are, respectively:

$$C_t^{-1}\left(1 + \eta(B_{H,t+1} - B_H)\right) = \beta\left(1 + r_{t+1}\right)\mathbb{E}_t\left(C_{t+1}^{-1}\right), \tag{11}$$

and

$$C_t^{-1}\left(1 + \eta(B^*_{H,t+1} - B^*_H)\right) = \beta\left(1 + r^*_{t+1}\right)\mathbb{E}_t\left(\frac{Q_{t+1}}{Q_t}C_{t+1}^{-1}\right). \tag{12}$$

The Euler equation for optimal share holdings implies:

$$\tilde{v}_t = \beta\left(1 - \delta\right)\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\tilde{v}_{t+1} + \tilde{d}_{t+1}\right)\right]. \tag{13}$$

Forward iteration of this equation and the relevant transversality condition imply the expression for $\tilde{v}_t$ in the free-entry condition above, thus establishing the general equilibrium link between firm entry decisions and household decisions regarding the financing of entry.

Finally, the first-order conditions for optimal supply of labor to the gas and consumption sectors are $\kappa_G L_{G,t} = w_{G,t}/C_t$ and $\kappa L_t = w_t/C_t$.

2.5. Market Clearing and Aggregate Accounting

The price of usable gas, $\rho_{G,t}$, is determined by gas market clearing conditions:

$$G_N L_{G,t} = G_{H,t}, \tag{14}$$

$$G^*_N L^*_{G,t} = G^*_{H,t} + G^*_{F,t}, \tag{15}$$

where $G_{H,t} = N_{D,t}G_{H,t}\left(\tilde{z}_D\right) + N_{X,t}G_{H,t}\left(\tilde{z}_{X,t}\right)$, $G^*_{H,t} = N_{D,t}G^*_{H,t}\left(\tilde{z}_D\right) + N_{X,t}G^*_{H,t}\left(\tilde{z}_{X,t}\right)$, and $G^*_{F,t} = N^*_{D,t}G^*_{F,t}\left(\tilde{z}_D\right) + N^*_{X,t}G^*_{F,t}\left(\tilde{z}^*_{X,t}\right)$.

Market clearing for individual goods requires $y_t\left(z\right) = c_{H,t}\left(z\right) + c_{F,t}\left(z\right)$ for the product of a Home firm with specific productivity z and $y^*_t\left(z\right) = c^*_{H,t}\left(z\right) + c^*_{F,t}\left(z\right)$ for the product of a Foreign firm with the same productivity.

Labor market clearing in gas production in Home and Foreign requires $L_{G,t} = w_{G,t}/(\kappa_G C_t)$ and $L^*_{G,t} = w^*_{G,t}/(\kappa_G C^*_t)$, respectively. Since $w_{G,t} = \rho_{G,t}G_N$ and $w^*_{G,t} = \rho^*_{G,t}G^*_N$, it follows that $L_{G,t} = \rho_{G,t} G_N / (\kappa_G C_t)$ and $L^*_{G,t} = \rho^*_{G,t} G^*_N / (\kappa_G C^*_t) = \rho_{G,t} G^*_N / (\kappa_G Q_t C^*_t)$, where the last equality uses the fact that $\rho_{G,t} = \tau_{G,t} Q_t \rho^*_{G,t}$. Ceteris paribus, the amount of labor employed in gas production in each country is larger the larger the country's endowment of natural gas and the higher the price of gas; instead, labor in the gas sector is smaller the higher the country's consumption and, intuitively, the higher the weight of the disutility of labor. Since a real depreciation of the Home currency (an increase in Q_t) causes a higher real price of usable gas in Home, it causes a decrease in gas-sector employment in Foreign, as there is an incentive to shift production to Home.

Labor market clearing in the consumption sectors of the two countries requires

$$N_{D,t} L_t \left(\tilde{z}_D \right) + N_{X,t} L_t \left(\tilde{z}_{X,t} \right) + N_{E,t} \frac{f_{E,t}}{Z_t} + N_{X,t} \frac{f_{X,t}}{Z_t} = \frac{w_t}{\kappa C_t}, \tag{16}$$

$$N^*_{D,t} L^*_t \left(\tilde{z}_D \right) + N^*_{X,t} L^*_t \left(\tilde{z}^*_{X,t} \right) + N^*_{E,t} \frac{f^*_{E,t}}{Z^*_t} + N^*_{X,t} \frac{f^*_{X,t}}{Z^*_t} = \frac{w^*_t}{\kappa C^*_t}. \tag{17}$$

Market clearing for bonds issued by Home requires $B_{H,t+1} + B_{F,t+1} = B_{H,t} + B_{F,t} = 0$ in every period, and for bonds issued by Foreign: $B^*_{H,t+1} + B^*_{F,t+1} = B^*_{H,t} + B^*_{F,t} = 0$ in every period. Stock market clearing in each country requires $x_{t+1} = x_t = 1$ and $x^*_{t+1} = x^*_t = 1$ in every period. Since costs of adjusting bond holdings away from zero are rebated back to households in equilibrium, imposing equilibrium conditions on the household budget constraint yields:

$$C_t + \tilde{v}_t N_{E,t} + B_{H,t+1} + Q_t B^*_{H,t+1} = (1 + r_t) B_{H,t} + Q_t (1 + r^*_t) B^*_{H,t} + w_{G,t} L_{G,t} + w_t L_t + N_{D,t} \tilde{d}_t, \tag{18}$$

in Home and:

$$C^*_t + \tilde{v}^*_t N^*_{E,t} + \frac{B_{F,t+1}}{Q_t} + B^*_{F,t+1} = \frac{(1 + r_t) B_{F,t}}{Q_t} + (1 + r^*_t) B^*_{F,t} + w^*_{G,t} L^*_{G,t} + w^*_t L^*_t + N^*_{D,t} \tilde{d}^*_t. \tag{19}$$

These two equations together, and bond market equilibrium, imply that Home net foreign assets obey the law of motion:

$$\begin{aligned} & B_{H,t+1} + Q_t B^*_{H,t+1} \\ & = (1 + r_t) B_{H,t} + Q_t (1 + r^*_t) B^*_{H,t} + \tfrac{1}{2} \left(w_{G,t} L_{G,t} - Q_t w^*_{G,t} L^*_{G,t} \right) + \tfrac{1}{2} \left(w_t L_t - Q_t w^*_t L^*_t \right) \\ & \qquad + \tfrac{1}{2} \left(N_{D,t} \tilde{d}_t - Q_t N^*_{D,t} \tilde{d}^*_t \right) - \tfrac{1}{2} \left(C_t - Q_t C^*_t \right) - \tfrac{1}{2} \left(\tilde{v}_t N_{E,t} - Q_t \tilde{v}^*_t N^*_{E,t} \right), \end{aligned} \tag{20}$$

or that Home's current account is determined by:

$$CA_t \equiv B_{H,t+1} + Q_t B^*_{H,t+1} - \left(B_{H,t} + Q_t B^*_{H,t}\right) = r_t B_{H,t} + Q_t r^*_t B^*_{H,t} + TB_t, \tag{21}$$

where TB_t is the trade balance:

$$\begin{aligned} TB_t \equiv \; & \tfrac{1}{2}\left(w_{G,t}L_{G,t} - Q_t w^*_{G,t}L^*_{G,t}\right) + \tfrac{1}{2}\left(w_t L_t - Q_t w^*_t L^*_t\right) \\ & + \tfrac{1}{2}\left(N_{D,t}\tilde{d}_t - Q_t N^*_{D,t}\tilde{d}^*_t\right) - \tfrac{1}{2}\left(C_t - Q_t C^*_t\right) - \tfrac{1}{2}\left(\tilde{v}_t N_{E,t} - Q_t \tilde{v}^*_t N^*_{E,t}\right) \end{aligned} \tag{22}$$

Finally, the trade balance can be rewritten as:

$$TB_t \equiv \frac{1}{2}\left(Y_t - Q_t Y^*_t\right) - \frac{1}{2}\left(C_t - Q_t C^*_t\right) - \frac{1}{2}\left(\tilde{v}_t N_{E,t} - Q_t \tilde{v}^*_t N^*_{E,t}\right). \tag{23}$$

Once we recognize that $w_{G,t}L_{G,t} + w_t L_t + N_{D,t}\tilde{d}_t$ is total Home income from labor and dividends (or Home GDP, Y_t) and $w^*_{G,t}L^*_{G,t} + w^*_t L^*_t + N^*_{D,t}\tilde{d}^*_t$ is total Foreign income from labor and dividends (or Foreign GDP, Y^*_t). Home and Foreign current accounts and trade balances are such that $CA_t + Q_t CA^*_t = TB_t + Q_t TB^*_t = 0$.

3. Analytical Insights

Like the GM model we build on, our model cannot be fully solved analytically. It, however, is possible to obtain intermediate analytical results on key variables of interest. We present some of these results below, focusing on two prices: the price of gas and the real exchange rate.

3.1. Gas Price

Using gas market clearing conditions, production functions, optimal prices, and marginal cost expressions, it is possible to express the price of gas, $\rho_{G,t}$, as:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}\,\kappa_G C_t}{G_N^2}\left\{\frac{1 + \xi_t + \tau^{-1}_{G,t}\left[1 - (1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t}{\left[1 + \tau^{-1}_{G,t}\left[1 - (1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t\right](1+\xi_t)}\right\} \tag{24}$$

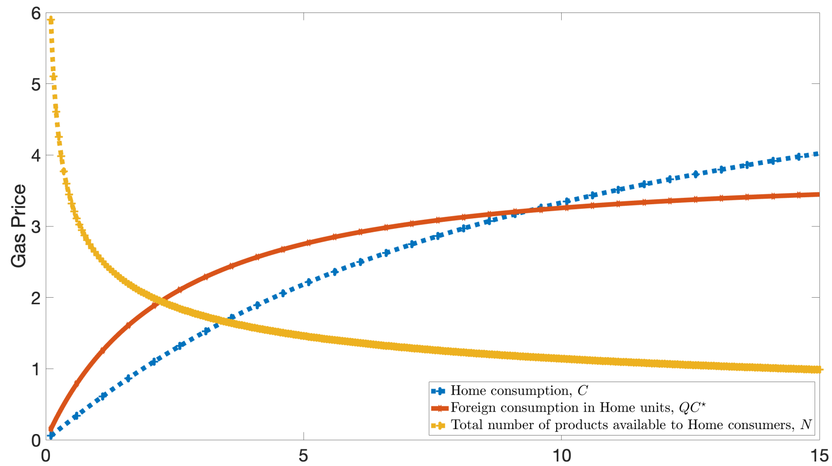

Figure 2: Decomposition of the gas price, $\tilde{\rho}_G$

where $\xi_t \equiv (G_N^*/G_N)^2 [\kappa_G/(\kappa_G^*\tau_{G,t})] [C_t/(Q_tC_t^*)]$.[7] For given level of gas trade cost, $\tau_{G,t}$, fluctuations in the price of gas paid by Home consumption-sector firms are driven by fluctuations in Home consumption and in the extent to which the relation between Home and Foreign consumptions deviates from the complete markets outcome (under complete markets, the ratio $C_t/(Q_tC_t^*)$ would be constant, and changes in $\tau_{G,t}$ would be the only reason for ξ_t to move).

Figure 2 plots the impact of a change in Home consumption, Foreign consumption (in Home units) and the total number products available to Home consumers on gas price, using equation (24). The figure shows that, *ceteris paribus*, an increase in Home consumption or Foreign consumption generates an increase in Home gas price. Total number of products available to Home consumers is negatively related with the data-consistent gas price, all else equal.

To build intuition for the implications of equation (24), suppose that markets are indeed complete, so that, up to a constant, $C_t = Q_tC_t^*$. Suppose also that $\tau_{G,t} = 1$, $G_N = G_N^*$, and $\kappa_G = \kappa_G^*$. then, equation (24) becomes:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}\,\kappa_G C_t}{G_N^2}\left\{\frac{3-(1-\alpha)^{(1-\alpha)(\theta-1)}}{2\left[2-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]}\right\}. \tag{25}$$

The expression in curly brackets is smaller than 1. It tends to 1 if the share of gas in consumption production, α, tends to 0 or 1. Interestingly, both the cases in which there is no international

[7]See Appendix 1 for details.

trade in gas ($\alpha \to 0$) or there is the highest need for Home to import gas ($\alpha \to 1$) imply that the price of gas tends to $\kappa_G C_t / G_N^2$. We show in Appendix 2 that there is a non-monotonicity in gas price behavior as the share of gas in consumption production varies. For given Home consumption, if α is sufficiently high, further increases in α cause a higher gas price. If instead α is sufficiently low, increases in α have the opposite effect on $\rho_{G,t}$. When α is high, the effect of rising α on gas demand prevails, resulting in a higher price. If α is low, demand does not increase enough to offset the effect of substitution toward labor, and the price of gas falls.

The effects of κ_G, C_t, and G_N on $\rho_{G,t}$ in equation (25) are also consistent with intuition: If the weight of the disutility of supplying labor to gas production increases, the price of gas increases as agents reduce gas labor supply. If consumption increases, the price of gas increases, because there is more demand for consumption goods. If efficiency in gas production (or the endowment of natural gas) increases, the price of gas decreases as its supply rises.

In the general case in which $\rho_{G,t}$ is determined by equation (24), we can build intuition by considering the version of equation (24) that is obtained by log-linearizing it around the steady state. We show in Appendix 3 that it is:

$$\hat{\rho}_{G,t} = \mathsf{C}_t + [\Gamma_1 - \Gamma_2] \left(\mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}_t^* \right) \tag{26}$$

where Sans Serif fonts denote percentage deviations from the steady state, and the coefficients Γ_1 and Γ_2 are given by, respectively.

$$\Gamma_1 \equiv \frac{\left(1 + \tau_G^{-1} A\right) \left(1 + \tau_G^{-1} A\xi\right) (1 + \xi)\, \xi}{\left(1 + \xi + \tau_G^{-1} A\xi\right) (1 + \xi) \left(1 + \tau_G^{-1} A\xi\right)}, \tag{27}$$

$$\Gamma_2 \equiv \frac{\left(\tau_G^{-1} A(1 + \xi) + (1 + \tau_G^{-1} A\xi)\right) \xi}{(1 + \tau_G^{-1} A\xi)(1 + \xi)}. \tag{28}$$

In these expressions, $A \equiv 1 - (1 - \alpha)^{(1-\alpha)(\theta-1)}$, and we denote steady-state levels of variables by dropping the time subscript and using an overbar. If $-1 < \Gamma_1 - \Gamma_2 < 0$, the effects of Home consumption, the real exchange rate, foreign consumption, and the iceberg cost of gas trade on the gas price paid by Home firms are intuitive: Higher C_t causes higher demand of gas for production by Home firms, hence a higher price of gas. The effect of Q_t in equation (26) is tied to the role of the real exchange rate in international risk sharing and is best understood in conjunction with that of $\mathsf{C}_t{}^*$.[8] Higher $\mathsf{Q}_t + \mathsf{C}_t^*$ implies an increase in gas demand by Foreign firms relative to Home (given a share on non-traded consumption goods larger than $1/2$). Supply of Foreign gas to Home firms decrease because it has more domestic use. The latter implies that Home consumption good producers substitute towards Home gas, generating an increase in Home gas price. Any policy action (including sanctions) that causes Home consumption, the real exchange rate, Foreign consumption, and/or the iceberg cost of gas trade to change will have an effect on the price of gas facing Home consumption-sector firms that can be understood based on these results.[9]

A final observation on the gas price $\rho_{G,t}$ concerns its measurement: $\rho_{G,t}$ is measured in units of consumption, i.e., in welfare-consistent units. It can fluctuate because of pure variety effects that are not accounted for in available data on the price index P_t. This implies that, while understanding the dynamics of $\rho_{G,t}$ is important to understand the welfare-effects of sanctions through their impact on the price of gas, if we want to have a model-implied measure of real gas price that can be compared to data, we must deflate the nominal price of gas $p_{G,t}$ using a measure of the Home price index that has been purged of pure variety effects. As in Feenstra (1994) and GM, this measure of the Home price level is given by $\tilde{P}_t \equiv N_t^{\frac{1}{\theta-1}} P_t$, where $N_t \equiv N_{D,t} + N_{X,t}^*$ is the total number of products available to Home consumers. Deflating $p_{G,t}$ with $\tilde{P}_t$ yields the data-consistent gas price $\tilde{\rho}_{G,t} \equiv p_{G,t}/\tilde{P}_t$. Notice that this gas price is such that $\tilde{\rho}_{G,t} = N_t^{\frac{1}{1-\theta}} \rho_{G,t}$. Hence, the log-linear equation for $\tilde{\rho}_{G,t}$ follows immediately from this relation and equation (26) as:

$$\tilde{\rho}_{G,t} = \mathsf{C}_t + [\Gamma_1 - \Gamma_2] (\mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}_t^*) - \frac{1}{\theta - 1} \mathsf{N}_t \tag{29}$$

In addition to the effects through $\rho_{G,t}$, policy actions affect the data-consistent gas price by changing the number of products available to Home consumers. Actions that reduce product variety in the Home country cause $\tilde{\rho}_{G,t}$ to depreciate. The reason follows from the effect of product variety on welfare via the price index P_t. Holding product prices constant, this price index decreases if product variety expands, implying that consumers can buy more consumption (and hence obtain more welfare) by spending a given nominal amount. The data-consistent price index $\tilde{P}_t$ removes

[8] With complete markets, we would have $\mathsf{C}_t - \mathsf{C}_t^* = \mathsf{Q}_t$, which would imply that the ceteris paribus scenario of a change in Q_t in equation (26) without at least one between C_t and C_t^* also moving would be impossible.

[9] If Home imposes a full embargo on Foreign gas, there no longer is any arbitrage force that ensures the condition $\rho_{G,t} = \tau_{G,t} Q_t \rho_{G,t}^*$, which is used in obtaining equation (24). In case of a full embargo, the price of gas in Home is determined solely by $\rho_{G,t} = \frac{w_{G,t}}{G_N}$.

this variety effect by augmenting with $N_t^{\frac{1}{\theta-1}}$. Since $\tilde{\rho}_{G,t}$ is obtained by deflating $p_{G,t}$ with $\tilde{P}_t$, it follows that higher N_t would cause $\tilde{\rho}_{G,t}$ to decrease, and lower N_t causes it to increase.

3.2. Real Exchange Rate

Similar to the gas price $\rho_{G,t}$, the real exchange rate Q_t is in welfare-consistent units that are not comparable to data because of unmeasured variety effects. As in GM, the data-consistent real exchange rate $\tilde{Q}_t$ is related to Q_t by the equation:

$$\tilde{Q}_t = \left(\frac{N_t^*}{N_t}\right)^{\frac{1}{\theta-1}} Q_t, \tag{30}$$

where $N_t^* \equiv N_{D,t}^* + N_{X,t}$ is the total number of products available to Foreign consumers.

Using price index equations and optimal price setting by Home and Foreign consumption-sector firms yields:

$$\tilde{Q}_t^{\,1-\theta} = \frac{\frac{N_{D,t}^*}{N_t^*}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z_t^*}\right)^{\alpha}\frac{\tilde{z}_D}{\tilde{z}_D^*}\right]^{1-\theta} + \frac{N_{X,t}}{N_t^*}\left[\frac{\tau\tilde{z}_D}{\tilde{z}_{X,t}}\right]^{1-\theta}}{\frac{N_{D,t}}{N_t} + \frac{N_{X,t}^*}{N_t}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z_t^*}\right)^{\alpha}\frac{\tau^*\tilde{z}_D}{\tilde{z}_{X,t}^*}\right]^{1-\theta}} \tag{31}$$

where $TOL_t \equiv Q_t(w_t^*/Z_t^*)/(w_t/Z_t)$. As in GM, this variable measures the relative cost of effective labor in the two countries. Interestingly, gas prices do not enter the real exchange rate expression directly. Factor prices enter the equation through cross-country ratios of variables. The ratio of Home to Foreign gas prices is such that $\rho_{G,t}/(Q_t\rho_{G,t}^*) = \tau_{G,t}$. Hence, only the iceberg cost paid by Home (the importer) appears in equation (31). In addition to the terms of labor and the iceberg cost of gas trade, the real exchange rate can change because of changes in the total number of products available to Home and Foreign consumers, in the numbers of producers serving the domestic or export market, and in average export productivities.

Consider a permanent decline in Home gas imports, a scenario that we study below as resulting from gas sanctions. In response to lower Home demand of Foreign gas, resources in the Foreign economy will be shifted toward production of consumption goods in order to sustain exports by increasing consumption-sector output. This translates into an increase in labor demand by Foreign consumption good producers, which puts upward pressure on consumption good sector wages. In

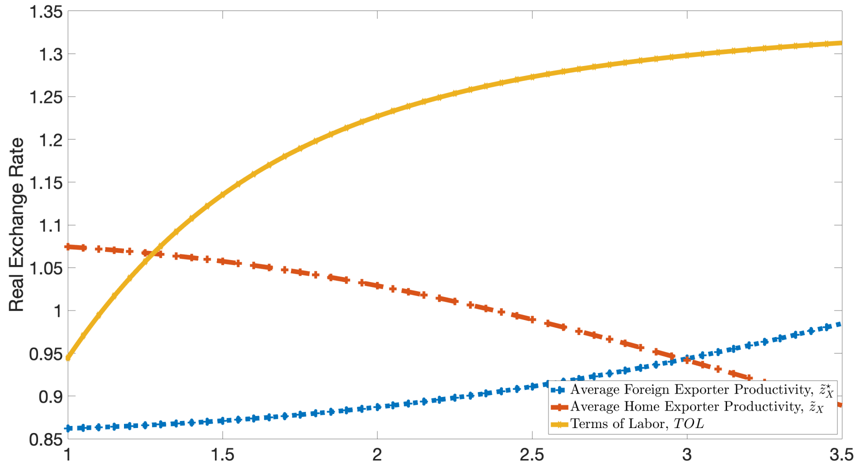

Figure 3: Decomposition of the real exchange rate, $\tilde{Q}$ (1/2)

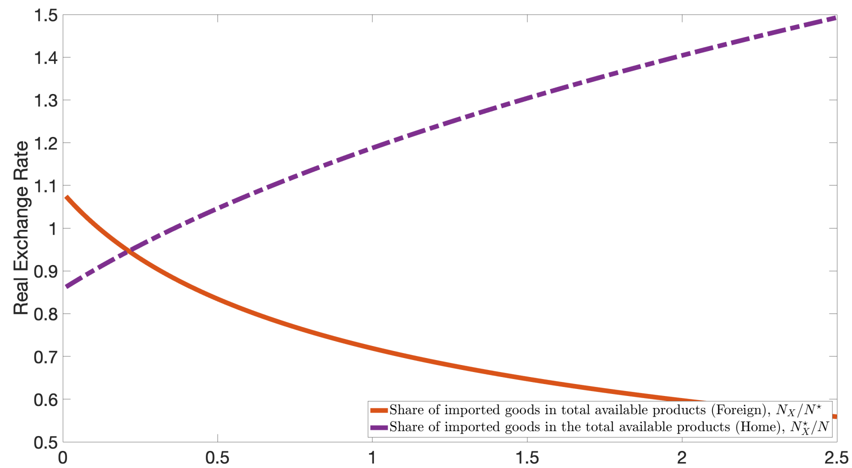

Figure 4: Decomposition of the real exchange rate, $\tilde{Q}$ (2/2)

turn, this leads to a depreciation (an increase) in TOL_t. We will show below that, to a first order, terms of labor depreciation is associated with depreciation of $\tilde{Q}_t$.

Figures 3 and 4 plot the relationship between the real exchange rate and average Foreign exporter productivity, average Home exporter productivity, term of labor, share of imported goods in total available products in Foreign and in Home, respectively. All of the lines in both figures show the relationship while all other variables stay constant. We observe that while the average Foreign exporter productivity, terms of labor, and share of imported goods in the total available products in Home is positively related with the real exchange rate ($i.e.$, an increase in these variables, *ceteris paribus*, depreciate the exchange rate), average Home exporter productivity and the share of imported goods in the total available products in Foreign are negatively related.

As for the gas price, we can build intuition on the determinants of the real exchange rate by considering the log-linear version of equation (31). Letting NUM_t denote the numerator of the expression in equation (31) and DEN_t the denominator, it is:

$$\tilde{\mathsf{Q}}_t = \frac{\overline{NUM} \cdot dDEN_t - \overline{DEN} \cdot dNUM_t}{(\theta - 1) \cdot \overline{NUM} \cdot \overline{DEN}} \tag{32}$$

where d is the differentiation operator. Hence, up to the constant $\frac{1}{(\theta-1)\cdot\overline{NUM}\cdot\overline{DEN}}$, the behavior of $\tilde{\mathsf{Q}}_t$ is determined by $\overline{NUM} \cdot dDEN_t - \overline{DEN} \cdot dNUM_t$. We show in Appendix 4 that:

$$\begin{aligned}
\overline{NUM} \cdot dDEN_t - \overline{DEN} \cdot dNUM_t \;=\;& (\theta - 1)(\Phi_1 - \Phi_2)[(1-\alpha)\mathsf{TOL}_t + \alpha(\mathsf{Z}_t - \mathsf{Z}_t^* - \tau_{G,t})] \\
&+(\theta - 1)(\Phi_2 + \Phi_4)\tilde{z}^*_{X,t} - (\Phi_2 + \Phi_3)(\tilde{z}_{X,t} - \tau_t) \\
&+\Phi_1[\mathsf{N_{D,t}} - \mathsf{N_t} - (\mathsf{N^*_{D,t}} - \mathsf{N^*_t})] \\
&+\Phi_2[\mathsf{N^*_{X,t}} - \mathsf{N_t} - (\mathsf{N_{X,t}} - \mathsf{N^*_t})] \\
&-\Phi_3[\mathsf{N_{X,t}} - \mathsf{N^*_t} - (\mathsf{N_{D,t}} - \mathsf{N_t})] \\
&+\Phi_4[\mathsf{N^*_{X,t}} - \mathsf{N_t} - (\mathsf{N^*_{D,t}} - \mathsf{N^*_t})],
\end{aligned} \tag{33}$$

where $\Phi_1 \equiv \chi_1 \left(\bar{N}_D/\bar{N}\right)^2 \left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\right)^{1-\theta} > 0$, $\Phi_2 \equiv \gamma\chi_1 \left(\bar{N}^*_X/\bar{N}\right)^2 \left(\overline{TOL}^{1-\alpha}\bar{\tau}_G^{-\alpha}\chi_2\right)^{1-\theta}$, and we assumed $\bar{Z} = \bar{Z}^* = 1$. In the expressions above, the parameters χ_1, χ_2, and γ are defined implicitly by:

$$\frac{\bar{N}^*_D}{\bar{N}^*} = \chi_1 \frac{\bar{N}_D}{\bar{N}}, \quad \frac{\bar{N}_X}{\bar{N}^*} = \gamma\chi_1 \frac{\bar{N}^*_X}{\bar{N}}, \quad \text{and} \left(\frac{\tau\tilde{z}_D}{\bar{\tilde{z}}_X}\right)^{1-\theta} = \left(\chi_2 \frac{\tau^*\tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{1-\theta}.$$

Equation (33) (or, more precisely, the equation that follows from combining equations 32 and 33) is a more complicated version of the log-linear equation that is central to understanding real exchange rate dynamics in GM. Our version of the equation is more complicated because of the two-sector structure of production in each country and the fact that the steady state of the model is not symmetric. Nevertheless, it is still possible to obtain an equation that, to a first order, disentangles the different determinants of the real exchange rate that are at work in our model.

Consider the effect of TOL_t. We show in Appendix 4 that $\Phi_1 - \Phi_2 > 0$ if and only if:

$$\left(\frac{\bar{N}_D}{\bar{N}_X^*}\right)^2 > \gamma \chi_2^{1-\theta} \left(\frac{\tau^* \tilde{z}_D}{\tilde{z}_X^*}\right)^{2(1-\theta)}.$$

This condition is satisfied for all plausible calibrations of our model. It follows that, ceteris paribus, appreciation of the terms of labor (a downward movement in TOL_t) causes appreciation of the data-consistent real exchange rate (negative $\tilde{\mathsf{Q}}_t$) as in GM.

Higher average productivity of Foreign exporters (higher $\tilde{\mathsf{z}}^*_{X,t}$) causes $\tilde{\mathsf{Q}}_t$ to depreciate because it implies a lower domestic price index $\tilde{P}_t$, as more productive Foreign exporters charge lower prices.

The last four parts of equation (33) capture the effects of changes in the composition of consumption baskets in Home and Foreign. The first term measures the relative share of domestic goods in the total numbers of products available in Home and Foreign. The second term measures the relative share of imported goods in the total numbers of products available in Home and Foreign. If the share of imported goods in total Home variety rises relative to Foreign, the real exchange rate depreciates. An increase in Foreign exporter representation in the Home consumption basket relative to Home exporter representation in the Foreign consumption basket implies a lower price level $\tilde{P}_t$ in Home and a higher price level $\tilde{P}_t^*$ in Foreign because, on average, exporters charge lower prices. Hence, depreciation of $\tilde{\mathsf{Q}}_t$. The third and fourth terms measure the relative share of imported goods in total available variety versus domestic goods in total variety abroad in the two countries. If this share rises for Home, the real exchange rate depreciates; if it rises for Foreign, the real exchange rate appreciates. Consider, for example, the third term: If imported products representation in total variety available in Foreign rises relative to domestic products representation in total variety available in Home, $\tilde{P}_t^*$ falls and $\tilde{P}_t$ rise because, on average, exporters charge lower prices than non-exporters. Similarly, but with opposite effects on $\tilde{\mathsf{Q}}_t$ for the fourth term.

The results in the previous paragraphs help us understand the results of policy actions (including sanctions) that cause changes in the determinants of the real exchange rate. We use these results and those for the price of gas above to guide our interpretation of the numerical exercises in the next section.

4. Model Calibration and Dynamics

In this section, we calibrate and solve our model numerically to provide illustrations in response to productivity changes in our model.[10]

4.1. Calibration

We calibrate the model with parameter values that are widely used in the literature. This allows us to assess the implications of sanctions without the risk of our findings being the product of an unusual calibration. We set the discount factor and firm exit rates to $\beta = 0.99$ and $\delta = 0.025$, respectively. The disutility parameter from working is $\kappa = \kappa_G = 0.75$ to normalize the consumption good sector labor supply to 1. The scale parameter for the costs of adjusting bond/share holdings, η, is 0.025, which is sufficient to induce stationarity. This value implies that this adjustment cost has a negligible impact on model dynamics, other than pinning down the non-stochastic steady state and ensuring mean reversion when shocks are transitory. Following Ghironi and Melitz (2005), we set the elasticity of substitution of varieties θ to 3.8. We assume that firm-level productivity z is drawn from a Pareto distribution with lower bound z_{min} and shape parameter k. We set k to 3.4 and normalize z_{min} and f_E to 1. Our calibration implies that Pareto shape parameter of (domestic) sales distribution is 1.21. Also, the top 5 percent exporters (top 1 percent firms) contribute to 60% of total exports when 20% firms export.[11] This calibration ensures that z_{min} is smaller than the exporter cutoff, $z_{X,t}$. The fixed cost of exporting, $f_X = 0.0085$, implies that in the initial steady state 17 and 26 percent of Home and Foreign firms export their good, respectively. We follow Kim, Ozhan, and Schembri (2021) to set the share of gas in consumption good production and set α to 0.1. We set the iceberg costs for consumption good trade, τ and τ^*, to 1.3, and the iceberg cost for gas trade, τ_G, to 1.1.

An important dimension of our model is the asymmetry between the two countries. We deviate from the symmetric two-country standard parameterization when calibrating cross-country productivities and natural gas endowments. First, we assume that Home is a gas importer and Foreign

[10] We solve the model as a nonlinear, forward-looking, deterministic system using Dynare's own nonlinear equation solver with line search. This method solves simultaneously all equations for each period, without relying on low-order, local approximations.

[11] According to Mayer and Ottaviano (2008), the share of top 5 percent exporters in total exports is 81, 73, 69, 59, 73, and 81 percent in Germany, France, UK, Italy, Belgium, and Norway, respectively.

is a gas exporter country. Second, Home's consumption good producing sector is subject to lower sunk costs of firm entry. Third, Home is endowed with smaller natural gas resources than Foreign. Without loss of generality, we set $Z = 1.5$, $Z^* = 1$ and $G_N = 1$, $G_N^* = 1.5$. Our calibration indicates that Home GDP is about 56% larger than the Foreign GDP in the initial steady state, $i.e.$, without sanctions. On the financial front, Foreign has a positive initial NFA position. We set Home households' initial holdings of Home and Foreign bonds to $B_H = -5$ and $B_H^* = 3$, respectively. Our calibration implies that the value of Foreign households' initial holdings of Home bonds is 118% of Foreign GDP and Foreign NFA position is at 38% of Foreign GDP.

4.2. Effects of a Change in Aggregate Home Technology

First, we study how our model reacts to a permanent positive change in Home technology, before studying the sanctions imposed on Foreign economy. Our experiment with Home technology enables us to compare our model outcome with models in the international trade and macroeconomic dynamics literature that do not include an energy sector and/or assume symmetric countries. To make our comparison clearer, we also simulate our model with varying degrees of gas share in consumption good producer sector.

Figure 5 shows responses to a 1% permanent increase in Home consumption good sector aggregate productivity. Blue, green, and red lines indicate simulations when the share of gas in consumption good production is 20%, 10%, and 1%, respectively, i.e., $\alpha \in \{0.2, 0.1, 0.01\}$. The varying share of gas in consumption good production affects all model variables only quantitatively.

After the permanent change in productivity, Home becomes a more attractive business environment, which leads to an increase in Home entry in consumption good producer sector. Because of high productivity, marginal costs of production in the consumption good sector goes down in the long run. Consumption good producers demand more labor to expand production and wages for labor in consumption goods sector increase. There is a temporary increase in marginal costs because of higher factor demands after the realization of the shock. The cutoff productivity goes down for the least productive exporter in the short run because more producers are productive enough to cover the fixed export cost and the labor cost in the short run.

On the household side, appreciation in the wages for those employed in consumption good

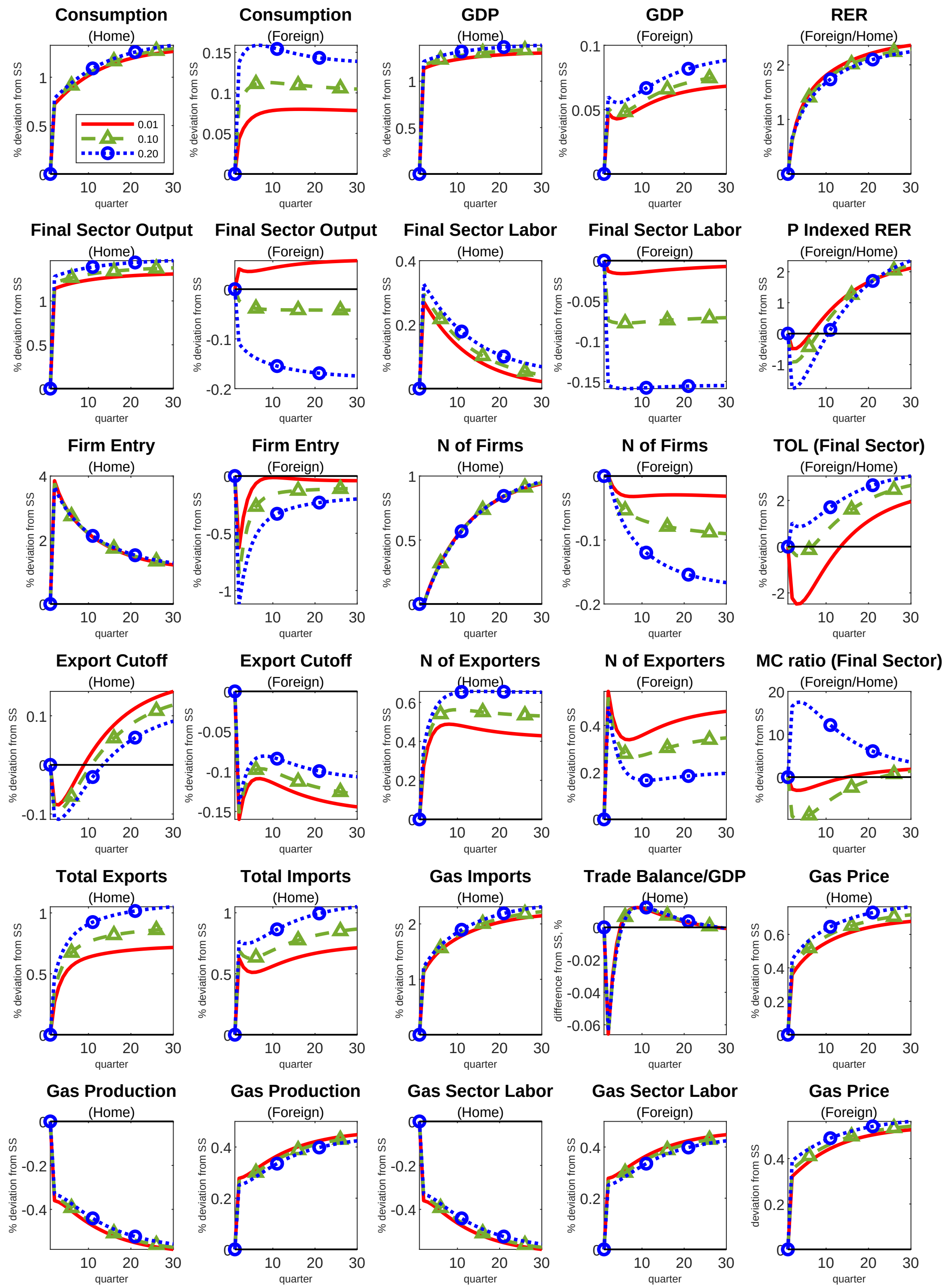

Figure 5: Responses to a 1% Permanent Increase in Home Productivity

production generates an expansion of labor supply towards that sector but a reduction in the labor supply to gas producing sector. Therefore, the amount of Home produced gas diminishes. This is compensated by importing more Foreign gas in the consumption good production. The increase in gas demand by consumption good firms increase the Home gas price more than Foreign gas price, causing real exchange rate to depreciate (from Home perspective). It is important to note that the short run exchange rate depreciation is independent from variety effects, which is a result of the relative changes in labor supply to consumption good producing and gas producing sectors. In addition, Foreign becomes more concentrated in the gas sector than the consumption good sector as the share of gas gets larger in consumption good production.

5. The Sanctions

We assume that sanctions are imposed by Home, and we consider three types of sanctions: consumption good trade sanctions, financial sanctions, and gas trade sanctions. We focus on transition dynamics in the short, medium, and long term. The intuition from the numerical results are following the mechanisms from our analytical results.

We introduce consumption good trade sanctions by preventing trade for consumption good producers with productivity above a certain threshold. The idea is that sanctions imply a reduction in the trade of larger producers. Under financial market sanctions, a fraction of Foreign households is excluded from international bond trading and, in the limit, all Foreign households are excluded. To study the effects of gas trade sanctions, we conduct simulations with a permanent fall in gas imports from Foreign that takes place in the first period. The simulations describe the reaction to the shocks until the system returns to a new state of equilibrium.

5.1. Consumption Good Trade Sanctions

We introduce sanctions on consumption good trade through imposing another productivity cutoff, $z^S_{X,t}$, for Home producers exporting to Foreign. The sanction in consumption good trade is in two forms. First, Home consumption good producers with higher productivity level than the sanction cutoff level stop exporting to Foreign. Second, Home stops importing from the most productive Foreign producers. We set the sanction cutoff as a function of the number of consumption good exporters in the initial steady state. For example, in our simulations, we pin down the sanction cutoff productivity level by assuming that the top 5 percent most productive consumption good producers stop exporting. While the top 5% Home firms dominate around 80 percent of aggregate Home exports to Foreign, the export ban on them increases the export participation of other (less productive) firms, and the actual decrease in aggregate final good export amount is less than 80 percent.

Figure 6 presents transition dynamics after the introduction of consumption good trade sanctions. Green lines indicate simulations after the introduction of export sanctions (EXS)–Home top 5% productive firms stop exporting to Foreign. Blue lines indicate simulations after the introduction of import sanctions (IMS)–Foreign top 5% productive firms cannot export to Home. Red lines indicate simulations when both import and export sanctions are in place simultaneously (TS).

Our first observation is that, under both export and import sanctions, consumption falls more in Foreign than in Home. Following export sanctions, the most productive producers in the Home export market drop from international trade, but Home economy still faces an external demand due to its comparative advantage of producing consumption goods. After the exclusion of most productive Home exporters, productivity cutoff level for the least Home exporter falls down, making less productive Home producers to join the export market. Therefore, the average price of Home exporters increase. The change in the average price of Home exports pushes exchange rate up, implying depreciation from Home perspective. Foreign economy reallocates labor towards the consumption good sector to produce more and compensate for the lost imports from Home. Moreover, most productive Home exporters dropping from international trade implies less gas usage in Home and a decrease of gas imports from Foreign. Hence, the price of gas in Foreign goes down.

In response to import sanctions, top 5% productive Foreign consumption good producers drop from international trade. Less productive Foreign producers start to export, implying a fall in the cutoff productivity level of the least productive Foreign exporter. Number of exporters in Foreign increases and Foreign consumption exports become more expensive on average due to fall in Foreign exporter average productivity. The latter implies an appreciation of real exchange rate from Home perspective. Shrinking number of consumption good producers implies less demand for labor in the consumption sector. Foreign economy reallocates labor to the gas sector. Entry in Home consumption sector goes up to compensate for the lost imports from Foreign. The latter

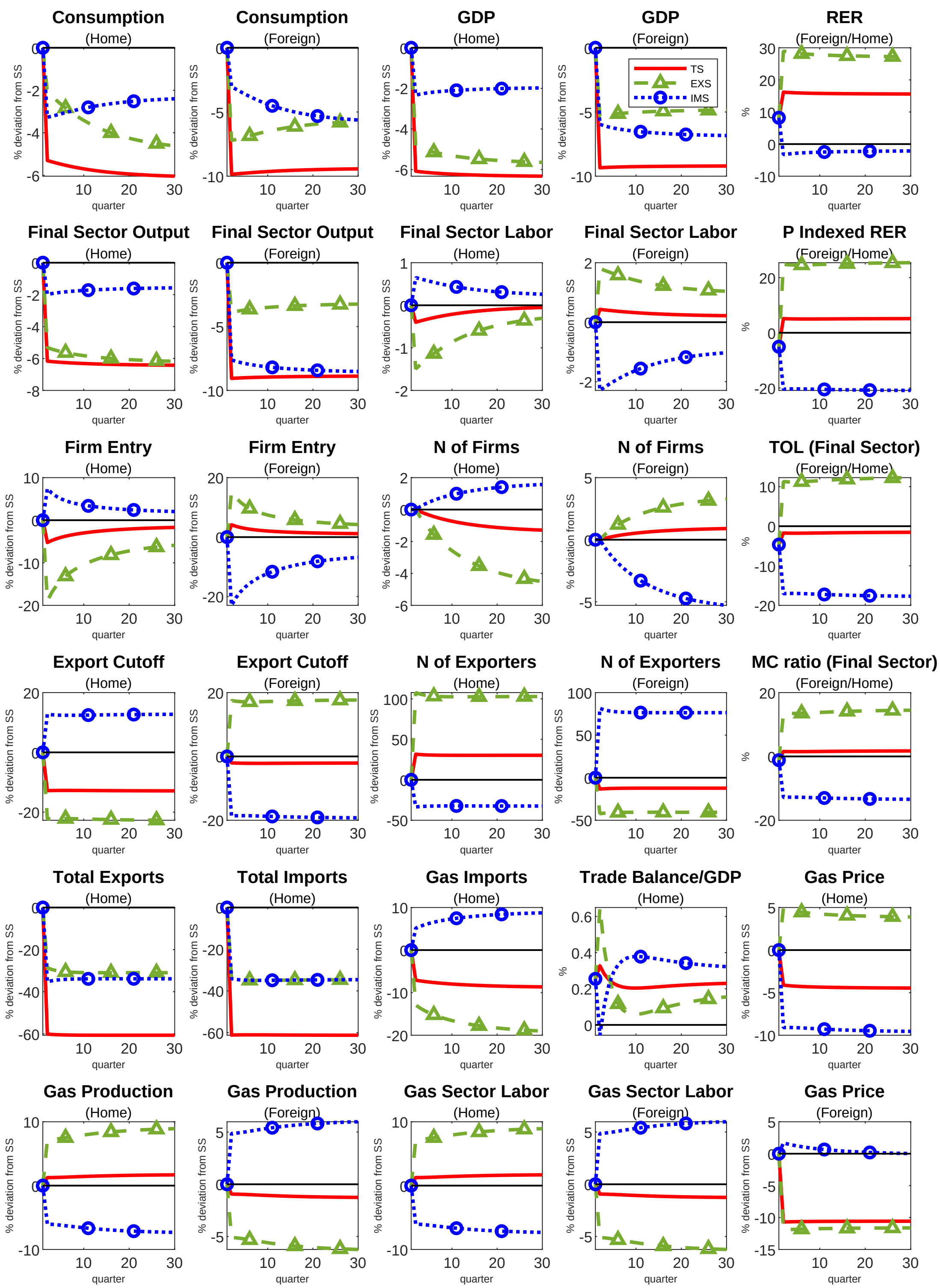

Figure 6: Transition Dynamics after Consumption Good Export and Import Sanctions

implies more gas demand from Home consumption sector and Foreign gas price increases.

We observe Home export sanctions generate a larger fall in consumption in Foreign. The reason is the asymmetricity between the two regions. Namely, the relative advantage in producing consumption goods in Home makes export sanctions more effective than import sanctions.

5.2. Financial Sanctions

In this subsection, we, first, describe the changes in key relationships after Foreign agents are excluded from trading in international financial markets, and then, discuss the simulations under financial sanctions.

When Home imposes financial sanctions on Foreign, a fraction $\lambda > 0$ of Foreign households are excluded from participating in international financial markets. After the imposition of sanctions, these households can only trade Foreign bonds and shares with other Foreign households. When the entire Foreign economy is subject to financial sanctions with $\lambda = 1$, Foreign operates under financial autarky. In a two-country world, this means that also Home must live under financial autarky, a situation that we relax in a model extension below.

Once financial sanctions are imposed, Foreign population is divided into two groups of households: λ of them who are subject to the sanctions and $1 - \lambda$ who are not. The budget constraint of the representative sanctioned household becomes:

$$\begin{gathered} C^*_{S,t} + B^*_{S,F,t+1} + \tfrac{\eta}{2}(B^*_{S,F,t+1} - B^*_F)^2 + \tfrac{\eta}{2}\tilde{v}^*_t N^*_t (x^*_{S,t+1} - 1)^2 + \tilde{v}^*_t N^*_t x^*_{S,t+1} \\ = (1 + r^*_t)\, B^*_{S,F,t} + w^*_{G,t} L^*_{S,G,t} + w^*_t L^*_{S,t} + \left(\tilde{d}^*_t + \tilde{v}^*_t\right) N^*_{D,t} x^*_{S,t} + T^{*f}_{S,t}. \end{gathered} \tag{34}$$

The subscript S denotes households that are subject to sanctions. The sanctioned household cannot trade Home bonds. They can still trade Foreign bonds, but its terminal steady state bond holding is zero, i.e., $B^*_{S,F} = 0$.

The budget constraint of the representative non-sanctioned household is:

$$\begin{gathered} C^*_{NS,t} + \tfrac{B_{F,t+1}}{Q_t} + \tfrac{\eta}{2Q_t}(B_{NS,F,t+1} - B_{NS,F})^2 + B^*_{NS,F,t+1} + \tfrac{\eta}{2}(B^*_{NS,F,t+1} - B^*_{NS,F})^2 \\ + \tfrac{\eta}{2}\tilde{v}^*_t N^*_t \left[x^*_{NS,t+1} - 1\right]^2 + \tilde{v}^*_t N^*_t x^*_{t+1} \\ = (1 + r_t)\, \tfrac{B_{NS,F,t}}{Q_t} + (1 + r^*_t)\, B^*_{NS,F,t} + w^*_{G,t} L^*_{NS,G,t} + w^*_t L^*_{NS,t} + \left(\tilde{d}^*_t + \tilde{v}^*_t\right) N^*_{D,t} x^*_t + T^{*f}_{NS,t}. \end{gathered} \tag{35}$$

The subscript NS denotes non-sanctioned households. In the terminal steady state, non-sanctioned Foreign households' bond holdings are zero after financial sanctions, $i.e.$, $B_{NS,F} = B^*_{NS,F} = 0$, but they can always trade bonds during the transition. Market clearing conditions for bonds and shares in the presence of financial market sanctions are as follows:

$$B_{H,t+1} + (1-\lambda)\,B_{NS,F,t+1} = 0 = B_{H,t} + (1-\lambda)\,B_{NS,F,t} \tag{36}$$

$$B^*_{H,t+1} + (1-\lambda)\,B^*_{NS,F,t+1} + \lambda B^*_{S,F,t+1} = 0 = B^*_{H,t} + (1-\lambda)\,B^*_{NS,F,t} + \lambda B^*_{S,F,t} \tag{37}$$

$$x_{t+1} = 1 = x_t \tag{38}$$

$$\lambda x^*_{S,t+1} + (1-\lambda)\,x^*_{NS,t+1} = 1 = \lambda x^*_{S,t} + (1-\lambda)\,x^*_{NS,t}. \tag{39}$$

Figures 7 and 8 present transition dynamics under financial sanctions. The figures plot transition dynamics from the initial steady state in which Foreign has a positive NFA position to the terminal steady state in which Foreign has zero NFA position. The blue, green, and red lines show simulations for this transition behavior when 99%, 90%, and 80% of Foreign households are excluded from international financial transactions, respectively ($i.e.$, $\lambda \in \{0.8, 0.9, 0.99\}$).

The immediate observation is that financial sanctions generate a more pronounced drop in Foreign consumption in the short to medium run, if a larger proportion of Foreign households is sanctioned. In the long term, Home consumption increases and Foreign consumption decreases under every scenario due to the change in the long-term NFA positions.

In response to 90% exclusion of Foreign households from international bond markets, Foreign incentive to front-load entry increases and Foreign expands borrowing from abroad by non-sanctioned Foreign households. Proceedings of borrowing from Home is used for facilitating entry in new Foreign consumption good producers. The increase in entry in consumption good production translates into more labor demand and depreciation of the terms of labor (from Home perspective). Real exchange rate also depreciates (from Home perspective) in response to depreciation of the terms of labor. It is observed from Figure 8 that sanctioned households increase supply of labor to both consumption good production and gas production sector to compensate for the fall in their financial income.

When greater share of households are financially sanctioned, international borrowing dampens and Foreign producer entry slows down. This is due to limitation of generating resources to facil-

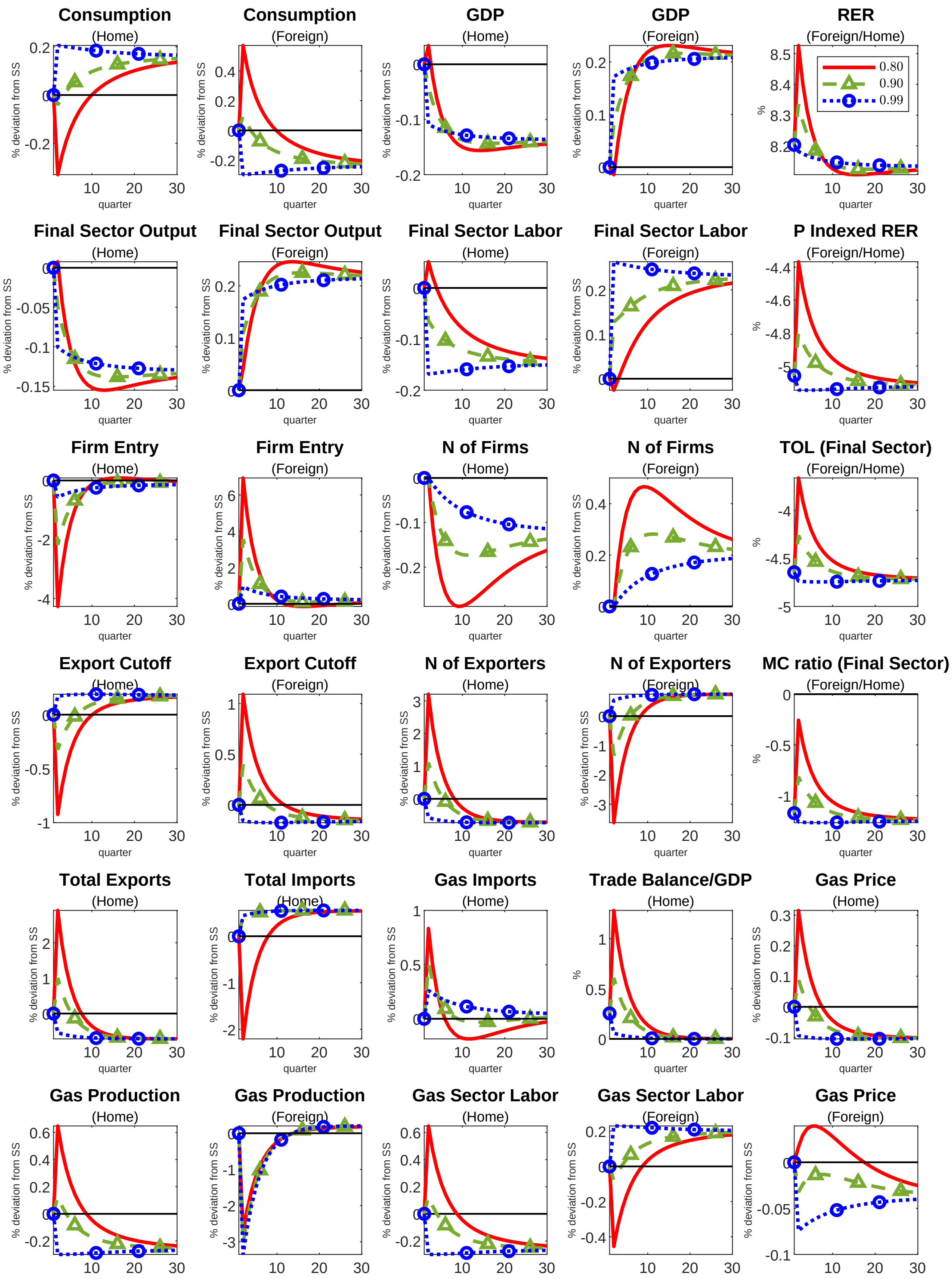

Figure 7: Transition Dynamics after Financial Sanctions

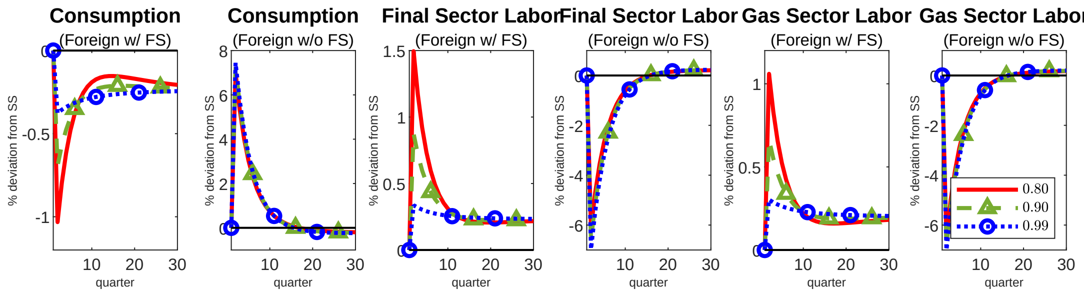

Figure 8: Foreign Household Consumption and Labor (Financial Sanctions)

itate entry. Under this scenario, the increase in sanctioned households' labor supply is amplified. When smaller fraction of Foreign households are sanctioned, Foreign aggregate consumption increases in the short run. This is because non-sanctioned households increase their consumption while transitioning into a lower NFA steady state and reduce their savings. Sanctioned households cannot expand consumption during the transition. Therefore, if a smaller share in Foreign is sanctioned, Foreign trade balance deteriorates and the number of exporters in Foreign decreases (blue lines in Figures 7 and 8). Hence, fewer Foreign producers start to export and Foreign average export price decreases, leading to a depreciation of the exchange rate (from Home perspective).

5.3. Gas Sanctions

We study gas sanctions by generating a permanent drop in Home imports of gas from Foreign in period 1. Figure 9 shows the behavior of several variables when gas sanctions are in place (blue lines). For comparison purposes, we also plot the transition dynamics under trade sanctions (green lines) and financial sanctions (red lines). Gas sanctions are not as effective as combined import and export sanctions of consumption good trade in terms of reducing Foreign consumption. However, in response to gas sanctions Foreign consumption drops more than Home consumption, even in the absence of Home economy's ability to substitute toward gas imports from other regions/countries.

The fall in demand for Foreign gas induces a drop in gas production in Foreign and a subsequent jump in gas prices in Home. The gas price in Home economy rises because consumption good producing firms demand more domestic gas to compensate for the lost imported gas. With rising gas prices, marginal costs of production in consumption good sector increase. Rising costs are translated into less entrants in Home and the total number of producers decline. Home households increase labor supply to gas production and decrease labor supply to consumption good production.

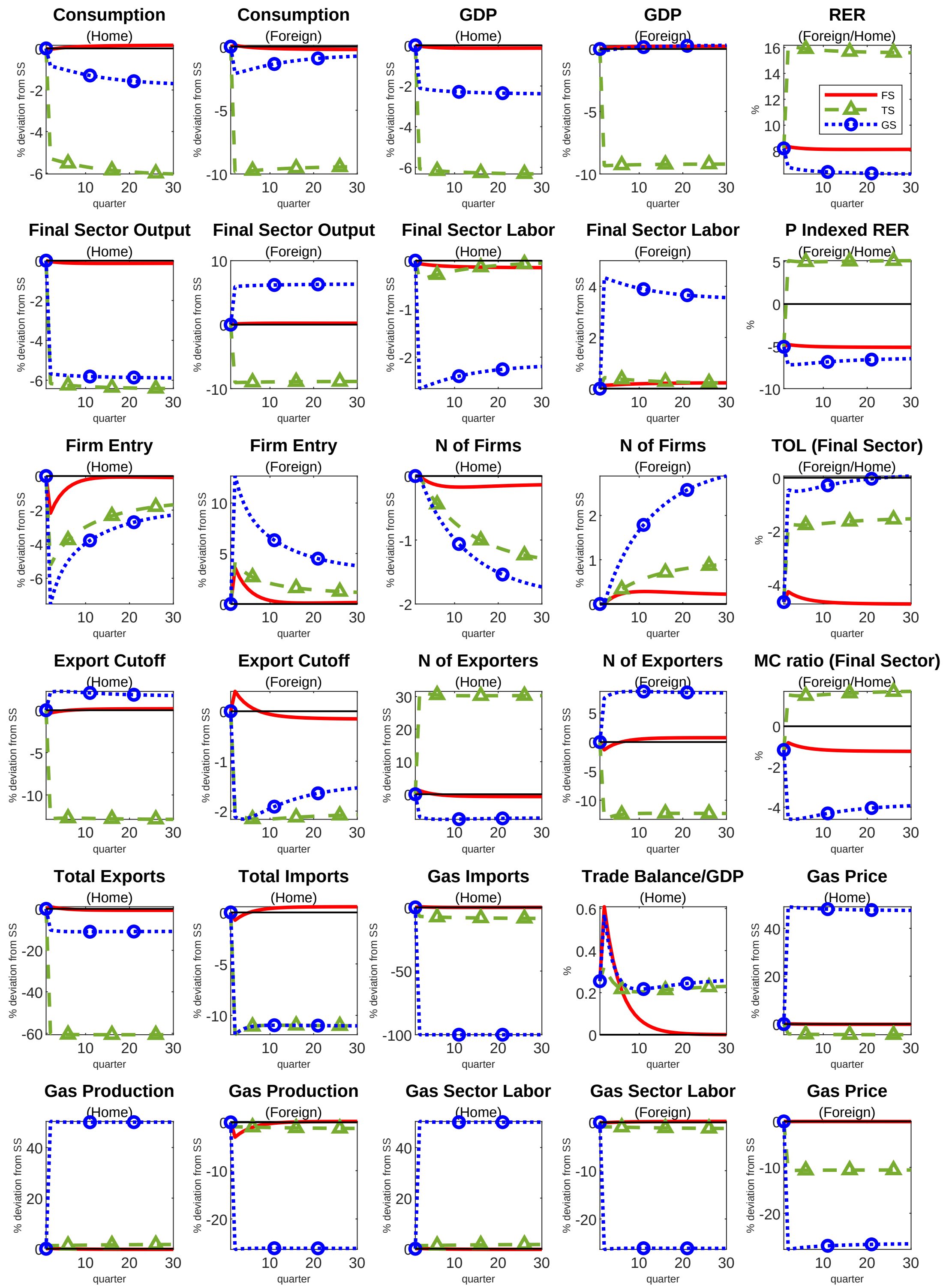

Figure 9: Transition Dynamics after Trade, Financial, and Gas Sanctions

Foreign economy rebalances itself in the opposite way. To compensate for the loss of gas exports, the economy shifts toward producing more in consumption good sector and import more of consumption goods. Consumption good producers increase demand for labor and wages rise. Whereas the fall in gas production means less labor is needed in the gas sector, and wages in gas sector decrease. The shift in the economy facilitates more entrants into the consumption good sector. Therefore, the number of producers in Foreign consumption good sector increases. To compensate for the loss of energy exports, more firms in the consumption goods sector export, and the cutoff productivity level for the least efficient exporter in Foreign goes down. The move in the exporter productivity cutoff translates into higher average export prices in Foreign, appreciating real exchange rate from Home perspective.

5.4. Combinations of Sanctions

In this subsection, we present the combined impact of several sanctions that are introduced simultaneously. In particular, we consider three cases: (1) combination of financial sanctions and consumption good import and export sanctions (FS&TS), (2) combination of financial and gas sanctions (FS&GS) and, (3) combination of financial, consumption good trade, and gas sanctions, altogether (FS&TS&GS).

Figures 10 and 11 show transition dynamics when several combination of sanctions are in place. The negative impact on Foreign consumption gets amplified if all sanctions are applied simultaneously. The quantitative impact similar when all sanctions are applied simultaneously (FS&TS&GS, red solid lines) or when only consumption good trade and financial sanctions (FS&TS, green dashed lines with triangles) applied simultaneously. It is also important to note that all of the sanctions generate a fall in Home consumption and GDP, although the fall is not as large as in Foreign. The impact of consumption good export sanctions generates the most pronounced fluctuations. Therefore, Home exchange rate depreciates when trade sanctions are combined with any other sanction.

Gas sanctions damage gas exporters (Foreign) more than gas importers (Home). Foreign economy rebalances towards consumption good production and, therefore, Foreign GDP stays stable. Gas sanctions contribute to a long term drop in Home consumption whereas the long term impact on Foreign consumption is relatively small. The combination of financial and consumption good

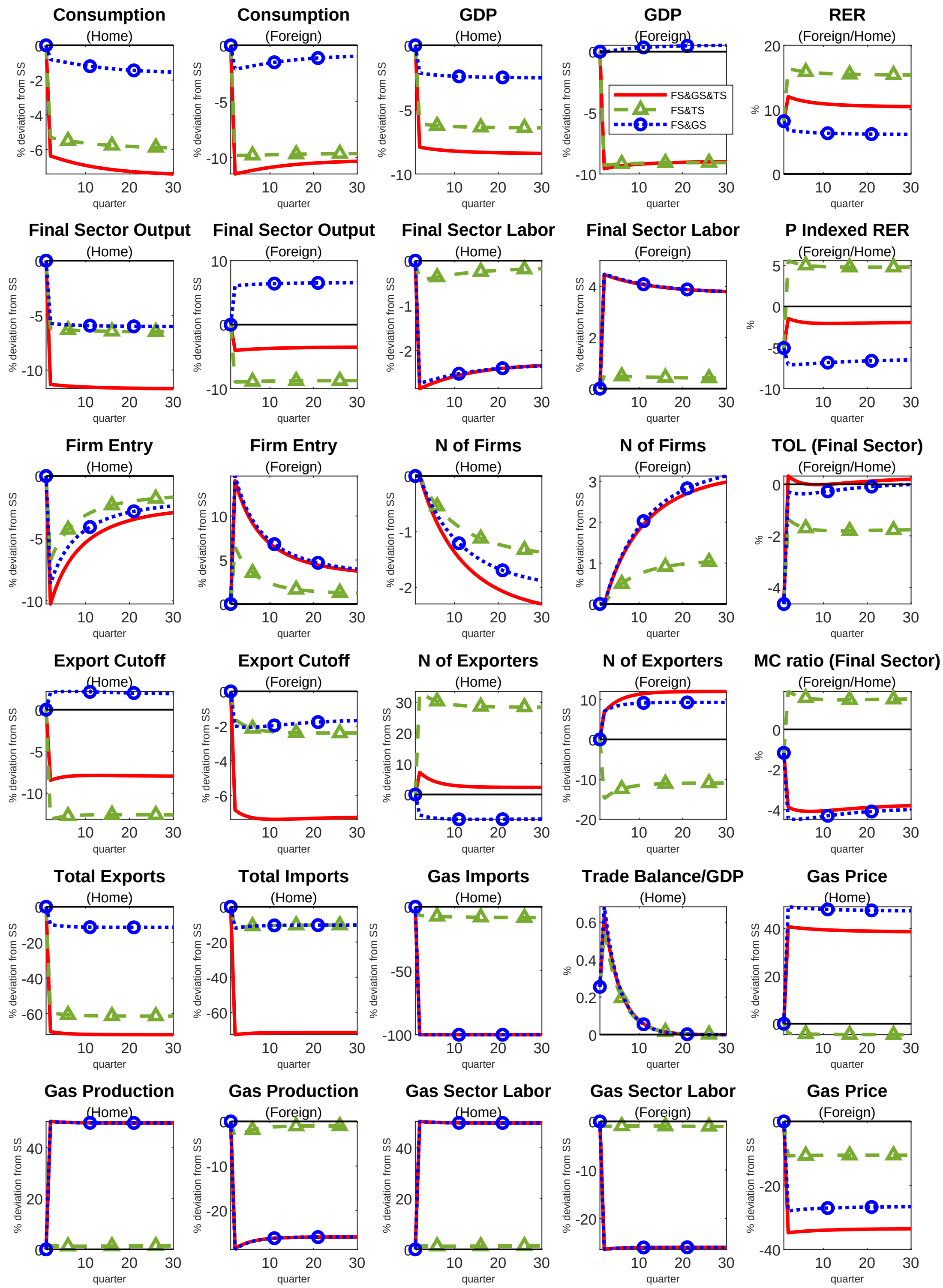

Figure 10: Transition Dynamics with Combinations of Sanctions

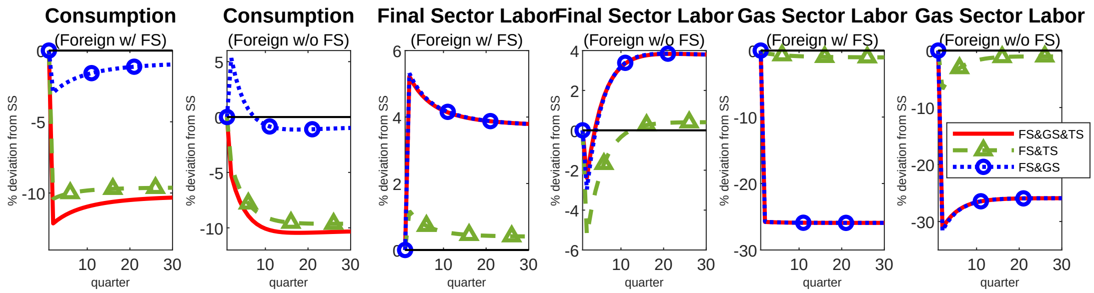

Figure 11: Foreign Household Consumption and Labor (under combinations of sanctions)

trade sanctions (FS&TS) looks favorable to dampen the negative impact on Home economy but to amplify the negative impact on Foreign economy, compared to the other combination of sanctions. On the other hand, Figures 10 and 11 show that gas sanctions could be used as a by Foreign to generate a fall in Home GDP. To be more concrete on the evaluation of the success of sanctions, we provide welfare results in the next subsection.

5.5. Welfare

We calculate welfare as the lifetime utility from consumption and disutility from labor. We incorporate the impact of sanctions after they are imposed in the first period, $i.e.$, $t = 1$, include transition dynamics until $t = 201$, and the terminal impact. Simulating our model for 200 periods is enough for the economy to reach its new steady state. In particular,

$$\begin{aligned} W_0^{Sanction} &= \sum_{t=0}^{\infty} \beta^t \left(logC_t - \frac{\kappa}{2} L_t^2 - \frac{\kappa_G}{2} L_{G,t}^2 \right) \\ &= \sum_{t=0}^{200} \beta^t \left(logC_t - \frac{\kappa}{2} L_t^2 - \frac{\kappa_G}{2} L_{G,t}^2 \right) + \frac{\beta^{201}}{1-\beta} \left(logC_{201} - \frac{\kappa}{2} L_{201}^2 - \frac{\kappa_G}{2} L_{G,201}^2 \right) \end{aligned} \quad (40)$$

To evaluate the impact of sanctions we calculate the welfare gain by taking the difference from the welfare metric without sanctions:

$$\hat{W}_0 = W_0^{Sanction} / W_0^{NoSanction} - 1, \quad (41)$$

Table 1: Change in Welfare after Sanctions

	Type of Sanctions	% of Welfare Gain	
		Home	Foreign
Individual Sanctions	Gas sanction	–1.37	–0.75
	C-good export sanction	–3.56	–7.15
	C-good import sanction	–1.87	–6.85
	C-good trade sanction	–4.73	–11.88
	Financial sanction	0.21	–0.50
Sanction Combinations	Gas + Financial sanctions	–1.15	–1.27
	C-good trade + Financial sanctions	–4.53	–12.35
	Gas + C-good trade + Financial sanctions	–5.82	–13.14

Notes: Foreign welfare gain is calculated from the weighted sum of financially sanctioned and unsanctioned foreign households.

where the lifetime welfare without sanction is

$$W_0^{NoSanction} = \frac{\left(logC_0 - \frac{\kappa}{2}L_0^2 - \frac{\kappa_G}{2}L_{G,0}^2\right)}{1-\beta}. \tag{42}$$

To measure the aggregate Foreign welfare under sanctions, we calculate the deviations from the weighted average of welfare of sanctioned and non-sanctioned households:

$$W_0^{Sanction*} = \lambda W_{S,0}^{Sanction*} + (1-\lambda) W_{NS,0}^{Sanction*}. \tag{43}$$

Table 1 presents welfare gains in Home and Foreign from sanctions. Consumption good trade sanctions generate the most welfare loss both in Home and Foreign with a more pronounced impact in Foreign. Gas sanctions generate more welfare losses in Home than in Foreign, although the quantitative impact is much smaller than consumption good trade sanctions. Financial sanctions generate a small amount of welfare gain in Home because of Home's transition from a negative net foreign position to zero.

In line with our analysis in the previous section about economies' rebalancing towards different sectors, sanctioning the less efficient sector in Foreign while enforcing the Foreign economy to reallocate resources towards the less efficient sector generates the most welfare losses in Foreign. After accounting for the interaction terms, combining consumption good trade sanctions with financial sanctions generates little additional loss in Foreign.

6. International Coordination with a Third Country

Our analysis, so far, excludes from substitution to a third region in response to sanctions. To account for the impact of a third country, we extend our model to a three-country setting in this section.

Inclusion of the third country, which we label as the rest of the world (RoW), enables us to investigate the impact of international coordination when sanctioning Foreign. For clarity purposes, we assume that Home and RoW are symmetric. That is, Home and RoW are subject to smaller entry costs in consumption good sector and have less gas endowment than Foreign. Our calibration follows the numerical values that we set in Section 1 except for the financial variables and the fixed export costs. To maintain a positive net foreign position of Foreign, we set Home (also RoW) households' initial holdings of Home and Foreign bonds to $B_H = -5/2$ and $B_H^* = 3/2$, respectively (half of the value in the two-country model). Symmetric Home and RoW implies that net foreign asset position between Home and RoW is zero. Furthermore, the fixed cost of exporting is set to 0.0072, which implies that the percent of Home firms export their good to RoW and from RoW firms to Home is 21, the percent of RoW firms export to Foreign and from Home firms to Foreign is 21, the percent of Foreign firms export to Home and from Foreign firms to RoW is 23 in the initial steady state.

Figures 12 and 13 present the transition dynamics from the three-country model when all financial, consumption good and financial sanctions are in place. Green lines indicate dynamics when only Home sanctions Foreign ($i.e.$, uncoordinated sanctions) whereas red lines indicate dynamics when both Home and RoW sanction Foreign ($i.e.$, coordinated sanctions).

Under uncoordinated sanctions, Home and Foreign consumption fall while RoW increases. RoW GDP is also positively affected under uncoordinated sanctions because of the substitution effects. RoW reallocates its economy towards consumption good production to match the additional demand coming from Foreign after Home introduces sanctions. RoW expands exports to Foreign while increasing its gas imports from Foreign. Therefore, the impact of Home sanctions on Foreign generates a milder drop in Foreign consumption vis-a-vis the two-country model.

Under coordinated sanctions, the negative effects of sanctions on Home is dampened and on Foreign is amplified. It almost doubled the loss in Foreign consumption and GDP in both short and

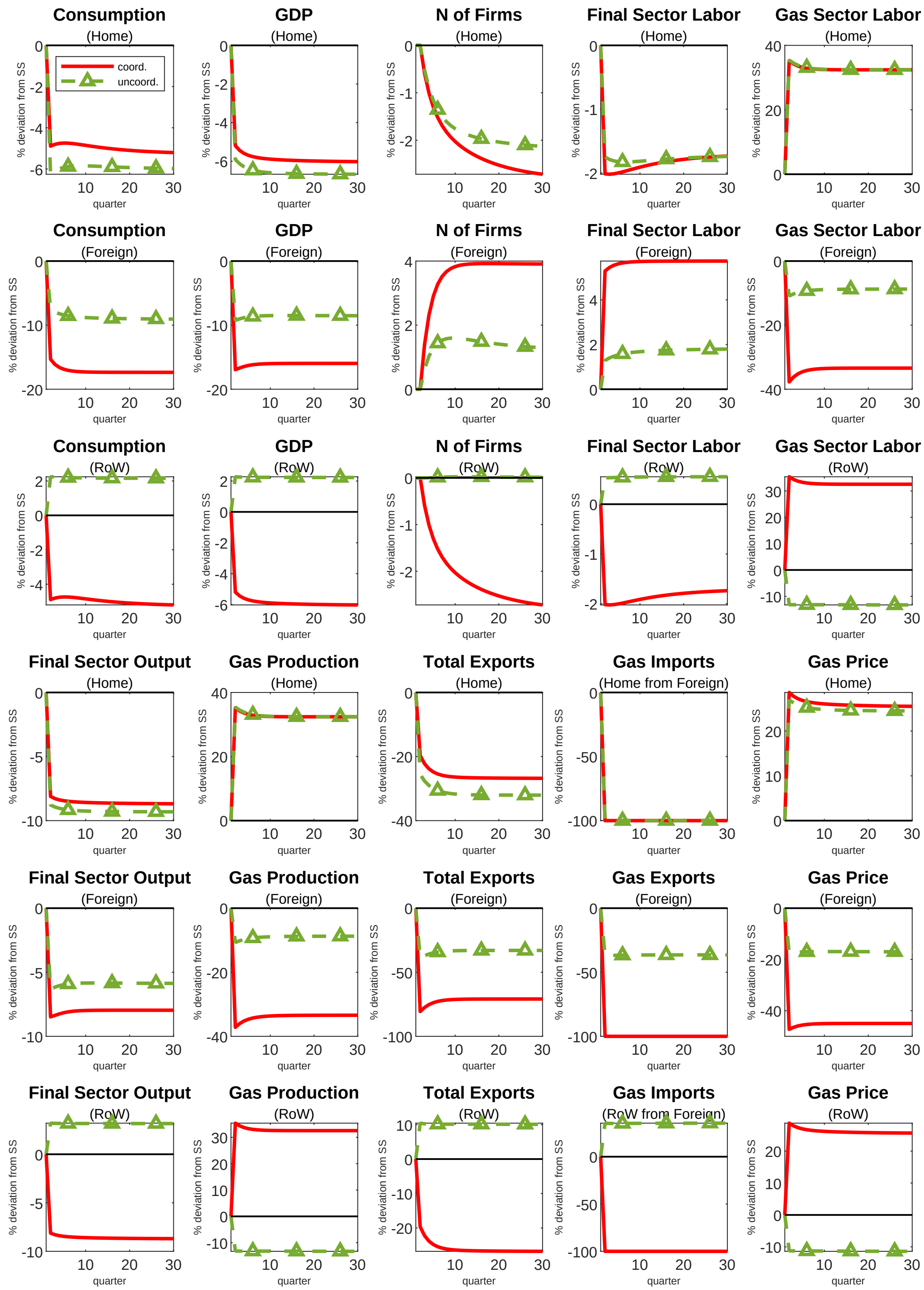

Figure 12: Transition Dynamics from the Three-Country Model (1/2)

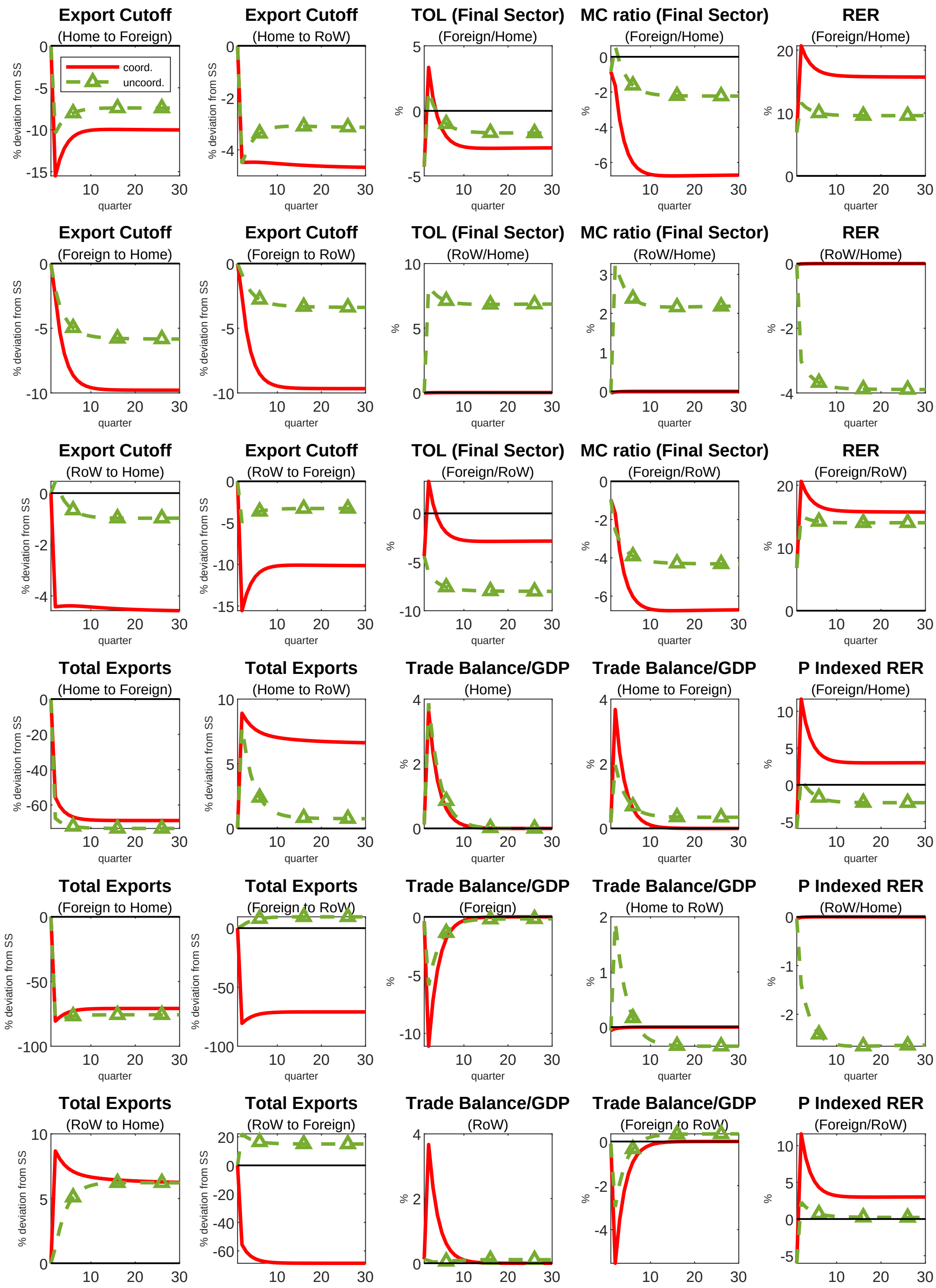

Figure 13: Transition Dynamics from the Three-Country Model (2/2)

Table 2: Change in Welfare after Sanctions with International Coordination

Type of Sanctions	International Coordination	% of Welfare Gain		
		Home	Foreign	RoW
Gas + Financial sanctions	Coordinated	–1.62	–2.58	–1.62
	Uncoordinated	–1.56	–1.85	–0.43
C-good trade + Financial sanctions	Coordinated	–3.97	–19.09	–3.97
	Uncoordinated	–4.79	–9.87	0.28
Gas + C-good trade + Financial sanctions	Coordinated	–4.74	–20.99	–4.74
	Uncoordinated	–5.34	–10.90	0.69

Notes: Foreign welfare gain is calculated from the weighted sum of financially sanctioned and unsanctioned foreign households.

long term. Coordinated sanctions are costly for RoW. RoW consumption and GDP is similarly affected from the sanctions with Home. Under coordinated sanctions, RoW rebalances the economy towards gas production. Convergence towards zero net foreign asset position implies that Home exporters substitute towards RoW, generating a positive trade balance.

The effects are seen clearer when comparing welfare of the three regions. Table 2 presents welfare gains from sanctions under coordinated and uncoordinated scenarios and for different combinations of sanctions. The immediate observation is in line with the analysis above: coordinated sanctions generate more welfare losses in Foreign and RoW. The impact of sanctions in Foreign is almost doubled under every scenario. In line with our analysis from the two-country model, consumption good trade sanctions generate the most welfare losses in Foreign. We observe that when there is no coordination between Home and RoW, consumption good trade sanctions can benefit RoW, although in small amount. Under the uncoordinated scenario, after Home introduces gas sanctions to Foreign, RoW welfare increases, accounting for the substitution of Foreign gas imports to RoW.

The main takeaway from this section is that coordinated sanctions generate the most pronounced welfare losses in Foreign at a smaller cost on RoW.

7. Conclusions

We studied sanctions in a model of international trade and macroeconomic dynamics. We examined how sanctions would work and their impact on international relative prices, balances, standard macroeconomic aggregates. Our analysis focuses on the impact of sanctions both in the country they are imposed and in the country which imposes them.

Product variety effects are central to the transmission of sanctions. In response to sanctions by prohibiting consumption good exports of Home producers, average Home exporter price increases. Foreign households shift demand to domestically produces goods. Home exchange rate depreciates. In response to gas sanction that is introduced by prohibiting imported Foreign gas, Foreign economy rebalances itself by moving resources to consumption good sector. Number of exporters in Foreign increases, leading to higher average exporter price. Home exchange rate appreciates. Exchange rate is not a useful metric to evaluate the effectiveness of sanctions. The exchange rate depreciates or appreciates, reflecting the type of the sanction and the way that economies rebalance themselves by moving resources in between sectors.

The calibrated model's welfare analysis shows that consumption good trade sanctions are generating the most pronounced welfare losses in Foreign. Sanctioning less effective sectors while enforcing the sanctioned economy to rebalance towards less effective sectors generates most drop in welfare in Foreign. Also, our results from the three-country version of our model indicate that coordination with a third country when imposing sanctions almost doubles the welfare losses in the sanctioned economy. Coordination is costly for the third country because of lost gains from substitution when there are sanctions between the first two countries.

These results are relevant for the ongoing discussions on geopolitical tensions affecting the global economy. Our analysis provides a roadmap on how several sanctions might work. A natural extension of our model would account for the effects of sanctions on the consolidated budget constraint of the targeted country's government sector. When this is considered, we can study the impact of government revenue from taxation and government spending, when fiscal revenues rely substantially on international trade. We leave this extension for future work.

References

Albrizio, Silvia, John C Bluedorn, Christoffer Koch, Andrea Pescatori and Martin Stuermer, 2022, "Market size and supply disruptions: Sharing the pain of a potential gas shut-off to the European Union," IMF Working Paper No. 2022/143 (Washington, DC, International Monetary Fund).

Auray, Stephane and Aurelien Eyquem, 2011, "Endogenous entry, international business cycles, and welfare," GATE Working Paper No. 1114.
http://dx.doi.org/10.2139/ssrn.1800748

Bachmann, Rudiger, David Baqaee, Christian Bayer, Moritz Kuhn, Andreas Loschel, Benjamin Moll, Andreas Peichl, Karen Pittel and Moritz Schularick, 2022, "What if? The economic effects for Germany of a stop of energy imports from Russia" (unpublished, University of Notre Dame).

Bergin, Paul, and Giancarlo Corsetti, 2019, "Beyond competitive devaluations: The monetary dimensions of comparative advantage," American Economic Journal: Macroeconomics, Vol. 12, No. 4, pp. 246-86.
http://dx.doi.org/10.1257/mac.20160094

Bergina, Paul, Ling Fengb and Ching-Yi Lin, 2021, "Trade and firm financing," Journal of International Economics, Vol. 131. No. 103461.
https://doi.org/10.1016/j.jinteco.2021.103461

Bianchi, Javier, and Cesar Sosa-Padilla, 2022, "On wars, sanctions and sovereign default" (unpublished, University of Notre Dame).

Bilbiie, Florin O., Fabio Ghironi, and Marc Melitz, 2021, "Endogenous entry, product variety, and business cycles," Journal of Political Economy, Vol. 120. pp. 304-345.
https://doi.org/10.1086/665825

Blackwill, Robert D., and, Jennifer M. Harris, 2016, War by other means geoeconomics and statecarft (Cambridge, Massachusetts, Harvard University Press).

Cacciatore, Matteo, 2014, "International trade and macroeconomic dynamics with labor market frictions," Journal of International Economics, Vol. 93, pp. 17-30.
https://doi.org/10.1016/j.jinteco.2014.01.008

Cacciatore, Matteo, and Fabio Ghironi, 2021, "Trade, unemployment, and monetary policy," Journal of International Economics, Elsevier, Vol. 132, No. 103488.
https://doi.org/10.1016/j.jinteco.2021.103488

Corsetti, Giancarlo, Philippe Martin and Paolo Pesenti, 2007, "Productivity, terms of trade and the 'home market effect'," Journal of International Economics, Vol. 73, pp. 99-127.
https://doi.org/10.1016/j.jinteco.2006.08.005

Corsetti, Giancarlo, Philippe Martin and Paolo Pesenti, 2013, “Varieties and the transfer problem,” Journal of InternationalEconomics, vol. 89, pp. 1-12.
https://doi.org/10.1016/j.jinteco.2012.05.011

Dekle, Robert, Hyeok Jeong and Nobuhiro Kiyotaki, 2015, “Dynamics of firms and trade in general equilibrium,” USC-INET Research Paper No. 15-12 (Los Angeles, Califonia, USC Dornsife INET Research).
http://dx.doi.org/10.2139/ssrn.2574454

Eichengreen, Barry, Massimo Ferrari Minesso, Arnaud Mehl and Isabel Vansteenkiste, 2022, “Sanctions and the exchange rate in time,” CEPR Discussion Paper No. 17641 (London, UK, CEPR).

Feenstra, Robert C., 1994, “New Product Varieties and the Measurement of International Prices,” American Economic Review, Vol. 84, No.1, pp. 157-177.

Ghironi, Fabio, and Mark Melitz, 2005, “International Trade and Macroeconomic Dynamics with Heterogeneous Firms,” The Quarterly Journal of Economics, Vol. 120, pp. 865-915. https://doi.org/10.1093/qje/120.3.865

Hamano, Masashige, and Francesco Zanetti, 2017, “Endogenous product turnover and macroeconomic dynamics,” Review of Economic Dynamics, Vol.26, pp. 263-279.
https://doi.org/10.1016/j.red.2017.06.003

Imura, Yuko and Malik Shukayev, 2019, “The extensive margin of trade and monetary policy,” Journal of Economic Dynamics and Control, Vol. 100, Pages 417-441.
https://doi.org/10.1016/j.jedc.2019.01.002

Itskhoki, Oleg, and Dmitry Mukhin, 2022, “Sanctions and the exchange rate,” NBER Working Paper No. 30009 (Cambridge, Massachusetts, National Bureau of Economic Research).
https://doi.org/10.3386/w30009

Kim, Daisoon, 2021, “Economies of Scale and Business Cycles,” Journal of International Economics, Vol. 131, No. 103459.
https://doi.org/10.1016/j.jinteco.2021.103459

Kim, Daisoon, Galip Kemal Ozhan and Larry Schembri, 2021, “Monetary policy, trade, and commodity price fluctuations” (unpublished, Bank of Canada).

Korhonen, Iikka, 2019, “Economic sanctions on Russia and their effects,” CESifo Forum, Volume 20, pp. 19-22.

Lorenzoni, Guido, and Ivan Werning, 2022, “A minimalist model for the Ruble during the Russian invasion of Ukraine,” NBER Working Paper No. 29929 (Cambridge, Massachusetts, National Bureau of Economic Research).
https://doi.org/10.3386/w29929

Melitz, Mark, 2003, “The impact of trade on intra-industry reallocations and aggregate industry,” Econometrica, Vol. 71, pp. 1695-1725.
https://doi.org/10.1111/1468-0262.00467

Mulder, Nicholas, 2022, “The Economic Weapon: The rise of sanctions as a yool of modern war” (New Haven, Connecticut, Yale University Press).

Patureau, Lise, and Celine Poilly, 2019, “Reforms and the real exchange rate: The role of pricing-to-market,” Journal of International Economics, Vol. 119, pp. 150-168.
https://doi.org/10.1016/j.jinteco.2019.04.005

Rodriguez-Lopez, Jose Antonio, 2011, “Prices and exchange rates: A theory of disconnect,” The Review of Economic Studies, Vol. 78, pp. 1135-1177.
https://doi.org/10.1093/restud/rdq031

Sonnenfeld, Jeffrey, Steven Tian, Franek Sokolowski, Michal Wyrebkowski and Mateusz Kasprowicz, 2022, “Business retreats and sanctions are crippling the Russian economy” (unpublished, Yale School of Management).
http://dx.doi.org/10.2139/ssrn.4167193

Sturm, John, 2022, “A Note on Designing Economic Sanctions” (unpublished, MIT).

van Bergeijk, 2021, Peter A.G., “Introduction to the Research Handbook on Economic Sanctions,” in Research Handbook on Economic Sanctions, ed by Peter A.G. Bergeijk (Cheltenham, UK: Edward Elgar Publishing).
https://doi.org/10.4337/9781839102721.00006

Zlate, Andrei, 2016, “Offshore Production and Business Cycle Dynamics with Heterogeneous Firms,” Journal of International Economics, Vol. 100, pp. 34-49.
https://doi.org/10.1016/j.jinteco.2016.01.004

Appendix

A. Mathematical Derivations

A.1. Gas Price Determination

Total demand for labor in the final Home sector can be written as $N_{D,t}L_t(\tilde{z}_D) + N_{X,t}L_t(\tilde{z}_{X,t})$. Since our assumptions are such that Home is the gas importer, total demand of gas by Home has to be equal to Home gas production ($G_N L_{G,t}$) plus Home imports of gas from Foreign ($\frac{G^*_{H,t}}{\tau_{G,t}}$). Optimal input demand conditions in Home then imply:

$$w_t\left(N_{D,t}L_t(\tilde{z}_D) + N_{X,t}L_t(\tilde{z}_{X,t})\right) = \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G^*_{H,t}}{\tau_{G,t}}\right).$$

Using final sector production functions, this equation can be rewritten as:

$$\begin{aligned} & w_t\left[N_{D,t}\left(\frac{y_t(\tilde{z}_D)}{Z_t\tilde{z}_D}\right)^{\frac{1}{1-\alpha}} G_t(\tilde{z}_D)^{-\frac{\alpha}{1-\alpha}} + N_{X,t}\left(\frac{y_t(\tilde{z}_{X,t})}{Z_t\tilde{z}_{X,t}}\right)^{\frac{1}{1-\alpha}} G_t(\tilde{z}_{X,t})^{-\frac{\alpha}{1-\alpha}}\right] \qquad \text{(A1)} \\ = \; & \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G^*_{H,t}}{\tau_{G,t}}\right). \end{aligned}$$

Next, note that optimal gas demand by a firm with productivity $\tilde{z}_D$ and market clearing for its output are such that $G_t(\tilde{z}_D) = \left(\frac{\alpha}{1-\alpha}\frac{w_t}{\rho_{G,t}}\right)^{1-\alpha}\frac{y_t(\tilde{z}_D)}{Z_t\tilde{z}_D}$ and $y_t(\tilde{z}_D) = \rho_{H,t}(\tilde{z}_D)^{-\theta}C_t$. Similarly, optimal gas demand by a firm with productivity $\tilde{z}_D$ and market clearing for its output satisfy $G_t(\tilde{z}_{X,t}) = \left(\frac{\alpha}{1-\alpha}\frac{w_t}{\rho_{G,t}}\right)^{1-\alpha}\frac{y_t(\tilde{z}_{X,t})}{Z_t\tilde{z}_{X,t}}$ and $y_t(\tilde{z}_{X,t}) = \rho_{H,t}(\tilde{z}_{X,t})^{-\theta}C_t + \tau_t\rho_{X,t}(\tilde{z}_{X,t})^{-\theta}C^*_t$. Substituting these equations into (A1) and rearranging yields:

$$\begin{aligned} & \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\rho^{\alpha}_{G,t}w^{1-\alpha}_t\left[N_{D,t}\frac{\rho_{H,t}(\tilde{z}_D)^{-\theta}C_t}{Z_t\tilde{z}_D} + N_{X,t}\frac{\rho_{H,t}(\tilde{z}_{X,t})^{-\theta}C_t + \tau_t\rho_{X,t}(\tilde{z}_{X,t})^{-\theta}C^*_t}{Z_t\tilde{z}_{X,t}}\right] \\ = \; & \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G^*_{H,t}}{\tau_{G,t}}\right). \qquad \text{(A2)} \end{aligned}$$

Optimal price setting by Home final sector firms and the expression for final sector marginal cost imply:

$$\rho_{H,t}\left(\tilde{z}_D\right) = \left(\frac{\theta}{\theta-1}\right)\frac{\rho_{G,t}^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\tilde{z}_D Z_t}, \tag{A3}$$

$$\rho_{F,t}\left(\tilde{z}_{X,t}\right) = \frac{\tau_t}{Q_t}\frac{\tilde{z}_D}{\tilde{z}_{X,t}}\rho_{H,t}\left(\tilde{z}_D\right), \tag{A4}$$

$$\rho_{H,t}\left(\tilde{z}_{X,t}\right) = \frac{\tilde{z}_D}{\tilde{z}_{X,t}}\rho_{H,t}\left(\tilde{z}_D\right), \tag{A5}$$

Substituting equations (A3)-(A5) into equation (A2) and rearranging yields:

$$\alpha\left(\frac{\theta-1}{\theta}\right)^{\theta} Z_t^{\theta-1}\left\{N_{D,t}\tilde{z}_D^{\theta-1}C_t + N_{X,t}\tilde{z}_{X,t}^{\theta-1}\left[C_t + \left(\frac{\tau_t}{Q_t}\right)^{1-\theta} Q_t C_t^*\right]\right\}\left[\frac{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}{\rho_{G,t}^{\alpha}w_t^{1-\alpha}}\right]^{\theta-1}$$
$$= \left(\frac{1-\alpha}{\alpha}\right)\rho_{G,t}\left(G_N L_{G,t} + \frac{G_{H,t}^*}{\tau_{G,t}}\right). \tag{A6}$$

This equation can be solved for w_t as:

$$w_t = \left(\frac{\alpha}{\rho_{G,t}}\right)^{\frac{1+\alpha(\theta-1)}{(1-\alpha)(\theta-1)}}\left\{\frac{Z_t^{\theta-1}\left\{N_{D,t}\tilde{z}_D^{\theta-1}C_t + N_{X,t}\tilde{z}_{X,t}^{\theta-1}\left[C_t + \left(\frac{\tau_t}{Q_t}\right)^{1-\theta} Q_t C_t^*\right]\right\}}{\left(\frac{\theta}{\theta-1}\right)^{\theta}\left(G_N L_{G,t} + \frac{G_{H,t}^*}{\tau_{G,t}}\right)}\right\}^{\frac{1}{(1-\alpha)(\theta-1)}}. \tag{A7}$$

Working in a similar way for the Foreign economy yields:

$$w_t^* = \left(\frac{\alpha\tau_{G,t}Q_t}{\rho_{G,t}}\right)^{\frac{1+\alpha(\theta-1)}{(1-\alpha)(\theta-1)}}\left\{\frac{Z_t^{*\theta-1}\left\{N_{D,t}^*\tilde{z}_D^{\theta-1}C_t^* + N_{X,t}^*\tilde{z}_{X,t}^{*\theta-1}\left[C_t^* + \left(\tau_t^* Q_t\right)^{1-\theta}\frac{C_t}{Q_t}\right]\right\}}{\left(\frac{\theta}{\theta-1}\right)^{\theta} G_N^* L_{G,t}^*}\right\}^{\frac{1}{(1-\alpha)(\theta-1)}}. \tag{A8}$$

To economize on notation in the following steps, rewrite the last two equations as:

$$w_t = f\left(\rho_{G,t}\right) \quad \text{and} \quad w_t^* = f^*\left(\rho_{G,t}\right). \tag{A9}$$

Equilibrium in the world market for gas requires total supply to be equal to demand. Hence, using production functions and optimal demand conditions:

$$G_N L_{G,t} + G_N^* L_{G,t}^* = N_{D,t}\left(\frac{1-\alpha}{\alpha}\frac{w_t}{\rho_{G,t}}\right)^{1-\alpha}\frac{y_t(\tilde{z}_D)}{\tilde{z}_D Z_t} + N_{X,t}\left(\frac{1-\alpha}{\alpha}\frac{w_t}{\rho_{G,t}}\right)^{1-\alpha}\frac{\tau_t y_t(\tilde{z}_{X,t})}{\tilde{z}_{X,t} Z_t} \tag{A10}$$

$$+N_{D,t}^*\left(\frac{1-\alpha}{\alpha}\frac{w_t^*}{\rho_{G,t}^*}\right)^{1-\alpha}\frac{y_t^*(\tilde{z}_D)}{\tilde{z}_D Z_t^*} + N_{X,t}^*\left(\frac{1-\alpha}{\alpha}\frac{w_t^*}{\rho_{G,t}^*}\right)^{1-\alpha}\frac{\tau_t^* y_t^*(\tilde{z}_{X,t}^*)}{\tilde{z}_{X,t}^* Z_t^*} \tag{A11}$$

Optimal labor supply for Home and Foreign gas production is given by, respectively:

$$L_{G,t} = \frac{w_{G,t}}{\kappa_G C_t} \quad \text{and} \quad L_{G,t}^* = \frac{w_{G,t}^*}{\kappa_G^* C_t^*}.$$

Therefore, it is:

$$L_{G,t} = \frac{\rho_{G,t} G_N}{\kappa_G C_t}, \tag{A12}$$

and:

$$L_{G,t}^* = \frac{\rho_{G,t} G_N^*}{\kappa_G^* \tau_{G,t} Q_t C_t^*}. \tag{A13}$$

Substituting $w_t = f(\rho_{G,t})$, $w_t^* = f^*(\rho_{G,t})$, and equations (A12) and (A13) into equation (A10), using market clearing conditions for Home and Foreign final sector products, and rearranging yields:

$$\begin{aligned}
&\rho_{G,t}\left(\frac{G_N^2}{\kappa_G C_t} + \frac{G_N^{*2}}{\tau_{G,t}\kappa_G^* Q_t C_t^*}\right) \\
= \;& N_{D,t}\left(\frac{\alpha}{1-\alpha}\frac{f(\rho_{G,t})}{\rho_{G,t}}\right)^{1-\alpha}(\tilde{z}_D Z_t \rho_{H,t}(\tilde{z}_D))^{-\theta}(\tilde{z}_D Z_t)^{\theta-1} C_t + \\
& N_{X,t}\left(\frac{\alpha}{1-\alpha}\frac{f(\rho_{G,t})}{\rho_{G,t}}\right)^{1-\alpha}\tau_t(\tilde{z}_D Z_t \rho_{H,t}(\tilde{z}_D))^{-\theta}(\tilde{z}_{X,t} Z_t)^{\theta-1}\left[C_t + \left(\frac{\tau_t}{Q_t}\right)^{1-\theta} Q_t C_t^*\right] + \\
& N_{D,t}^*\left(\frac{\alpha}{1-\alpha}\frac{f(\rho_{G,t})/X_t}{\rho_{G,t}/(\tau_{G,t}Q_t)}\right)^{1-\alpha}\left(\tilde{z}_D Z_t^* \rho_{F,t}^*(\tilde{z}_D)\right)^{-\theta}(\tilde{z}_D Z_t^*)^{\theta-1} C_t^* + \\
& N_{X,t}^*\left(\frac{\alpha}{1-\alpha}\frac{f(\rho_{G,t})/X_t}{\rho_{G,t}/(\tau_{G,t}Q_t)}\right)^{1-\alpha}\tau_t^*\left(\tilde{z}_D Z_t^* \rho_{F,t}^*(\tilde{z}_D)\right)^{-\theta}\left(\tilde{z}_{X,t}^* Z_t^*\right)^{\theta-1}\left[\begin{array}{c} C_t^* + \\ (\tau_t^* Q_t)^{1-\theta}\frac{C_t}{Q_t}\end{array}\right]
\end{aligned} \tag{A14}$$

In this equation, we used the fact that $f^*(\rho_{G,t}) = f(\rho_{G,t})/X_t$, with:

$$X_t \equiv (\tau_{G,t} Q_t)^{-\frac{1+\alpha(\theta-1)}{(1-\alpha)(\theta-1)}} \left\{ \frac{\frac{Z_t^{\theta-1}\left\{N_{D,t}\tilde{z}_D^{\theta-1}C_t + N_{X,t}\tilde{z}_{X,t}^{\theta-1}\left[C_t + \left(\frac{\tau_t}{Q_t}\right)^{1-\theta} Q_t C_t^*\right]\right\}}{\left(\frac{\theta}{\theta-1}\right)^\theta \left(G_N L_{G,t} + \frac{G_{H,t}^*}{\tau_{G,t}}\right)}}{\frac{Z_t^{*\theta-1}\left\{N_{D,t}^*\tilde{z}_D^{\theta-1}C_t^* + N_{X,t}^*\tilde{z}_{X,t}^{*\theta-1}\left[C_t^* + (\tau_t^* Q_t)^{1-\theta}\frac{C_t}{Q_t}\right]\right\}}{\left(\frac{\theta}{\theta-1}\right)^\theta G_N^* L_{G,t}^*}} \right\}^{\frac{1}{(1-\alpha)(\theta-1)}}.$$

Notice that X_t does not depend directly on $\rho_{G,t}$.

The expressions for optimal $\rho_{H,t}(\tilde{z}_D)$ and $\rho_{F,t}^*(\tilde{z}_D)$ and tedious manipulation then make it possible to rewrite equation (A14) as:

$$\begin{aligned} & \rho_{G,t}\left[\frac{G_N}{\kappa_G C_t} + \left(\frac{G_N^*}{G_N}\right)\frac{G_N^*}{\tau_{G,t}\kappa_G^* Q_t C_t^*}\right] \\ = \; & \frac{\alpha}{G_N \rho_{G,t}}\left(\frac{\theta}{\theta-1}\right)^{-\theta}\left[\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\rho_{G,t}^\alpha f(\rho_{G,t})^{1-\alpha}}\right]^{\theta-1} \times \\ & \left\{ \begin{array}{l} N_{D,t}(\tilde{z}_D Z_t)^{\theta-1} C_t + N_{X,t}\tau_t(\tilde{z}_{X,t}Z_t)^{\theta-1}\left[C_t + \left(\frac{\tau_t}{Q_t}\right)^{1-\theta} Q_t C_t^*\right] + \\ \tau_{G,t} Q_t \left[(\tau_{G,t}Q_t)^\alpha X_t^{1-\alpha}\right]^{\theta-1}\left[\begin{array}{l} N_{D,t}^*(\tilde{z}_D Z_t^*)^{\theta-1} C_t^* + \\ N_{X,t}^*\tau_t^*(\tilde{z}_{X,t}^* Z_t^*)^{\theta-1}\left[C_t^* + (\tau_t^* Q_t)^{1-\theta}\frac{C_t}{Q_t}\right] \end{array}\right] \end{array} \right\} \end{aligned} \quad \text{(A15)}$$

Equation (A7) implies:

$$f(\rho_{G,t})^{-(1-\alpha)(\theta-1)} = \left(\frac{\alpha}{\rho_{G,t}}\right)^{-[1+\alpha(\theta-1)]}\left(\frac{\theta}{\theta-1}\right)^\theta \frac{G_N L_{G,t} + \frac{G_{H,t}^*}{\tau_{G,t}}}{N_{D,t} Z_t^{\theta-1} A_t}. \quad \text{(A16)}$$

where:

$$A_t \equiv \tilde{z}_D^{\theta-1} C_t + \tau_t (\nu z_{\min})^k \tilde{z}_{X,t}^{-[k-(\theta-1)]}\left[C_t + \left(\frac{\tau_t}{Q_t}\right)^{1-\theta} Q_t C_t^*\right]. \quad \text{(A17)}$$

In this expression, we used the relation between $N_{X,t}$ and $N_{D,t}$ implied by the assumption of a Pareto distribution of firm-specific productivity draws: $N_{X,t} = \left(\frac{\nu z_{\min}}{\tilde{z}_{X,t}}\right)^k N_{D,t}$.

It is also possible to verify that:

$$X_t^{(1-\alpha)(\theta-1)} = (\tau_{G,t} Q_t)^{-[1+\alpha(\theta-1)]}\left(\frac{N_{D,t} Z_t^{\theta-1} A_t}{G_N L_{G,t} + \frac{G_{H,t}^*}{\tau_{G,t}}}\right)\left(\frac{G_N^* L_{G,t}^*}{N_{D,t}^* Z_t^{*\theta-1} B_t}\right), \quad \text{(A18)}$$

where:

$$B_t \equiv \tilde{z}_D^{\theta-1} C_t^* + \tau_t^* \left(\nu z_{\min}\right)^k \tilde{z}_{X,t}^{*-[k-(\theta-1)]} \left[C_t^* + \left(\tau_t^* Q_t\right)^{1-\theta} \frac{C_t}{Q_t} \right]. \tag{A19}$$

Equation (A15) can be rewritten as:

$$\begin{aligned} & \rho_{G,t} \left[\frac{G_N}{\kappa_G C_t} + \left(\frac{G_N^*}{G_N} \right) \frac{G_N^*}{\tau_{G,t} \kappa_G^* Q_t C_t^*} \right] \\ = \; & \frac{\alpha}{G_N \rho_{G,t}} \left(\frac{\theta}{\theta-1} \right)^{-\theta} \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{\rho_{G,t}^\alpha} \right]^{\theta-1} f\left(\rho_{G,t}\right)^{-(1-\alpha)(\theta-1)} \\ & \times \left[N_{D,t} Z_t^{\theta-1} A_t + \left(\tau_{G,t} Q_t\right)^{1+\alpha(\theta-1)} X_t^{(1-\alpha)(\theta-1)} N_{D,t}^* Z_t^{*\theta-1} B_t \right]. \end{aligned} \tag{A20}$$

Then, substituting equations (A16)-(A19) into equation (A20) yields:

$$\rho_{G,t} \left[\frac{G_N}{\kappa_G C_t} + \left(\frac{G_N^*}{G_N} \right) \frac{G_N^*}{\tau_{G,t} \kappa_G^* Q_t C_t^*} \right] = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}}{G_N} \left(1 + \frac{G_N^* L_{G,t}^*}{G_N L_{G,t} + \frac{G_{H,t}^*}{\tau_{G,t}}} \right). \tag{A21}$$

Finally, using $L_{G,t} = \frac{G_N \rho_{G,t}}{\kappa_G C_t}$ and $L_{G,t}^* = \frac{G_N^* \rho_{G,t}}{\tau_{G,t} \kappa_G^* Q_t C_t^*}$ and rearranging gives us:

$$\begin{aligned} & \rho_{G,t} \frac{G_N}{\kappa_G C_t} \left[1 + \left(\frac{G_N^*}{G_N} \right)^2 \left(\frac{\kappa_G}{\kappa_G^* \tau_{G,t}} \right) \left(\frac{C_t}{Q_t C_t^*} \right) \right] \\ = \; & \frac{(1-\alpha)^{(1-\alpha)(\theta-1)}}{G_N} \left[1 + \left(\frac{G_N^*}{G_N} \right)^2 \left(\frac{\kappa_G}{\kappa_G^* \tau_{G,t}} \right) \left(\frac{C_t}{Q_t C_t^*} \right) \left(\frac{\rho_{G,t}}{\rho_{G,t} + \kappa_G \tau_{G,t}^{-1} G_N^{-2} C_t G_{H,t}^*} \right) \right] \end{aligned} \tag{A22}$$

Home imports of Foreign gas are given by:

$$G_{H,t}^* = G_N^* L_{G,t}^* - G_{F,t}^* = \frac{\rho_{G,t} G_N^{*2}}{\kappa_G^* \tau_{G,t} Q_t C_t^*} - G_{F,t}^*, \tag{A23}$$

where the second equality follows from using equation (A13).

Optimal input demands by Foreign final sector firms and the relation $N_{X,t}^* = \left(\frac{\nu z_{\min}}{\tilde{z}_{X,t}^*} \right)^k N_{D,t}^*$ imply:

$$G_{F,t}^* = N_{D,t}^* \left(\frac{\alpha}{1-\alpha} \frac{w_t^*}{\rho_{G,t}^*} \right)^{1-\alpha} \left[\frac{y_t^*\left(\tilde{z}_D\right)}{\tilde{z}_D Z_t^*} + \left(\frac{\nu z_{\min}}{\tilde{z}_{X,t}^*} \right)^k \tau_t^* \frac{y_t^*\left(\tilde{z}_{X,t}^*\right)}{\tilde{z}_{X,t}^* Z_t^*} \right]. \tag{A24}$$

Substituting market clearing conditions for Foreign final sector products and optimal price setting by Foreign firms into equation (A24) yields:

$$\begin{aligned} G^*_{F,t} &= \left(\frac{\theta-1}{\theta}\right)^{\theta} N^*_{D,t} \left(\frac{\alpha Q_t \tau_{G,t}}{\rho_{G,t}}\right)^{1+\alpha(\theta-1)} \left(\frac{1-\alpha}{w^*_t}\right)^{(1-\alpha)(\theta-1)} Z_t^{*\theta-1} \cdot \\ & \quad \tilde{z}_D^{\theta-1} C^*_t + \tau^*_t \left(\nu z_{\min}\right)^k \tilde{z}_{X,t}^{*-[k-(\theta-1)]} \left[C^*_t + \left(\tau^*_t Q_t\right)^{1-\theta} \frac{C_t}{Q_t}\right]. \end{aligned} \tag{A25}$$

Finally, substituting $N^*_{X,t} = \left(\frac{\nu z_{\min}}{\tilde{z}^*_{X,t}}\right)^k N^*_{D,t}$ and equation (A13) into equation (A8), and plugging the resulting expression for w^*_t into equation (A25) makes it possible to obtain:

$$G^*_{F,t} = (1-\alpha)^{(1-\alpha)(\theta-1)} \frac{G_N^{*2} \rho_{G,t}}{\tau_{G,t} \kappa^*_G Q_t C^*_t}. \tag{A26}$$

Equations (A23) and (A26) then imply:

$$G^*_{H,t} = \left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right] \frac{G_N^{*2} \rho_{G,t}}{\tau_{G,t} \kappa^*_G Q_t C^*_t}. \tag{A27}$$

This expression can be substituted into equation (A22). Then, defining $\xi_t \equiv \left(\frac{G^*_N}{G_N}\right)^2 \left(\frac{\kappa_G}{\kappa^*_G \tau_{G,t}}\right) \left(\frac{C_t}{Q_t C^*_t}\right)$ and rearranging the resulting equation, we have:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)} \kappa_G C_t}{G_N^2} \left\{ \frac{1+\xi_t+\tau_{G,t}^{-1}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t}{\left[1+\tau_{G,t}^{-1}\left[1-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]\xi_t\right](1+\xi_t)} \right\}. \tag{A28}$$

A.2. Gas Price and Gas Share

The gas price equation in the special case of complete markets, $\tau_{G,t} = 1$, $G_N = G^*_N$, and $\kappa_G = \kappa^*_G$ is reproduced below for your convenience:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)} \kappa_G C_t}{G_N^2} \left\{ \frac{3-(1-\alpha)^{(1-\alpha)(\theta-1)}}{2\left[2-(1-\alpha)^{(1-\alpha)(\theta-1)}\right]} \right\}. \tag{A29}$$

Let $\psi \equiv (1-\alpha)^{(1-\alpha)(\theta-1)}$. The derivative of ψ with respect to α is given by:

$$\psi_\alpha = -(\theta-1)(1-\alpha)^{(1-\alpha)(\theta-1)}[1+ln(1-\alpha)].$$

Now let $\Lambda_t \equiv \frac{\kappa_G C_t}{G_N^2}$. Then, equation (A29) can be rewritten as:

$$\rho_{G,t} = \psi \Lambda_t \left[\frac{3-\psi}{2(2-\psi)} \right]. \tag{A30}$$

Our interest is in determining how $\psi \left[\frac{3-\psi}{2(2-\psi)} \right]$ varies with α. Taking the derivative and rearranging yields:

$$\frac{\partial \psi \left[\frac{3-\psi}{2(2-\psi)} \right]}{\partial \alpha} = \frac{\psi_\alpha \left[2(3-2\psi) + \psi^2 \right]}{2(2-\psi)^2}.$$

The definition of ψ, $0 \leq \alpha \leq 1$, and $\theta > 1$ imply $3 > 2\psi$. Thus, the sign of the derivative we are interested in is determined by the sign of ψ_α. Since $\theta > 1$, the sign of ψ_α depends on the sign of $1 + \ln(1-\alpha)$. This expression is a monotonically decreasing function of α. It is positive if α is smaller than (approximately) 0.63. It is negative if α is higher than this number. It follows that $\psi \left[\frac{3-\psi}{2(2-\psi)} \right]$ is a monotonically decreasing function of α if $0 \leq \alpha \leq 0.63$, and it increases with α if $0.63 < \alpha \leq 1$. Since $\psi \left[\frac{3-\psi}{2(2-\psi)} \right] = 1$ when $\alpha = 0$ and $\alpha = 1$, the relation between the price of gas and its share in production of final goods when markets are complete, countries are fully symmetric, and there is no iceberg cost of gas trade is U-shaped.

A.3. The Log-Linear Gas Price Equation

The non-linear equation for the gas price $\rho_{G,t}$ is reproduced below for your convenience:

$$\rho_{G,t} = \frac{(1-\alpha)^{(1-\alpha)(\theta-1)} \kappa_G C_t}{G_N^2} \left\{ \frac{1 + \xi_t + \tau_{G,t}^{-1} \left[1 - (1-\alpha)^{(1-\alpha)(\theta-1)} \right] \xi_t}{\left[1 + \tau_{G,t}^{-1} \left[1 - (1-\alpha)^{(1-\alpha)(\theta-1)} \right] \xi_t \right] (1 + \xi_t)} \right\} \tag{A31}$$

where $\xi_t \equiv \left(\frac{G_N^*}{G_N} \right)^2 \left(\frac{\kappa_G}{\kappa_G^* \tau_{G,t}} \right) \left(\frac{C_t}{Q_t C_t^*} \right)$.

Taking logarithms and applying first-order Taylor approximation, the left hand side of the above equation yields

$$log \bar{\rho}_G + \frac{1}{\bar{\rho}_G} \left(\rho_{G,t} - \bar{\rho}_G \right) \equiv log \bar{\rho}_G + \hat{\rho}_{G,t}$$

where $\hat{\rho}_{G,t}$ is the percentage deviation of $\rho_{G,t}$ from the steady state.

Proceeding similarly, taking logs and applying first-order Taylor approximation, the right hand side of equation (A31) becomes

$$log\bar{\rho}_G + \mathsf{C}_t + \frac{\left(1+\tau_G^{-1}A\right)\left(1+\tau_G^{-1}A\xi\right)(1+\xi)\,\xi - \left(1+\xi+\tau_G^{-1}A\xi\right)\left(\tau_G^{-1}A(1+\xi)+(1+\tau_G^{-1}A\xi)\right)\xi}{\left((1+\tau_G^{-1}A\xi)(1+\xi)\right)^2}\hat{\xi}_t$$

where $A \equiv 1-(1-\alpha)^{(1-\alpha)(\theta-1)}$, C_t is the percentage deviation of C_t from the steady state: $\mathsf{C}_t \equiv \frac{dC_t}{C}$, d is the differentiation operator, and Sans Serif variables in equations below are defined similarly.

It is trivial to show that $\hat{\xi}_t = \mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}_t^*$. Thus;

$$\hat{\rho}_{G,t} = \mathsf{C}_t + [\Gamma_1 - \Gamma_2]\left(\mathsf{C}_t - \mathsf{Q}_t - \mathsf{C}_t^*\right) \tag{A32}$$

with

$$\Gamma_1 \equiv \frac{\left(1+\tau_G^{-1}A\right)\left(1+\tau_G^{-1}A\xi\right)(1+\xi)\,\xi}{\left(1+\xi+\tau_G^{-1}A\xi\right)(1+\xi)\left(1+\tau_G^{-1}A\xi\right)}$$

and

$$\Gamma_2 \equiv \frac{\left(\tau_G^{-1}A(1+\xi)+(1+\tau_G^{-1}A\xi)\right)\xi}{(1+\tau_G^{-1}A\xi)(1+\xi)}.$$

It follows that $\hat{\rho}_{G,t}$ is positively related with both C_t and $\mathsf{Q}_t + \mathsf{C}_t^*$ if and only if $-1 < \Gamma_1 - \Gamma_2 < 0$.

A.3.1. On Γ_1 and Γ_2

$\Gamma_1 - \Gamma_2 > -1$ if and only if:

$$-(1+\xi+\tau_G^{-1}A\xi) < \xi(1+\tau_G^{-1}A\xi) - \frac{\tau_G^{-1}A\xi(1+\xi+\tau_G^{-1}A\xi)}{1+\tau_G^{-1}A\xi} - \frac{\xi(1+\xi+\tau_G^{-1}A\xi)}{1+\xi}$$

Tedious algebra shows that this inequality implies:

$$-1 < \xi\left[1-\left(1+\xi+\tau_G^{-1}A\xi\right)\left[1+2\tau_G^{-1}A\xi\right]-\left(\tau_G^{-1}A\right)^2\xi\right].$$

The term in the squared brackets is negative and scales ξ with less than unity for the parameterization in the paper. $\Gamma_1 - \Gamma_2 < 0$ if and only if:

$$\left(\xi\tau_G^{-1}A\right)^2 + \left(1+\xi+\tau_G^{-1}A\xi\right)\left[1+2\tau_G^{-1}A\xi\right] > 0$$

which is ensured for $A, \xi, \tau_G > 0$.

A.4. The Log-Linear Real Exchange Rate Equation

The non-linear equation for the data-consistent real exchange rate $\tilde{Q}_t$ is reproduced below for your convenience:

$$\tilde{Q}_t^{1-\theta} = \frac{\frac{N^*_{D,t}}{N^*_t}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z^*_t}\right)^{\alpha}\frac{\tilde{z}_D}{\tilde{z}^*_D}\right]^{1-\theta} + \frac{N_{X,t}}{N^*_t}\left[\frac{\tau\tilde{z}_D}{\tilde{z}_{X,t}}\right]^{1-\theta}}{\frac{N_{D,t}}{N_t} + \frac{N^*_{X,t}}{N_t}\left[TOL_t^{1-\alpha}\left(\frac{Z_t}{\tau_{G,t}Z^*_t}\right)^{\alpha}\frac{\tau^*\tilde{z}_D}{\tilde{z}^*_{X,t}}\right]^{1-\theta}}. \tag{A33}$$

Let NUM_t denote the numerator of this equation and DEN_t the denominator. Then, the log-linear version of equation (A33) can be written as:

$$\tilde{\mathsf{Q}}_t = \frac{\overline{NUM}\cdot dDEN_t - \overline{DEN}\cdot dNUM_t}{(\theta-1)\overline{NUMDEN}} \tag{A34}$$

Assume $\bar{Z} = \bar{Z}^* = 1$. Differentiating NUM_t and using the definitions of log-linearized variables yields:

$$\begin{aligned} dNUM_t = \; & \frac{\bar{N}^*_D}{\bar{N}^*}\left(\mathsf{N}^*_{\mathsf{D,t}} - \mathsf{N}^*_{\mathsf{t}}\right)\left(\overline{TOL}^{1-\alpha}\overline{\tau}_G^{-\alpha}\right)^{1-\theta} \\ & +(1-\theta)\frac{\bar{N}^*_D}{\bar{N}^*}\left[\mathsf{TOL_t}\overline{TOL}^{1-\alpha}\overline{\tau}_G^{-\alpha} + \alpha\overline{TOL}^{1-\alpha}\overline{\tau}_G^{-\alpha}(\mathsf{Z}_t - \mathsf{Z}^*_t - \tau_{G,t})\right](\overline{TOL}^{1-\alpha}\overline{\tau}_G^{-\alpha})^{-\theta} \\ & +\frac{\bar{N}_X}{\bar{N}^*}\left(\mathsf{N_{X,t}} - \mathsf{N}^*_{\mathsf{t}}\right)\left(\frac{\overline{\tau}\tilde{z}_D}{\overline{\tilde{z}}_X}\right)^{1-\theta} + \frac{\bar{N}_X}{\bar{N}^*}\frac{\overline{\tau}\tilde{z}_D}{\overline{\tilde{z}}_X}(\tau_t - \overline{\tilde{z}}_{X,t})\left(\frac{\overline{\tau}\tilde{z}_D}{\overline{\tilde{z}}_X}\right)^{-\theta}, \end{aligned}$$

or, after rearranging:

$$\begin{aligned} dNUM_t = \; & \frac{\bar{N}^*_D}{\bar{N}^*}\left(\overline{TOL}^{1-\alpha}\overline{\tau}_G^{-\alpha}\right)^{1-\theta}\left\{\mathsf{N}^*_{\mathsf{D,t}} - \mathsf{N}^*_{\mathsf{t}} + (1-\theta)[(1-\alpha)\mathsf{TOL_t} + \alpha(\mathsf{Z}_t - \mathsf{Z}^*_t - \tau_{G,t})]\right\} \\ & +\frac{\bar{N}_X}{\bar{N}^*}\left(\frac{\overline{\tau}\tilde{z}_D}{\overline{\tilde{z}}_X}\right)^{1-\theta}\left(\mathsf{N_{X,t}} - \mathsf{N}^*_{\mathsf{t}} + \tau_t - \overline{\tilde{z}}_{X,t}\right). \end{aligned} \tag{A35}$$

Proceeding similarly with DEN_t yields:

$$\begin{aligned} dDEN_t = \; & \frac{\bar{N}_D}{\bar{N}}\left(\mathsf{N_{D,t}} - \mathsf{N_t}\right) \\ & +\frac{\bar{N}^*_X}{\bar{N}}\left(\overline{TOL}^{1-\alpha}\overline{\tau}_G^{-\alpha}\frac{\overline{\tau}^*\tilde{z}_D}{\overline{\tilde{z}}^*_X}\right)^{1-\theta}\left\{\mathsf{N}^*_{\mathsf{X,t}} - \mathsf{N_t} + (1-\theta)\begin{bmatrix} -\overline{\tilde{z}}^*_{X,t} + (1-\alpha)\mathsf{TOL_t} \\ +\alpha(\mathsf{Z}_t - \mathsf{Z}^*_t - \tau_{G,t}) \end{bmatrix}\right\}. \end{aligned} \tag{A36}$$

Let the parameters, χ_1, χ_2, and γ be defined implicitly by: $\frac{\bar{N}^*_D}{\bar{N}^*} = \chi_1\frac{\bar{N}_D}{\bar{N}}$, $\frac{\bar{N}_X}{\bar{N}^*} = \gamma\chi_1\frac{\bar{N}^*_X}{\bar{N}}$, and

$\left(\frac{\tau \tilde{z}_D}{\bar{\tilde{z}}_X}\right)^{1-\theta} = \left(\chi_2 \frac{\tau^* \tilde{z}_D}{\bar{\tilde{z}}_X^*}\right)^{1-\theta}$. Then, equation (A35) can be written as:

$$\begin{aligned} dNUM_t = \;& \chi_1 \tfrac{\bar{N}_D}{\bar{N}} \left(\overline{TOL}^{1-\alpha} \bar{\tau}_G^{-\alpha}\right)^{1-\theta} \left\{\mathsf{N}^*_{\mathsf{D,t}} - \mathsf{N}^*_{\mathsf{t}} + (1-\theta)[(1-\alpha)\mathsf{TOL}_{\mathsf{t}} + \alpha(\mathsf{Z}_t - \mathsf{Z}^*_t - \tau_{G,t})]\right\} \\ & + \gamma\chi_1 \tfrac{\bar{N}^*_X}{\bar{N}} \left(\chi_2 \tfrac{\tau^* \tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{1-\theta} \left(\mathsf{N}_{\mathsf{X,t}} - \mathsf{N}^*_{\mathsf{t}} + \tau_t - \bar{\tilde{z}}_{X,t}\right). \end{aligned} \tag{A37}$$

Substituting equations (A36) and (A37) and the expressions for $\overline{NUM}$ and $\overline{DEN}$ into $\overline{NUM} \cdot dDEN_t - \overline{DEN} \cdot dNUM_t$ (the numerator of the expression for $\tilde{\mathsf{Q}}_t$ in equation A34), and rearranging yields:

$$\begin{aligned} & \overline{NUM} \cdot dDEN_t - \overline{DEN} \cdot dNUM_t \\ & = (\theta-1)(\Phi_1 - \Phi_2)[(1-\alpha)\mathsf{TOL}_t + \alpha(\mathsf{Z}_t - \mathsf{Z}^*_t - \tau_{G,t})] \\ & \quad + (\theta-1)(\Phi_2 + \Phi_4)\tilde{z}^*_{X,t} - (\Phi_2 + \Phi_3)(\tilde{z}_{X,t} - \tau_t) \\ & \quad + \Phi_1[\mathsf{N}_{\mathsf{D,t}} - \mathsf{N}_{\mathsf{t}} - (\mathsf{N}^*_{\mathsf{D,t}} - \mathsf{N}^*_{\mathsf{t}})] + \Phi_2[\mathsf{N}^*_{\mathsf{X,t}} - \mathsf{N}_{\mathsf{t}} - (\mathsf{N}_{\mathsf{X,t}} - \mathsf{N}^*_{\mathsf{t}})] \\ & \quad - \Phi_3[\mathsf{N}_{\mathsf{X,t}} - \mathsf{N}^*_{\mathsf{t}} - (\mathsf{N}_{\mathsf{D,t}} - \mathsf{N}_{\mathsf{t}})] + \Phi_4[\mathsf{N}^*_{\mathsf{X,t}} - \mathsf{N}_{\mathsf{t}} - (\mathsf{N}^*_{\mathsf{D,t}} - \mathsf{N}^*_{\mathsf{t}})] \end{aligned} \tag{A38}$$

where

$$\begin{aligned} \Phi_1 &\equiv \chi_1 \left(\frac{\bar{N}_D}{\bar{N}}\right)^2 \left(\overline{TOL}^{1-\alpha} \bar{\tau}_G^{-\alpha}\right)^{1-\theta} > 0, \\ \Phi_2 &\equiv \gamma\chi_1 \left(\frac{\bar{N}^*_X}{\bar{N}}\right)^2 \left(\overline{TOL}^{1-\alpha} \bar{\tau}_G^{-\alpha} \chi_2\right)^{1-\theta} \left(\frac{\tau^* \tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{2(1-\theta)} > 0, \\ \Phi_3 &\equiv \gamma\chi_1 \frac{\bar{N}_D \bar{N}^*_X}{\bar{N}^2} \left(\chi_2 \frac{\tau^* \tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{1-\theta} > 0, \\ \Phi_4 &\equiv \chi_1 \frac{\bar{N}_D \bar{N}^*_X}{\bar{N}^2} \left(\frac{\tau^* \tilde{z}_D}{\bar{\tilde{z}}^*_X}\right)^{1-\theta} \left(\overline{TOL}^{1-\alpha} \bar{\tau}_G^{-\alpha}\right)^{2(1-\theta)} > 0. \end{aligned}$$