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A SIMULATION MODEL FOR EVALUATING BUFFER STOCK PROGRAMS

by

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Introduction*

1. Growth in output of foodgrains and potential self-sufficiency in foodgrain supplies in a number of developing countries have stimulated a lively interest in large investment in grain storage facilities. The only kind of storage function considered in this paper is year-to-year storage which is motivated by a desire to reduce fluctuations in consumption in the face of unpredictable annual fluctuations in grain supplies. Buffer stocks are defined as those quantities of grain withdrawn from consumption when production happens to be unusually plentiful and re-introduced into the market when harvests happen to be less than normal. 1/

2. Unlike Joseph in Egypt, 2,500 years ago, the modern-day planner has no prophetic insights which would make it possible for him to predict production in any particular future year. Instead, we assume he can reasonably well guess the probability distribution of the deviations of annual production about a trend over a long period. As to the sequence of "surplus" and "deficit" years, we have assumed that they are randomly "selected" from that probability distribution. Since buffer stock storage benefits are very much affected by the sequence of deviations, any worthwhile prediction of their magnitude ought to be based on a large sample of production series. Prediction of benefits and good storage planning can only be made in a probability context.

3. Stochastic simulation with the aid of a computer has proved well-suited for the analysis of buffer stock storage plans. While simulation does not yield the optimum storage rules and capacity, as a difficult-to-construct dynamic programming routine might, simulation can be used to predict how a variety of assumptions affects a number of different storage benefit indices, giving both the expected values and their standard deviations. The models involved are simple to construct and the storage rules easy to follow.

4. An important conclusion derived from initial empirical investigations using simulation is that many less rigorous evaluations of the problem tend to overstate the net benefits and the extent of stabilization derivable from buffer stock schemes.

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* The paper abstracts from a larger study on cost-benefit methodology for evaluation storage investments. Messrs. N. Wilde and P. King assisted in developing and running the computer models. The views expressed are those of the author and not necessarily those of his colleagues or of the IBRD.

1/ When supplies relative to demand are known to be different from one period to another, storage benefits are best analyzed by a temporal equilibrium model. (For reference, see: Evaluation of Benefits from Season-to-Season Grain Storage Programs, by Sholo Reutlinger and Norman Wilde (Consultant), IBRD Economics Department Working Paper No. 66, dated February 17, 1970.)

The Model

5. The stochastic simulation model presented in this paper consists of four parts: (a) generation of a sample of n sets of m annual, sequentially ordered production values, (b) calculation of the storage activity and related costs and benefits for each production value, (c) calculation of the discounted present value of various benefits and costs and a measure of stability for each set of m annual production values, and (d) calculation of the expected value and standard deviations of the present values and measures of stability over the sample of n sets.

6. The time-ordered production values are randomly selected from a probability distribution of production, assuming zero serial correlation.1/

7. Storage activity is determined by the level of production and storage rules, and is constrained by available vacant storage capacity, and the amount of grain available in storage, respectively. The storage rules express the desired level of stabilization, i.e., the quantity of grain consumption not to be exceeded, Q_B, and the minimal quantity of grain consumption to be made available, Q_S.

8. The desired level of storage is calculated as follows: if production Q is greater than Q_B, the desired amount of grain to be put into storage is $(Q - Q_B)$. If Q is between Q_B and Q_S, no storage activity is desired. If Q is less than Q_S, the desired amount of grain to be taken out of storage is $(Q_S - Q)$. The actual amount of grain put into storage can, of course, never exceed the size of available storage capacity, and the actual amount of grain taken out of storage cannot exceed the amount of grain available in storage. Available storage space and the amount of grain in storage depend on the maximum available storage capacity and the storage activity in prior years. Hence, the estimates of storage activity are specific to a given sequentially ordered production series, the given storage rules and the given maximum storage capacity.

9. Several kinds of benefits and costs are calculated for a given level of storage activity. They can be best explained by reference to Figure 1. First, consider social benefits and costs. If production is Q_1 and $(Q_1 - Q_1')$ is put into storage, consumer surplus sacrificed is the amount of grain stored multiplied by the average of the prices which would prevail without and with storage, i.e., $(Q_1 - Q_1')(P_1 + P_1')/2$. To this negative benefit is added any variable storage cost incurred at the time grain is put into storage. If grain production is Q_2 and $(Q_2' - Q_2)$ is taken out of storage, consumer surplus is the amount of grain taken out of storage multiplied by the average of the prices which would prevail with and without storage activity, i.e., $(Q_2' - Q_2)(P_2 + P_2')/2$. From this is subtracted any variable cost incurred in taking grain out of storage.

1/ Such a probability distribution can be usually estimated on the basis of historical production data.

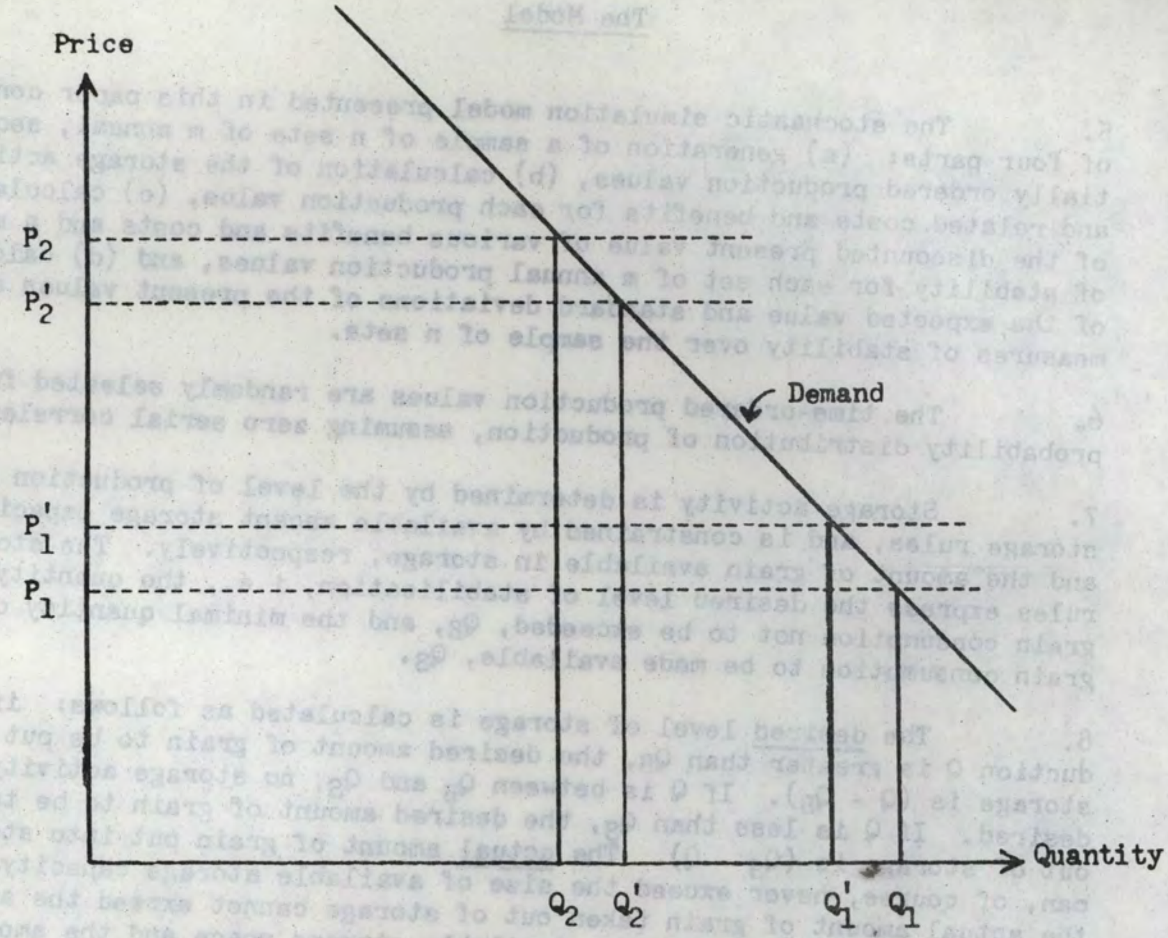

Figure 1

10. The financial benefits and costs to a public storage corporation are similarly calculated, except the market prices of grain which prevail with storage -- P_1' when grain is put into storage, and P_2' when grain is taken out of storage -- are used to evaluate the quantities of grain. The cost of grain put into storage is $(Q_1 - Q')P_1'$ and the revenue of grain taken out of storage is $(Q_2' - Q_2)P_2'$. Variable storage costs are included as in the previous paragraph.

11. When production is Q_1 and an amount $(Q_1 - Q')$ of grain is put into storage, farm income increases by $(P_1' - P_1) Q_1$. When production is Q_2 and an amount of $(Q_2' - Q_2)$ of grain is taken out of storage, farm income decreases by $(P_2 - P_2')Q_2$. Note that a long holding period is favorable to the storage effect on farm income as the benefit preceeds the cost, whereas for consumers, extended storage reduces the discounted value of benefits.

12. There are secondary effects to a reduction in price fluctuation which are not quantified in this model. Grain prices may affect wages and if wages are sticky, a price rise in grain in one year may mean higher wages (more consumption and less savings) forever after. On the supply side, price fluctuations result in price uncertainty and in retrospect less than rational allocation of resources. Calculation of these beneficial effects of buffer stocks remain a challenge for further work. However, in the meantime, the simulation model has been used to estimate the effect of storage on reducing price fluctuations and farm income fluctuations as measured by their standard deviations.

13. Having calculated the social and financial costs and benefits and farm income changes as well as the price and farm income for each year in the set of m annual observations, the next step is to calculate the present value of social and financial benefits and farm income changes using a desired discount rate, and the standard deviation of price and farm income based on the m observations in the set.

14. Finally, since any measure of benefits is very specific to sampling variation in the frequency distribution of production and particularly in the sequential ordering of production levels, n values of each of the above measures, derived from different, randomly chosen m-year-long production sequences, are used to derive estimates of their expected values and standard deviations based on the sample of n sets.

Illustrative Conditional Projections

15. To illustrate even more concretely how the model is used, consider the following data:

(a) A triangular probability distribution of production with a mean of 40 million tons and a range from 34 to 46 million tons.

(b) A linear demand function with price elasticity of - 0.5 at the mean production and mean of price ($ 100), i.e.,

$$P = 300 - 5Q$$

where P is the price per ton and Q is grain available for consumption in million tons.

(c) Variable storage cost is $ 1.00 per ton at time of loading and the discount rate is 6 percent.

(d) Maximum storage capacity is one million tons and the storage rules (Q_B, Q_S) are as follows:

Q_B = 41 (i.e., the intention is not to let consumption exceed 41 million tons),

Q_S = 39 (i.e., the intention is not to let consumption be less than 39 million tons).

16. Sample calculations of the storage activity and some benefit indices for a number of years are shown in Table 1. Clearly, the results are very specific to the chance-controlled production series. Particularly the sequential ordering will determine the total amount of grain stored (capacity utilization) and the cost of holding grain. If, for instance, five years have elapsed between time grain is put into and taken out of storage and a 6 percent discount rate is used, the net gain would have to exceed 25 percent for the discounted storage benefits to be postive.

Table 1: Illustration of Year-to-Year Simulation of Buffer Stock Operations

(Maximum Storage Capacity: 1 Million tons)
(Storage Rules: Q_B = 41, Q_S = 39)

Year	(1) Production (randomly selected from P.D.)	(2) Opening Stock	(3) Grain Stored	(4) Economic Benefit (Consumer Surplus)	(5) Price Without Storage	(6) Price With Storage
	(Million Tons)			(Million $)	($)	($)
1	45.8	0	1.0	- 74	71	76
2	44.8	1	0	0	75	75
3	41.2	1	0	0	94	94
4	37.6	1	- 1.0	109	112	107
5	37.2	0	0	0	113	113
6	43.1	0	1.0	- 88	85	90
7	38.2	1	- .8	86	109	105
8	40.0	.2	0	0	100	100
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
30	.	.	.	.	.	.

17. Table 2 shows the expected values of a number of benefit indices obtained from simulating buffer stock operations when the above stated assumptions apply, but the storage rules and maximum storage capacities are varied. Each expected value in the table is obtained from a sample of 300 runs of 30 years duration. Index (2), the present value of net benefits, is obtained by subtracting \$ 25 per ton for construction costs of the storage facilities minus their discounted salvage value after 30 years.1/ Percent capacity utilization measures the average annual amount of grain put into storage per 100 tons of storage capacity.

18. Particularly noteworthy are the declines in marginal benefits as storage capacity increases. Beyond a point it would not pay to increase storage activity even if the facilities could be obtained without cost.

19. Also noteworthy is the large gain in farm income from the buffer stock operations. This explains why farmers' lobbies would advocate large buffer stock programs. The time dimension of storage works against the consumer; he bears the cost when grain is withdrawn from consumption and gets a benefit several years later, when grain is withdrawn from storage. The farmers' time dimension is the mirror image of the consumers': farmers benefit now and pay later. This phenomenon suggests that developing countries should perhaps pay more attention to storing buffer stocks of export commodities than of domestically consumed grain. Even after subtracting the negative financial benefits from the gain in farm income, the net gain (Index 6) is large.

20. Finally, it should be noted that the expected reduction in price fluctuations is not all as impressive as advocates of large buffer stock operations would have us believe. While the storage rules reflect the desire to practically eliminate the large price fluctuations as reflected in the size of the standard deviation (\$ 17), the fluctuations are reduced only to \$ 13 (24 percent) even when the storage capacity is fairly large.

21. Table 3 shows the standard deviations of several of the calculated benefit indices. As expected, the dispersion of discounted benefits stemming from the incidence of differing sequences of production levels is quite large. This adds a dimension of risk to storage investment for buffer stock operations which is above and beyond the risk arising from the imprecise predictability of all other investment-related events.

22. Table 4 shows what storage benefits would be if the price elasticity is - 0.25 rather than 0.5. The benefits are generally much larger, but even under such an extreme assumption, the optimum capacity (3 million tons) is less than the standard deviation of production (3.7 million tons).

1/ One finds in the literature recommendations to provide for buffer stock operations of the order of one standard deviation of production fluctuation. Our analysis suggests that a more optimal size in this specific case is one-third of a standard deviation (1-2 million tons).

Table 2 : Expected Indices of Storage Benefits

(Price Elasticity: - 0.5)

Storage Rules (Q_H, Q_S)	Indices of Benefits	Storage Capacity (million tons) 0	.5	1	2	3	4	10
		---(million dollars)---						
	(1) P.V. of Gross Eco. Ben.		28	48	70	**72**	61	
	(2) P.V. of Net Eco. Ben.		16	**23**	20	- 5	- 39	
	(3) P.V. of Gross Fin. Ben.		24	34	22	- 18	- 68	
	(4) P.V. of Net Fin. Ben.		**12**	9	- 28	- 93	-168	
40.5, 39.5	(5) P.V. of △ Farm Inc.		102	202	397	575	733	
	(6) (4) + (5)		114	211	269	482	565	
	(7) St. Dev. Price ($)	17.4	16.4	15.6	14.1	13.0	12.3	
	(8) St. Dev. Farm Inc.	358	321	293	263	260	270	
	(9) Cap. Utili.		22	22	20	18	16	
	(1) P.V. of Gross Eco. Ben.		27	47	67	**68**	59	- 47
	(2) P.V. of Net Eco. Ben.		15	**22**	17	- 7	- 41	- 297
	(3) P.V. of Gross Fin. Ben.		24	35	25	- 9	- 48	- 313
	(4) P.V. of Net Fin. Ben.		**12**	10	- 25	- 84	-148	- 463
41, 39	(5) P.V. of △ Farm Inc.		101	200	384	552	696	1,265
	(6) (4) + (5)		113	210	359	468	548	802
	(7) St. Dev. Price ($)	17.4	16.5	15.6	14.2	13.2	12.5	10.2
	(8) St. Dev. Farm Inc.	358	321	293	260	253	253	227
	(9) Cap. Utili.		21	20	18	16	14	
	(1) P.V. of Gross Eco. Ben		23	39	**53**	51	39	
	(2) P.V. of Net Eco. Ben.		11	**14**	3	- 24	- 61	
	(3) P.V. of Gross Fin. Ben.		21	29	23	0	- 24	
	(4) P.V. of Net Fin. Ben.		**9**	4	- 27	- 75	-124	
42, 38	(5) P.V. of △ Farm Inc.		94	183	347	484	597	
	(6) (4) + (5)		103	187	320	409	473	
	(7) St. Dev. Price ($)	17.4	16.6	15.8	14.6	13.9	13.3	
	(8) St. Dev. Farm Inc.	358	323	299	267	252	240	
	(9) Cap. Utili.		16	15	13	12	10	

NOTE: The boxed figures are the maximum values of the respective indices.

Table 3: Standard Deviations of Indices of Storage Benefits

Storage Rules (Q_B,Q_S)	Index of Benefits	Storage Capacity (million tons)				
		.5	1	2	3	4
		Million Dollars				
40.5,39.5	(1) P.V. of Eco. Ben.	13	24	41	52	61
	(2) P.V. of Fin. Ben.	13	23	35	42	51
	(3) P.V. of △ Farm Inc.	19	38	76	116	155
41, 39	(1) P.V. of Eco. Ben.	13	23	39	51	60
	(2) P.V. of Fin. Ben.	12	21	33	41	49
	(3) P.V. of △ Farm Inc.	20	39	79	119	157
42, 38	(1) P.V. of Eco. Ben.	12	23	38	49	58
	(2) P.V. of Fin. Ben.	12	21	32	41	50
	(3) P.V. of △ Farm Inc.	20	41	83	124	164

Table 4: Expected Values of Indices of Storage Benefits
(Price Elasticity: - 0.25)

Storage Rules (Q_B,Q_S)	Index of Benefits	Storage Capacity (million tons)					
		0	.5	1	2	3	4
		Million Dollars					
40.5,39.5	(1) P.V. of Gross Eco. Ben.		77	139	225	268	285
	(2) P.V. of Net Eco. Ben.		65	114	175	193	185
	(3) P.V. of Gross Eco. Ben.		69	111	129	89	27
	(4) P.V. of Net Fin. Ben.		57	86	79	14	- 73
	(5) P.V. of △ Farm Inc.		205	407	794	1,150	1,467
	(6) (4) + (5)		262	493	873	1,164	1,394
	(7) St. Dev. Price ($)	34.7	32.7	30.9	27.9	25.7	24.1
	(8) St. Dev. Farm Inc.	1,047	968	900	796	732	694
	(9) Cap. Utili. (%)		22	22	20	18	16

Table 2a: Expected Indices of Storage Benefits

(Price Elasticity: -0.5)

Storage Rules (Q_B,Q_S)	Indices of Benefits	Storage Capacity (illion ons)				
		0	.5	1	2	3
		(Million Dollars)				
	(1) P.V. of Gross Eco. Bene.		10	14	11	- 5
	(2) P.V. of Net Eco. Ben.		-2	- 11	- 31	- 80
	(3) P.V. of Gross Fin. Ben.		7	3	- 23	- 60
	(4) P.V. of Net Fin. Ben.		-5	- 22	- 27	-135
40.5,39.5	(5) P.V. of △ Farm Inc.		85	166	315	444
	(6) (4) + (5)		80	144	288	309
	(7) St. Dev. Price ($)	12.5	4.6	10.9	9.7	9.0
	(8) St. Dev. Farm Inc.	260	228	206	186	186
	(9) Cap. Utili.		20	19	16	14
	(1) P.V. of Gross Eco. Ben.		9	14	9	- 7
	(2) P.V. of Net Eco. Ben.		-3	- 11	- 41	- 82
	(3) P.V. of Gross Fin. Ben.		7	5	- 18	- 50
	(4) P.V. of Net Fin. Ben.		-5	- 20	- 68	-125
41, 39	(5) P.V. of △ Farm Inc.		81	157	294	413
	(6) (4) + (5)		76	137	226	288
	(7) St. Dev. Price ($)	12.5	11.7	11.0	10.0	9.3
	(8) St. Dev. Farm Inc.	260	229	206	185	178
	(9) Cap. Utili.		16	15	13	11
	(1) P.V. of Gross Eco. Ben.		5	6	- 4	- 20
	(2) P.V. of Net Eco. Ben.		-7	- 19	- 54	- 90
	(3) P.V. of Gross Fin. Ben.		4	0	- 19	- 43
	(4) P.V. of Net Fin. Ben.		-8	- 25	- 69	-118
42, 38	(5) P.V. of △ Farm Inc.		69	133	247	346
	(6) (4) + (5)		61	108	178	228
	(7) St. Dev. Price ($)	12.5	11.9	11.3	10.6	10.1
	(8) St. Dev. Farm Inc.	260	235	217	195	183
	(9) Cap. Utili.		10	10	8	7

The benefit estimates are too high if one considers it unrealistic that a country would isolate itself from trade at any price rather than experience further price fluctuations. In the case used for illustration it was implied, for instance, that the value of storing a ton of grain from a time when production is one standard deviation above normal to a time when production is one standard deviation less than normal is approximately $ 70. With trade being a realistic alternative to storing, the value of storing would never exceed the difference between the export and import price.

Evaluation of Model

23. A major shortcoming of the simulation model is that it does not yield optimal storage rules, which could be obtained by using dynamic programming. The offsetting advantage is that the storage rules are simple and more likely to be followed in practice. Furthermore, given the low computing cost, it is feasible to experiment with different storage rules until optimal rules are approached. For instance, we could let the amount of desired storage be proportional to the level of production and the amount of grain in storage, with the proportionality coefficients to be determined by experimentation.

24. The great advantage of the simulation approach is the ease with which it is possible to calculate the effect of different storage capacities and rules on a large number of measures of benefits. For instance, the measured social benefits do not necessarily account for the difficult-to-measure indirect effects of a reduction in price fluctuations. It is useful to see, therefore, the trade-off between measurable benefits and the standard deviation of prices as storage capacity increases.

25. Another advantage of the simulation over the analytical optimization approach is the ease with which it is possible to calculate not only expected values but also measures of dispersion of the benefit indices. Intuitively, the benefits will be quite different in the case where an investment in storage is followed by seven "bad" years and then by seven "good" years, from the case where the "good" and "bad" years alternate over the life of the investment. One would like to have, therefore, a measure of this kind of risk attached to a storage investment.

26. Now we have a few words about the nature of some of the implied assumptions underlying the model and their implications. First, the distribution of production deviations is usually quite difficult to estimate. Historical production data contain trends, cycles and random fluctuations which are difficult to sort out. Usually we make an assumption about the nature of the random fluctuations and then proceed to estimate trends and cycles. Deviations of production about a least-sqaures trend lines often look more uniformly than normally distributed. This is, of course, because production fluctuations are as often cyclical as random.

27. We have used a triangular distribution, partly because of convenience and partly because we have assumed zero year-to-year correlation. If production were distributed uniformly, storage benefits would tend to be larger than what we have found; however, if there are production cycles

and we use simple storage rules abstracting from any knowledge of future cycles, storage benefits would tend to be smaller. We are counting, therefore, that the bias introduced by the choice of distribution will be compensated by the bias resulting from assuming zero correlation.

28. The model implies that storage operations from "good" to "bad" production years are the only alternative to fluctuating consumption. This is clearly untenable when production fluctuations are large, the price elasticity is low, and a country enjoys proximity to export and import markets. When trade is feasible and advantageous, storage benefits will usually be less than those measured by assuming the only alternative is to let domestic consumption vary with domestic production.

29. The social benefit does not include a measure of the benefits from shifts in the supply and demand functions which may result from operating a price stabilization scheme. This is clearly a shortcoming, but not so much of the model as of empirical knowledge about the magnitude of such shifts. We hope that the measure of stabilization calculated will be, nevertheless, an additional useful indicator of the benefits of a storage program of a given size. As our illustrations have shown, there is a large difference between the intended and achievable extent of stabilization with a given storage capacity.

30. Many extensions or variants of the simulation model discussed are possible. We have already experimented with introducing a growth trend into annual supply and demand and supplementing the standard deviation of price by indices of dispersion which would give larger weights to extreme deviations, as a measure of how much storage might eliminate undesirable effects of supply fluctuations. We have also proposed a model which measures the benefits of buffer stock storage when the function of such storage is to eliminate fluctuations in import requirements. Finally, we are prepared to invest effort in exploring whether more complex storage rules could substantially increase the expected storage benefits, and we generally need to further test the sensitivity of such models.

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Errata and Addendum

Change: in paragraph 15 (a) Instead of : "........ a triangular ...." "........ a uniform........"

Delete: Paragraph 27.

Add: Tables 2a and 4a given simulation results when all the assumptions are as given for Tables 2 and 4, except a triangular distribution of production is substituted for the uniform distribution.

Table 4a: Expected Values of Indicies of Storage Benefits
(Price Elasticity: -0.25)

Storage Rules (Q_B,Q_S)	Index of Benefits	Storage Capacity (million tons)					
		0	.5	1	2	3	4
		Million Dollars					
	(1) P.V. of Gross Eco. Ben.		42	71	102	/107/	97
	(2) P.V. of Net Eco. Ben.		30	46	/52/	32	- 3
	(3) P.V. of Gross Fin. Ben.		35	48	35	- 4	- 51
	(4) P.V. of Net Fin. Ben.		23	/23/	- 15	- 79	-151
40.5,39.5	(5) P.V. of △ Farm Inc.		170	332	629	888	1,121
	(6) (4) + (5)		193	355	614	809	970
	(7) St. Dev. Price ($)	24.8	23.0	21.4	19.0	17.4	16.3
	(8) St. Dev. Farm Inc.	749	677	620	537	589	461
	(9) Cap. Utili. (%)		20	19	16	14	12