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PROJECT EVALUATION: THE RETURN ON CAPITAL

by Benjamin B. King

1. Introduction

To every investment of capital there corresponds a subsequent stream of receipts from the goods or services produced and of expenses for labor, materials and so forth associated with production. The investor looks to the difference between these, which we may call the gross returns, for the recovery of his initial capital plus what we may term a net return on his investment.

These terms are not without ambiguity. In this session, we shall try to strip some of the ambiguities away and arrive at a consistent and useful meaning for the net* return on capital. We shall also try to see how useful the idea is in evaluating an investment, in particular in deciding which of two alternative investments to make.

The return on his capital is the principal criterion, though not necessarily the only one, by which a private investor judges an investment. In the context of the whole economy, there may be other factors of social significance to be taken into account. We shall not be concerned with these for the time being, but will try to graft them on in subsequent sessions.

The outline that follows is divided into three parts: first, a discussion of what the rate of return is; secondly, some problems arising particularly in connection with physical investments; and finally, a discussion of the rate of return as a criterion. Three appendices are added. One is a brief note on the effects inflation may have on the calculations discussed. The second is a comparison with cost-benefit analysis for those who may be acquainted with this type of evaluation. The third is a note on uncertainty and risk premiums.

This paper was designed as a basis for discussion of Project Evaluation in seminars at the Economic Development Institute. It is not designed to cover all aspects of the problem, nor is it in any sense an expression of official Bank policy. It is essential that it be used in connection with John G. McLean, How to Evaluate New Capital Investments, Harvard Business Review, November-December 1958, which is reproduced in this library in Investment Criteria and Project Appraisal, Articles and Papers, Volume 3.

* In future, return on capital or investment will mean net return, unless qualified by the word "gross".

2. The "internal" rate of return

The rate of return on financial investments

In Table 1 are listed three types of security, each having a life of five years and each yielding 6%. The first is an annuity; the investor's initial outlay is $1,000 and he receives a gross return of $237 each year for five years. The second is an ordinary bond with a face-value of $1,000 and an interest rate of 8%; the investor receives $80 a year for five years plus the face-value of $1,000 at the end of the fifth year; in the current market he pays a premium of $84 for this bond. The third is a savings bond, for which the investor pays $747; in return he receives $1,000 at the end of the five years, but nothing in between.

The table has been set out in such a way as to reveal the characteristic common to the three. Let us take the annuity as an example. In the first year the investor receives $237, of which $60 constitutes the return of 6% on his capital of $1,000 and the balance ($177) is the repayment of capital in that year. The next year his outstanding investment is only $823, the net return is correspondingly less and the repayment of capital correspondingly more. At the end of the fifth year, the capital outstanding is exactly zero.

At 6%, the same thing happens with the other two, although the savings bond appears as rather an oddity. In effect, the investor at the same time receives his interest or net return and puts it back into the security. The common characteristic of the three securities is this: if each year we subtract from the gross return a net return of 6% on the outstanding capital, the annual repayment of capital (which may be negative in some years) reduces the investment to zero at the end of the period of five years; the investor's stake in his investment changes from year to year, but his return is always six per cent on the stake he still has in it. 6% is the "internal" rate of return.

We may put the same point slightly differently. If in the first case, the investor borrows 1000 and, each year, uses the gross returns to pay interest at 6% on the debt outstanding and to reduce the debt, he will extinguish the debt exactly at the end of 5 years. The same is true in the other two cases, except that, in the third, he continues to borrow.

The characteristic which distinguishes the three is the way in which the outstanding capital (or debt, if we think of it in those terms) changes. In the first case it falls steadily, though not by the same amount each year. In the second it falls slowly with a "balloon" in the fifth year. In the third it increases until it is wiped out abruptly at the end.

As between the three securities, there is nothing to choose as regards the rate of return. But all other things may not be equal. In particular, the pattern of recovery of capital may become important, unless the investor can be sure of reinvesting at the same rate — no more, no less. But, of this, more later.

For the moment, we may turn to the question how the rate of return is determined.

Discounting

With the aid of the right kind of tables, an example of which is shown in Table II, it is a simple though laborious matter to find the appropriate rate. For any given year and any given rate, there is a discount factor. For example, for three years and 6%, it is roughly 0.8396. This means that 83.96 will grow to 100 in three years at compound interest at 6%. If we can earn 6%, then the present value (or worth) of 100 three years hence is 83.96.

If each of the annual gross returns is multiplied by the appropriate discount factor at a particular rate of interest and the products are added, the total is the value of these returns discounted to the present time. If this "present value" is less than the original investment, the rate chosen is too high; if it is more, the rate is too low. A certain amount of trial and error may be necessary to get the right rate.* There are numerous examples in McLean's article and it is not necessary to repeat them here.

If there is a uniform return for a number of years (e.g. Case A and the 280 p.a. in Case B), there is a short cut. One can use the table of present values of annuities (see Table III). For example, the present value of 237 p.a. for five years at 6% is 237 x 4.212 = 1000.

The rate of return on physical investments

There is no real difference in principle between a financial investment in a security and a physical investment in a machine. Each requires a capital outlay at the beginning and yields a series of gross returns. The problem of calculating the rate of return is the same for each, although certain practical problems may intervene in the case of physical investments which do not apply or at least do not apply so critically to financial ones. One of the differences is that, whereas financial investments are made at a single point of time and the returns come in on certain dates, both physical investment and the subsequent returns are spread over time. As this fact makes little difference in principle (though some in practice), we shall ignore it.

The fact that the problem is fundamentally the same for financial and physical investments is liable to be beclouded by the conventional allocation of depreciation, which is simply the recovery of capital spread over the life of the machine in a predetermined way. One way - but, by no means,

* If the right rate is not a round figure or fraction given in the tables, one will get two present values (at, say, 7 and 7½%) which "bracket" the original investment. It is then necessary to interpolate.

the only way - is straight-line depreciation, i.e. an equal amount each year. It is instructive to see what happens, if we do this for a machine costing $1,000 and yielding gross returns of $237 for five years (i.e. comparable to Case A in Table I). The following shows the annual net return on this basis in absolute figures and as a percentage of outstanding capital:

	Year 1	Year 2	Year 3	Year 4	Year 5
Capital Outstanding	1000	800	600	400	200
Gross Return	237	237	237	237	237
Depreciation	200	200	200	200	200
Net Return	37	37	37	37	37
% Return	3.7	4.6	6.2	9.3	18.5

Plainly, this does not give a uniform rate of return. The only way of arriving at a uniform rate of return in this case is to allocate depreciation or recovery of capital as in Case A of Table I. Any predetermined method of allocating depreciation is likely to give variable and meaningless rates. Thus, we can say that the discounted cash-flow method of calculating the rate of return enables us to allocate "depreciation" so as to achieve a uniform rate of return on the outstanding investment.*

The element of time

It is clear that the discounted cash-flow method gives a different answer from the average return on initial investment or on average investment. But does this mean that it is in any way superior?

First of all, let us take the example of the annuity quoted above. The average return on the initial investment of 1000 would be 37 or 3.7%; as calculated on the average investment of 500 it would be just double that, namely, 7.4%. These figures are different from the discounted cash-flow figure of 6%, but we may ask whether in choosing between two different projects, one method would give a different answer from the other.

In the case of level annual gross returns, there is some difference but not too much. Table IV illustrates this. If we consider three investments with lives of 2, 25 and 50 years for all of which the average income divided by initial investment is 3.8%, we see that by the discounted cash-flow method the 2-year and the 50-year investment yields 5%, but the 25-year one yields 6%. In other words, there is some difference.

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* Commercial experience in a particular industry over a period of time may, of course, suggest to entrepreneurs a pattern of depreciation for plant and equipment, which is a close rule-of-thumb approximation to the discounted-cash-flow pattern.

Nevertheless, if we are choosing between two investments of roughly the same life and we are using the rate of return as a criterion of choice, the two methods will, broadly speaking, indicate the same investment as the one to be preferred (although the figures will be different). This statement is subject to the very important proviso that we are still talking of investments of the annuity type with level annual returns.

It is when the returns are not level, but going up or going down, that quite different results can be obtained. Exhibit III in McLean's article gives an instance of this. The reason is that calculating the average return (whether on the basis of the initial or the average investment) takes no account of time. Time is an essential element. The quicker the returns come in, the sooner they can be reinvested to create additional income.

3. Special aspects of physical investments

Life of a project

The life of a financial investment is usually determined quite easily. In the three cases in Table I, there would not be much quarrel that the life of each investment is five years. Physical investments cause more trouble. A physical investment may consist of land (including artificially created land, such as cuttings and reclamations), which has a virtually perpetual life; buildings with a comparatively long life; and machines with a comparatively short life.

How are we to proceed? There is no rigid rule and one must exercise judgment. Let us take, first, the case of an engineering plant with machines lasting 15 years in a factory lasting 30 years. The market may be difficult to foresee 15 years ahead, but certainly much more so thereafter. It might be sensible in this case to take a life of 15 years for the investment. At the end of 15 years, the factory building would have a sale value, which would be part of the gross returns in the fifteenth year.

In contrast to this, let us take a hydroelectric plant, the dam lasting 60 years, but the generators and turbines only 20. In this case we would reasonably expect the plant to continue producing electricity for the whole 60 years. It would be unrealistic to think of selling the dam after 20 years - except for the purpose of producing electricity. We should think, therefore, of a 60-year project with investments not only at the beginning but also in the 20th and 40th year. We come to this question of renewals in the next section.

It is worth noting that, when we talk of physical life, this is not necessarily the same as economic life. This should, of course, come out "in the wash", because, as soon as economic returns cease, the physical condition of the remaining plant becomes irrelevant (except for sale purposes). The principal point to bear in mind is that obsolescence can shorten the economic life of a machine.

Renewals

In principle, it is not difficult to deal with the problem of renewals which have to be made at periodic intervals during the life of the project. When there is no subsequent investment, we will have at the appropriate rate:

Investment = Future returns discounted

When there is future investment, then we will have:

Present investment plus discounted future investments = future returns discounted.

An example is shown in Table V. It is similar to the case of the hydroelectric plant mentioned above except that we have foreshortened the life to 6 years instead of 60 in order to make the example manageable. The first part of the table shows the calculation with the (correct) discount factors of 10% for both sides of the equation. The second part checks the result in the same way as in Table I.

An interesting aspect of this example is the fact that we could just as well subtract the renewal expenditures from the gross returns in the 2nd and 4th years as though they were ordinary current expenditure.* The returns would then be:

1st year	2nd year	3rd year	4th year	5th year	6th year
100	10	100	10	100	100

Discounting this set of returns at 10% will, as shown in the table, give a figure of 300. This is an instance of the fact that the distinction between capital and current expenditure is sometimes a semantic one.

Supplementary investments

It may happen that an investment is undertaken at one time, which makes possible a supplementary investment later. An obvious instance is that of a large hydroelectric dam, controlling a river with variable flow. The existence of this storage dam may make possible a relatively low-cost hydroelectric plant further down the river, but building the second has to wait for demand for electricity to increase sufficiently. How, then should we proceed?

Let us take a numerical example, again foreshortened to a period of 6 years. An original investment of 400 is expected to give gross returns of 79 per annum for six years. It also makes possible a supplementary investment of 400 yielding gross returns of 126 from the third to sixth years.

The details of these investments are set out in Table VI. The original investment turns out to yield a net return of only 5%, while the supplementary investment yields 10%. It is clear that, even if we consider the return on the original investment inadequate, the probability that the supplementary one will go forward later would help to justify it. We should, at least, consider the combination of the two. The yield of the combined investment is 7%, somewhat less than the average of the two. This is because the more advantageous investment and the returns from it occur, on the whole, further off in time than the less advantageous one.

* i.e., the above equation can be written: Present Investment = discounted future returns minus investments.

Uncertainty *

All investments, whether financial or physical, are subject to some measure of uncertainty. One can only predict the gross returns as best one can in the knowledge that there will be a margin of error on either side. Thus, in effect, an investment may have the expectation of yielding not a single return but a range of returns. Whether one range is somehow preferable to another is not a question which can be answered by rigid rules. Would one prefer, for example, a range of 0% to 25% to one of 6% to 10%? The first promises the whole gamut from no return at all to rich rewards. The second is a conservative kind of investment with the average expectation considerably lower than the gamble. The answer depends on circumstances and taste.

We can, however, make the observation that the greatest uncertainties are usually those most distant in time. It is a happy conjunction that these uncertainties matter least, because they are discounted most heavily. In Table VIII we give an example of this. We take the case of an investment, costing 1,211 which is expected to yield gross returns of 100 for 45 years. This is equivalent to 8%.

Suppose that the returns turn out to be lower during a particular period of 15 years, reducing the rate of return to 7½%. How much lower will they be, if the period is (a) the first, (b) the middle or (c) the last fifteen years? The answer shows a striking difference. A reduction of revenue from 100 to 92 (i.e. by 8%) in the first 15 years will reduce the overall rate of return from 8% to 7½%. But it takes a reduction from 100 to 30 (i.e. by 70%) in the last 15 years to give the same result. The explanation, ofcourse, lies in the much lower discount factor. Distant losses are less costly.

Expenses already incurred.

Sometimes a decision will have to be made whether to proceed with a project on which some expense has already been incurred. The expense may be preparatory work necessary to the making of a decision or the project may have been started and a promising alternative has come to light. Should these past expenses be included in deciding on the rate of return?

It is important to realize that it is only the future expenses (investment and current) which are relevant to the decision to go ahead or not. One cannot undo the past expenses (except to the extent that they can be recovered by sale of any assets created); whether one goes on with the project or not, they will have to be borne. Consequently in making a decision on incurring new expenses, past expenses must be ignored.

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* See also Appendix III.

4. The rate of return as a criterion

Suppose that we have a collection of possible projects and we line them up in order of the rate of return that each yields, as in the table on the next page. To what extent can we use this as a criterion of priority? First of all, let us consider the possible limitations, which make it necessary to cut off one's investment at a particular point. The limitation might be:

(i) The rate at which one can borrow. If this is 4%, say, one would clearly undertake A to H, but no more.
(ii) A capital budget of, say, 8000. Then the cut-off point would appear to come at project J, which would (near enough) employ all one's capital. In other words, the cut-off rate of return would be 4%. Some in this group are included; some not.
(iii) The counter pull of other uses, e.g. consumption. The investor may feel that, after investing 8000, he needs more than a 4% return to justify any further investment of his own capital.

To a considerable extent, all three amount to the same thing, though with somewhat different emphasis. A balance is somehow reached between the amount to be invested and the "marginal" rate at which this capital is invested. In this paper, however, we shall consider only the second approach. We have a capital budget of 8000; how best can it be invested?

Given this approach, is it reasonable to suppose that the answer suggested above under (ii) is the right one, that we undertake projects A to J? It is, provided that the cut-off point in the future is also likely to be at about 4%. We shall henceforward assume that this is so.*

We may then ask ourselves whether the order A to J represents the order of priority of these projects. This is, of course, an academic question, if one in any case intends to undertake all of them. But, suppose some of these projects are mutually exclusive alternatives; we can undertake D or G, but not both. Which is to be preferred? Is D necessarily better than G? It is not, as we shall see in the next section. We shall also see that the "marginal" rate of return expected in the future (in our example, 4%) plays an important part in the choice.

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* It would complicate the issue considerably to assume anything else, although it would be possible to make the necessary adjustments. As, in many cases, both the present and the future cut-off points are likely to be extremely vague, the extra complication seems hardly worthwhile.

List of available projects

		Rate of return	Capital required	
Yielding over 4%				
Project	A	20%	600	
"	B	17%	300	
"	C	12%	800	
"	D	8%	1000	
"	E	8%	527	
"	F	8%	534	
"	G	6½%	1000	
"	H	5%	1400	6161
Yielding 4%				
Project	I	4%	1200	
"	J	4%	640	
"	K	4%	500	
"	L	4%	500	
"	M	4%	530	3370
Yielding under 4%				
Project	N.	3%	500	
	etc.	etc.	etc.	

The marginal rate of return and the choice between alternatives.

In Table IX, we give one additional piece of information on projects D and G, namely the life of the project. D is a three-year project and G a six-year one. Does this affect our choice?

Suppose that, as the gross returns come in, they are reinvested in full*, and that the possible return on the reinvestment (at the margin) is 4%. Then we can see, from section 2 of the table, that the amount of the reinvestment accumulated over 6 years (the same period for both) is larger for G (the 6½% project).

In order to see which is the better of two projects in the circumstances we have described, it is not necessary to make up a long table such as we have under IX (2). We can discount both sets of returns at the prevailing rate (here 4%) and see which is the higher, as is done in IX (3). This gives us the "present value" of the returns in each case.** The "bonus" for doing G is greater than that for doing D.

The reason for the apparent anomaly that the project with a higher return is inferior is not hard to find. Both projects give a considerably better net return than the general run of alternatives. But, while D on the one hand has a greater advantage as long as it lasts, G on the other hand lasts longer. It is the hare and the tortoise over again.

We may ask whether, in real life, there do exist mutually exclusive alternatives with different economic lives. One category would be the exploitation of a natural asset such as a mineral deposit or a forest. If it is a particularly rich asset, it may (or may not) pay to use it up slowly, even though the rate of return is less.

In Table X we have a second example in which the capital cost of two projects (E and G) is different, but the life is the same. E is a smaller project than G and yields a higher return. An example of this might be a dam for hydroelectric power or irrigation, whose capacity can be extended, but the extension is more costly than the original structure in terms of the yield in power or water. This suggests the fact that what one is really considering is the return on the addition. In cases where the addition is a concrete physical structure, this may be obvious. But in others, where we are comparing two quite different ways of utilizing the same resource, it may not be so clear.

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* It makes no difference, in fact, if a part is withheld for, say, consumption, as long as it is the same for both projects.

** At compound interest at 4%, the present value under (3) will become in six years the ultimate figure under (2). For example, 1077 x 1.265 (the compound interest factor) = 1362.

In Table X, we see that the difference yields 5%. This is better than the general run of alternatives, which we have assumed yield only 4%. So G is preferred to E in spite of the lower overall yield. Again the reason is not hard to find. The capital saved by undertaking E cannot be invested as well as in G. One fairly good project may — or may not — be better than one very good and one average project. As before, we can come to the same answer by discounting the future returns of the two projects.

In Table XI there is a third example. Here we compare two projects with the same annual gross return, but project F lasts for only 3 years instead of 6. The initial expenditure on F is much lower, but it must be renewed after three years. We might, for example, compare temporary housing with more permanent structures.

As before, F earns 8% as against G's 6½%. But comparing the two accumulated totals after 6 years, F is not as good as G. Here again, we can take a short cut. Since the returns are the same, it is only necessary to compare the two capital investments. Under XI (3) we see that investment in F (discounted at 4%) is more than that in G. To put it another way, it is not worth saving 466 in capital now, if the available investments will not be sufficiently rewarding to build it up to 534 in three years' time.

Joint Costs:

A problem may arise where a project has more than one purpose. The typical case is a dam which can be used to provide water for irrigation and electric power. Of all the works carried out in such a project, some are exclusively needed for irrigation, some exclusively for power and some are not identifiable with either purpose. The cost of the latter is a joint cost.

In Table XIII we give an example of such a case. We have made an unrealistic assumption, that all the gross returns come in in one year. This is done to simplify exposition; it has no bearing on the principle involved.

In this example, irrigation and hydro power would each have a capital cost of 100, if the project were built for one purpose alone. The returns would be very low. But if a multipurpose project were built, there would be joint costs of 25, so that the total cost would be only 175. The returns on the whole project would be much higher, in this example 17%. Clearly, it is advantageous to build the joint project, as long as one's marginal rate of return is within reason.

But now let us introduce a complication. There exists an alternative to hydroelectric power, thermal power. The rate of return on this is very high - 29%. But the real alternatives are not thermal power versus hydro power (D and B); they are thermal power versus hydro power plus irrigation (D and C).

The question we now have to ask is whether the extra cost of providing irrigation is worthwhile. This extra cost and the returns on it are shown on the line marked (C-D). The rate of return is 7%, which may or may not be good enough, depending on what the marginal rate of return is. If it is 6%, the multipurpose project should be undertaken; if it is 10%, then thermal power and no irrigation.

But let us now postulate a slightly different situation. A government, say, wishes to irrigate, regardless of the cost. Whether this decision is right or wrong, it is a fact of life. Then we have a choice between adding hydro instalations to the dam we are building anyway (C - A) or a thermal station (D). Here we see that the capital cost of adding the hydro installations to the dam being built anyway is only 75 compared with 80 for thermal power. Hydro power is always cheaper.

Very often problems involving joint costs are more complicated than this. Making a sensible decision depends on putting the right questions. What one should not do (though it is done) is to allocate the joint costs in some arbitrary way to the different purposes to be fulfilled.

Conclusion

It is, of course, possible to multiply examples of this kind ad infinitum. It is, perhaps, worth repeating that we have assumed that the "marginal" opportunity for investment is the same today and at any time in the future. If this is not the expectation, the problem becomes that much more complicated. In fact, it is difficult enough to say what the "marginal" rate of return is. In the case of the private investor, there may be certain market criteria. But for a whole economy, the problem is a great deal harder.

It may be useful, in some circumstances, to turn the question upside down. If we are faced with a choice between two alternatives, at what rate of discount does there appear to be nothing to choose between them? If the rate is implausibly high (or low), we should incline towards that alternative which appears better at a lower rate (or, in the opposite case, higher rate). If there is no a priori reason to feel that the rate is high or low, then the return on capital is not a decisive criterion and other considerations may have more weight.

Appendix I

Inflation

What difference does the prospect of inflation make to calculations of the return on a project? This can become a very complicated question and we shall only allude to a few aspects here. The first answer to the question is that a general inflation makes no difference, but this is subject to qualifications. By a general inflation we mean one which affects all costs and prices equally.*

If inflation is proceeding at a rate of 10% per annum, the returns the investor gets and the proceeds of reinvestment will have a much higher money value than before, but the same real value. It is not easy to demonstrate this simply. In Table XII we have taken a highly oversimplified example; the principle involved can, however, be extended more generally.

In the first case, without inflation, the investor invests 100 and obtains 110 after two years. This is a return of 5%. In the second case, with inflation at 10%, the money value of the returns goes up by 21%, though the real value remains at 110. In real terms the return is just the same.

There is one major qualification that we must make to this. Inflation usually affects prices in different ways. They range from the very "sticky" to the very volatile. Thus, the returns tend to be higher in those projects where the product's price is volatile and the costs are sticky and to be lower where the position is the other way around. This is really what we mean by inflation "distorting" the economy. General inflation which affects everything equally does not distort in this way, though it may create other difficulties, if indeed it ever exists in practice.

There is one important cost that can be sticky. It is foreign exchange, since exchange rates often lag behind the inflation. Foreign equipment and materials bought at an overvalued exchange rate can make a bad project look good. Conversely a good project providing something for export can appear bad.

Furthermore, in inflation the rate of return in terms of current money is very high. So, if a foreign loan is obtained for a project at a moderate rate of interest and the interest and principal are paid off at an overvalued exchange rate, a very bad project can appear a very good one under inflation.

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* This does not mean that they all go up at an equal rate. Some would go up or down, relative to each other, without any inflation.

Appendix II

Relationship to cost-benefit analysis

It may be useful to those who have previously been acquainted with cost-benefit analysis to point out the relationship between this type of analysis and that discussed in the outline. The term cost-benefit analysis covers a variety of approaches to project evaluation, but here we shall only mention those most closely relevant.

Essentially, the method consists in taking all the costs and all the benefits in a project and comparing the two in some way. Let us take first the example in Table IX. We could, as in this example, discount the benefits (gross returns) and the costs (investment) and obtain a ratio of the two for each product. In this case the ratios can be deduced directly from IX (3):

	Project D	Project G
Benefits	1077	1085
Costs	1000	1000
Ratio	1.077	1.085

However, it is more usual in cost-benefit analysis not to "net" out the current operating costs, but to include both current and capital costs on the cost side and total output on the benefit side. Suppose that in the same example (a) the operating costs of D were 388 per year and the output 736 (the difference being 388) and (b) the operating costs of G were 414 per year and output 621 (the difference being 207). The costs and benefits would be as follows:

	Project D	Project G
Benefits	2 x 1077 = 2154	3 x 1085 = 3255
Costs	1000 + 1077 = 2077	1000 + 2170 = 3170
Ratio	1.037	1.027

In other words, we get not only a different answer, but a different order of preference. D now looks better than G. As long as the costs in each represent true costs to an investor, it is difficult to see why one should prefer D on the grounds that it has, in effect, a higher "bonus" per unit of output. The investor is really concerned with the return on his capital. It is true that, in the case of an economy as a whole, money costs might not be real costs, but, in this case, the costs need adjusting.

We can, perhaps, bring this out a little more forcefully, if we consider the example in Table X. Here, even if we do "net" out the current costs, we get a different result:

	Project G	Project E
Benefits	1085	598
Costs	1000	527
Ratio	1.085	1.135

Here E looks better than G. Why? It is true that, per unit of capital, E does produce better results than G, but is that the point? We must consider what we can do with the capital saved in carrying out E instead of G. We can invest it at 4%, which will give us no "bonus" at all. The right comparison is between investing 1000 in G and investing 1000 in E plus something else. Then we discover that the first gives a "bonus" of 85 and the second one of 71. Ergo, G is better.

Thus the use of ratios of this kind is full of pitfalls. For a much more exhaustive description of them see McKean, op. cit., pp. 107 ff.

Appendix III

Uncertainty and risk premiums

The notion of a premium for risk is a common one. An investor who wants a rate of return of 8% for a fairly certain project will want something more for a less certain one. This is perhaps tantamount to saying that one wants a higher return on the risky projects that are successful to compensate for those that are not.

Suppose we have four five-year projects, each costing 100 and that we wish to earn as a minimum 6% on the total investment of 400. This is equivalent to a gross return of 95 for the four (see Table VII A).

We also assume that one out of the four will go sour and earn no more than enough to recover its capital (i.e. 20 p.a.). Then the remaining three will have to earn the remaining 75. It turns out that this is a yield of about 8% on the capital invested in them of 300 (see Table VII B). The result is approximately (not exactly) one-third more, as one might guess. Three earning 8% plus one earning 0% is about the same as four earning 6%.

But now let us suppose that the failure is so bad as to be a total loss, no capital at all being recovered. In this case the three good projects must together earn the whole 95. This means that they must each earn 17½% in order to compensate for the loss and also earn 6% on the total 400 invested (see Table VII C) (This may explain, to some extent, the conduct of moneylenders).

Table 1

Three types of security yielding 6%

	Year 1	Year 2	Year 3	Year 4	Year 5
A. Annuity					
Capital outstanding (beginning)	1000	823	635	435	224
Gross return	237	237	237	237	237
Net return	60	49	38	26	13
Recovery of capital	177	188	199	211	224
B. 8% coupon bond at a premium					
Capital outstanding (beginning)	1084	1069	1054	1037	1019
Gross return	80	80	80	80	1080
Net return	65	64	63	62	61
Recovery of capital	15	16	17	18	1019
C. Savings bond					
Capital outstanding (beginning)	747	792	840	890	943
Gross return	0	0	0	0	1000
Net return	45	48	50	53	57
Recovery of capital	-45	-48	-50	-53	943

TABLE II

PRESENT VALUE $1/(1+i)^n$

Years	Rate i							
n	4%	5%	6%	6½%	7%	7½%	8%	10%
1	.962	.952	.943	.939	.935	.930	.926	.909
2	.925	.907	.890	.882	.873	.865	.857	.826
3	.889	.864	.840	.828	.816	.805	.794	.751
4	.855	.823	.792	.777	.763	.749	.735	.683
5	.822	.784	.747	.730	.713	.697	.681	.621
6	.790	.746	.705	.685	.666	.648	.630	.564
7	.760	.711	.665	.644	.623	.603	.583	.513
8	.731	.677	.627	.604	.582	.561	.540	.467
9	.703	.645	.592	.567	.544	.522	.500	.424
10	.676	.614	.558	.533	.508	.485	.463	.386
15	.555	.481	.417	.389	.362	.338	.315	.239
20	.456	.377	.312	.284	.258	.235	.215	.149
25	.375	.295	.233	.207	.184	.164	.146	.092
30	.308	.231	.174	.151	.131	.114	.099	.057
40	.208	.142	.097	.081	.067	.055	.046	.022
50	.141	.087	.054	.043	.034	.027	.021	.009

TABLE III

PRESENT VALUE OF AN ANNUITY

$$\left[1 - (1+i)^{-n}\right] / i$$

Years n	Rate i 4%	5%	6%	6½%	7%	7½%	8%	10%
1	.962	.952	.943	.939	.935	.930	0.926	0.909
2	1.866	1.859	1.833	1.821	1.808	1.796	1.783	1.736
3	2.775	2.723	2.673	2.648	2.624	2.601	2.577	2.487
4	3.630	3.546	3.465	3.426	3.387	3.349	3.312	3.170
5	4.452	4.329	4.212	4.156	4.100	4.046	3.993	3.791
6	5.242	5.076	4.917	4.841	4.767	4.694	4.623	4.355
7	6.002	5.786	5.582	5.485	5.389	5.297	5.206	4.868
8	6.733	6.463	6.210	6.089	5.971	5.857	5.747	5.335
9	7.435	7.108	6.802	6.656	6.515	6.379	6.247	5.759
10	8.111	7.722	7.360	7.189	7.024	6.864	6.710	6.145
15	11.118	10.380	9.712	9.403	9.108	8.827	8.559	7.606
20	13.590	12.462	11.470	11.019	10.594	10.194	9.818	8.514
25	15.622	14.094	12.783	12.198	11.654	11.147	10.675	9.077
30	17.292	15.372	13.765	13.059	12.409	11.810	11.258	9.427
40	19.793	17.159	15.046	14.116	13.332	12.594	11.925	9.779
50	21.482	18.256	15.762	14.725	13.801	12.975	12.233	9.915

TABLE IV

The following table shows, for a project with equal annual gross returns (before depreciation), the return as calculated by two methods:

(a) Annual income after straight-line depreciation divided by original investment.

(b) Discounted cash-flow.

The discounted cash-flow figures are at the head of each column. The corresponding figures for the other method are shown in each column for five different life-spans of the project.

	If the return on discounted cash-flow basis is:				
	2%	5%	6%	7%	20%
Corresponding return by method (a) is:					
For 1 year	2.0	5.0	6.0	7.0	20.0
" 2 years	1.5	3.8	4.5	5.3	15.5
" 10 "	1.1	3.0	3.6	4.2	13.9
" 25 "	1.1	3.1	3.8	4.6	16.2
" 50 "	1.2	3.8	4.3	5.2	18.0

TABLE V

Discounting renewals*

Details of project

Investment: 300 initially
90 in 2nd and 4th years

Returns: 100 annually for 6 years

Discount calculation (at 10%)

Using appropriate discount factors for 2nd year and 4th year in the case of investments and the present value of an annuity for 6 years in the case of the returns, we have:

	Amount	Discount Factor	Present Value
Investment:	300	1	300
	90	.826	74
	90	.683	62
			436
Returns:	100	4.36	436.

Check year by year

	1st year	2nd year	3rd year	4th year	5th year	6th
Capital outstanding (beginning)	300	230	243	167	174	91
Gross returns	100	100	100	100	100	100
Net returns (@ 10%)	30	23	24	17	17	9
Recovery of capital	70	77	76	83	83	91
New investment	-	90	-	90	-	-
Net recovery of capital	70	-13	76	-7	83	91

Discounting future returns less future investments

	Amount	Discount Factor	Present Value
1st year	100	.909	91
2nd "	10	.826	8
3rd "	100	.751	75
4th "	10	.683	7
5th "	100	.621	62
6th "	100	.564	56
			300

* See also note on following page.

NOTE TO TABLE V

Discounting renewals

If an investment of 100 has to be made every 5 years for 40 years (say), there is a short cut to obtaining the discounted value. In this case, when the renewal is made seven times (i.e. 5th to 35th year), the discounted value (at 8%) -

$$100 \times \frac{\text{Present value of an annuity of 1 for 35 yrs}}{\text{Amount of annuity of 1 after 5 years.*}} = \frac{11.6555}{5.867} \times 100 = 199$$

Check, using discount factors for each separate investment.

5th year	100	x	.6806	=	68.1
10th "	"		.4632	=	46.3
15th "	"		.3152	=	31.5
20th "	"		.2145	=	21.5
25th "	"		.1460	=	14.6
30th "	"		.0994	=	9.9
35th "	"		.0676	=	6.8
					198.7

* Tables exist for this, but are not shown here.

TABLE VI

Original & Supplementary Investments

A. Original Inv.	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Capital outstanding	400	341	280	215	147	75
Gross return	79	79	79	79	79	79
Net return (@ 5%)	20	17	14	11	7	4
Recovery of capital	59	62	65	68	72	75

B. Supplementary Inv.						
Capital outstanding	--	--	400	314	219	115
Gross return	--	--	126	126	126	126
Net return (@ 10%)	--	--	40	31	22	11
Recovery of capital	--	--	86	95	104	115

C. Combined Inv.						
Capital outstanding	400	349	694	538	371	192
Gross return	79	79	205	205	205	205
Net return (@ 7%)	28	24	49	38	26	13
Recovery of capital	51	55	156	167	179	192

TABLE VII

Uncertainty and risk-premium

A. Return on the total investment	1st year	2nd year	3rd year	4th year	5th year
Capital outstanding	400	329	254	174	89
Gross returns	95	95	95	95	95
Net returns (@ 6%)	24	20	15	10	5
Recovery of capital	71	75	80	85	90

B. Return on three projects, if the fourth earns 20 p.a.					
Capital outstanding	300	249	194	135	71
Gross returns	75	75	75	75	75
Net returns (@ 8%)	24	20	16	11	6
Recovery of capital	51	55	59	64	69

C. Return on three projects, if the fourth earns nothing					
Capital outstanding	300	258	208	150	81
Gross returns	95	95	95	95	95
Net returns (@ 17½%)	53	45	37	26	14
Recovery of capital	42	50	58	69	81

TABLE VIII

Uncertainty and Time

Investment	:	1211
Annual gross returns expected	:	100
Life	:	45 years
Rate of return	:	8%

How much would these returns have to be reduced in order to bring the r ate of return down to 7½%, if the reduction is confined to (a) the first 15 years, (b) the middle 15 years or (c) the last 15 years? For this we need the discount factors for an annuity over each period of 15 years.

	Discount factor (at 7½%)	Case (a) Annual Return	Case (a) Discounted	Case (b) Annual Return	Case (b) Discounted	Case (c) Annual Return	Case (c) Discounted
Years 1-15	8.827	92	812	100	883	100	883
Years 16-30	2.983	100	298	76	227	100	298
Years 31-45	1.009	100	101	100	101	30	30
			1211		1211		1211
Possible reduction		8%		24%		70%	

TABLE IX

Proceeds of reinvestment from two projects

1. Characteristics of the two projects

	Project D.	Project G.
Investment :	1000	1000
Gross annual return:	388	207
Life :	3 yrs.	6 yrs.
Rate of return* :	8%	6½%

2. Accumulated returns with reinvestment at 4%.

Project D.	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Reinvestment from previous years	—	388	792	1211	1260	1310
Return at 4%	—	16	32	48	50	52
Project gross return	388	388	388	—	—	—
Total (end year)	388	792	1211	1260	1310	1362

Project G.	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Reinvestment from previous years	—	207	422	646	879	1121
Return at 4%	—	8	17	26	35	45
Project gross return	207	207	207	207	207	207
Total (end year)	207	422	646	879	1121	1373

3. Gross returns discounted at 4%.

	Annual return	Discount factor (annuity at 4%)	Total discounted returns	"Bonus"
Project D.	388	2.775 (3 years)	1077	77
Project G.	207	5.242 (6 years)	1085	85

* Compare Table III: 388 x 2.577 = 1000 and 207 x 4.841 = 1000 (approx.)

TABLE X

1. Characteristics of the two projects

	Project E	Project G	Difference (G-E)
Investment :	527	1000	473
Gross annual return :	114	207	93
Life :	6	6	6
Rate of return* (approx.):	8%	6½%	5%

2. Accumulated returns with reinvestment at 4%

Project E	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Reinvestment from previous years	474**	607	745	889	1039	1194
Return at 4%	19	24	30	36	41	48
Project gross return	114	114	114	114	114	114
Total (end year)	607	745	889	1039	1194	1356

** It is assumed that the difference in capital cost (1000-526) is invested at 4%.

Project G - As in Table IX (final total = 1373)

3. Gross returns discounted at 4%

	Annual return	Discount factor (annuity at 4%)	Total discounted returns	"Bonus"
Project G	207	5.242 (6 years)	1085	85
Project E	114	5.242 (6 years)	598	71
Difference	93	5.242 (6 years	487	14

* G (as in Table IX); E 114 x 4.623 = 527; Difference 93 x 5.076 = 472

TABLE XI

	Project F	Project G
Investment :	534	1000
Gross annual return:	207	207
Life :	3 yrs.	6 yrs.
Rate of return* :	8%	6½%

2. Accumulated returns with reinvestment at 4%

Project F	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Reinvestment from previous years**	466	692	926	637	871	1113
Return of 4%	19	28	37	26	35	44
Project gross return	207	207	207	207	207	207
Total (end year)	692	926	1171	871	1113	1364

** In year 1, balance (1000-534) is invested at 4%; at end of year 3 there is "repeat" investment in the project of 534 (1171-637).

Project G: As in Table IX (Final total = 1373)

3. Cost of investment compared.

Cost	Discount factor (@ 4%)	Project F	Project G
Now	1.000	534	1000
3 years hence	.889	475	--
Total	.....	1009	1000

* As in Table IX; F is D reduced in size.

TABLE XII

A. Investment without inflation

Investment	100
Gross returns (all in the second year):	110 at current prices
	110 at real prices
Rate of return	5%

B. Investment with inflation

Investment	100
Gross returns (all in the second year):	133 at current prices
	110 at real prices
Rate of return	5%

TABLE XIII

Projects with joint costs

Project	Capital cost	Gross returns*	Internal rate of return	Returns discounted at 6%	at 10%
A. Irrigation alone	100	102	2%	96	92
B. Hydro power alone	100	103	3%	97	93
C. Irrigation plus hydro power	175	205	17%	193	185
D. Thermal power	80	103	29%	97	93
(C –)	95	102	7%	96	92
(C –)	75	103	37%	97	93

* The simplifying assumption is made that all the returns come in one year after construction.