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THE MEASUREMENT OF ECONOMIC CHANGE*

by

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THE MEASUREMENT OF ECONOMIC CHANGE

Objective

The phenomenon known as economic growth has become exceedingly important since the post-war era. There is hardly a country today that does not consider the maximization of economic growth a prime domestic goal. However, there is substantial disagreement as to the actual mechanics of measuring growth. There exist today about seven different methods for computing a rate of growth. Some of these are more widely accepted than others. Pesek /1 has given us an excellent review of these methods and has provided criterion by which each method can be evaluated. Also, he has developed two formulas for measuring growth and has put them forward as superior to any other method. It is the objective of this paper to utilize the information provided by Pesek and to offer some new arguments which it is hoped will demonstrate that Pesek's methods are not quite so superior and that one other method, in theory and in practice, appears to be as reliable in measuring growth.

Fundamentally, growth, or relative change, is the ratio of the amount of change over a given period of time relative to some base. In symbols this definition of the basis of growth looks like this:

$$(1)\quad \text{Rate of Growth (G)} = \frac{\Delta x_1}{x_1}$$

when: x_1 = a base figure

Δx_1 = change in x_1 from T to T=1, when Time changes from T to T+1

In other words, the rate of growth (G) is derived from the actual changes in the variable over time. The rate of growth then seeks to find some

/1 Boris P. Pesek, "Economic Growth and its Measurement," Economic Development and Cultural Change, April 1961.

quantitative measure of the persistent and sustained trend (amount of change) exhibited by the variable. Therefore, growth is a function of the persistent or sustained trend in the variable over time. /2 Clearly a method for computing growth should not yield an amount of change different from that which did evolve in real life. Should a particular method yield an amount of change different from that which actually occurred, then what does that method measure? It measures a trend, of course, but certainly one very different from that which would reflect the actual amount of change. This implies, therefore, that the theoretical values for each time period obtained from the fitted equation should fluctuate around the actual values rather than lying consistently above or below. Any method which gives theoretical values of the latter type would certainly fail to measure the correct amount of growth. It is for these major reasons that the method used to measure the rate of growth should: (1) Reflect the trend with reasonable accuracy and (2) yield the same amount of change as did actually occur in the variable over the time in question (that is, $\sum y_c = \sum y$, where "y_c" is the computed or trend value and "y" is the actual value).

Summary of Findings

The method put forward equally in this paper as reliable as the Pesek methods and better than other available methods for measuring growth is known as Glover's method. This technique meets the basic requirements for measuring growth as stated above and is relatively simple to compute. The methods put forward by Pesek as superior are subject to certain theoretical and practical faults, as will be subsequently shown. The Glover method is an exponential growth formula and is of the form:

$$y_c = ar^x$$

/2 Other authors define economic growth similarly. Phyllis Deane in the October, 1961 issue of the Malayan Review in her article, "The Long Term Trends in World Economic Growth" argues that "Economic growth is defined as a sustained secular increase..." Professor Simon Kuznets in an October, 1961 lecture at the Economic Development Institute defined economic growth in terms of a sustained trend exclusive of cyclical or annual ups and downs.

where:

y_c = computed values

r = rate of growth (expressed as 1 + the growth percentage)

x = number of times an observation is removed from the origin

a = the constant or trend value for the first observation (origin)

Discussion

Four specific methods of measuring change have been evaluated in this paper. They are the terminal method, the exponential curve fit to logarithms, by the least squares method, Pesek's method (referred to hereafter as Pesek Method VI), and finally Glover's Method. Pesek has examined other methods, and the reader is referred to his article.

The terminal method is probably one of the best-known and most-frequently used techniques for measuring growth. It involves using only two observations - one at the beginning, and one at the end. Given the number of observations in between these two terminal points (although the actual observations for the time periods in between are not utilized by this method) and a compound interest table, a rate of growth can be calculated quickly. Under this technique, the "a," or first value, is the actual beginning observation; in other words, with this method the growth rate is anchored to a specific initial value. The arbitrary exclusion of the intervening values violates the criteria for measuring growth set forward earlier in this paper. The trend values which result from the terminal points method reflect what would have happened in the intervening years had there been no cyclical or irregular influences in these years. In other words, it is hypothesized according to the terminal points method that the only factor influencing the given value of a economic variable in a given year is a hypothetical trend factor. However, any time series value (annual) is composed of trend, cyclical and irregular influences, and accordingly, any given value is the product of trend times cyclical times irregular. /3 Moreover, there is nothing inherent in the terminal points formula to ensure that the theoretical or trend amount of change equals the actual amount of change. The amount of change data are derived from the sum of trend values, sum of actual values and sum of values assuming no change in the data. The source of the first two is obvious but the third sum requires some explanation. The sum of values assuming no change for any given variable is obtained by taking the actual first value and multiplying it by the number of observations. The trend amount of change is derived by taking the difference between the sum of trend values and sum of values assuming no change. The actual amount of change is obtained similarly. /4 Accordingly, since the terminal points method does not meet either one of the criteria for measuring growth established earlier, its use should be limited to those circumstances wherein it is the only formula that can be applied.

The exponential curve fit to logarithms by the least squares method is sometimes used to extract a rate of growth. However, it is open to several objections. The major objection revolves around the use of logarithms rather than actual observations; and being a least squares fit, the sum of the logarithms of the actual values is made equal to the sum of the logarithms of the trend values. This means that this technique equates the product of the actual values with the product of the trend values. Of course, "both of these products and their equality from the standpoint of economics are meaningless," and "the economically

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/3 The problem is much more complicated than is indicated. The question of the time period selected is important regardless of the method of computing growth. Care must be exercised so as not to choose a time period which contains a fractional number of cycles. The accepted method of avoiding this pitfall is to select boundry dates so that the end values are approximately at the same cyclical phase, and in this way avoid cyclical bias in the trend. This issue is extremely important since growth is defined as the persistent trend over time -- and this persistent trend should be measured exclusive of cyclical influences. For further discussion see Arthur F. Burns "Production Trends in the U.S. since 1870," 1934, pages 38-42; Frederick C. Mills, "Economic Tendencies in the U.S.," 1932, pages 46-48; Croxton and Cowden, "Applied General Statistics," Second edition, pages 278-279 and Chapter 16.

/4 Symbolically, $\sum y_c - \sum y_1 n = \sum y - \sum y_1 n$ where y_c, y as defined above, and $y_1 n$ is the actual value in period 1 multiplied by n, which is defined as sum of values assuming no change.

meaninful requirement that the sum of actual outputs be equal to the sum of the estimated outputs is not satisfied." /5 In other words, this method yields the best fit to the logarithms rather than to the actual observations themselves. Nevertheless despite these conceptual shortcomings, the exponential curve fit to logs by least squares often yields growth rates which are close to those obtained under Pesek's formulas and Glover's method.

Glover's method and the Pesek methods will be discussed simultaneously now. Pesek's chief criticism of the Glover method is based on what he call an "arbitrary set of weights" /6. However, it appears that Pesek has misinterpreted the use of the "x" values in the Glover method, which Pesek refers to as the "arbitrary set of weights." In reality, the "x" values are the number of times a given observation is removed from the origin or first observation. The formula, as we recall, for Glover's method is $y_c = ar^x$. The first step in solving for r (the rate of growth) is to solve for what Glover calls the Mean Value (M)./7 The specific formula for the Mean Value is $M = \frac{\sum yx}{\sum y}$. The Mean Value therefore is the weighted average of the "x" series or number of times removed from origin. In effect then, the "y" values, or actual observations, are the real weights rather than the "x" values as indicated by Pesek. The "x" values then are not as arbitrary as contended by Pesek; they indicate how far removed a given observation is from the origin observation. Since growth is based upon the amount of change in the variable divided by the change in time, the number of times that a given observation is removed from the origin seems rather important in solving for the rate of growth. Since it is possible to determine the change in the

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/5 Ibid., page 301-302.

/6 Ibid., page 303.

/7 Glover's Mean Value Tables are shown at the end of the appendix. Two sets of tables are shown. The first set is the original Glover's Tables reproduced as they appear in the 1951 edition of "Tables of Applied Mathematics in Finance, Insurance, Statistics," pages 470-483, developed by Mr. James W. Glover. I wish to thank the publisher, the George Wahr Publishing Company, for permission to reproduce these tables. The second set is an extension of the original Glover's Mean Value Tables. This set was developed by Mr. Charles Wittmann of the U.S. Department of Agriculture and published by the National Bureau of Economic Research. I wish to thank Mr. Wittmann and the National Bureau for permission to reproduce these extensions.

variable (y) as well as the number of times removed from the origin (x), the combined use of the actual observation and the number of time that it is removed from the origin does in fact have economic as well as practical meaning in measuring the rate of growth. Moreover, Pesek's belief that the "arbitrary set of weights" gives undue weight to the observations furthest removed from the origin is unwarranted, again due to the fact that the "x" values are not the weights. The "a" value in the Glover method is the theoretical origin value, thus this technique is not secured to any given beginning value. Also, the Glover technique yields a computed amount of growth equal to the actual amount of growth. Examination of Charts I and II reveal that Glover's method approximates a least squares fit since actual values tend to fall rather evenly on either side of the Glover's trend. Further evidence of this trait comes from the fact that the sum of the Glover's computed or trend values is equal to the sum of the actual values (in least squares the sum of trend equals sum of actual also). Table 5 pinpoints this fact by showing the sum of deviations from Glover's trend, and these are, in most cases, close to zero.

Referring now to Pesek's article, his Method VI does yield a growth rate which ensures that the sum of the trend values be equal to the sum of actual values. However, it does require that the "estimated output in period 1 should be equal to the actual output in period 1; in other words the base year must be selected in the case of this method" /8. This is of course, a disadvantage as Pesek himself admits in that the selection of the base period is arbitrary. The most efficient technique of finding the growth rate under this method according to Pesek is as follows:

(a) Calculate

$$\frac{\sum_{t=1}^{n} P_t}{P_1} = x$$

(b) find this number x in the body of a table showing amount of 1 Per Annum at Compound Interest ($S_{\overline{n|}}\, i$) in period n;

/8 Ibid., page 304.

(c) the rate of interest shown by the table for the known x and n is the rate g_6.

However, this "most efficient method" becomes less efficient when it is necessary to calculate a negative rate of change since the table showing the amount of 1 Per Annum at Compound Interest is available (as far as this author can determine) only for use in finding a positive rate of change. It seems desirable that any method be able to provide estimates of a negative rate of change as efficiently as it does provide a positive rate of change. For these theoretical and practical reasons, the use of Pesek's Method VI seems undesirable. Pesek's Method VII is substantially better than his Method VI in that the estimated base year value is not preselected, but rather that it provides a theoretical initial trend value. Moreover, it is a least squares fit which means that a standard error of estimate can be computed to measure the goodness of fit. That is,

$$\sum_{t=1}^{n}\left(\hat{P}_1 b^{t-1} - P_t\right)^2 = \min$$

where:

$\hat{P}_1$ is the fictitious base year value,

P_t is the actual value,

b is the growth rate.

After differentiating and solving for $\hat{P}_1$, Pesek arrives at the final equation

$$\frac{b^n + 1}{b + 1} = \frac{\sum_{t=1}^{n} P_t b^{t-1}}{\sum_{t=1}^{n} P_t}$$

and the growth rate is equal to (b - 1) x 100.

However, the use of this method is rather cumbersome and does require a substantial computational effort such that Pesek has found a need for a computor program as an aid in its use. This seems to be a serious practical disadvantage inasmuch as the computational work required to calculate a growth rate should be reasonably efficient.

Practical Applications

For purposes of this paper, data on wool production in various countries covering the period 1948-1961 were used. Also as tests for longer periods the gross national product of the United States and net electrical energy production in the United States covering the period 1920-1955 were used. Tables 1 and 2 which follow show these data. Table 3 summarizes the calculations for wool production, while table 4 summarizes the calculations for net electrical energy and GNP. /9 The results are evaluated according to the basic growth theorem set out above which is based upon the concept that the stream of trend values should equal the stream of actual values. In other words, the trend amount of change obtained from the application of any specific formula should equal the actual amount of change. Tables 3 and 4 are organized with this principle in mind.

Without fail Pesek's Method VI and Glover's method consistently yield an amount of growth which is equal to the actual. Although Pesek Method VII was not tested empirically here, Pesek's article shows that his Method VII and the Glover method result in a theoretical or trend stream of data equal to the actual; in other words, the sum of output under Method VII and the sum of output under Glover both equal the sum of actual output /10 (that is, $\sum y_c = \sum y$, where "y_c" is the computed or trend value and "y" is the actual value). On the other hand, the terminal points method rarely comes close to the actual amount of growth. A rather curious result occurs with the exponential curve fit to logarithms, namely, despite the theoretical objections to this method there are many instances where the growth derived from this technique does equal the actual amount of growth; this tendency was also noted by Pesek). The specific growth rates obtained under each of the four methods do, of course, vary rather considerably; this is to be expected. However, Pesek shows that his Method VII and the Glover method yield almost identical growth rates for long as well as short periods of time. Curi-

/9 The reader is referred to Pesek's article and especially to his sections called Comparison of Performance for additional empirical results.

/10 Pesek Method VII was not used since the author does not have a copy of the required computor program. The reader is referred to Pesek's article for the empirical results under Pesek Method VII, especially Table 2.

ously enough, the growth rates obtained from Glover's method and the exponential curve fit to logs are very close to one another (which occurs by the way in the results by Pesek). The growth rates obtained from the terminal points method are in most instances different from the preferred Glover's method. Charts I and II demonstrate graphically the results obtained under the four methods discussed in this paper.

In addition to the basic requirement for the calculation of a rate of growth (the total actual output be equal to the total computed output), it seems reasonable to ask of any method that it yield a rate of growth which faithfully reflects the stream of output over the period of time covered. The terminal points method fails to meet either one of these requirements. The exponential curve fit to logarithms by the least squares method fails to meet the former requirement as well as the latter requirement (it uses the logarithms of the observations rather than the observations themselves to fit a linear trend). Pesek's Method VI suffers from an **arbitrary selection of a base** period, while Pesek Method VII can be obtained only after a substantial amount of computational effort or with the use of a computor.

Accordingly, Pesek Method VII and the Glover method are submitted as the most efficient techniques for calculating a rate of growth. Both methods meet the basic requirements of the growth rate calculation as outlined above. In the opinion of the author, however, the Glover method, since it yields precisely the same measurements and is easier to compute, appears to be a more usable method than Pesek VII.

The Stability of Growth

Deviation from regularity of growth for any economic variable is a significant question which needs to be answered if the growth rate derived is to be interpreted intelligently and utilized wisely. A growth rate can be calculated for any series of economic data, but what of the departures from the trend or norm? These deviations from the trend values (which would have been expected had the variable grown with a smooth and regular movement) merit measurement. A device which has been offered as a basis for judgment on this matter is the index of instability of growth. /11 The index as conceived by Mills is the mean percentage deviation about the trend line measuring the rate of change. It takes the form:

$$I = \frac{\sum \left| (y/y_c - 1) \right|}{N} \cdot 100$$

where: y = actual values
y_c = trend or computed values
N = number of observations

The more stable the persistent change in a given variable, the smaller the index, according to Mills. The index does not measure the variability of a given economic series; other statistical resources are designed to accomplish this. However, if information on the stability of the persistent trend (growth) is required, then the measures which are relevant are those "defining departures from values which would have been recorded if a given rate of change had prevailed with absolute regularity". /12

According to Mills the proper interpretation of the Index is that it measures the average annual (if annual data are used) percentage deviation from the trend values. The trend values represent the smooth and regular change of a variable over time. However, economic progress is uneven in most instances and the Index is intended to measure the degree to which the progress has been uneven. But what level of the degree of uneveness represents stability or instability in the growth of a variable? No answer has been available to the question up to now although Mills has suggested that an Index value of 3.7 percent is "rather wide... in contrast to the relatively smooth and regular growth of population". /13 This, however, smacks of too much subjective judgment and seems to me only part of the answer. Moreover, the Index provides no clue as to the distribution of the dif-

/11 Due to Frederick C. Mills, in his "Economic Tendencies in the United States," 1932, pages 48-49.

/12 Ibid., p. 49.

/13 Mills, page 2-3. The index value for population quoted by Mills is 0.3 percent and refers to the population of the United States. The index value of 3.7 percent is for the physical Volume of Production in the United States, 1901-1913.

ferences and the extent of their dispersion. I wish therefore, to submit a statistical test which will aid in determining:

1. The level of the Index which suggests instability in the growth of a variable and,

2. the validity of the linear exponential calculation as applied to any given variable.

A number of alternative tests presented themselves, but for one or more reasons the chi-square test was chosen to complete the answer to the question of stability of growth. /14 Probably the most common use of the chi-square distribution is as a means of testing the consistency between expected and actual observations. In this type of hypothesis the expected (in our case, trend) observations can be obtained from any one of several sources, including theoretical frequency distributions such as the normal and Poisson distributions, or they may be based upon empirical data of various kinds. In other words, it is not necessary that the expected or trend values be derived from a particular distribution in order to use the chi-square test. Hence, although the Glover method is not based upon a particular distribution chi-square can be used to test for significant difference between the two distributions of data; /15 the pothesis, namely, that there is no significant difference between the two distributions. Next we set the probability level for chi-square which is acceptable, say 5%, or 2.5% on either side for a two-tail test. Then, at the given level of probability, if the chi-square obtained from the two distributions is higher than that which is given in a chi-square table,

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/14 The t and F tests for evaluating the differences between means (in this case the difference between the mean percentage deviation and zero) are inadequate and not much better than the Index of Instability of Growth since the big issue in the problem of the stability of growth is the distribution of actual data with respect to the trend value. The standard error cannot be used since Glover's method is not, strictly speaking, a pure least squares calculation. Moreover, the SE cannot be referred to a normal probability table for the purpose of making probability statements unless the residuals are distributed as the normal curve with zero mean and variance equal to σ^2.

/15 The general form of the chi-square test is:

$$\sum \frac{(f - f_e)^2}{f_e}$$

where f is an actual frequency, f_e is an expected frequency, and the summation is over the several frequencies being tested. The particular equation used in this paper is for 2xr table and takes the form:

$$\frac{N^2}{N_a N_b}\left[\left(\frac{a_1^2}{N_1} + \frac{a_2^2}{N_2} + \dots + \frac{a_n^2}{N_n}\right) - \frac{Na^2}{N}\right]$$

we reject the null hypothesis. On the other hand, if the calculated chi-square is lower than that given in the table the null hypothesis is accepted. In this latter instance, we would judge, therefore, that the two distributions differ primarily (statistically speaking) due to chance. However, it does not follow that the low chi-square complements the Index of Instability of Growth. On the contrary, there is essentially no relationship between the size of the Index and the size of the chi-square. This holds regardless of the number of observations. Chi-square is a function of variance, or absolute differences, while the Index is a function of relative differences. Furthermore, with the Index there is no formal statistical limit or range which will help us decide what is stable or unstable growth. The reason being that there is not associated with the relative differences as used in the Index a statistical measure which provides a basis for judging a stable or unstable Index level. On the other hand, the use of absolute deviations from trend, in other words the concept of variance coupled with a critical level of acceptance (probability that a given χ^2 could have arisen by chance), provides us with sufficient information to make a decision on the null hypothesis.

The chi-square test then, rather than the Index of Instability of Growth, enables us to assess the validity of the growth rate calculation and at the same time make a decision on the stability or instability of the growth trend. If a calculated chi-square is below a given chi-square at the selected confidence level, then the difference between actual and trend are due, essentially, to chance. In other words, there is no reason to suspect that the fluctuations of the actual about the trend line are any more than random fluctuations. Hence, the growth of the particular variable involved would be judged stable; in other words, a sufficiently strong and persistent exponential growth trend is present in the given variable.

Conversely, a calculated chi-square higher than the given one indicates that there is a significant difference between the actual and the trend. It follows that the dispersion about the trend line is too wide to be due, essentially, to chance. Accordingly, it can be inferred that the growth of the given variable is too unstable to be measured with an exponential growth rate; or that there are other problems involved, such as poor original data. In short, a significant chi-square should serve as a warning signal not to accept the calculated growth rate, but to investigate further the original data themselves.

However, the Index is not altogether useless; it can be a very valuable guide in using the growth rate for projection purposes. If it is desired to project a given variable by means of its growth rate, knowledge of the level of the Index provides a clue as to the proportion of deviation that can be expected from the projected trend on the basis of past experience. Allowance can be made either in the growth rate or in the Index for changes from past experience anticipated for the projection period. The chi-square test cannot be used to determine the possible deviation from trend for a projected period.

Indices of the Instability of Growth have been calculated only for the growth rates derived from Glover's method. These results are contained in Table 5 which also lists the answers from the chi-square tests as well as the sum of the absolute deviations from the Glover trend line.

TABLE 1: WOOL PRODUCTION, GREASY BASIS, SELECTED COUNTRIES AND WORLD

(Million Pounds)

Year	Australia	New Zealand	South Africa	Argentina	Uruguay	U.S.A.	U.K.
1948	1,057	367	227	425	144	294	81
1949	1,142	390	230	415	163	273	88
1950	1,118	390	245	430	185	271	89
1951	1,080	407	251	408	188	272	93
1952	1,281	418	274	406	190	290	95
1953	1,246	426	283	397	203	304	100
1954	1,283	455	306	364	198	310	105
1955	1,417	462	314	390	195	305	102
1956	1,564	491	321	388	190	311	105
1957	1,434	496	299	409	200	296	114
1958	1,591	540	314	421	175	293	119
1959	1,689	577	319	423	159	319	124
1960	1,616	580	305	409	181	324	120
1961 /a	1,692	599	322	410	195	325	125

Year	Others	Free World	Communist Bloc	World Total
1948	771	3,366	472	3,838
1949	741	3,442	494	3,936
1950	761	3,489	533	4,022
1951	791	3,490	614	4,104
1952	821	3,775	684	4,459
1953	862	3,821	748	4,569
1954	870	3,891	756	4,647
1955	891	4,076	828	4,904
1956	885	4,255	850	5,105
1957	879	4,127	923	5,050
1958	892	4,345	1,010	5,355
1959	909	4,519	1,093	5,612
1960	908	4,443	1,106	5,549
1961 /a	902	4,570	1,090	5,660

/a Preliminary.

Source: Commonwealth Economic Committee.

TABLE 2: UNITED STATES; GROSS NATIONAL PRODUCT IN 1929 PRICES AND NET PRODUCTION OF ELECTRICAL ENERGY

Year	Net Production of Electrical Energy (Billions kWh)	Gross National Product (Billions US$)
1920	56.6	73.3
1921	53.1	71.6
1922	61.2	75.8
1923	71.4	85.8
1924	75.9	88.4
1925	84.7	90.5
1926	94.2	96.4
1927	101.4	97.3
1928	108.1	98.5
1929	116.7	104.4
1930	114.6	95.1
1931	109.4	89.5
1932	99.4	76.4
1933	102.7	74.2
1934	110.4	80.8
1935	118.9	91.4
1936	136.0	100.9
1937	146.5	109.1
1938	141.9	103.2
1939	161.3	111.0
1940	179.9	121.0
1941	208.3	138.7
1942	233.1	154.7
1943	267.5	170.2
1944	279.5	183.6
1945	271.2	180.9
1946	269.6	166.8
1947	307.4	165.6
1948	336.8	174.4
1949	345.1	171.1
1950	388.7	187.1
1951	433.4	199.9
1952	463.0	206.7
1953	514.2	215.3
1954	544.6	212.6
1955	629.0	230.8

Source: Historical Statistics of the U.S.: Colonial Times to the Present, 1957.

TABLE 3: WOOL PRODUCTION; SUMMARY OF GROWTH RATES AND AMOUNT OF GROWTH

Page 1

Country	Method of Computing Growth Rate /a	Growth Rate %	Actual Amount of Change	Change from Each Method
Argentina	Glover	0.00	- 243	- 244
	Pesek No. 6	/b	- 243	n.a.
	Exponential Curve	- 0.058	- 243	- 247
	Terminal Points	- 0.46	- 243	- 74
Uruguay	Glover	0.70	550	549
	Pesek No. 6	3.62	550	550
	Exponential Curve	0.72	550	549
	Terminal Points	2.36	550	340
U.S.A.	Glover	1.11	66	66
	Pesek No. 6	0.24	66	64
	Exponential Curve	1.12	66	62
	Terminal Points	0.61	66	167
U.K.	Glover	3.44	332	334
	Pesek No. 6	3.84	332	332
	Exponential Curve	3.44	332	330
	Terminal Points	3.71	332	318
Australia	Glover	3.95	4,414	4,414
	Pesek No. 6	3.91	4,414	4,418
	Expo nential Curve	3.98	4,414	4,397
	Terminal Points	3.69	4,414	4,176
New Zealand	Glover	4.01	1,468	1,469
	Pesek No. 6	3.77	1,468	1,470
	Exponential Curve	3.96	1,468	1,466
	Terminal Points	3.84	1,468	1,502
South Africa	Glover	2.76	844	846
	Pesek No. 6	3.53	844	843
	Exponential Curve	2.87	844	742
	Terminal Points	2.73	844	631
Others	Glover	1.64	1,114	1,113
	Pesek No. 6	1.49	1,114	1,111
	Exponential Curve	1.67	1,114	1,111
	Terminal Points	1.40	1,114	1,040
Free World	Glover	2.57	8,544	8,543
	Pesek No. 6	2.52	8,544	8,555
	Exponential Curve	2.58	8,544	8,537
	Terminal Points	2.40	8,544	8,106

/a Time period covered is 1948-1961 for all growth rates.

/b Unable to compute negative growth by Pesek's Method VI.

TABLE 3: WOOL PRODUCTION; SUMMARY OF GROWTH RATES AND AMOUNT OF GROWTH (CONT.)

Page 2

Country	Method of Computing Growth Rate /a	Growth Rate %	Actual Amount of Change	Change from Each Method
Communist Block	Glover	7.01	4,611	4,612
	Pesek No. 6	7.74	4,611	4,611
	Exponential Curve	7.25	4,611	4,613
	Terminal Points	6.67	4,611	3,792
World Total	Glover	3.30	13,155	13,151
	Pesek No. 6	3.29	13,155	13,149
	Exponential Curve	3.32	13,155	13,148
	Terminal Points	3.05	13,155	12,068

/a Time period covered is 1948-1961 for all growth rates.
Exponential curve refers to "exponential curve fit to logs by the least squares method."

TABLE 4: NET ELECTRICAL ENERGY AND GROSS NATIONAL PRODUCT; SUMMARY OF GROWTH RATES AND AMOUNT OF GROWTH

Method of Computing Growth Rate /a	Growth Rate (%)	Actual Amount of Change	Change from Each Method
Net Electrical Energy (billion kWh)			
Glover	7.13	5,700	5,692
Pesek No. 6	6.62	5,700	5,696
Exponential Curve	6.73	5,700	5,556
Terminal Points	7.1	5,700	6,579
Gross National Product (billion dollars, 1929 prices)			
Glover	3.50	2,054	2,050
Pesek No. 6	3.06	2,054	2,051
Exponential Curve	3.37	2,054	2,669
Terminal Points	3.3	2,054	2,284

/a Exponential curve refers to "exponential curve fit to logs by the least squares method."

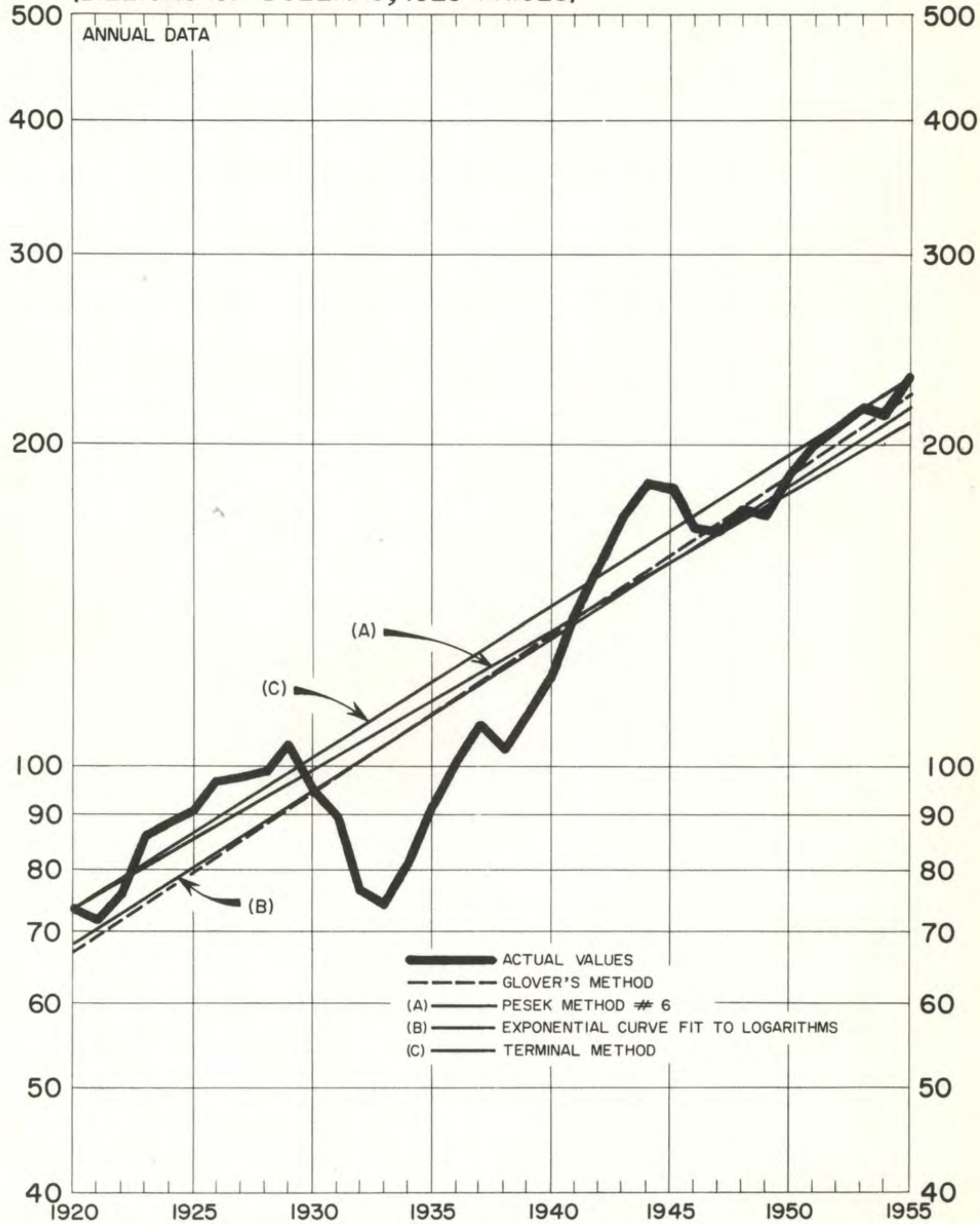
TABLE 5: SUM OF DEVIATIONS FROM GLOVER'S TREND, CHI-SQUARE TEST AND INDEX OF INSTABILITY OF GROWTH

	Sum of Deviations from Glover's Trend	Chi-square Test	Index of Instability of Growth (%)
World Production (million pounds)			
Australia	0	19.40	3.9
South Africa	+ 2	6.40	4.7
New Zealand	+ 1	1.52	1.6
Argentina	+ 1	4.80	3.1
United Kingdom	+ 2	0.80	2.2
Uruguay	- 1	9.60	7.2
United States	0	2.00	3.3
Others	- 1	4.80	2.3
Free World	- 1	10.80	1.6
Communist Bloc	+ 1	9.60	4.0
World Total	- 4	13.20	3.8
Gross National Product (billions US$)	- 4	34.24	10.4
Net Electrical Energy (billions kWh)	- 8	35.68	10.6

Note: All chi-square tests are not significant at the 2.5% probability level; therefore, the null hypothesis is accepted.

UNITED STATES GROSS NATIONAL PRODUCT: ACTUAL AND TREND VALUES

(BILLIONS OF DOLLARS, 1929 PRICES)



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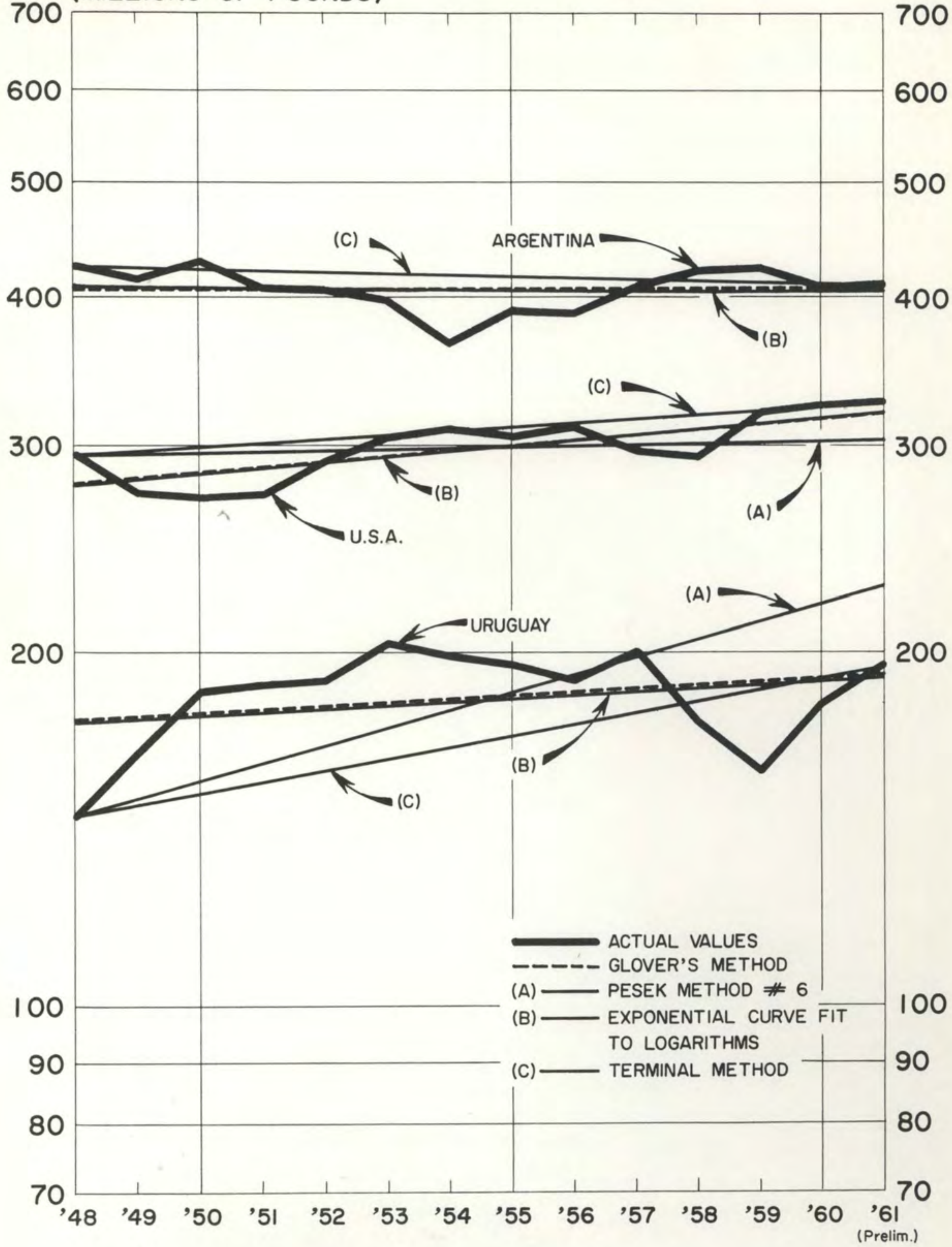
WOOL PRODUCTION: ACTUAL AND TREND VALUES FOR SELECTED COUNTRIES

(MILLIONS OF POUNDS)



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APPENDIX

A P P E N D I X

Calculation of the Glover Method

In order to provide a basis for a thorough understanding of the Glover Method and, most importantly, its application to the measurement of economic growth, an examination of the computational steps involved will prove helpful.

The first step is to calculate the mean value, or M. The formula for this is:

$$M = \frac{\sum xy}{\sum y}$$

where: x = number of times a given observation is removed from the origin time period, beginning with zero for the first observation.

y = actual observation value

For the first observation, "x" takes the value of zero, since the observation in the first time period is the origin. Thereafter, "x" proceeds arithmetically; 1, 2, 3, 4, ... n-1. The resultant mean value (M) represent, as noted earlier, the weighted mean of the "x" series, or number of times removed from the origin. Economically speaking, this makes a great deal of sense. For, given two variables such as, say, GNP and Electricity output each with the same number of observations, the one with the larger mean value (M) would naturally have the larger growth rate. The mean value is not tied to any specific kind of data and, in a sense, is not influenced by the kind of variable used (which is important in a growth rate calculation). Conversely, given two variables each with the same mean value (M) but with different number of observations, the one with the fewer number of observations would have the larger growth rate.

The next step is the calculation of "a," or the trend value for the point of origin:

$$(2) \qquad a = \frac{r - [(n - M) \quad (r - 1)]}{n} \sum y$$

As can be seen "r" is required for the solution of "a." Once "M" is obtained, the rate of growth, "r," can be secured by finding the corresponding "r" in the mean value table shown by Glover /16. This table is set up as follows:

Growth rate r	Number of observations (n) 2	3	4	5	. . .
1.000					
1.001		MEAN	VALUES		
1.002					
.					
.					
.					
.					

Given the mean value and the number of observations, it becomes a simple task to pick off the corresponding growth rate.

In the event of a negative rate of change the following modification is made in the solution for "r" /17.

/16 See Mean Value Tables at the end of this appendix.

/17 I wish to thank Mr. Charles Wittmann of the U.S. Department of Agriculture for drawing my attention to this modification.

a) $n - 1 - M = M'$

b) find a hypothetical "r" using M'

c) then $\frac{1}{r}$ (this is used to obtain "a")

d) $\frac{1}{r} - 1$ = rate of decline

Solving for "a" in a negative rate of change problem requires the use of "$\frac{1}{r}$" from step c above.

MEAN VALUE TABLES

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

$$\mathbf{M} = \sum_{0}^{n-1} x_i \cdot {}_o y_i \Big/ \sum_{0}^{n-1} {}_o y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$n=2$	$n=3$	$n=4$	r	$n=2$	$n=3$	$n=4$
1.001	0.5002	1.0007	1.5012	1.051	0.5124	1.0331	1.5621
2	0.5005	1.0013	1.5025	52	0.5127	1.0338	1.5633
3	0.5007	1.0020	1.5037	53	0.5129	1.0344	1.5645
4	0.5010	1.0027	1.5050	54	0.5131	1.0350	1.5657
5	0.5013	1.0033	1.5062	55	0.5134	1.0357	1.5669
6	0.5015	1.0040	1.5075	56	0.5136	1.0363	1.5681
7	0.5018	1.0047	1.5087	57	0.5139	1.0369	1.5692
8	0.5020	1.0053	1.5100	58	0.5141	1.0376	1.5704
9	0.5022	1.0060	1.5112	59	0.5143	1.0382	1.5716
10	0.5025	1.0066	1.5124	60	0.5146	1.0388	1.5728
1.011	0.5027	1.0073	1.5137	1.061	0.5148	1.0395	1.5739
12	0.5030	1.0079	1.5149	62	0.5150	1.0401	1.5751
13	0.5032	1.0086	1.5161	63	0.5153	1.0407	1.5763
14	0.5035	1.0093	1.5174	64	0.5155	1.0413	1.5775
15	0.5037	1.0099	1.5186	65	0.5157	1.0420	1.5786
16	0.5040	1.0106	1.5198	66	0.5160	1.0426	1.5798
17	0.5042	1.0112	1.5211	67	0.5162	1.0432	1.5810
18	0.5045	1.0119	1.5223	68	0.5164	1.0438	1.5821
19	0.5047	1.0125	1.5235	69	0.5167	1.0444	1.5833
20	0.5050	1.0132	1.5248	70	0.5169	1.0451	1.5845
1.021	0.5052	1.0139	1.5260	1.071	0.5171	1.0457	1.5856
22	0.5054	1.0145	1.5272	72	0.5174	1.0463	1.5868
23	0.5057	1.0152	1.5284	73	0.5176	1.0469	1.5879
24	0.5059	1.0158	1.5296	74	0.5178	1.0476	1.5891
25	0.5062	1.0165	1.5309	75	0.5181	1.0482	1.5903
26	0.5064	1.0171	1.5321	76	0.5183	1.0488	1.5914
27	0.5067	1.0178	1.5333	77	0.5185	1.0494	1.5926
28	0.5069	1.0184	1.5345	78	0.5188	1.0500	1.5937
29	0.5071	1.0191	1.5357	79	0.5190	1.0506	1.5949
30	0.5074	1.0197	1.5369	80	0.5192	1.0513	1.5960
1.031	0.5076	1.0203	1.5382	1.081	0.5195	1.0519	1.5972
32	0.5079	1.0210	1.5394	82	0.5197	1.0525	1.5983
33	0.5081	1.0216	1.5406	83	0.5199	1.0531	1.5995
34	0.5084	1.0223	1.5418	84	0.5202	1.0537	1.6006
35	0.5086	1.0229	1.5430	85	0.5204	1.0543	1.6018
36	0.5088	1.0236	1.5442	86	0.5206	1.0549	1.6029
37	0.5091	1.0242	1.5454	87	0.5208	1.0556	1.6041
38	0.5093	1.0249	1.5466	88	0.5211	1.0562	1.6052
39	0.5096	1.0255	1.5478	89	0.5213	1.0568	1.6064
40	0.5098	1.0261	1.5490	90	0.5215	1.0574	1.6075
1.041	0.5100	1.0268	1.5502	1.091	0.5218	1.0580	1.6086
42	0.5103	1.0274	1.5514	92	0.5220	1.0586	1.6098
43	0.5105	1.0281	1.5526	93	0.5222	1.0592	1.6109
44	0.5108	1.0287	1.5538	94	0.5224	1.0598	1.6120
45	0.5110	1.0293	1.5550	95	0.5227	1.0604	1.6132
46	0.5112	1.0300	1.5562	96	0.5229	1.0610	1.6143
47	0.5115	1.0306	1.5574	97	0.5231	1.0616	1.6154
48	0.5117	1.0312	1.5586	98	0.5234	1.0622	1.6166
49	0.5120	1.0319	1.5598	99	0.5236	1.0628	1.6177
50	0.5122	1.0325	1.5609	1.100	0.5238	1.0634	1.6188

$$a = \frac{r-(n-\mathbf{M})(r-1)}{n} \sum_{0}^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

$$M = \sum_{0}^{n-1} x_i \cdot {}_o y_i / \sum_{0}^{n-1} {}_o y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$n=5$	$n=6$	$n=7$	r	$n=5$	$n=6$	$n=7$
1.001	2.0020	2.5029	3.0040	1.051	2.0994	2.6449	3.1986
2	2.0040	2.5058	3.0080	52	2.1013	2.6476	3.2023
3	2.0060	2.5087	3.0120	53	2.1032	2.6504	3.2061
4	2.0080	2.5117	3.0160	54	2.1051	2.6531	3.2099
5	2.0100	2.5145	3.0200	55	2.1069	2.6559	3.2137
6	2.0120	2.5175	3.0239	56	2.1088	2.6586	3.2174
7	2.0140	2.5204	3.0279	57	2.1107	2.6613	3.2212
8	2.0159	2.5232	3.0319	58	2.1126	2.6641	3.2249
9	2.0179	2.5261	3.0358	59	2.1145	2.6669	3.2287
10	2.0199	2.5290	3.0398	60	2.1164	2.6696	3.2324
1.011	2.0219	2.5319	3.0438	1.061	2.1182	2.6723	3.2362
12	2.0239	2.5348	3.0477	62	2.1201	2.6751	3.2399
13	2.0258	2.5377	3.0517	63	2.1220	2.6778	3.2436
14	2.0278	2.5405	3.0556	64	2.1239	2.6805	3.2473
15	2.0298	2.5434	3.0595	65	2.1257	2.6832	3.2511
16	2.0317	2.5463	3.0635	66	2.1276	2.6859	3.2548
17	2.0337	2.5492	3.0674	67	2.1295	2.6887	3.2585
18	2.0357	2.5520	3.0713	68	2.1313	2.6914	3.2622
19	2.0376	2.5549	3.0753	69	2.1332	2.6941	3.2659
20	2.0396	2.5577	3.0792	70	2.1351	2.6968	3.2696
1.021	2.0416	2.5606	3.0831	1.071	2.1369	2.6995	3.2733
22	2.0435	2.5635	3.0870	72	2.1388	2.7022	3.2770
23	2.0455	2.5663	3.0909	73	2.1406	2.7049	3.2807
24	2.0474	2.5692	3.0948	74	2.1425	2.7076	3.2844
25	2.0494	2.5720	3.0987	75	2.1443	2.7103	3.2880
26	2.0513	2.5748	3.1026	76	2.1462	2.7129	3.2917
27	2.0533	2.5777	3.1065	77	2.1480	2.7156	3.2954
28	2.0552	2.5805	3.1104	78	2.1498	2.7183	3.2990
29	2.0572	2.5833	3.1143	79	2.1517	2.7210	3.3027
30	2.0591	2.5862	3.1181	80	2.1535	2.7237	3.3063
1.031	2.0610	2.5890	3.1220	1.081	2.1554	2.7263	3.3100
32	2.0630	2.5918	3.1259	82	2.1572	2.7290	3.3136
33	2.0649	2.5946	3.1298	83	2.1590	2.7317	3.3173
34	2.0668	2.5975	3.1336	84	2.1609	2.7343	3.3209
35	2.0688	2.6003	3.1375	85	2.1627	2.7370	3.3245
36	2.0707	2.6031	3.1413	86	2.1645	2.7396	3.3281
37	2.0726	2.6059	3.1452	87	2.1663	2.7423	3.3318
38	2.0745	2.6087	3.1490	88	2.1682	2.7449	3.3354
39	2.0765	2.6115	3.1528	89	2.1700	2.7476	3.3390
40	2.0784	2.6143	3.1567	90	2.1718	2.7502	3.3426
1.041	2.0803	2.6171	3.1605	1.091	2.1736	2.7528	3.3462
42	2.0822	2.6199	3.1643	92	2.1754	2.7555	3.3498
43	2.0841	2.6227	3.1682	93	2.1772	2.7581	3.3534
44	2.0860	2.6254	3.1720	94	2.1791	2.7607	3.3570
45	2.0880	2.6282	3.1758	95	2.1809	2.7634	3.3605
46	2.0899	2.6310	3.1796	96	2.1827	2.7660	3.3641
47	2.0918	2.6338	3.1834	97	2.1845	2.7686	3.3677
48	2.0937	2.6366	3.1872	98	2.1863	2.7712	3.3713
49	2.0956	2.6393	3.1910	99	2.1881	2.7738	3.3748
50	2.0975	2.6421	3.1948	1.100	2.1899	2.7764	3.3784

$$a = \frac{r - (n - M)(r-1)}{n} \sum_{0}^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y=ar^x$ to n Observed Statistical Ordinates

$$\mathbf{M}=\sum_{0}^{n-1} x_i \cdot {}_o y_i \Big/ \sum_{0}^{n-1} {}_o y_i = n/(1-r^{-n})-1/(1-r^{-1})$$

r	$n=8$	$n=9$	$n=10$	r	$n=8$	$n=9$	$n=10$
1.001	3.5052	4.0067	4.5082	1.051	3.7604	4.3305	4.9087
2	3.5105	4.0133	4.5165	52	3.7654	4.3368	4.9164
3	3.5157	4.0200	4.5247	53	3.7703	4.3430	4.9242
4	3.5210	4.0266	4.5329	54	3.7746	4.3493	4.9319
5	3.5262	4.0333	4.5412	55	3.7802	4.3555	4.9396
6	3.5314	4.0399	4.5494	56	3.7851	4.3618	4.9473
7	3.5366	4.0465	4.5576	57	3.7901	4.3680	4.9550
8	3.5418	4.0531	4.5657	58	3.7950	4.3742	4.9627
9	3.5470	4.0597	4.5739	59	3.7999	4.3805	4.9703
10	3.5522	4.0663	4.5821	60	3.8048	4.3867	4.9780
1.011	3.5574	4.0729	4.5902	1.061	3.8097	4.3929	4.9856
12	3.5626	4.0795	4.5984	62	3.8146	4.3991	4.9933
13	3.5678	4.0861	4.6065	63	3.8195	4.4052	5.0009
14	3.5730	4.0927	4.6147	64	3.8243	4.4114	5.0085
15	3.5781	4.0992	4.6228	65	3.8292	4.4176	5.0161
16	3.5833	4.1058	4.6309	66	3.8341	4.4237	5.0237
17	3.5885	4.1123	4.6390	67	3.8389	4.4299	5.0313
18	3.5936	4.1189	4.6471	68	3.8438	4.4360	5.0388
19	3.5988	4.1254	4.6552	69	3.8486	4.4421	5.0464
20	3.6039	4.1319	4.6633	70	3.8535	4.4483	5.0539
1.021	3.6091	4.1385	4.6713	1.071	3.8583	4.4544	5.0615
22	3.6142	4.1450	4.6794	72	3.8631	4.4605	5.0690
23	3.6193	4.1515	4.6874	73	3.8679	4.4666	5.0765
24	3.6244	4.1580	4.6955	74	3.8727	4.4727	5.0840
25	3.6296	4.1645	4.7035	75	3.8775	4.4787	5.0915
26	3.6347	4.1710	4.7115	76	3.8823	4.4848	5.0989
27	3.6398	4.1774	4.7195	77	3.8871	4.4908	5.1064
28	3.6449	4.1839	4.7275	78	3.8919	4.4969	5.1138
29	3.6500	4.1904	4.7355	79	3.8967	4.5029	5.1213
30	3.6550	4.1968	4.7435	80	3.9015	4.5090	5.1287
1.031	3.6601	4.2033	4.7515	1.081	3.9062	4.5150	5.1361
32	3.6652	4.2097	4.7594	82	3.9110	4.5210	5.1435
33	3.6703	4.2161	4.7674	83	3.9158	4.5270	5.1509
34	3.6753	4.2226	4.7753	84	3.9205	4.5330	5.1583
35	3.6804	4.2290	4.7832	85	3.9252	4.5390	5.1656
36	3.6854	4.2354	4.7912	86	3.9300	4.5450	5.1730
37	3.6905	4.2418	4.7991	87	3.9347	4.5509	5.1803
38	3.6955	4.2482	4.8070	88	3.9394	4.5569	5.1876
39	3.7005	4.2545	4.8149	89	3.9441	4.5628	5.1949
40	3.7056	4.2609	4.8227	90	3.9488	4.5688	5.2022
1.041	3.7106	4.2673	4.8306	1.091	3.9535	4.5747	5.2095
42	3.7156	4.2736	4.8385	92	3.9582	4.5806	5.2168
43	3.7206	4.2800	4.8463	93	3.9629	4.5865	5.2241
44	3.7256	4.2863	4.8541	94	3.9676	4.5924	5.2313
45	3.7306	4.2927	4.8620	95	3.9723	4.5983	5.2385
46	3.7356	4.2990	4.8698	96	3.9769	4.6042	5.2458
47	3.7406	4.3053	4.8776	97	3.9816	4.6101	5.2530
48	3.7456	4.3116	4.8854	98	3.9862	4.6159	5.2602
49	3.7508	4.3179	4.8931	99	3.9909	4.6218	5.2674
50	3.7555	4.3242	4.9009	1.100	3.9955	4.6276	5.2745

$$a=\frac{n-(n-\mathbf{M})(r-1)}{n}\sum_{0}^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

$$M = \sum_{0}^{n-1} x_i \cdot {}_o y_i \Big/ \sum_{0}^{n-1} {}_o y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$n=11$	$n=12$	$n=13$	r	$n=11$	$n=12$	$n=13$
1.001	5.0100	5.5119	6.0140	1.051	5.4949	6.0892	6.6916
2	5.0200	5.5238	6.0280	52	5.5043	6.1004	6.7046
3	5.0299	5.5357	6.0419	53	5.5137	6.1115	6.7176
4	5.0399	5.5476	6.0560	54	5.5230	6.1226	6.7306
5	5.0500	5.5594	6.0698	55	5.5323	6.1337	6.7436
6	5.0598	5.5713	6.0837	56	5.5416	6.1447	6.7565
7	5.0698	5.5831	6.0977	57	5.5509	6.1557	6.7694
8	5.0797	5.5949	6.1115	58	5.5602	6.1668	6.7823
9	5.0896	5.6068	6.1254	59	5.5695	6.1778	6.7952
10	5.0995	5.6186	6.1393	60	5.5787	6.1887	6.8080
1.011	5.1094	5.6303	6.1531	1.061	5.5879	6.1997	6.8208
12	5.1192	5.6421	6.1669	62	5.5972	6.2106	6.8336
13	5.1291	5.6539	6.1807	63	5.6064	6.2216	6.8464
14	5.1390	5.6656	6.1945	64	5.6156	6.2325	6.8592
15	5.1488	5.6773	6.2083	65	5.6247	6.2434	6.8719
16	5.1587	5.6890	6.2221	66	5.6339	6.2542	6.8846
17	5.1685	5.7007	6.2358	67	5.6430	6.2651	6.8973
18	5.1783	5.7124	6.2495	68	5.6522	6.2759	6.9099
19	5.1881	5.7241	6.2632	69	5.6613	6.2867	6.9226
20	5.1979	5.7358	6.2769	70	5.6704	6.2975	6.9352
1.021	5.2076	5.7474	6.2906	1.071	5.6795	6.3083	6.9477
22	5.2174	5.7590	6.3042	72	5.6885	6.3190	6.9603
23	5.2272	5.7706	6.3179	73	5.6976	6.3297	6.9728
24	5.2369	5.7822	6.3315	74	5.7066	6.3404	6.9853
25	5.2466	5.7938	6.3451	75	5.7156	6.3511	6.9978
26	5.2563	5.8054	6.3587	76	5.7246	6.3618	7.0102
27	5.2660	5.8169	6.3722	77	5.7336	6.3724	7.0227
28	5.2757	5.8285	6.3858	78	5.7426	6.3831	7.0351
29	5.2854	5.8400	6.3993	79	5.7516	6.3937	7.0474
30	5.2951	5.8515	6.4128	80	5.7605	6.4043	7.0598
1.031	5.3047	5.8630	6.4263	1.081	5.7694	6.4148	7.0721
32	5.3144	5.8745	6.4397	82	5.7783	6.4254	7.0844
33	5.3240	5.8859	6.4532	83	5.7872	6.4359	7.0967
34	5.3336	5.8974	6.4666	84	5.7961	6.4464	7.1089
35	5.3432	5.9088	6.4800	85	5.8050	6.4569	7.1211
36	5.3528	5.9202	6.4934	86	5.8138	6.4673	7.1333
37	5.3623	5.9316	6.5068	87	5.8226	6.4778	7.1455
38	5.3719	5.9430	6.5201	88	5.8315	6.4882	7.1576
39	5.3815	5.9543	6.5334	89	5.8403	6.4986	7.1698
40	5.3910	5.9657	6.5467	90	5.8490	6.5090	7.1818
1.041	5.4005	5.9770	6.5600	1.091	5.8578	6.5193	7.1939
42	5.4100	5.9883	6.5732	92	5.8666	6.5297	7.2059
43	5.4195	5.9996	6.5865	93	5.8753	6.5400	7.2179
44	5.4290	6.0108	6.5997	94	5.8840	6.5503	7.2299
45	5.4384	6.0221	6.6129	95	5.8927	6.5606	7.2419
46	5.4479	6.0333	6.6260	96	5.9014	6.5708	7.2538
47	5.4573	6.0445	6.6392	97	5.9100	6.5810	7.2657
48	5.4668	6.0558	6.6523	98	5.9187	6.5912	7.2776
49	5.4762	6.0669	6.6654	99	5.9273	6.6014	7.2894
50	5.4856	6.0781	6.6785	1.100	5.9359	6.6116	7.3012

$$a = \frac{r - (n-M)(r-1)}{n} \sum_{0}^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

$$M = \sum_0^{n-1} x_i \cdot {}_o y_i \Big/ \sum_0^{n-1} {}_o y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$n=14$	$n=15$	$n=16$	r	$n=14$	$n=15$	$n=16$
1.001	6.5163	7.0187	7.5212	1.051	7.3018	7.9200	8.5460
2	6.5325	7.0373	7.5425	52	7.3169	7.9372	8.5656
3	6.5487	7.0559	7.5636	53	7.3319	7.9545	8.5851
4	6.5649	7.0745	7.5848	54	7.3470	7.9716	8.6046
5	6.5810	7.0931	7.6060	55	7.3620	7.9888	8.6240
6	6.5972	7.1117	7.6271	56	7.3769	8.0059	8.6434
7	6.6133	7.1302	7.6482	57	7.3919	8.0230	8.6628
8	6.6295	7.1487	7.6693	58	7.4068	8.0400	8.6821
9	6.6456	7.1672	7.6903	59	7.4216	8.0571	8.7013
10	6.6616	7.1857	7.7114	60	7.4365	8.0740	8.7206
1.011	6.6777	7.2041	7.7324	1.061	7.4513	8.0910	8.7398
12	6.6937	7.2225	7.7533	62	7.4661	8.1079	8.7589
13	6.7098	7.2410	7.7743	63	7.4808	8.1247	8.7780
14	6.7258	7.2593	7.7952	64	7.4956	8.1415	8.7970
15	6.7418	7.2777	7.8161	65	7.5103	8.1583	8.8160
16	6.7577	7.2960	7.8369	66	7.5249	8.1751	8.8350
17	6.7737	7.3143	7.8578	67	7.5396	8.1918	8.8539
18	6.7896	7.3326	7.8786	68	7.5542	8.2085	8.8727
19	6.8055	7.3509	7.8994	69	7.5687	8.2251	8.8916
20	6.8214	7.3691	7.9201	70	7.5833	8.2417	8.9103
1.021	6.8372	7.3873	7.9408	1.071	7.5978	8.2583	8.9290
22	6.8531	7.4055	7.9615	72	7.6123	8.2748	8.9477
23	6.8689	7.4236	7.9821	73	7.6267	8.2913	8.9663
24	6.8847	7.4418	8.0028	74	7.6411	8.3077	8.9849
25	6.9005	7.4599	8.0234	75	7.6555	8.3241	9.0034
26	6.9162	7.4779	8.0439	76	7.6699	8.3405	9.0219
27	6.9319	7.4960	8.0644	77	7.6842	8.3568	9.0404
28	6.9476	7.5140	8.0849	78	7.6985	8.3731	9.0587
29	6.9633	7.5320	8.1054	79	7.7127	8.3893	9.0771
30	6.9790	7.5500	8.1258	80	7.7269	8.4055	9.0954
1.031	6.9946	7.5679	8.1462	1.081	7.7411	8.4217	9.1136
32	7.0102	7.5858	8.1665	82	7.7553	8.4378	9.1318
33	7.0258	7.6037	8.1868	83	7.7694	8.4539	9.1499
34	7.0413	7.6215	8.2071	84	7.7835	8.4700	9.1680
35	7.0569	7.6393	8.2274	85	7.7976	8.4860	9.1861
36	7.0724	7.6571	8.2476	86	7.8116	8.5019	9.2041
37	7.0879	7.6748	8.2677	87	7.8256	8.5178	9.2220
38	7.1033	7.6926	8.2879	88	7.8396	8.5337	9.2399
39	7.1187	7.7103	8.3079	89	7.8535	8.5495	9.2577
40	7.1341	7.7279	8.3280	90	7.8674	8.5654	9.2755
1.041	7.1495	7.7455	8.3480	1.091	7.8812	8.5811	9.2933
42	7.1649	7.7631	8.3680	92	7.8951	8.5968	9.3110
43	7.1802	7.7807	8.3879	93	7.9089	8.6125	9.3286
44	7.1955	7.7982	8.4078	94	7.9226	8.6282	9.3462
45	7.2108	7.8157	8.4277	95	7.9364	8.6437	9.3637
46	7.2260	7.8332	8.4475	96	7.9500	8.6593	9.3812
47	7.2412	7.8506	8.4673	97	7.9637	8.6748	9.3987
48	7.2564	7.8680	8.4870	98	7.9773	8.6903	9.4160
49	7.2716	7.8854	8.5067	99	7.9909	8.7057	9.4334
50	7.2867	7.9027	8.5264	1.100	8.0045	8.7211	9.4507

$$a = \frac{r - (n - M)(r - 1)}{n} \sum_0^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

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r	$n=17$	$n=18$	$n=19$	r	$n=17$	$n=18$	$n=19$
1.001	8.0240	8.5269	9.0300	1.051	9.1798	9.8213	10.4704
2	8.0480	8.5538	9.0599	52	9.2018	9.8459	10.4977
3	8.0719	8.5806	9.0899	53	9.2237	9.8704	10.5249
4	8.0958	8.6074	9.1198	54	9.2457	9.8948	10.5521
5	8.1197	8.6342	9.1496	55	9.2675	9.9192	10.5791
6	8.1435	8.6610	9.1794	56	9.2893	9.9436	10.6061
7	8.1674	8.6877	9.2092	57	9.3111	9.9679	10.6330
8	8.1912	8.7144	9.2390	58	9.3328	9.9921	10.6598
9	8.2149	8.7411	9.2687	59	9.3544	10.0162	10.6866
10	8.2387	8.7677	9.2983	60	9.3760	10.0403	10.7133
1.011	8.2624	8.7943	9.3280	1.061	9.3976	10.0643	10.7399
12	8.2861	8.8208	9.3575	62	9.4191	10.0883	10.7664
13	8.3097	8.8474	9.3871	63	9.4405	10.1122	10.7929
14	8.3334	8.8738	9.4166	64	9.4619	10.1360	10.8192
15	8.3569	8.9003	9.4461	65	9.4832	10.1597	10.8455
16	8.3805	8.9267	9.4755	66	9.5045	10.1834	10.8717
17	8.4040	8.9530	9.5048	67	9.5257	10.2071	10.8979
18	8.4275	8.9794	9.5342	68	9.5468	10.2306	10.9239
19	8.4510	9.0056	9.5635	69	9.5679	10.2541	10.9499
20	8.4744	9.0319	9.5927	70	9.5890	10.2775	10.9758
1.021	8.4977	9.0581	9.6219	1.071	9.6100	10.3009	11.0016
22	8.5211	9.0842	9.6510	72	9.6309	10.3242	11.0274
23	8.5444	9.1104	9.6801	73	9.6518	10.3474	11.0530
24	8.5677	9.1364	9.7091	74	9.6726	10.3705	11.0786
25	8.5909	9.1625	9.7381	75	9.6933	10.3936	11.1041
26	8.6141	9.1884	9.7670	76	9.7140	10.4166	11.1295
27	8.6372	9.2144	9.7959	77	9.7347	10.4396	11.1548
28	8.6603	9.2403	9.8247	78	9.7553	10.4624	11.1801
29	8.6834	9.2661	9.8534	79	9.7758	10.4852	11.2052
30	8.7064	9.2919	9.8821	80	9.7963	10.5080	11.2303
1.031	8.7294	9.3176	9.9108	1.081	9.8167	10.5306	11.2553
32	8.7524	9.3433	9.9394	82	9.8370	10.5532	11.2802
33	8.7753	9.3690	9.9679	83	9.8573	10.5757	11.3050
34	8.7981	9.3946	9.9963	84	9.8775	10.5982	11.3298
35	8.8209	9.4201	10.0247	85	9.8977	10.6206	11.3544
36	8.8437	9.4456	10.0531	86	9.9178	10.6429	11.3790
37	8.8665	9.4710	10.0814	87	9.9378	10.6651	11.4035
38	8.8891	9.4963	10.1096	88	9.9578	10.6872	11.4279
39	8.9118	9.5217	10.1378	89	9.9778	10.7093	11.4523
40	8.9344	9.5470	10.1658	90	9.9976	10.7311	11.4764
1.041	8.9569	9.5722	10.1939	1.091	10.0174	10.7533	11.5006
42	8.9794	9.5974	10.2218	92	10.0372	10.7752	11.5246
43	9.0019	9.6225	10.2497	93	10.0569	10.7969	11.5486
44	9.0243	9.6475	10.2776	94	10.0765	10.8187	11.5725
45	9.0466	9.6725	10.3053	95	10.0960	10.8403	11.5963
46	9.0690	9.6975	10.3330	96	10.1155	10.8619	11.6200
47	9.0912	9.7224	10.3606	97	10.1350	10.8834	11.6436
48	9.1134	9.7472	10.3882	98	10.1543	10.9048	11.6671
49	9.1356	9.7719	10.4157	99	10.1737	10.9262	11.6905
50	9.1577	9.7966	10.4431	1.100	10.1929	10.9474	11.7139

$$a = \frac{r - (n - M)(r-1)}{n} \sum_{0}^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y=ar^x$ to n Observed Statistical Ordinates

$$M=\sum_0^{n-1} x_i \cdot {}_o y_i \Big/ \sum_0^{n-1} {}_o y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$n=20$	$n=21$	$n=22$	r	$n=20$	$n=21$	$n=22$
1.001	9.5332	10.0367	10.5402	1.051	11.1272	11.7915	12.4632
2	9.5664	10.0733	10.5804	52	11.1573	11.8245	12.4992
3	9.5996	10.1110	10.6206	53	11.1873	11.8574	12.5352
4	9.6327	10.1464	10.6607	54	11.2172	11.8902	12.5710
5	9.6658	10.1828	10.7007	55	11.2470	11.9229	12.6067
6	9.6989	10.2193	10.7407	56	11.2768	11.9555	12.6423
7	9.7319	10.2557	10.7807	57	11.3064	11.9880	12.6777
8	9.7648	10.2920	10.8206	58	11.3360	12.0204	12.7131
9	9.7978	10.3283	10.8604	59	11.3655	12.0527	12.7483
10	9.8306	10.3646	10.9002	60	11.3949	12.0849	12.7834
1.011	9.8635	10.4008	10.9399	1.061	11.4241	12.1170	12.8184
12	9.8962	10.4369	10.9796	62	11.4534	12.1490	12.8532
13	9.9290	10.4730	11.0192	63	11.4825	12.1809	12.8879
14	9.9617	10.5090	11.0587	64	11.5115	12.2126	12.9225
15	9.9943	10.5450	11.0982	65	11.5404	12.2443	12.9570
16	10.0269	10.5809	11.1376	66	11.5693	12.2759	12.9914
17	10.0594	10.6168	11.1769	67	11.5980	12.3073	13.0256
18	10.0919	10.6526	11.2162	68	11.6267	12.3386	13.0597
19	10.1243	10.6883	11.2554	69	11.6552	12.3699	13.0937
20	10.1567	10.7240	11.2945	70	11.6837	12.4010	13.1275
1.021	10.1890	10.7596	11.3336	1.071	11.7121	12.4320	13.1612
22	10.2213	10.7952	11.3726	72	11.7403	12.4629	13.1948
23	10.2535	10.8306	11.4115	73	11.7685	12.4937	13.2283
24	10.2856	10.8660	11.4503	74	11.7966	12.5243	13.2616
25	10.3177	10.9014	11.4890	75	11.8246	12.5549	13.2948
26	10.3497	10.9366	11.5277	76	11.8525	12.5853	13.3279
27	10.3817	10.9718	11.5662	77	11.8803	12.6157	13.3608
28	10.4136	11.0069	11.6047	78	11.9080	12.6459	13.3936
29	10.4454	11.0420	11.6431	79	11.9356	12.6760	13.4263
30	10.4771	11.0769	11.6814	80	11.9631	12.7060	13.4588
1.031	10.5088	11.1118	11.7196	1.081	11.9905	12.7358	13.4912
32	10.5405	11.1466	11.7578	82	12.0178	12.7656	13.5235
33	10.5720	11.1813	11.7958	83	12.0450	12.7952	13.5556
34	10.6035	11.2160	11.8337	84	12.0721	12.8248	13.5876
35	10.6349	11.2505	11.8716	85	12.0991	12.8542	13.6195
36	10.6662	11.2850	11.9093	86	12.1260	12.8835	13.6512
37	10.6975	11.3194	11.9470	87	12.1528	12.9126	13.6828
38	10.7287	11.3537	11.9845	88	12.1795	12.9417	13.7143
39	10.7598	11.3879	12.0220	89	12.2061	12.9706	13.7456
40	10.7909	11.4221	12.0593	90	12.2326	12.9994	13.7768
1.041	10.8218	11.4561	12.0966	1.091	12.2589	13.0281	13.8078
42	10.8527	11.4901	12.1337	92	12.2852	13.0567	13.8387
43	10.8836	11.5239	12.1708	93	12.3114	13.0852	13.8695
44	10.9143	11.5578	12.2077	94	12.3375	13.1135	13.9002
45	10.9450	11.5914	12.2445	95	12.3635	13.1418	13.9307
46	10.9755	11.6250	12.2813	96	12.3894	13.1699	13.9610
47	11.0060	11.6585	12.3179	97	12.4152	13.1979	13.9913
48	11.0364	11.6919	12.3544	98	12.4409	13.2257	14.0213
49	11.0668	11.7252	12.3908	99	12.4664	13.2535	14.0513
50	11.0970	11.7584	12.4270	1.100	12.4919	13.2811	14.0811

$$a=\frac{r-(n-M)(r-1)}{n}\sum_0^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y=ar^x$ to n Observed Statistical Ordinates

$$M=\sum_{0}^{n-1} x_i \cdot {}_oy_i \Big/ \sum_{0}^{n-1} {}_oy_i = n/(1-r^{-n})-1/(1-r^{-1})$$

r	n=23	n=24	n=25	r	n=23	n=24	n=25
1.001	11.0440	11.5479	12.0520	1.051	13.1423	13.8286	14.5222
2	11.0879	11.5957	12.1039	52	13.1815	13.8711	14.5680
3	11.1318	11.6435	12.1558	53	13.2205	13.9133	14.6136
4	11.1756	11.6913	12.2076	54	13.2594	13.9555	14.6590
5	11.2194	11.7389	12.2593	55	13.2982	13.9974	14.7043
6	11.2631	11.7865	12.3110	56	13.3369	14.0393	14.7493
7	11.3068	11.8341	12.3626	57	13.3754	14.0809	14.7942
8	11.3504	11.8816	12.4141	58	13.4138	14.1224	14.8389
9	11.3940	11.9290	12.4655	59	13.4520	14.1638	14.8835
10	11.4374	11.9263	12.5169	60	13.4901	14.2049	14.9278
1.011	11.4809	12.0236	12.5682	1.061	13.5280	14.2460	14.9720
12	11.5242	12.0708	12.6194	62	13.5659	14.2868	15.0159
13	11.5675	12.1179	12.6705	63	13.6035	14.3275	15.0597
14	11.6107	12.1649	12.7215	64	13.6410	14.3680	15.1033
15	11.6538	12.2119	12.7724	65	13.6784	14.4084	15.1467
16	11.6969	12.2588	12.8232	66	13.7157	14.4486	15.1899
17	11.7399	12.3055	12.8740	67	13.7528	14.4886	15.2330
18	11.7828	12.3522	12.9246	68	13.7897	14.5285	15.2758
19	11.8256	12.3988	12.9751	69	13.8265	14.5681	15.3184
20	11.8683	12.4453	13.0255	70	13.8631	14.6077	15.3609
1.021	11.9110	12.4917	13.0759	1.071	13.8996	14.6470	15.4032
22	11.9535	12.5380	13.1260	72	13.9360	14.6862	15.4452
23	11.9960	12.5842	13.1761	73	13.9722	14.7252	15.4871
24	12.0384	12.6303	13.2261	74	14.0083	14.7640	15.5288
25	12.0807	12.6763	13.2759	75	14.0442	14.8027	15.5702
26	12.1229	12.7222	13.3256	76	14.0799	14.8412	15.6115
27	12.1650	12.7680	13.3752	77	14.1155	14.8795	15.6526
28	12.2070	12.8136	13.4247	78	14.1509	14.9176	15.6935
29	12.2489	12.8592	13.4740	79	14.1862	14.9556	15.7341
30	12.2907	12.9046	13.5232	80	14.2214	14.9934	15.7746
1.031	12.3324	12.9499	13.5723	1.081	14.2564	15.0310	15.8150
32	12.3739	12.9951	13.6212	82	14.2912	15.0684	15.8550
33	12.4154	13.0402	13.6700	83	14.3259	15.1057	15.8949
34	12.4568	13.0851	13.7187	84	14.3604	15.1428	15.9345
35	12.4981	13.1299	13.7672	85	14.3948	15.1797	15.9740
36	12.5392	13.1746	13.8155	86	14.4290	15.2164	16.0133
37	12.5803	13.2192	13.8637	87	14.4630	15.2530	16.0524
38	12.6212	13.2636	13.9118	88	14.4969	15.2894	16.0913
39	12.6620	13.3079	13.9597	89	14.5307	15.3256	16.1300
40	12.7027	13.3521	14.0075	90	14.5643	15.3616	16.1684
1.041	12.7433	13.3961	14.0551	1.091	14.5977	15.3974	16.2067
42	12.7837	13.4400	14.1025	92	14.6310	15.4331	16.2447
43	12.8241	13.4838	14.1498	93	14.6641	15.4686	16.2826
44	12.8643	13.5274	14.1969	94	14.6970	15.5039	16.3203
45	12.9044	13.5709	14.2439	95	14.7299	15.5390	16.3577
46	12.9444	13.6142	14.2907	96	14.7625	15.5739	16.3950
47	12.9842	13.6574	14.3373	97	14.7950	15.6087	16.4321
48	13.0239	13.7004	14.3838	98	14.8273	15.6433	16.4686
49	13.0635	13.7433	14.4301	99	14.8595	15.6777	16.5056
50	13.1029	13.7860	14.4762	1.100	14.8915	15.7119	16.5420

$$a=\frac{r-(n-M)\,(r-1)}{n}\sum_{0}^{n-1} {}_oy_i$$

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

$$M = \sum_0^{n-1} x_i \cdot {}_0y_i \Big/ \sum_0^{n-1} {}_0y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$n=26$	$n=27$	$n=28$	r	$n=26$	$n=27$	$n=28$
1.001	12.5562	13.0606	13.5652	1.051	15.2229	15.9306	16.6452
2	12.6124	13.1212	13.6304	52	15.2721	15.9833	16.7016
3	12.6685	13.1817	13.6954	53	15.3211	16.0358	16.7577
4	12.7245	13.2421	13.7604	54	15.3699	16.0881	16.8136
5	12.7805	13.3025	13.8253	55	15.4185	16.1402	16.8691
6	12.8364	13.3627	13.8901	56	15.4670	16.1920	16.9245
7	12.8922	13.4229	13.9549	57	15.5152	16.2437	16.9796
8	12.9479	13.4830	14.0195	58	15.5632	16.2950	17.0344
9	13.0035	13.5430	14.0840	59	15.6110	16.3462	17.0889
10	13.0591	13.6029	14.1484	60	15.6586	16.3971	17.1432
1.011	13.1145	13.6627	14.2127	1.061	15.7059	16.4477	17.1972
12	13.1699	13.7224	14.2769	62	15.7531	16.4981	17.2509
13	13.2252	13.7820	14.3410	63	15.8000	16.5483	17.3044
14	13.2803	13.8415	14.4049	64	15.8468	16.5983	17.3576
15	13.3354	13.9008	14.4687	65	15.8933	16.6480	17.4105
16	13.3903	13.9600	14.5323	66	15.9396	16.6974	17.4632
17	13.4452	14.0191	14.5959	67	15.9857	16.7466	17.5156
18	13.4999	14.0781	14.6592	68	16.0316	16.7956	17.5677
19	13.5545	14.1370	14.7225	69	16.0772	16.8443	17.6195
20	13.6090	14.1957	14.7855	70	16.1227	16.8928	17.6711
1.021	13.6634	14.2542	14.8485	1.071	16.1679	16.9410	17.7223
22	13.7176	14.3127	14.9112	72	16.2129	16.9890	17.7733
23	13.7717	14.3709	14.9738	73	16.2577	17.0367	17.8240
24	13.8257	14.4291	15.0362	74	16.3022	17.0842	17.8745
25	13.8795	14.4870	15.0985	75	16.3465	17.1314	17.9246
26	13.9332	14.5448	15.1606	76	16.3906	17.1784	17.9745
27	13.9867	14.6025	15.2225	77	16.4345	17.2251	18.0241
28	14.0401	14.6600	15.2842	78	16.4782	17.2716	18.0734
29	14.0934	14.7173	15.3457	79	16.5216	17.3178	18.1224
30	14.1465	14.7745	15.4070	80	16.5648	17.3637	18.1711
1.031	14.1995	14.8314	15.4682	1.081	16.6078	17.4094	18.2196
32	14.2523	14.8882	15.5291	82	16.6506	17.4549	18.2677
33	14.3049	14.9449	15.5898	83	16.6931	17.5001	18.3156
34	14.3574	15.0013	15.6504	84	16.7354	17.5450	18.3632
35	14.4097	15.0576	15.7107	85	16.7775	17.5897	18.4105
36	14.4619	15.1136	15.7708	86	16.8193	17.6342	18.4576
37	14.5139	15.1695	15.8307	87	16.8609	17.6783	18.5043
38	14.5657	15.2252	15.8904	88	16.9023	17.7223	18.5508
39	14.6173	15.2807	15.9498	89	16.9435	17.7659	18.5970
40	14.6688	15.3360	16.0091	90	16.9844	17.8094	18.6429
1.041	14.7201	15.3911	16.0681	1.091	17.0252	17.8525	18.6885
42	14.7712	15.4460	16.1269	92	17.0656	17.8954	18.7338
43	14.8221	15.5007	16.1854	93	17.1059	17.9381	18.7788
44	14.8729	15.5552	16.2438	94	17.1459	17.9805	18.8236
45	14.9235	15.6095	16.3018	95	17.1857	18.0226	18.8681
46	14.9738	15.6635	16.3597	96	17.2253	18.0645	18.9123
47	15.0240	15.7174	16.4173	97	17.2647	18.1062	18.9562
48	15.0740	15.7710	16.4746	98	17.3038	18.1475	18.9998
49	15.1238	15.8244	16.5318	99	17.3427	18.1887	19.0432
50	15.1734	15.8776	16.5886	1.100	17.3814	18.2296	19.0863

$$a = \frac{r - (n - M)(r - 1)}{n} \sum_0^{n-1} {}_0y_i$$

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

$$M = \sum_{0}^{n-1} x_i \cdot {}_o y_i \Big/ \sum_{0}^{n-1} {}_o y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$n=29$	$n=30$	$n=31$	r	$n=29$	$n=30$	$n=31$
1.001	14.0700	14.5749	15.0800	1.051	17.3667	18.0950	18.8299
2	14.1399	14.6497	15.1598	52	17.4268	18.1588	18.8976
3	14.2097	14.7244	15.2396	53	17.4866	18.2223	18.9649
4	14.2794	14.7990	15.3193	54	17.5461	18.2855	19.0319
5	14.3490	14.8735	15.3988	55	17.6052	18.3484	19.0985
6	14.4185	14.9479	15.4783	56	17.6642	18.4109	19.1647
7	14.4880	15.0222	15.5576	57	17.7228	18.4731	19.2306
8	14.5573	15.0964	15.6368	58	17.7811	18.5350	19.2961
9	14.6265	15.1704	15.7159	59	17.8391	18.5966	19.3612
10	14.6956	15.2443	15.7948	60	17.8968	18.6578	19.4260
1.011	14.7645	15.3181	15.8735	1.061	17.9542	18.7187	19.4904
12	14.8333	15.3917	15.9521	62	18.0114	18.7792	19.5544
13	14.9020	15.4652	16.0305	63	18.0682	18.8394	19.6180
14	14.9706	15.5385	16.1088	64	18.1247	18.8993	19.6813
15	15.0390	15.6117	16.1869	65	18.1809	18.9588	19.7442
16	15.1072	15.6847	16.2648	66	18.2368	19.0180	19.8067
17	15.1753	15.7575	16.3425	67	18.2924	19.0768	19.8688
18	15.2433	15.8302	16.4200	68	18.3477	19.1354	19.9306
19	15.3110	15.9026	16.4973	69	18.4026	19.1935	19.9919
20	15.3786	15.9749	16.5743	70	18.4573	19.2513	20.0529
1.021	15.4460	16.0470	16.6512	1.071	18.5117	19.3088	20.1135
22	15.5133	16.1188	16.7278	72	18.5657	19.3659	20.1737
23	15.5803	16.1905	16.8043	73	18.6194	19.4227	20.2336
24	15.6472	16.2619	16.8804	74	18.6728	19.4791	20.2930
25	15.7139	16.3332	16.9564	75	18.7259	19.5352	20.3521
26	15.7803	16.4042	17.0321	76	18.7787	19.5909	20.4107
27	15.8466	16.4750	17.1075	77	18.8312	19.6463	20.4690
28	15.9127	16.5455	17.1827	78	18.8834	19.7013	20.5269
29	15.9785	16.6158	17.2576	79	18.9352	19.7560	20.5844
30	16.0442	16.6859	17.3322	80	18.9867	19.8103	20.6416
1.031	16.1096	16.7558	17.4066	1.081	19.0379	19.8643	20.6983
32	16.1748	16.8253	17.4807	82	19.0888	19.9179	20.7547
33	16.2398	16.8947	17.5545	83	19.1394	19.9712	20.8106
34	16.3045	16.9638	17.6280	84	19.1897	20.0241	20.8662
35	16.3690	17.0326	17.7013	85	19.2396	20.0767	20.9214
36	16.4333	17.1011	17.7742	86	19.2892	20.1289	20.9763
37	16.4973	17.1694	17.8468	87	19.3386	20.1808	21.0307
38	16.5611	17.2374	17.9192	88	19.3876	20.2323	21.0847
39	16.6247	17.3051	17.9912	89	19.4363	20.2835	21.1384
40	16.6880	17.3726	18.0629	90	19.4846	20.3343	21.1917
1.041	16.7510	17.4397	18.1343	1.091	19.5327	20.3848	21.2446
42	16.8138	17.5066	18.2054	92	19.5804	20.4350	21.2971
43	16.8763	17.5732	18.2761	93	19.6279	20.4848	21.3493
44	16.9385	17.6395	18.3465	94	19.6750	20.5342	21.4011
45	17.0005	17.7055	18.4166	95	19.7218	20.5833	21.4525
46	17.0623	17.7712	18.4863	96	19.7683	20.6321	21.5035
47	17.1237	17.8366	18.5558	97	19.8145	20.6805	21.5541
48	17.1849	17.9016	18.6248	98	19.8603	20.7286	21.6044
49	17.2458	17.9664	18.6935	99	19.9059	20.7764	21.6543
50	17.3064	18.0309	18.7619	1.100	19.9511	20.8238	21.7038

$$a = \frac{r - (n - M)(r - 1)}{n} \sum_{0}^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y = ar^x$ to n Observed Statistical Ordinates

$$M = \sum_0^{n-1} x_i \cdot {}_o y_i \Big/ \sum_0^{n-1} {}_o y_i = n/(1-r^{-n}) - 1/(1-r^{-1})$$

r	$=32$	$n=33$	$n=34$	r	$n=32$	$n=33$	$n=34$
1.001	15.5852	16.0906	16.5962	1.051	19.5714	20.3194	21.0738
2	15.6703	16.1811	16.6923	52	19.6430	20.3950	21.1533
3	15.7553	16.2715	16.7883	53	19.7142	20.4701	21.2324
4	15.8402	16.3619	16.8841	54	19.7850	20.5448	21.3111
5	15.9250	16.4520	16.9798	55	19.8554	20.6190	21.3892
6	16.0097	16.5420	17.0754	56	19.9254	20.6928	21.4668
7	16.0942	16.6319	17.1708	57	19.9950	20.7661	21.5440
8	16.1786	16.7216	17.2660	58	20.0641	20.8390	21.6206
9	16.2628	16.8112	17.3610	59	20.1329	20.9115	21.6968
10	16.3468	16.9005	17.4559	60	20.2012	20.9834	21.7724
1.011	16.4307	16.9897	17.5506	1.061	20.2692	21.0550	21.8476
12	16.5144	17.0787	17.6450	62	20.3367	21.1261	21.9223
13	16.5980	17.1675	17.7392	63	20.4038	21.1967	21.9964
14	16.6813	17.2561	17.8332	64	20.4705	21.2668	22.0700
15	16.7645	17.3445	17.9269	65	20.5368	21.3365	22.1432
16	16.8474	17.4326	18.0204	66	20.6027	21.4058	22.2158
17	16.9301	17.5205	18.1137	67	20.6681	21.4745	22.2879
18	17.0126	17.6082	18.2066	68	20.7331	21.5429	22.3596
19	17.0949	17.6956	18.2993	69	20.7977	21.6107	22.4307
20	17.1770	17.7828	18.3917	70	20.8619	21.6781	22.5013
1.021	17.2588	17.8697	18.4839	1.071	20.9257	21.7450	22.5714
22	17.3403	17.9563	18.5757	72	20.9890	21.8115	22.6409
23	17.4216	18.0426	18.6672	73	21.0519	21.8774	22.7100
24	17.5027	18.1286	18.7583	74	21.1144	21.9430	22.7786
25	17.5834	18.2144	18.8492	75	21.1764	22.0080	22.8466
26	17.6639	18.2998	18.9397	76	21.2381	22.0726	22.9142
27	17.7442	18.3849	19.0299	77	21.2993	22.1367	22.9812
28	17.8241	18.4698	19.1197	78	21.3600	22.2004	23.0477
29	17.9037	18.5542	19.2091	79	21.4204	22.2636	23.1137
30	17.9831	18.6384	19.2982	80	21.4803	22.3263	23.1792
1.031	18.0621	18.7222	19.3869	1.081	21.5398	22.3886	23.2443
32	18.1408	18.8057	19.4753	82	21.5989	22.4504	23.3088
33	18.2192	18.8888	19.5632	83	21.6576	22.5117	23.3727
34	18.2973	18.9716	19.6508	84	21.7158	22.5726	23.4362
35	18.3751	19.0540	19.7380	85	21.7736	22.6330	23.4992
36	18.4525	19.1360	19.8247	86	21.8310	22.6929	23.5617
37	18.5296	19.2177	19.9111	87	21.8880	22.7524	23.6237
38	18.6064	19.2990	19.9970	88	21.9446	22.8115	23.6852
39	18.6828	19.3799	20.0825	89	22.0007	22.8701	23.7462
40	18.7589	19.4604	20.1676	90	22.0564	22.9282	23.8067
1.041	18.8346	19.5406	20.2522	1.091	22.1117	22.9859	23.8667
42	18.9099	19.6203	20.3364	92	22.1666	23.0431	23.9263
43	18.9849	19.6996	20.4202	93	22.2211	23.0999	23.9853
44	19.0596	19.7786	20.5035	94	22.2752	23.1562	24.0438
45	19.1338	19.8571	20.5863	95	22.3288	23.2121	24.1019
46	19.2077	19.9352	20.6687	96	22.3820	23.2675	24.1595
47	19.2812	20.0129	20.7507	97	22.4349	23.3225	24.2166
48	19.3544	20.0902	20.8322	98	22.4873	23.3770	24.2732
49	19.4271	20.1670	20.9132	99	22.5393	23.4311	24.3294
50	19.4995	20.2434	20.9937	1.100	22.5909	23.4848	24.3851

$$a = \frac{r-(n-M)(r-1)}{n} \sum_0^{n-1} {}_o y_i$$

Mean Value Table to Fit Exponential Growth Curve $y=ar^x$ to n Observed Statistical Ordinates

$$M=\sum_{0}^{n-1} x_i \cdot {}_oy_i \Big/ \sum_{0}^{n-1} {}_oy_i = n/(1-r^{-n})-1/(1-r^{-1})$$

r	$n=35$	$n=36$	$n=37$	r	$n=35$	$n=36$	$n=37$
1.001	17.1019	17.6079	18.1140	1.051	21.8344	22.6012	23.3741
2	17.2038	17.7156	18.2278	52	21.9181	22.6890	23.4660
3	17.3055	17.8232	18.3414	53	22.0012	22.7762	23.5574
4	17.4071	17.9307	18.4549	54	22.0838	22.8628	23.6481
5	17.5085	18.0380	18.5683	55	22.1659	22.9489	23.7381
6	17.6097	18.1451	18.6814	56	22.2474	23.0343	23.8276
7	17.7108	18.2520	18.7943	57	22.3284	23.1192	23.9163
8	17.8117	18.3587	18.9071	58	22.4088	23.2035	24.0045
9	17.9124	18.4652	19.0195	59	22.4887	23.2872	24.0919
10	18.0130	18.5715	19.1318	60	22.5681	23.3702	24.1787
1.011	18.1132	18.6776	19.2438	1.061	22.6469	23.4527	24.2649
12	18.2132	18.7833	19.3555	62	22.7251	23.5346	24.3504
13	18.3130	18.8889	19.4669	63	22.8029	23.6159	24.4353
14	18.4125	18.9941	19.5780	64	22.8800	23.6965	24.5195
15	18.5118	19.0991	19.6888	65	22.9566	23.7766	24.6030
16	18.6108	19.2037	19.7992	66	23.0326	23.8561	24.6859
17	18.7095	19.3081	19.9094	67	23.1081	23.9349	24.7682
18	18.8079	19.4121	20.0191	68	23.1831	24.0132	24.8497
19	18.9061	19.5158	20.1285	69	23.2574	24.0908	24.9306
20	19.0039	19.5191	20.2375	70	23.3313	24.1679	25.0109
1.021	19.1013	19.7221	20.3462	1.071	23.4045	24.2443	25.0905
22	19.1985	19.8247	20.4544	72	23.4772	24.3201	25.1695
23	19.2953	19.9270	20.5622	73	23.5494	24.3954	25.2477
24	19.3917	20.0288	20.6696	74	23.6210	24.4700	25.3254
25	19.4878	20.1303	20.7765	75	23.6920	24.5440	25.4024
26	19.5835	20.2313	20.8830	76	23.7625	24.6174	25.4787
27	19.6789	20.3319	20.9891	77	23.8324	24.6902	25.5544
28	19.7738	20.4321	21.0946	78	23.9018	24.7625	25.6294
29	19.8684	20.5319	21.1997	79	23.9707	24.8341	25.7038
30	19.9625	20.6312	21.3043	80	24.0389	24.9051	25.7775
1.031	20.0562	20.7301	21.4084	1.081	24.1067	24.9755	25.8506
32	20.1496	20.8285	21.5120	82	24.1738	25.0454	25.9231
33	20.2424	20.9264	21.6151	83	24.2405	25.1146	25.9949
34	20.3349	21.0239	21.7177	84	24.3066	25.1833	26.0661
35	20.4269	21.1209	21.8197	85	24.3721	25.2513	26.1367
36	20.5185	21.2173	21.9212	86	24.4371	25.3188	26.2066
37	20.6096	21.3133	22.0221	87	24.5016	25.3857	26.2759
38	20.7003	21.4089	22.1225	88	24.5655	25.4520	26.3445
39	20.7904	21.5037	22.2223	89	24.6289	25.5177	26.4126
40	20.8802	21.5982	22.3216	90	24.6917	25.5829	26.4800
1.041	20.9694	21.6921	22.4203	1.091	24.7540	25.6475	26.5468
42	21.0581	21.7855	22.5184	92	24.8158	25.7115	26.6130
43	21.1464	21.8783	22.6159	93	24.8771	25.7750	26.6786
44	21.2342	21.9706	22.7128	94	24.9378	25.8379	26.7436
45	21.3215	22.0624	22.8091	95	24.9980	25.9002	26.8080
46	21.4082	22.1536	22.9048	96	25.0577	25.9619	26.8718
47	21.4945	22.2443	22.9999	97	25.1169	26.0232	26.9350
48	21.5802	22.3343	23.0944	98	25.1756	26.0838	26.9976
49	21.6655	22.4239	23.1882	99	25.2338	26.1439	27.0598
50	21.7502	22.5128	23.2814	1.100	25.2914	26.2035	27.1213

$$a=\frac{r-(n-M)(r-1)}{n}\sum_{0}^{n-1} {}_oy_i$$

Mean Value Table to Fit Exponential Growth Curve $y=ar^x$ to n Observed Statistical Ordinates

$$M=\sum_{0}^{n-1} x_i \cdot {}_o y_i \Big/ \sum_{0}^{n-1}{}_o y_i = n/(1-r^{-n})-1/(1-r^{-1})$$

r	$n=38$	$n=39$	$n=40$	r	$n=38$	$n=39$	$n=40$
1.001	18.6202	19.1266	19.6332	1.051	24.1529	24.9376	25.7281
2	18.7402	19.2531	19.7662	52	24.2491	25.0380	25.8328
3	18.8601	19.3793	19.8991	53	24.3446	25.1377	25.9367
4	18.9799	19.5055	20.0317	54	24.4394	25.2367	26.0398
5	19.0994	19.6314	20.1642	55	24.5335	25.3348	26.1421
6	19.2187	19.7570	20.2964	56	24.6269	25.4323	26.2435
7	19.3378	19.8825	20.4283	57	24.7196	25.5289	26.3441
8	19.4567	20.0077	20.5600	58	24.8116	25.6248	26.4439
9	19.5753	20.1326	20.6913	59	24.9029	25.7199	26.5429
10	19.6937	20.2572	20.8224	60	24.9935	25.8143	26.6410
1.011	19.8118	20.3815	20.9531	1.061	25.0834	25.9079	26.7383
12	19.9295	20.5055	21.0835	62	25.1725	26.0007	26.8348
13	20.0470	20.6292	21.2135	63	25.2610	26.0927	26.9304
14	20.1641	20.7525	21.3431	64	25.3487	26.1840	27.0252
15	20.2809	20.8754	21.4723	65	25.4357	26.2745	27.1192
16	20.3973	20.9979	21.6010	66	25.5220	26.3642	27.2124
17	20.5133	21.1200	21.7294	67	25.6076	26.4532	27.3047
18	20.6290	21.2417	21.8572	68	25.6925	26.5414	27.3962
19	20.7442	21.3629	21.9846	69	25.7767	26.6288	27.4868
20	20.8591	21.4837	22.1115	70	25.8602	26.7155	27.5767
1.021	20.9735	21.6041	22.2379	1.071	25.9429	26.8014	27.6657
22	21.0875	21.7239	22.3637	72	26.0250	26.8865	27.7538
23	21.2010	21.8432	22.4890	73	26.1063	26.9709	27.8412
24	21.3140	21.9621	22.6138	74	26.1869	27.0545	27.9278
25	21.4266	22.0804	22.7380	75	26.2669	27.1373	28.0135
26	21.5387	22.1982	22.8616	76	26.3461	27.2194	28.0984
27	21.6502	22.3154	22.9846	77	26.4246	27.3007	28.1825
28	21.7613	22.4321	23.1070	78	26.5024	27.3813	28.2659
29	21.8718	22.5482	23.2287	79	26.5796	27.4612	28.3484
30	21.9818	22.6637	23.3498	80	26.6560	27.5403	28.4301
1.031	22.0913	22.7786	23.4703	1.081	26.7317	27.6186	28.5110
32	22.2002	22.8929	23.5901	82	26.8068	27.6962	28.5911
33	22.3085	23.0066	23.7093	83	26.8812	27.7731	28.6705
34	22.4163	23.1196	23.8277	84	26.9548	27.8492	28.7490
35	22.5235	23.2321	23.9455	85	27.0278	27.9246	28.8268
36	22.6301	23.3438	24.0625	86	27.1002	27.9991	28.9038
37	22.7360	23.4550	24.1789	87	27.1718	28.0733	28.9801
38	22.8414	23.5654	24.2945	88	27.2428	28.1466	29.0556
39	22.9462	23.6752	24.4094	89	27.3137	28.2191	29.1303
40	23.0503	23.7843	24.5235	90	27.3828	28.2910	29.2043
1.041	23.1538	23.8927	24.6369	1.091	27.4518	28.3621	29.2775
42	23.2567	24.0004	24.7495	92	27.5201	28.4325	29.3500
43	23.3589	24.1075	24.8614	93	27.5878	28.5023	29.4217
44	23.4605	24.2138	24.9725	94	27.6549	28.5713	29.4927
45	23.5614	24.3194	25.0828	95	27.7213	28.6397	29.5630
46	23.6617	24.4242	25.1923	96	27.7871	28.7074	29.6326
47	23.7613	24.5284	25.3011	97	27.8522	28.7744	29.7014
48	23.8602	24.6318	25.4090	98	27.9167	28.8408	29.7696
49	23.9584	24.7345	25.5162	99	27.9806	28.9065	29.8370
50	24.0560	24.8364	25.6225	1.100	28.0438	28.9715	29.9038

$$a=\frac{r-(n-M)(r-1)}{n}\sum_{0}^{n-1}{}_o y_i$$

Mean Value Table for Computational Use
for fitting exponential curve $y = ar^x$

National Bureau of Economic Research, Inc.

A simplified method for fitting the exponential growth curve $y = ar^x$ is found in Tables of Applied Mathematics in Finance, Insurance, Statistics, James W. Glover, pp. 468-481. This carries a mean value table through n = 40 for values of r from 1.001 to 1.100. The American Telephone and Telegraph Co. extended Glover's table for the same values of n thru r = 1.150 (at intervals of .005). Charles H. Wittmann computed the "mean value table" for rates of change from r = 1.150 to r = 2.00 for values of n from 2 to 20; r = 1.150 to r = 1.800 for values of n from 21 to 44; and r = 1.000 to r = 1.100 for values of n from 41 to 64.

The author expresses his appreciation to the National Bureau and to Mr. Wittmann for permission to present these extensions.

Contents

Mean Value Table to fit curve $y = ar^x$

r	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.100	.5238	1.0634	1.6188	2.1899	2.7764	3.3784	3.9955	4.6276
1.105	.5249	1.0665	1.6245	2.1988	2.7894	3.3961	4.0186	4.6567
1.110	.5261	1.0694	1.6300	2.2077	2.8024	3.4137	4.0415	4.6856
1.115	.5272	1.0724	1.6356	2.2166	2.8152	3.4312	4.0643	4.7142
1.120	.5283	1.0754	1.6411	2.2254	2.8280	3.4485	4.0869	4.7426
1.125	.5294	1.0783	1.6467	2.2342	2.8406	3.4658	4.1093	4.7707
1.130	.5305	1.0813	1.6521	2.2429	2.8532	3.4829	4.1315	4.7986
1.135	.5316	1.0842	1.6576	2.2515	2.8657	3.4999	4.1535	4.8263
1.140	.5327	1.0871	1.6630	2.2601	2.8782	3.5168	4.1754	4.8537
1.145	.5338	1.0900	1.6684	2.2687	2.8905	3.5335	4.1971	4.8808
1.150	.5349	1.0929	1.6737	2.2772	2.9028	3.5502	4.2187	4.9078
1.155	.5360	1.0957	1.6791	2.2856	2.9150	3.5666	4.2400	4.9345
1.160	.5370	1.0986	1.6844	2.2940	2.9271	3.5831	4.2613	4.9609
1.165	.5381	1.1014	1.6897	2.3024	2.9392	3.5994	4.2823	4.9871
1.170	.5392	1.1042	1.6949	2.3107	2.9511	3.6155	4.3030	5.0130
1.175	.5402	1.1070	1.7001	2.3190	2.9630	3.6315	4.3237	5.0387
1.180	.5413	1.1098	1.7053	2.3272	2.9747	3.6474	4.3442	5.0642
1.185	.5423	1.1126	1.7105	2.3353	2.9865	3.6632	4.3645	5.0894
1.190	.5434	1.1154	1.7156	2.3434	2.9981	3.6789	4.3846	5.1143
1.195	.5444	1.1181	1.7207	2.3515	3.0096	3.6944	4.4046	5.1391
1.200	.5455	1.1209	1.7258	2,3595	3.0212	3.7098	4.4244	5.1636
1.205	.5465	1.1236	1.7308	2.3675	3.0327	3.7252	4.4440	5.1878
1.210	.5475	1.1263	1.7359	2.3754	3.0439	3.7403	4.4634	5.2118
1.215	.5485	1.1290	1.7409	2.3832	3.0551	3.7554	4.4827	5.2356
1.220	.5495	1.1317	1.7458	2.3910	3.0663	3.7703	4.5017	5.2591
1.225	.5506	1.1343	1.7508	2.3988	3.0775	3.7852	4.5207	5.2824
1.230	.5516	1.1370	1.7557	2.4065	3.0884	3.7999	4.5395	5.3054
1.235	.5526	1.1397	1.7606	2.4142	3.0994	3.8145	4.5580	5.3282
1.240	.5536	1.1423	1.7654	2.4218	3.1102	3.8289	4.5764	5.3508
1.245	.5546	1.1449	1.7703	2.4294	3.1209	3.8433	4.5947	5.3731
1.250	.5556	1.1475	1.7751	2.4369	3.1317	3.8576	4.6128	5.3952
1.255	.5565	1.1501	1.7798	2.4444	3.1423	3.8717	4.6307	5.4171
1.260	.5575	1.1527	1.7846	2.4519	3.1528	3.8857	4.6484	5.4387
1.265	.5585	1.1553	1.7893	2.4592	3.1633	3.8996	4.6660	5.4601
1.270	.5595	1.1578	1.7940	2.4666	3.1737	3.9134	4.6834	5.4813
1.275	.5604	1.1604	1.7987	2.4739	3.1840	3.9271	4.7006	5.5023
1.280	.5614	1.1629	1.8034	2.4811	3.1943	3.9406	4.7177	5.5230
1.285	.5624	1.1654	1.8080	2.4883	3.2045	3.9541	4.7346	5.5435
1.290	.5633	1.1680	1.8126	2.4955	3.2145	3.9674	4.7513	5.5638
1.295	.5643	1.1704	1.8172	2.5026	3.2246	3.9806	4.7680	5.5839
1.300	.5652	1.1729	1.8217	2.5097	3.2346	3.9938	4.7844	5.6037

Mean Value Table to fit curve $y = ar^x$

r	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.300	.5652	1.1729	1.8217	2.5097	3.2346	3.9938	4.7844	5.6037
1.310	.5671	1.1779	1.8307	2.5237	3.2543	4.0196	4.8168	5.6428
1.320	.5690	1.1827	1.8397	2.5375	3.2736	4.0450	4.8485	5.6810
1.330	.5708	1.1876	1.8485	2.5512	3.2928	4.0700	4.8798	5.7183
1.340	.5726	1.1924	1.8572	2.5647	3.3116	4.0946	4.9103	5.7549
1.350	.5745	1.1971	1.8659	2.5780	3.3302	4.1189	4.9403	5.7906
1.360	.5763	1.2018	1.8744	2.5911	3.3485	4.1426	4.9696	5.8256
1.370	.5781	1.2065	1.8829	2.6041	3.3665	4.1660	4.9984	5.8597
1.380	.5798	1.2111	1.8912	2.6169	3.3842	4.1890	5.0267	5.8932
1.390	.5816	1.2157	1.8995	2.6295	3.4016	4.2115	5.0544	5.9258
1.400	.5833	1.2202	1.9077	2.6420	3.4189	4.2336	5.0815	5.9578
1.410	.5851	1.2247	1.9157	2.6543	3.4359	4.2554	5.1081	5.9890
1.420	.5868	1.2291	1.9237	2.6665	3.4526	4.2768	5.1341	6.0195
1.430	.5885	1.2335	1.9316	2.6785	3.4690	4.2979	5.1597	6.0493
1.440	.5902	1.2379	1.9395	2.6903	3.4852	4.3185	5.1847	6.0785
1.450	.5918	1.2422	1.9472	2.7020	3.5012	4.3388	5.2093	6.1070
1.460	.5935	1.2464	1.9548	2.7136	3.5169	4.3588	5.2333	6.1349
1.470	.5951	1.2507	1.9624	2.7250	3.5324	4.3784	5.2569	6.1622
1.480	.5968	1.2549	1.9699	2.7362	3.5476	4.3976	5.2800	6.1888
1.490	.5984	1.2590	1.9773	2.7473	3.5627	4.4166	5.3026	6.2149
1.500	.6000	1.2632	1.9846	2.7583	3.5774	4.4352	5.3248	6.2404
1.510	.6016	1.2672	1.9919	2.7691	3.5920	4.4534	5.3466	6.2653
1.520	.6032	1.2713	1.9990	2.7798	3.6064	4.4714	5.3679	6.2896
1.530	.6047	1.2753	2.0061	2.7903	3.6205	4.4890	5.3888	6.3135
1.540	.6063	1.2793	2.0131	2.8007	3.6344	4.5063	5.4093	6.3368
1.550	.6078	1.2832	2.0200	2.8110	3.6481	4.5234	5.4294	6.3595
1.560	.6094	1.2871	2.0269	2.8212	3.6616	4.5401	5.4491	6.3818
1.570	.6109	1.2909	2.0337	2.8312	3.6749	4.5566	5.4684	6.4036
1.580	.6124	1.2948	2.0404	2.8411	3.6880	4.5727	5.4873	6.4250
1.590	.6139	1.2986	2.0470	2.8508	3.7009	4.5886	5.5058	6.4458
1.600	.6154	1.3023	2.0536	2.8604	3.7136	4.6042	5.5240	6.4662
1.610	.6169	1.3060	2.0601	2.8699	3.7261	4.6195	5.5419	6.4862
1.620	.6183	1.3097	2.0665	2.8793	3.7385	4.6346	5.5594	6.5058
1.630	.6198	1.3134	2.0729	2.8886	3.7506	4.6494	5.5765	6.5249
1.640	.6212	1.3170	2.0791	2.8977	3.7626	4.6640	5.5934	6.5436
1.650	.6226	1.3206	2.0854	2.9068	3.7744	4.6783	5.6099	6.5619
1.660	.6241	1.3242	2.0915	2.9157	3.7860	4.6924	5.6260	6.5799
1.670	.6255	1.3277	2.0976	2.9245	3.7974	4.7062	5.6419	6.5974
1.680	.6269	1.3312	2.1036	2.9332	3.8087	4.7198	5.6575	6.6146
1.690	.6283	1.3347	2.1096	2.9418	3.8198	4.7331	5.6728	6.6315
1.700	.6296	1.3381	2.1155	2.9503	3.8307	4.7463	5.6878	6.6480

Mean Value Table to fit curve $y = ar^x$

r	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.700	.6296	1.3381	2.1155	2.9503	3.8307	4.7463	5.6878	6.6480
1.710	.6310	1.3415	2.1213	2.9586	3.8415	4.7592	5.7025	6.6641
1.720	.6323	1.3449	2.1271	2.9669	3.8521	4.7719	5.7169	6.6799
1.730	.6337	1.3482	2.1328	2.9751	3.8626	4.7844	5.7311	6.6954
1.740	.6350	1.3515	2.1385	2.9831	3.8729	4.7967	5.7450	6.7106
1.750	.6364	1.3548	2.1441	2.9911	3.8831	4.8088	5.7587	6.7255
1.760	.6377	1.3581	2.1496	2.9989	3.8931	4.8206	5.7721	6.7401
1.770	.6390	1.3613	2.1551	3.0067	3.9030	4.8323	5.7852	6.7544
1.780	.6403	1.3645	2.1605	3.0144	3.9127	4.8438	5.7981	6.7684
1.790	.6416	1.3677	2.1659	3.0219	3.9223	4.8551	5.8108	6.7821
1.800	.6429	1.3709	2.1712	3.0294	3.9318	4.8662	5.8233	6.7956
1.810	.6441	1.3740	2.1764	3.0368	3.9411	4.8772	5.8355	6.8088
1.820	.6454	1.3771	2.1816	3.0441	3.9502	4.8879	5.8475	6.8218
1.830	.6466	1.3801	2.186[illegible]	3.0513	3.9593	4.8985	5.8593	6.8345
1.840	.6479	1.3832	2.1918	3.0584	3.9682	4.9089	5.8709	6.8469
1.850	.6491	1.3862	2.1969	3.0654	3.9770	4.9192	5.8823	6.8591
1.860	.6503	1.3892	2.2019	3.0724	3.9857	4.9293	5.8934	6.8711
1.870	.6516	1.3922	2.2068	3.0792	3.9942	4.9392	5.9044	6.8829
1.880	.6528	1.3951	2.2117	3.0860	4.0027	4.9490	5.9152	6.8944
1.890	.6540	1.3980	2.2165	3.0927	4.0110	4.9586	5.9258	6.9057
1.900	.6552	1.4009	2.2213	3.0993	4.0192	4.9681	5.9363	6.9169
1.910	.6564	1.4038	2.2261	3.1059	4.0273	4.9774	5.9465	6.9278
1.920	.6575	1.4066	2.2308	3.1123	4.0353	4.9866	5.9566	6.9385
1.930	.6587	1.4095	2.2354	3.1187	4.0431	4.9956	5.9665	6.9490
1.940	.6599	1.4123	2.2400	3.1250	4.0509	5.0045	5.9762	6.9594
1.950	.6610	1.4150	2.2446	3.1312	4.0585	5.0133	5.9858	6.9695
1.960	.6622	1.4178	2.2491	3.1374	4.0661	5.0219	5.9952	6.9795
1.970	.6633	1.4205	2.2535	3.1435	4.0735	5.0304	6.0045	6.9893
1.980	.6644	1.4232	2.2580	3.1495	4.0808	5.0388	6.0136	6.9989
1.990	.6655	1.4259	2.2623	3.1554	4.0881	5.0470	6.0226	7.0083
2.000	.6667	1.4286	2.2667	3.1613	4.0952	5.0551	6.0314	7.0176

Mean Value Table to fit curve $y = ar^x$

r	n=10	n=11	n=12	n=13	n=14	n=15	n=16	n=17
1.100	5.2745	5.9359	6.6116	7.3012	8.0045	8.7211	9.4507	10.1929
1.105	5.3102	5.9788	6.6621	7.3599	8.0717	8.7973	9.5363	10.2882
1.110	5.3456	6.0212	6.7121	7.4178	8.1381	8.8725	9.6206	10.3820
1.115	5.3806	6.0632	6.7615	7.4751	8.2037	8.9467	9.7036	10.4741
1.120	5.4153	6.1047	6.8103	7.5317	8.2683	9.0197	9.7853	10.5647
1.125	5.4497	6.1459	6.8587	7.5876	8.3321	9.0917	9.8657	10.6537
1.130	5.4838	6.1866	6.9064	7.6427	8.3950	9.1625	9.9448	10.7411
1.135	5.5176	6.2269	6.9536	7.6972	8.4570	9.2323	10.0225	10.8269
1.140	5.5510	6.2667	7.0002	7.7509	8.5181	9.3010	10.0989	10.9112
1.145	5.5841	6.3061	7.0463	7.8039	8.5783	9.3685	10.1740	10.9938
1.150	5.6168	6.3451	7.0918	7.8562	8.6376	9.4350	10.2478	11.0749
1.155	5.6492	6.3836	7.1367	7.9078	8.6960	9.5004	10.3202	11.1544
1.160	5.6813	6.4217	7.1811	7.9587	8.7536	9.5648	10.3914	11.2324
1.165	5.7131	6.4593	7.2249	8.0089	8.8102	9.6280	10.4612	11.3089
1.170	5.7445	6.4965	7.2681	8.0583	8.8660	9.6901	10.5297	11.3838
1.175	5.7756	6.5333	7.3108	8.1071	8.9209	9.7513	10.5971	11.4572
1.180	5.8064	6.5697	7.3530	8.1551	8.9750	9.8113	10.6631	11.5292
1.185	5.8368	6.6056	7.3945	8.2025	9.0281	9.8703	10.7279	11.5996
1.190	5.8669	6.6410	7.4355	8.2491	9.0804	9.9283	10.7914	11.6686
1.195	5.8967	6.6761	7.4760	8.2950	9.1319	9.9852	10.8538	11.7362
1.200	5.9261	6.7107	7.5159	8.3403	9.1825	10.0412	10.9149	11.8024
1.205	5.9553	6.7449	7.5553	8.3849	9.2323	10.0961	10.9748	11.8672
1.210	5.9841	6.7787	7.5941	8.4288	9.2812	10.1500	11.0336	11.9306
1.215	6.0125	6.8120	7.6323	8.4720	9.3294	10.2029	11.0912	11.9927
1.220	6.0407	6.8449	7.6701	8.5145	9.3767	10.2549	11.1476	12.0535
1.225	6.0685	6.8774	7.7073	8.5564	9.4232	10.3059	11.2029	12.1129
1.230	6.0960	6.9095	7.7440	8.5977	9.4689	10.3559	11.2572	12.1711
1.235	6.1232	6.9411	7.7801	8.6383	9.5138	10.4051	11.3103	12.2281
1.240	6.1501	6.9724	7.8157	8.6782	9.5580	10.4533	11.3624	12.2838
1.245	6.1767	7.0032	7.8509	8.7176	9.6014	10.5006	11.4134	12.3383
1.250	6.2029	7.0337	7.8855	8.7563	9.6440	10.5470	11.4634	12.3916
1.255	6.2288	7.0637	7.9196	8.7943	9.6860	10.5926	11.5124	12.4438
1.260	6.2545	7.0934	7.9532	8.8318	9.7271	10.6372	11.5604	12.4949
1.265	6.2798	7.1226	7.9863	8.8687	9.7676	10.6811	11.6074	12.5448
1.270	6.3048	7.1515	8.0190	8.9049	9.8073	10.7241	11.6534	12.5937
1.275	6.3296	7.1800	8.0511	8.9406	9.8464	10.7663	11.6986	12.6415
1.280	6.3540	7.2081	8.0828	8.9757	9.8847	10.8077	11.7428	12.6883
1.285	6.3781	7.2358	8.1140	9.0103	9.9224	10.8483	11.7861	12.7340
1.290	6.4020	7.2631	8.1447	9.0442	9.9594	10.8881	11.8285	12.7788
1.295	6.4255	7.2901	8.1750	9.0777	9.9958	10.9272	11.8701	12.8226
1.300	6.4488	7.3167	8.2048	9.1105	10.0315	10.9656	11.9108	12.8655

Mean Value Table to **fit** curve $y = ar^x$

r	n=10	n=11	n=12	n=13	n=14	n=15	n=16	n=17
1.300	6.4488	7.3167	8.2048	9.1105	10.0315	10.9656	11.9108	12.8655
1.310	6.4945	7.3689	8.2632	9.1747	10.1010	11.0401	11.9898	12.9485
1.320	6.5390	7.4196	8.3197	9.2367	10.1682	11.1117	12.0656	13.0280
1.330	6.5825	7.4689	8.3746	9.2968	10.2329	11.1807	12.1384	13.1041
1.340	6.6249	7.5169	8.4279	9.3549	10.2954	11.2472	12.2083	13.1770
1.350	6.6662	7.5636	8.4795	9.4111	10.3557	11.3111	12.2754	13.2470
1.360	6.7066	7.6090	8.5296	9.4654	10.4139	11.3727	12.3399	13.3140
1.370	6.7459	7.6532	8.5782	9.5180	10.4700	11.4319	12.4019	13.3783
1.380	6.7842	7.6961	8.6254	9.5689	10.5242	11.4890	12.4614	13.4399
1.390	6.8216	7.7379	8.6711	9.6182	10.5766	11.5440	12.5187	13.4991
1.400	6.8581	7.7785	8.7155	9.6659	10.6271	11.5970	12.5738	13.5559
1.410	6.8937	7.8180	8.7585	9.7120	10.6760	11.6481	12.6268	13.6105
1.420	6.9283	7.8565	8.8003	9.7567	10.7231	11.6974	12.6778	13.6630
1.430	6.9621	7.8938	8.8408	9.7999	10.7687	11.7449	12.7269	13.7134
1.440	6.9951	7.9302	8.8801	9.8418	10.8127	11.7907	12.7742	13.7619
1.450	7.0272	7.9656	8.9183	9.8824	10.8553	11.8350	12.8198	13.8085
1.460	7.0586	8.0000	8.9554	9.9217	10.8965	11.8776	12.8637	13.8534
1.470	7.0892	8.0335	8.9914	9.9598	10.9363	11.9189	12.9061	13.8967
1.480	7.1190	8.0661	9.0263	9.9967	10.9748	11.9587	12.9469	13.9384
1.490	7.1481	8.0978	9.0602	10.0325	11.0120	11.9971	12.9863	13.9785
1.500	7.1765	8.1287	9.0932	10.0671	11.0481	12.0343	13.0244	14.0173
1.510	7.2042	8.1587	9.1252	10.1008	11.0830	12.0703	13.0611	14.0546
1.520	7.2312	8.1880	9.1563	10.1334	11.1169	12.1051	13.0967	14.0907
1.530	7.2575	8.2164	9.1866	10.1650	11.1496	12.1387	13.1310	14.1255
1.540	7.2832	8.2442	9.2160	10.1958	11.1814	12.1713	13.1642	14.1592
1.550	7.3083	8.2712	9.2445	10.2256	11.2122	12.2028	13.1962	14.1917
1.560	7.3328	8.2975	9.2723	10.2545	11.2420	12.2333	13.2273	14.2231
1.570	7.3567	8.3232	9.2994	10.2826	11.2710	12.2629	13.2574	14.2536
1.580	7.3801	8.3481	9.3256	10.3099	11.2991	12.2916	13.2865	14.2830
1.590	7.4029	8.3725	9.3512	10.3365	11.3263	12.3194	13.3147	14.3115
1.600	7.4251	8.3962	9.3761	10.3623	11.3528	12.3464	13.3420	14.3391
1.610	7.4468	8.4194	9.4003	10.3873	11.3785	12.3725	13.3685	14.3658
1.620	7.4681	8.4419	9.4239	10.4117	11.4034	12.3979	13.3942	14.3918
1.630	7.4888	8.4639	9.4469	10.4354	11.4277	12.4226	13.4191	14.4170
1.640	7.5091	8.4854	9.4693	10.4585	11.4513	12.4465	13.4433	14.4413
1.650	7.5288	8.5063	9.4911	10.4809	11.4742	12.4697	13.4668	14.4650
1.660	7.5482	8.5267	9.5123	10.5028	11.4965	12.4923	13.4897	14.4879
1.670	7.5671	8.5466	9.5330	10.5240	11.5181	12.5143	13.5118	14.5102
1.680	7.5856	8.5661	9.5532	10.5447	11.5392	12.5357	13.5334	14.5319
1.690	7.6036	8.5851	9.5729	10.5649	11.5598	12.5565	13.5543	14.5530
1.700	7.6213	8.6036	9.5921	10.5846	11.5797	12.5767	13.5747	14.5735

Mean Value Table to fit curve $y = ar^x$

r	n=10	n=11	n=12	n=13	n=14	n=15	n=16	n=17
1.700	7.6213	8.6036	9.5921	10.5846	11.5797	12.5767	13.5747	14.5735
1.710	7.6385	8.6217	9.6108	10.6037	11.5992	12.5963	13.5945	14.5934
1.720	7.6554	8.6394	9.6290	10.6224	11.6182	12.6155	13.6138	14.6128
1.730	7.6720	8.6567	9.6469	10.6406	11.6366	12.6342	13.6326	14.6317
1.740	7.6881	8.6736	9.6642	10.6584	11.6547	12.6524	13.6509	14.6500
1.750	7.7039	8.6900	9.6812	10.6757	11.6722	12.6701	13.6687	14.6679
1.760	7.7194	8.7062	9.6978	10.6926	11.6893	12.6873	13.6861	14.6854
1.770	7.7345	8.7219	9.7140	10.7091	11.7060	12.7042	13.7030	14.7023
1.780	7.7494	8.7373	9.7298	10.7252	11.7223	12.7206	13.7195	14.7189
1.790	7.7639	8.7524	9.7453	10.7409	11.7382	12.7366	13.7356	14.7350
1.800	7.7781	8.7671	9.7604	10.7562	11.7537	12.7522	13.7513	14.7508
1.810	7.7920	8.7816	9.7751	10.7712	11.7689	12.7675	13.7666	14.7661
1.820	7.8056	8.7957	9.7896	10.7859	11.7837	12.7824	13.7816	14.7811
1.830	7.8190	8.8095	9.8037	10.8002	11.7981	12.7969	13.7962	14.7958
1.840	7.8321	8.8230	9.8175	10.8142	11.8123	12.8111	13.8105	14.8101
1.850	7.8449	8.8362	9.8310	10.8279	11.8261	12.8250	13.8244	14.8240
1.860	7.8574	8.8492	9.8442	10.8413	11.8396	12.8386	13.8380	14.8376
1.870	7.8697	8.8618	9.8571	10.8544	11.8528	12.8518	13.8513	14.8510
1.880	7.8818	8.8743	9.8698	10.8672	11.8657	12.8648	13.8643	14.8640
1.890	7.8936	8.8864	9.8822	10.8797	11.8783	12.8775	13.8770	14.8767
1.900	7.9052	8.8983	9.8943	10.8920	11.8906	12.8899	13.8894	14.8892
1.910	7.9166	8.9100	9.9062	10.9040	11.9027	12.9020	13.9016	14.9014
1.920	7.9278	8.9215	9.9178	10.9157	11.9146	12.9139	13.9135	14.9133
1.930	7.9387	8.9327	9.9292	10.9273	11.9261	12.9255	13.9252	14.9250
1.940	7.9494	8.9437	9.9404	10.9385	11.9375	12.9369	13.9366	14.9364
1.950	7.9600	8.9545	9.9513	10.9496	11.9486	12.9480	13.9477	14.9476
1.960	7.9703	8.9650	9.9621	10.9604	11.9595	12.9590	13.9587	14.9585
1.970	7.9804	8.9754	9.9726	10.9710	11.9701	12.9696	13.9694	14.9692
1.980	7.9904	8.9856	9.9829	10.9814	11.9806	12.9801	13.9799	14.9797
1.990	8.0002	8.9956	9.9930	10.9916	11.9908	12.9904	13.9902	14.9900
2.000	8.0098	9.0054	10.0029	11.0016	12.0009	13.0005	14.0002	15.0001

Mean Value Table to fit curve $y = ar^x$

r	n=18	n=19	n=20	n=21	n=22	n=23	n=24	n=25
1.100	10.9474	11.7139	12.4919	13.2811	14.0811	14.8915	15.7119	16.5420
1.105	11.0527	11.8294	12.6178	13.4175	14.2281	15.0493	15.8804	16.7213
1.110	11.1561	11.9426	12.7410	13.5509	14.3717	15.2031	16.0445	16.8955
1.115	11.2576	12.0537	12.8617	13.6813	14.5119	15.3530	16.2041	17.0648
1.120	11.3573	12.1625	12.9798	13.8087	14.6486	15.4990	16.3594	17.2292
1.125	11.4550	12.2691	13.0953	13.9331	14.7819	15.6412	16.5103	17.3887
1.130	11.5509	12.3735	13.2083	14.0546	14.9119	15.7795	16.6569	17.5434
1.135	11.6449	12.4757	13.3187	14.1732	15.0386	15.9142	16.7993	17.6935
1.140	11.7370	12.5757	13.4266	14.2889	15.1619	16.0451	16.9376	17.8390
1.145	11.8273	12.6736	13.5320	14.4017	15.2820	16.1723	17.0719	17.9800
1.150	11.9157	12.7693	13.6349	14.5117	15.3990	16.2960	17.2021	18.1166
1.155	12.0023	12.8628	13.7353	14.6189	15.5128	16.4162	17.3285	18.2489
1.160	12.0870	12.9543	13.8334	14.7234	15.6235	16.5329	17.4510	18.3770
1.165	12.1700	13.0437	13.9291	14.8252	15.7312	16.6463	17.5698	18.5010
1.170	12.2512	13.1310	14.0224	14.9243	15.8359	16.7564	17.6851	18.6211
1.175	12.3307	13.2164	14.1134	15.0208	15.9378	16.8633	17.7968	18.7373
1.180	12.4084	13.2997	14.2022	15.1149	16.0368	16.9671	17.9050	18.8498
1.185	12.4844	13.3811	14.2888	15.2064	16.1330	17.0678	18.0100	18.9587
1.190	12.5587	13.4606	14.3732	15.2955	16.2266	17.1656	18.1117	19.0641
1.195	12.6314	13.5382	14.4555	15.3822	16.3175	17.2604	18.2102	19.1661
1.200	12.7025	13.6139	14.5357	15.4666	16.4059	17.3525	18.3057	19.2648
1.205	12.7719	13.6879	14.6138	15.5488	16.4917	17.4419	18.3983	19.3604
1.210	12.8398	13.7600	14.6900	15.6287	16.5752	17.5286	18.4881	19.4529
1.215	12.9062	13.8304	14.7642	15.7065	16.6563	17.6127	18.5750	19.5424
1.220	12.9711	13.8991	14.8365	15.7822	16.7351	17.6944	18.6593	19.6291
1.225	13.0344	13.9662	14.9070	15.8558	16.8117	17.7737	18.7410	19.7130
1.230	13.0964	14.0316	14.9757	15.9275	16.8861	17.8506	18.8203	19.7943
1.235	13.1569	14.0955	15.0426	15.9973	16.9585	17.9253	18.8971	19.8731
1.240	13.2160	14.1577	15.1078	16.0651	17.0288	17.9978	18.9716	19.9493
1.245	13.2738	14.2185	15.1714	16.1312	17.0971	18.0682	19.0438	20.0232
1.250	13.3302	14.2778	15.2333	16.1955	17.1635	18.1366	19.1139	20.0948
1.255	13.3854	14.3357	15.2936	16.2581	17.2281	18.2030	19.1819	20.1642
1.260	13.4392	14.3921	15.3524	16.3190	17.2909	18.2674	19.2478	20.2315
1.265	13.4919	14.4472	15.4097	16.3783	17.3520	18.3301	19.3118	20.2967
1.270	13.5433	14.5010	15.4656	16.4360	17.4114	18.3909	19.3740	20.3600
1.275	13.5936	14.5535	15.5200	16.4922	17.4691	18.4501	19.4343	20.4213
1.280	13.6427	14.6047	15.5731	16.5469	17.5253	18.5075	19.4929	20.4809
1.285	13.6907	14.6547	15.6248	16.6002	17.5800	18.5634	19.5498	20.5387
1.290	13.7376	14.7034	15.6753	16.6522	17.6332	18.6177	19.6051	20.5948
1.295	13.7834	14.7510	15.7245	16.7027	17.6850	18.6705	19.6588	20.6492
1.300	13.8282	14.7975	15.7725	16.7520	17.7354	18.7219	19.7110	20.7021

Mean Value Table to fit curve $y = ar^x$

r	n=18	n=19	n=20	n=21	n=22	n=23	n=24	n=25
1.300	13.8282	14.7975	15.7725	16.7520	17.7354	18.7219	19.7110	20.7021
1.310	13.9147	14.8872	15.8649	16.8468	17.8322	18.8205	19.8110	20.8035
1.320	13.9974	14.9727	15.9528	16.9369	17.9241	18.9138	19.9057	20.8992
1.330	14.0765	15.0543	16.0366	17.0225	18.0112	19.0023	19.9953	20.9897
1.340	14.1521	15.1322	16.1164	17.1039	18.0940	19.0863	20.0802	21.0754
1.350	14.2244	15.2065	16.1925	17.1814	18.1728	19.1660	20.1607	21.1567
1.360	14.2936	15.2775	16.2650	17.2552	18.2476	19.2418	20.2372	21.2337
1.370	14.3598	15.3454	16.3342	17.3256	18.3189	19.3138	20.3099	21.3068
1.380	14.4232	15.4103	16.4003	17.3927	18.3868	19.3824	20.3790	21.3764
1.390	14.4840	15.4724	16.4635	17.4568	18.4516	19.4477	20.4448	21.4425
1.400	14.5423	15.5318	16.5239	17.5179	18.5134	19.5100	20.5075	21.5056
1.410	14.5981	15.5888	16.5817	17.5764	18.5725	19.5695	20.5673	21.5656
1.420	14.6518	15.6434	16.6371	17.6324	18.6289	19.6263	20.6244	21.6229
1.430	14.7033	15.6957	16.6901	17.6859	18.6828	19.6806	20.6789	21.6777
1.440	14.7527	15.7459	16.7409	17 7372	18.7345	19.7325	20.7311	21.7300
1.450	14.8002	15.7941	16.7896	17.7864	18.7840	19.7822	20.7810	21.7801
1.460	14.8459	15.8404	16.8364	17.8335	18.8314	19.8299	20.8288	21.8280
1.470	14.8899	15.8849	16.8814	17.8788	18.8769	19.8756	20.8747	21.8740
1.480	14.9322	15.9277	16.9245	17.9222	18.9206	19.9195	20.9186	21.9181
1.490	14.9729	15.9689	16.9661	17.9640	18.9626	19.9616	20.9609	21.9604
1.500	15.0122	16.0086	17.0060	18.0042	19.0029	20.0020	21.0014	22.0010
1.510	15.0500	16.0468	17.0445	18.0429	19.0418	20.0410	21.0404	22.0401
1.520	15.0865	16.0836	17.0815	18.0801	19.0791	20.0784	21.0780	22.0776
1.530	15.1217	16.1191	17.1173	18.1160	19.1151	20.1145	21.1141	22.1138
1.540	15.1557	16.1533	17.1517	18.1506	19.1498	20.1493	21.1489	22.1487
1.550	15.1886	16.1864	17.1849	18.1839	19.1832	20.1828	21.1825	22.1822
1.560	15.2203	16.2184	17.2170	18.2161	19.2155	20.2151	21.2148	22.2147
1.570	15.2510	16.2492	17.2480	18.2472	19.2467	20.2463	21.2461	22.2459
1.580	15.2806	16.2791	17.2780	18.2773	19.2768	20.2765	21.2763	22.2761
1.590	15.3094	16.3079	17.3070	18.3063	19.3059	20.3056	21.3054	22.3053
1.600	15.3371	16.3359	17.3350	18.3344	19.3340	20.3338	21.3336	22.3335
1.610	15.3641	16.3629	17.3621	18.3616	19.3613	20.3611	21.3609	22.3608
1.620	15.3901	16.3891	17.3884	18.3879	19.3876	20.3874	21.3873	22.3872
1.630	15.4154	16.4145	17.4138	18.4134	19.4132	20.4130	21.4129	22.4128
1.640	15.4399	16.4391	17.4385	18.4381	19.4379	20.4378	21.4377	22.4376
1.650	15.4637	16.4629	17.4624	18.4621	19.4619	20.4618	21.4617	22.4616
1.660	15.4868	16.4861	17.4856	18.4853	19.4852	20.4850	21.4850	22.4849
1.670	15.5092	16.5086	17.5082	18.5079	19.5077	20.5076	21.5076	22.5075
1.680	15.5310	16.5304	17.5300	18.5298	19.5297	20.5296	21.5295	22.5295
1.690	15.5521	16.5516	17.5513	18.5511	19.5509	20.5509	21.5508	22.5508
1.700	15.5727	16.5722	17.5719	18.5717	19.5716	20.5715	21.5715	22.5715

Mean Value Table to fit curve $y = ar^x$

r	n=18	n=19	n=20	n=21	n=22	n=23	n=24	n=25
1.700	15.5727	16.5722	17.5719	18.5717	19.5716	20.5715	21.5715	22.5715
1.710	15.5927	16.5923	17.5920	18.5918	19.5917	20.5916	21.5916	22.5916
1.720	15.6121	16.6117	17.6115	18.6113	19.6113	20.6112	21.6112	22.6111
1.730	15.6311	16.6307	17.6305	18.6303	19.6303	20.6302	21.6302	22.6302
1.740	15.6495	16.6492	17.6490	18.6488	19.6488	20.6487	21.6487	22.6487
1.750	15.6674	16.6671	17.6669	18.6668	19.6668	20.6667	21.6667	22.6667
1.760	15.6849	16.6846	17.6845	18.6844	19.6843	20.6843	21.6842	22.6842
1.770	15.7019	16.7017	17.7015	18.7014	19.7014	20.7013	21.7013	22.7013
1.780	15.7185	16.7183	17.7181	18.7181	19.7180	20.7180	21.7180	22.7180
1.790	15.7347	16.7345	17.7344	18.7343	19.7342	20.7342	21.7342	22.7342
1.800	15.7505	16.7503	17.7502	18.7501	19.7501	20.7500	21.7500	22.7500
1.810	15.7658	16.7657	17.7656					
1.820	15.7809	16.7807	17.7806					
1.830	15.7955	16.7954	17.7953					
1.840	15.8098	16.8097	17.8096					
1.850	15.8238	16.8237	17.8236					
1.860	15.8375	16.8374	17.8373					
1.870	15.8508	16.8507	17.8506					
1.880	15.8638	16.8638	17.8637					
1.890	15.8766	16.8765	17.8765					
1.900	15.8891	16.8890	17.8889					
1.910	15.9013	16.9012	17.9011					
1.920	15.9132	16.9131	17.9131					
1.930	15.9249	16.9248	17.9248					
1.940	15.9363	16.9362	17.9362					
1.950	15.9475	16.9474	17.9474					
1.960	15.9584	16.9584	17.9584					
1.970	15.9692	16.9691	17.9691					
1.980	15.9797	16.9796	17.9796					
1.990	15.9900	16.9899	17.9899					
2.000	16.0001	17.0000	18.0000					

Mean Value Table to fit curve $y = ar^x$

r	n=26	n=27	n=28	n=29	n=30	n=31	n=32	n=33
1.100	17.3814	18.2296	19.0863	19.9511	20.8238	21.7038	22.5909	23.4848
1.105	17.5713	18.4302	19.2975	20.1728	21.0558	21.9460	22.8430	23.7467
1.110	17.7557	18.6246	19.5018	20.3869	21.2794	22.1790	23.0853	23.9979
1.115	17.9345	18.8129	19.6994	20.5935	21.4950	22.4033	23.3180	24.2388
1.120	18.1079	18.9951	19.8902	20.7929	21.7026	22.6189	23.5414	24.4698
1.125	18.2759	19.1714	20.0746	20.9851	21.9024	22.8261	23.7558	24.6910
1.130	18.4386	19.3418	20.2526	21.1704	22.0948	23.0253	23.9615	24.9029
1.135	18.5961	19.5066	20.4243	21.3489	22.2798	23.2165	24.1587	25.1059
1.140	18.7486	19.6658	20.5900	21.5209	22.4577	23.4002	24.3478	25.3002
1.145	18.8961	19.8196	20.7499	21.6865	22.6289	23.5766	24.5292	25.4862
1.150	19.0388	19.9681	20.9040	21.8459	22.7934	23.7459	24.7030	25.6643
1.155	19.1768	20.1115	21.0526	21.9994	22.9515	23.9084	24.8696	25.8348
1.160	19.3102	20.2500	21.1959	22.1472	23.1036	24.0644	25.0294	25.9981
1.165	19.4392	20.3837	21.3340	22.2895	23.2497	24.2142	25.1826	26.1545
1.170	19.5638	20.5127	21.4671	22.4264	23.3902	24.3581	25.3295	26.3042
1.175	19.6844	20.6372	21.5954	22.5582	23.5253	24.4962	25.4704	26.4477
1.180	19.8009	20.7575	21.7190	22.6851	23.6552	24.6288	25.6055	26.5851
1.185	19.9134	20.8735	21.8383	22.8073	23.7801	24.7562	25.7352	26.7169
1.190	20.0223	20.9855	21.9532	22.9249	23.9002	24.8785	25.8597	26.8432
1.195	20.1275	21.0936	22.0640	23.0382	24.0157	24.9962	25.9792	26.9644
1.200	20.2291	21.1980	22.1709	23.1473	24.1269	25.1092	26.0939	27.0806
1.205	20.3274	21.2988	22.2740	23.2525	24.2339	25.2179	26.2041	27.1922
1.210	20.4224	21.3961	22.3734	23.3538	24.3369	25.3225	26.3100	27.2994
1.215	20.5143	21.4901	22.4693	23.4515	24.4362	25.4231	26.4119	27.4023
1.220	20.6032	21.5809	22.5619	23.5456	24.5317	25.5199	26.5098	27.5012
1.225	20.6891	21.6687	22.6512	23.6364	24.6238	25.6131	26.6040	27.5963
1.230	20.7722	21.7535	22.7375	23.7240	24.7125	25.7029	26.6947	27.6878
1.235	20.8527	21.8354	22.8208	23.8085	24.7981	25.7894	26.7820	27.7759
1.240	20.9305	21.9147	22.9013	23.8901	24.8807	25.8728	26.8662	27.8606
1.245	21.0059	21.9913	22.9791	23.9689	24.9603	25.9532	26.9[illegible]72	27.9423
1.250	21.0788	22.0654	23.0543	24.0449	25.0372	26.0307	27.0[illegible]54	28.0209
1.255	21.1495	22.1372	23.1269	24.1185	25.1114	26.1056	27.1008	28.0968
1.260	21.2179	22.2066	23.1972	24.1895	25.1831	26.1778	27.1735	28.1699
1.265	21.2842	22.2738	23.2653	24.2582	25.2524	26.2476	27.2437	28.2405
1.270	21.3484	22.3389	23.3311	24.3246	25.3194	26.3151	27.3116	28.3087
1.275	21.4107	22.4019	23.3948	24.3889	25.3842	26.3803	27.3771	28.3745
1.280	21.4711	22.4630	23.4565	24.4511	25.4468	26.4433	27.4404	28.4381
1.285	21.5296	22.5222	23.5162	24.5114	25.5075	26.5043	27.5017	28.4996
1.290	21.5864	22.5796	23.5742	24.5697	25.5662	26.5633	27.5610	28.5591
1.295	21.6415	22.6353	23.6303	24.6263	25.6230	26.6204	27.6183	28.6167
1.300	21.6950	22.6893	23.6847	24.6811	25.6781	26.6758	27.6739	28.6724

Mean Value Table to fit curve $y = ar^x$

r	n=26	n=27	n=28	n=29	n=30	n=31	n=32	n=33
1.300	21.6950	22.6893	23.6847	24.6811	25.6781	26.6758	27.6739	28.6724
1.310	21.7974	22.7926	23.7888	24.7857	25.7833	26.7814	27.7799	28.7786
1.320	21.8941	22.8900	23.8868	24.8842	25.8822	26.8807	27.8794	28.8785
1.330	21.9854	22.9819	23.9792	24.9771	25.9755	26.9742	27.9732	28.9724
1.340	22.0717	23.0688	24.0666	25.0648	26.0634	27.0624	28.0616	29.0609
1.350	22.1535	23.1510	24.1491	25.1477	26.1465	27.1457	28.1450	29.1445
1.360	22.2310	23.2289	24.2273	25.2261	26.2252	27.2245	28.2239	29.2235
1.370	22.3045	23.3028	24.3015	25.3004	26.2997	27.2991	28.2986	29.2983
1.380	22.3744	23.3729	24.3718	25.3710	26.3703	27.3699	28.3695	29.3692
1.390	22.4409	23.4396	24.4387	25.4380	26.4374	27.4370	28.4367	29.4365
1.400	22.5041	23.5031	24.5023	25.5017	26.5012	27.5009	28.5007	29.5005
1.410	22.5644	23.5635	24.5628	25.5623	26.5620	27.5617	28.5615	29.5614
1.420	22.6219	23.6211	24.6206	25.6202	26.6199	27.6196	28.6195	29.6194
1.430	22.6768	23.6761	24.6757	25.6753	26.6751	27.6749	28.6748	29.6747
1.440	22.7293	23.7287	24.7283	25.7280	26.7278	27.7277	28.7275	29.7275
1.450	22.7794	23.7790	24.7786	25.7784	26.7782	27.7781	28.7780	29.7779
1.460	22.8275	23.8271	24.8268	25.8266	26.8264	27.8263	28.8263	29.8262
1.470	22.8735	23.8732	24.8729	25.8727	26.8726	27.8725	28.8725	29.8724
1.480	22.9176	23.9173	24.9171	25.9170	26.9169	27.9168	38.9168	29.9167
1.490	22.9600	23.9598	24.9596	25.9595	26.9594	27.9593	28.9593	29.9592
1.500	23.0007	24.0005	25.0003	26.0002	27.0002	28.0001	29.0001	30.0001
1.510	23.0398	24.0396	25.0395	26.0394	27.0393	28.0393	29.0393	30.0393
1.520	23.0774	24.0773	25.0771	26.0771	27.0770	28.0770	29.0770	30.0770
1.530	23.1136	24.1135	25.1134	26.1133	27.1133	28.1133	29.1132	30.1132
1.540	23.1485	24.1484	25.1483	26.1483	27.1482	28.1482	29.1482	30.1482
1.550	23.1821	24.1820	25.1819	26.1819	27.1819	28.1819	29.1818	30.1818
1.560	23.2145	24.2145	25.2144	26.2144	27.2143	28.2143	29.2143	30.2143
1.570	23.2458	24.2458	25.2457	26.2457	27.2457	28.2456	29.2456	30.2456
1.580	23.2760	24.2760	25.2759	26.2759	27.2759	28.2759	29.2759	30.2759
1.590	23.3052	24.3052	25.3051	26.3051	27.3051	28.3051	29.3051	30.3051
1.600	23.3335	24.3334	25.3334	26.3334	27.3334	28.3333	29.3333	30.3333
1.610	23.3608	24.3607	25.3607	26.3607	27.3607	28.3607	29.3607	30.3607
1.620	23.3872	24.3872	25.3871	26.3871	27.3871	28.3871	29.3871	30.3871
1.630	23.4128	24.4127	25.4127	26.4127	27.4127	28.4127	29.4127	30.4127
1.640	23.4376	24.4375	25.4375	26.4375	27.4375	28.4375	29.4375	30.4375
1.650	23.4616	24.4615	25.4615	26.4616	27.4615	28.4615	29.4615	30.4615
1.660	23.4849	24.4849	25.4849	26.4849	27.4849	28.4849	29.4849	30.4849
1.670	23.5075	24.5075	25.5075	26.5075	27.5075	28.5075	29.5075	30.5075
1.680	23.5294	24.5294	25.5294	26.5294	27.5294	28.5294	29.5294	30.5294
1.690	23.5507	24.5508	25.5507	26.5507	27.5507	28.5507	29.5507	30.5507
1.700	23.5714	24.5715	25.5714	26.5714	27.5714	28.5714	29.5714	30.5714

Mean Value Table to fit curve $y = ar^x$

r	n=26	n=27	n=28	n=29	n=30	n=31	n=32	n=33
1.700	23.5715	24.5714	25.5714	26.5714	27.5714	28.5714	29.5714	30.5714
1.710	23.5916	24.5916	25.5916	26.5916	27.5916	28.5916	29.5916	30.5915
1.720	23.6111	24.6111	25.6111	26.6111	27.6111	28.6111	29.6111	30.6111
1.730	23.6302	24.6301	25.6301	26.6301	27.6301	28.6301	29.6301	30.6301
1.740	23.6487	24.6487	25.6487	26.6487	27.6487	28.6486	29.6486	30.6486
1.750	23.6667	24.6667	25.6667	26.6667	27.6667	28.6667	29.6667	30.6667
1.760	23.6842	24.6842	25.6842	26.6842	27.6842	28.6842	29.6842	30.6842
1.770	23.7013	24.7013	25.7013	26.7013	27.7013	28.7013	29.7013	30.7013
1.780	23.7180	24.7180	25.7180	26.7180	27.7179	28.7179	29.7179	30.7179
1.790	23.7342	24.7342	25.7342	26.7342	27.7342	28.7342	29.7342	30.7342
1.800	23.7500	24.7500	25.7500	26.7500	27.7500	28.7500	29.7500	30.7500

Mean Value Table to fit curve $y = ar^x$

r	n=34	n=35	n=36	n=37	n=38	n=39	n=40	n=41
1.100	24.3851	25.2914	26.2035	27.1213	28.0438	28.9715	29.9038	30.8404
1.105	24.6565	25.5721	26.4933	27.4197	28.3510	29.2869	30.2272	31.1716
1.110	24.9164	25.8406	26.7700	27.7043	28.6433	29.5867	30.5341	31.4853
1.115	25.1653	26.0972	27.0340	27.9755	28.9214	29.8714	30.8252	31.7825
1.120	25.4035	26.3423	27.2859	28.2339	29.1859	30.1418	31.1012	32.0639
1.125	25.6314	26.5765	27.5262	28.4799	29.4375	30.3986	31.3630	32.3304
1.130	25.8493	26.8002	27.7552	28.7142	29.6767	30.6424	31.6112	32.5828
1.135	26.0577	27.0137	27.9737	28.9372	29.9041	30.8740	31.8467	32.8219
1.140	26.2569	27.2176	28.1820	29.1497	30.1204	31.0940	32.0700	33.0484
1.145	26.4474	27.4123	28.3805	29.3519	30.3261	31.3029	32.2820	33.2632
1.150	26.6295	27.5981	28.5699	29.5446	30.5219	31.5015	32.4832	33.4669
1.155	26.8036	27.7756	28.7506	29.7282	30.7082	31.6903	32.6743	33.6601
1.160	26.9701	27.9452	28.9229	29.9031	30.8855	31.8698	32.8559	33.8435
1.165	27.1294	28.1072	29.0874	30.0699	31.0544	32.0407	33.0285	34.0178
1.170	27.2818	28.2620	29.2445	30.2290	31.2153	32.2033	33.1927	34.1834
1.175	27.4276	28.4100	29.3944	30.3808	31.3688	32.3582	33.3490	34.3409
1.180	27.5672	28.5515	29.5377	30.5256	31.5151	32.5059	33.4978	34.4908
1.185	27.7009	28.6869	29.6747	30.6640	31.6547	32.6467	33.6397	34.6336
1.190	27.8289	28.8164	29.8056	30.7962	31.7881	32.7810	33.7749	34.7696
1.195	27.9516	28.9405	29.9309	30.9226	31.9155	32.9093	33.9040	34.8994
1.200	28.0692	29.0594	30.0509	31.0436	32.0373	33.0319	34.0272	35.0233
1.205	28.1820	29.1733	30.1657	31.1593	32.1538	33.1490	34.1450	35.1416
1.210	28.2903	29.2825	30.2758	31.2701	32.2653	33.2611	34.2576	35.2546
1.215	28.3942	29.3872	30.3813	31.3763	32.3721	33.3685	34.3654	35.3628
1.220	28.4940	29.4878	30.4826	31.4782	32.4744	33.4713	34.4686	35.4663
1.225	28.5899	29.5844	30.5797	31.5759	32.5726	33.5698	34.5675	35.5655
1.230	28.6820	29.6772	30.6731	31.6696	32.6667	33.6643	34.6623	35.6606
1.235	28.7707	29.7664	30.7627	31.7597	32.7572	33.7551	34.7533	35.7518
1.240	28.8560	29.8521	30.8489	31.8463	32.8440	33.8422	34.8407	35.8394
1.245	28.9381	29.9347	30.9319	31.9295	32.9276	33.9259	34.9246	35.9235
1.250	29.0172	30.0142	31.0117	32.0096	33.0079	34.0065	35.0053	36.0044
1.255	29.0935	30.0908	31.0886	32.0867	33.0852	34.0840	35.0830	36.0821
1.260	29.1670	30.1646	31.1626	32.1610	33.1597	34.1586	35.1577	36.1570
1.265	29.2379	30.2358	31.2340	32.2326	33.2314	34.2305	35.2297	36.2291
1.270	29.3063	30.3044	31.3029	32.3016	33.3006	34.2998	35.2991	36.2986
1.275	29.3724	30.3707	31.3694	32.3683	33.3674	34.3666	35.3660	36.3656
1.280	29.4363	30.4348	31.4335	32.4326	33.4318	34.4311	35.4306	36.4302
1.285	29.4980	30.4966	31.4956	32.4947	33.4940	34.4934	35.4930	36.4926
1.290	29.5576	30.5564	31.5555	32.5547	33.5541	34.5536	35.5532	36.5529
1.295	29.6153	30.6144	31.6134	32.6128	33.6122	34.6118	35.6115	36.6112
1.300	29.6712	30.6703	31.6695	32.6689	33.6684	34.6681	35.6678	36.6675

Mean Value Table to fit curve $y = ar^x$

r	n=34	n=35	n=36	n=37	n=38	n=39	n=40	n=41
1.300	29.6712	30.6703	31.6695	32.6689	33.6684	34.6681	35.6678	36.6675
1.310	29.7777	30.7769	31.7764	32.7759	33.7755	34.7752	35.7750	36.7748
1.320	29.8777	30.8771	31.8766	32.8763	33.8760	34.8758	35.8756	36.8755
1.330	29.9718	30.9713	31.9709	32.9707	33.9704	34.9703	35.9701	36.9700
1.340	30.0604	31.0601	32.0598	33.0596	34.0594	35.0593	36.0592	37.0591
1.350	30.1441	31.1438	32.1436	33.1434	34.1433	35.1433	36.1431	37.1430
1.360	30.2232	31.2230	32.2228	33.2226	34.2225	35.2225	36.2224	37.2224
1.370	30.2981	31.2979	32.2977	33.2976	34.2975	35.2975	36.2974	37.2974
1.380	30.3690	31.3689	32.3688	33.3687	34.3686	35.3686	36.3685	37.3685
1.390	30.4364	31.4362	32.4362	33.4361	34.4360	35.4360	36.4360	37.4360
1.400	30.5004	31.5003	32.5002	33.5001	34.5001	35.5001	36.5001	37.5000
1.410	30.5613	31.5612	32.5611	33.5611	34.5611	35.5610	36.5610	37.5610
1.420	30.6193	31.6192	32.6192	33.6191	34.6191	35.6191	36.6191	37.6191
1.430	30.6746	31.6745	32.6745	33.6745	34.6745	35.6745	36.6744	37.6744
1.440	30.7274	31.7274	32.7273	33.7273	34.7273	35.7273	36.7273	37.7273
1.450	30.7779	31.7779	32.7778	33.7778	34.7778	35.7778	36.7778	37.7778
1.460	30.8262	31.8261	32.8261	33.8261	34.8261	35.8261	36.8261	37.8261
1.470	30.8724	31.8724	32.8724	33.8724	34.8724	35.8724	36.8723	37.8723
1.480	30.9167	31.9167	32.9167	33.9167	34.9167	35.9167	36.9167	37.9167
1.490	30.9592	31.9592	32.9592	33.9592	34.9592	35.9592	36.9592	37.9592
1.500	31.0000	32.0000	33.0000	34.0000	35.0000	36.0000	37.0000	38.0000
1.510	31.0392	32.0392	33.0392	34.0392	35.0392	36.0392	37.0392	38.0392
1.520	31.0769	32.0769	33.0769	34.0769	35.0769	36.0769	37.0769	38.0769
1.530	31.1132	32.1132	33.1132	34.1132	35.1132	36.1132	37.1132	38.1132
1.540	31.1482	32.1482	33.1482	34.1482	35.1482	36.1482	37.1481	38.1481
1.550	31.1818	32.1818	33.1818	34.1818	35.1818	36.1818	37.1818	38.1818
1.560	31.2143	32.2143	33.2143	34.2143	35.2143	36.2143	37.2143	38.2143
1.570	31.2456	32.2456	33.2456	34.2456	35.2456	36.2456	37.2456	38.2456
1.580	31.2759	32.2759	33.2759	34.2759	35.2759	36.2759	37.2759	38.2759
1.590	31.3051	32.3051	33.3051	34.3051	35.3051	36.3051	37.3051	38.3051
1.600	31.3333	32.3333	33.3333	34.3333	35.3333	36.3333	37.3333	38.3333
1.610	31.3607	32.3607	33.3607	34.3607	35.3607	36.3607	37.3607	38.3607
1.620	31.3871	32.3871	33.3871	34.3871	35.3871	36.3871	37.3871	38.3871
1.630	31.4127	32.4127	33.4127	34.4127	35.4127	36.4127	37.4127	38.4127
1.640	31.4375	32.4375	33.4375	34.4375	35.4375	36.4375	37.4375	38.4375
1.650	31.4615	32.4615	33.4615	34.4615	35.4615	36.4615	37.4615	38.4615
1.660	31.4848	32.4848	33.4848	34.4848	35.4848	36.4848	37.4848	38.4848
1.670	31.5075	32.5075	33.5075	34.5075	35.5075	36.5075	37.5075	38.5075
1.680	31.5294	32.5294	33.5294	34.5294	35.5294	36.5294	37.5294	38.5294
1.690	31.5507	32.5507	33.5507	34.5507	35.5507	36.5507	37.5507	38.5507
1.700	31.5714	32.5714	33.5714	34.5714	35.5714	36.5714	37.5714	38.5714

Mean Value Table to fit curve $y = ar^x$

r	n=34	n=35	n=36	n=37	n=38	n=39	n=40	n=41
1.700	31.5714	32.5714	33.5714	34.5714	35.5714	36.5714	37.5714	38.5714
1.710	31.5915	32.5915	33.5915	34.5915	35.5915	36.5915	37.5915	38.5915
1.720	31.6111	32.6111	33.6111	34.6111	35.6111	36.6111	37.6111	38.6111
1.730	31.6301	32.6301	33.6301	34.6301	35.6301	36.6301	37.6301	38.6301
1.740	31.6486	32.6486	33.6486	34.6486	35.6486	36.6486	37.6486	38.6486
1.750	31.6667	32.6667	33.6667	34.6667	35.6667	36.6667	37.6667	38.6667
1.760	31.6842	32.6842	33.6842	34.6842	35.6842	36.6842	37.6842	38.6842
1.770	31.7013	32.7013	33.7013	34.7013	35.7013	36.7013	37.7013	38.7013
1.780	31.7179	32.7179	33.7179	34.7179	35.7179	36.7179	37.7179	38.7179
1.790	31.7342	32.7342	33.7342	34.7342	35.7342	36.7342	37.7342	38.7342
1.800	31.7500	32.7500	33.7500	34.7500	35.7500	36.7500	37.7500	38.7500

Mean Value Table to fit curve $y = ar^x$

r	n=42	n=43	n=44
1.100	31.7812	32.7259	33.6742
1.105	32.1198	33.0717	34.0269
1.100	32.4401	33.3983	34.3596
1.115	32.7431	33.7068	34.6733
1.120	33.0296	33.9981	34.9692
1.125	33.3006	34.2733	35.2484
1.130	33.5569	34.5333	35.5119
1.135	33.7994	34.7790	35.7606
1.140	34.0289	35.0014	35.9955
1.145	34.2463	35.2311	36.2175
1.150	34.4522	35.4391	36.4275
1.155	34.6474	35.6362	36.6261
1.160	34.8326	35.8229	36.8143
1.165	35.0083	35.9999	36.9926
1.170	35.1752	36.1680	37.1617
1.175	35.3338	36.3276	37.3222
1.180	35.4847	36.4793	37.4747
1.185	35.6283	36.6237	37.6197
1.190	35.7651	36.7611	37.7577
1.195	35.8955	36.8921	37.8891
1.200	36.0199	37.0169	38.0144
1.205	36.1386	37.1361	38.1340
1.210	36.2521	37.2499	38.2481
1.215	36.3606	37.3588	38.3572
1.220	36.4645	37.4629	38.4615
1.225	36.5639	37.5625	38.5614
1.230	36.6592	37.6580	38.6570
1.235	36.7506	37.7496	38.7488
1.240	36.8383	37.8375	38.8367
1.245	36.9226	37.9218	38.9212
1.250	37.0036	38.0029	39.0024
1.255	37.0815	38.0809	39.0804
1.260	37.1564	38.1559	39.1555
1.265	37.2286	38.2282	39.2278
1.270	37.2981	38.2978	39.2975
1.275	37.3652	38.3649	39.3646
1.280	37.4299	38.4296	39.4294
1.285	37.4923	38.4921	39.4919
1.290	37.5527	38.5525	39.5523
1.295	37.6110	38.6108	39.6107
1.300	37.6674	38.6672	39.6671

Mean Value Table to fit curve $y = ar^x$

r	n=42	n=43	n=44
1.300	37.6674	38.6672	39.6671
1.310	37.7747	38.7746	39.7745
1.320	37.8754	38.8753	39.8752
1.330	37.9700	38.9699	39.9699
1.340	38.0590	39.0590	40.0589
1.350	38.1430	39.1430	40.1429
1.360	38.2223	39.2223	40.2223
1.370	38.2974	39.2974	40.2973
1.380	38.3685	39.3685	40.3685
1.390	38.4359	39.4359	40.4359
1.400	38.5000	39.5000	40.5000
1.410	38.5610	39.5610	40.5610
1.420	38.6191	39.6191	40.6191
1.430	38.6744	39.6744	40.6744
1.440	38.7273	39.7273	40.7273
1.450	38.7778	39.7778	40.7778
1.460	38.8261	39.8261	40.8261
1.470	38.8723	39.8723	40.8723
1.480	38.9167	39.9167	40.9167
1.490	38.9592	39.9592	40.9592
1.500	39.0000	40.0000	41.0000
1.510	39.0392	40.0392	41.0392
1.520	39.0769	40.0769	41.0769
1.530	39.1132	40.1132	41.1132
1.540	39.1481	40.1481	41.1481
1.550	39.1818	40.1818	41.1818
1.560	39.2143	40.2143	41.2143
1.570	39.2456	40.2456	41.2456
1.580	39.2759	40.2759	41.2759
1.590	39.3051	40.3051	41.3051
1.600	39.3333	40.3333	41.3333
1.610	39.3607	40.3607	41.3607
1.620	39.3871	40.3871	41.3871
1.630	39.4127	40.4127	41.4127
1.640	39.4375	40.4375	41.4375
1.650	39.4615	40.4615	41.4615
1.660	39.4848	40.4848	41.4848
1.670	39.5075	40.5075	41.5075
1.680	39.5294	40.5294	41.5294
1.690	39.5507	40.5507	41.5507
1.700	39.5714	40.5714	41.5714

Mean Value Table to fit curve $y = ar^x$

r	n=42	n=43	n=44
1.700	39.5714	40.5714	41.5714
1.710	39.5915	40.5915	41.5915
1.720	39.6111	40.6111	41.6111
1.730	39.6301	40.6301	41.6301
1.740	39.6486	40.6486	41.6486
1.750	39.6667	40.6667	41.6667
1.760	39.6842	40.6842	41.6842
1.770	39.7013	40.7013	41.7013
1.780	39.7179	40.7179	41.7179
1.790	39.7342	40.7342	41.7342
1.800	39.7500	40.7500	41.7500

Mean Value Table to fit curve $y = ar^x$

r	n=41	n=42	n=43	n=44	n=45	n=46	n=47	n=48
1.000	20.0000	20.5000	21.0000	21.5000	22.0000	22.5000	23.0000	23.5000
1.001	20.1399	20.6468	21.1539	21.6612	22.1686	22.6762	23.1839	23.6918
1.002	20.2797	20.7935	21.3076	21.8221	22.3369	22.8521	23.3676	23.8834
1.003	20.4193	20.9400	21.4612	21.9829	22.5051	23.0278	23.5510	24.0747
1.004	20.5586	21.0862	21.6145	22.1434	22.6730	23.2032	23.7341	24.2657
1.005	20.6978	21.2322	21.7675	22.3036	22.8405	23.3783	23.9169	24.4563
1.006	20.8367	21.3779	21.9202	22.4635	23.0078	23.5530	24.0993	24.6465
1.007	20.9753	21.5234	22.0726	22.6231	23.1746	23.7273	24.2812	24.8362
1.008	21.1136	21.6685	22.2247	22.7822	23.3411	23.9013	24.4627	25.0255
1.009	21.2515	21.8132	22.3764	22.9410	23.5071	24.0747	24.6437	25.2142
1.010	21.3892	21.9576	22.5277	23.0994	23.6727	24.2476	24.8242	25.4024
1.011	21.5265	22.1016	22.6786	23.2573	23.8378	24.4201	25.0041	25.5900
1.012	21.6634	22.2452	22.8290	23.4147	24.0024	24.5919	25.1834	25.7769
1.013	21.7999	22.3884	22.9789	23.5716	24.1664	24.7632	25.3621	25.9631
1.014	21.9359	22.5310	23.1284	23.7280	24.3298	24.9338	25.5401	26.1486
1.015	22.0716	22.6733	23.2773	23.8838	24.4926	25.1038	25.7174	26.3333
1.016	22.2067	22.8150	23.4257	24.0390	24.6548	25.2731	25.8939	26.5173
1.017	22.3414	22.9561	23.5735	24.1936	24.8163	25.4417	26.0697	26.7004
1.018	22.4756	23.0968	23.7208	24.3476	24.9772	25.6095	26.2447	26.8826
1.019	22.6092	23.2368	23.8674	24.5009	25.1373	25.7766	26.4188	27.0640
1.020	22.7424	23.3763	24.0134	24.6535	25.2966	25.9429	26.5921	27.2444
1.021	22.8749	23.5152	24.1587	24.8054	25.4552	26.1083	26.7645	27.4239
1.022	23.0069	23.6535	24.3033	24.9565	25.6131	26.2729	26.9360	27.6023
1.023	23.1383	23.7911	24.4473	25.1070	25.7701	26.4366	27.1065	27.7798
1.024	23.2691	23.9280	24.5905	25.2566	25.9262	26.5994	27.2760	27.9562
1.025	23.3993	24.0643	24.7330	25.4054	26.0815	26.7612	27.4446	28.1315
1.026	23.5288	24.1999	24.8748	25.5535	26.2359	26.9222	27.6121	28.3057
1.027	23.6577	24.3348	25.0158	25.7007	26.3895	27.0821	27.7786	28.4788
1.028	23.7859	24.4689	25.1560	25.8470	26.5421	27.2410	27.9440	28.6508
1.029	23.9134	24.6023	25.2954	25.9925	26.6937	27.3990	28.1083	28.8215
1.030	24.0403	24.7350	25.4340	26.1371	26.8444	27.5559	28.2715	28.9911
1.031	24.1664	24.8669	25.5717	26.2808	26.9942	27.7117	28.4335	29.1595
1.032	24.2918	24.9980	25.7086	26.4236	27.1429	27.8665	28.5944	29.3256
1.033	24.4165	25.1283	25.8446	26.5654	27.2906	28.0202	28.7542	29.4924
1.034	24.5404	25.2578	25.9798	26.7063	27.4373	28.1728	28.9127	29.6570
1.035	24.6636	25.3865	26.1141	26.8462	27.5830	28.3243	29.0701	29.8203
1.036	24.7860	25.5144	26.2474	26.9852	27.7276	28.4746	29.2262	29.9823
1.037	24.9077	25.6414	26.3799	27.1231	27.8711	28.6238	29.3811	30.1429
1.038	25.0285	25.7675	26.5114	27.2601	28.0136	28.7718	29.5347	30.3022
1.039	25.1486	25.8928	26.6420	27.3961	28.1550	28.9187	29.6871	30.4602
1.040	25.2678	26.0172	26.7716	27.5310	28.2953	29.0644	29.8382	30.6168

Mean Value Table to fit curve $y = ar^x$

r	n=41	n=42	n=43	n=44	n=45	n=46	n=47	n=48
1.040	25.2678	26.0172	26.7716	27.5310	28.2953	29.0644	29.8382	30.6168
1.041	25.3862	26.1407	26.9003	27.6649	28.4344	29.2088	29.9880	30.7720
1.042	25.5038	26.2634	27.0280	27.7978	28.5725	29.3521	30.1366	30.9258
1.043	25.6206	26.3851	27.1548	27.9296	28.7094	29.4942	30.2838	31.0783
1.044	25.7366	26.5060	27.2806	28.0603	28.8452	29.6350	30.4297	31.2293
1.045	25.8517	26.6259	27.4054	28.1900	28.9798	29.7746	30.5743	31.3789
1.046	25.9659	26.7449	27.5291	28.3186	29.1133	29.9130	30.7176	31.5271
1.047	26.0793	26.8629	27.6519	28.4462	29.2456	30.0501	30.8596	31.6739
1.048	26.1918	26.9801	27.7737	28.5726	29.3768	30.1860	31.0002	31.8193
1.049	26.3035	27.0963	27.8945	28.6980	29.5067	30.3205	31.1394	31.9632
1.050	26.4143	27.2116	28.0143	28.8223	29.6356	30.4539	31.2774	32.1057
1.051	26.5242	27.3259	28.1330	28.9455	29.7632	30.5861	31.4139	32.2467
1.052	26.6332	27.4392	28.2507	29.0676	29.8897	30.7169	31.5492	32.3864
1.053	26.7414	27.5517	28.3674	29.1886	30.0149	30.8465	31.6830	32.5245
1.054	26.8487	27.6631	28.4831	29.3084	30.1390	30.9748	31.8156	32.6613
1.055	26.9550	27.7736	28.5977	29.4272	30.2619	31.1018	31.9467	32.7965
1.056	27.0605	27.8832	28.7113	29.5449	30.3837	31.2276	32.0766	32.9304
1.057	27.1651	27.9918	28.8239	29.6614	30.5042	31.3521	32.2050	33.0628
1.058	27.2688	28.0994	28.9354	29.7769	30.6236	31.4754	32.3321	33.1938
1.059	27.3716	28.2060	29.0459	29.8912	30.7417	31.5973	32.4579	33.3233
1.060	27.4736	28.3117	29.1555	30.0044	30.8587	31.7181	32.5823	33.4515
1.061	27.5746	28.4164	29.2638	30.1166	30.9745	31.8375	32.7054	33.5782
1.062	27.6747	28.5202	29.3712	30.2276	31.0891	31.9557	32.8272	33.7034
1.063	27.7739	28.6230	29.4776	30.3375	31.2026	32.0727	32.9476	33.8273
1.064	27.8722	28.7249	29.5829	30.4463	31.3148	32.1883	33.0667	33.9498
1.065	27.9697	28.8258	29.6873	30.5540	31.4259	32.3028	33.1845	34.0708
1.066	28.0662	28.9257	29.7905	30.6607	31.5359	32.4160	33.3009	34.1905
1.067	28.1619	29.0246	29.8928	30.7662	31.6446	32.5280	33.4161	34.3087
1.068	28.2566	29.1227	29.9940	30.8706	31.7522	32.6387	33.5299	34.4256
1.069	28.3505	29.2197	30.0943	30.9740	31.8587	32.7483	33.6425	34.5412
1.070	28.4435	29.3158	30.1935	31.0763	31.9640	32.8565	33.7537	34.6553
1.071	28.5356	29.4110	30.2917	31.1775	32.0682	32.9636	33.8637	34.7681
1.072	28.6268	29.5052	30.3889	31.2776	32.1712	33.0695	33.9724	34.8796
1.073	28.7172	29.5985	30.4851	31.3767	32.2731	33.1742	34.0798	34.9897
1.074	28.8066	29.6909	30.5803	31.4747	32.3739	33.2777	34.1860	35.0985
1.075	28.8952	29.7823	30.6745	31.5716	32.4735	33.3800	34.2909	35.2060
1.076	28.9830	29.8728	30.7677	31.6675	32.5721	33.4812	34.3946	35.3122
1.077	29.0698	29.9624	30.8599	31.7624	32.6695	33.5812	34.4971	35.4171
1.078	29.1558	30.0510	30.9512	31.8562	32.7659	33.6800	34.5983	35.5207
1.079	29.2410	30.1388	31.0415	31.9490	32.8612	33.7777	34.6984	35.6231
1.080	29.3253	30.2256	31.1308	32.0408	32.9553	33.8742	34.7972	35.7242

Mean Value Table to fit curve $y = ar^x$

r	n=41	n=42	n=43	n=44	n=45	n=46	n=47	n=48
1.080	29.3253	30.2256	31.1308	32.0408	32.9553	33.8742	34.7972	35.7242
1.081	29.4087	30.3115	31.2192	32.1316	33.0485	33.9696	34.8948	35.8240
1.082	29.4913	30.3966	31.3067	32.2214	33.1405	34.0639	34.9913	35.9226
1.083	29.5731	30.4807	31.3931	32.3101	33.2315	34.1571	35.0866	36.0200
1.084	29.6540	30.5640	31.4787	32.3979	33.3215	34.2491	35.1808	36.1161
1.085	29.7341	30.6464	31.5633	32.4847	33.4104	34.3401	35.2738	36.2111
1.086	29.8134	30.7279	31.6470	32.5705	33.4983	34.4300	35.3656	36.3048
1.087	29.8919	30.8085	31.7298	32.6554	33.5851	34.5189	35.4564	36.3974
1.088	29.9696	30.8883	31.8116	32.7393	33.6710	34.6066	35.5460	36.4889
1.089	30.0464	30.9673	31.8926	32.8222	33.7558	34.6934	35.6345	36.5792
1.090	30.1225	31.0454	31.9727	32.9042	33.8397	34.7790	35.7220	36.6683
1.091	30.1977	31.1226	32.0518	32.9852	33.9226	34.8637	35.8083	36.7563
1.092	30.2722	31.1990	32.1302	33.0654	34.0045	34.9473	35.8936	36.8432
1.093	30.3459	31.2746	32.2076	33.1446	34.0855	35.0299	35.9779	36.9290
1.094	30.4188	31.3494	32.2841	33.2229	34.1655	35.1116	36.0611	37.0137
1.095	30.4910	31.4233	32.3599	33.3003	34.2445	35.1922	36.1432	37.0974
1.096	30.5624	31.4965	32.4347	33.3768	34.3226	35.2719	36.2244	37.1800
1.097	30.6330	31.5688	32.5087	33.4525	34.3998	35.3506	36.3045	37.2615
1.098	30.7029	31.6404	32.5819	33.5272	34.4761	35.4283	36.3837	37.3420
1.099	30.7720	31.7112	32.6543	33.6011	34.5515	35.5051	36.4618	37.4215
1.100	30.8404	31.7812	32.7259	33.6742	34.6260	35.5810	36.5390	37.4999

Mean Value Table to fit curve $y = ar^x$

r	n=49	n=50	n=51	n=52	n=53	n=54	n=55	n=56
1.000	24.0000	24.5000	25.0000	25.5000	26.0000	26.5000	27.0000	27.5000
1.001	24.1999	24.7081	25.2165	25.7251	26.2339	26.7428	27.2518	27.7611
1.002	24.3995	24.9160	25.4328	25.9500	26.4674	26.9852	27.5034	28.0219
1.003	24.5989	25.1236	25.6488	26.1745	26.7007	27.2273	27.7545	28.2822
1.004	24.7979	25.3308	25.8643	26.3986	26.9334	27.4690	28.0052	28.5421
1.005	24.9965	25.5376	26.0795	26.6222	27.1657	27.7101	28.2553	28.8013
1.006	25.1947	25.7439	26.2941	26.8453	27.3975	27.9506	28.5048	29.0599
1.007	25.3924	25.9497	26.5082	27.0678	27.6286	28.1905	28.7536	29.3178
1.008	25.5896	26.1550	26.7217	27.2897	27.8590	28.4297	29.0016	29.5748
1.009	25.7862	26.3597	26.9346	27.5109	28.0887	28.6680	29.2488	29.8310
1.010	25.9822	26.5636	27.1467	27.7314	28.3177	28.9055	29.4951	30.0862
1.011	26.1776	26.7670	27.3581	27.9510	28.5457	29.1422	29.7404	30.3403
1.012	26.3722	26.9695	27.5687	28.1698	28.7729	29.3778	29.9846	30.5934
1.013	26.5662	27.1713	27.7785	28.3877	28.9990	29.6124	30.2278	30.8453
1.014	26.7593	27.3722	27.9873	28.6047	29.2242	29.8459	30.4698	31.0959
1.015	26.9516	27.5723	28.1953	28.8206	29.4483	30.0783	30.7106	31.3453
1.016	27.1431	27.7714	28.4022	29.0355	29.6713	30.3095	30.9502	31.5933
1.017	27.3337	27.9696	28.6081	29.2493	29.8930	30.5394	31.1883	31.8399
1.018	27.5233	28.1668	28.8130	29.4619	30.1136	30.7680	31.4251	32.0849
1.019	27.7120	28.3629	29.0167	29.6734	30.3329	30.9953	31.6604	32.3285
1.020	27.8997	28.5580	29.2193	29.8836	30.5509	31.2211	31.8943	32.5704
1.021	28.0864	28.7520	29.4207	30.0926	30.7675	31.4455	32.1266	32.8107
1.022	28.2720	28.9448	29.6209	30.3002	30.9827	31.6684	32.3572	33.0492
1.023	28.4565	29.1365	29.8198	30.5065	31.1965	31.8897	32.5863	33.2860
1.024	28.6398	29.3269	30.0174	30.7114	31.4088	32.1095	32.8136	33.5211
1.025	28.8220	29.5161	30.2137	30.9149	31.6195	32.3276	33.0392	33.7542
1.026	29.0031	29.7040	30.4087	31.1169	31.8287	32.5441	33.2631	33.9855
1.027	29.1829	29.8907	30.6022	31.3174	32.0363	32.7589	33.4851	34.2149
1.028	29.3614	30.0760	30.7943	31.5164	32.2423	32.9720	33.7053	34.4423
1.029	29.5388	30.2599	30.9850	31.7139	32.4467	33.1832	33.9236	34.6677
1.030	29.7148	30.4425	31.1742	31.9098	32.6493	33.3927	34.1400	34.8910
1.031	29.8895	30.6237	31.3619	32.1041	32.8502	33.6004	34.3544	35.1123
1.032	30.0629	30.8034	31.5480	32.2967	33.0494	33.8062	34.5669	35.3315
1.033	30.2349	30.9817	31.7326	32.4877	33.2469	34.0101	34.7774	35.5486
1.034	30.4056	31.1585	31.9157	32.6770	33.4425	34.2121	34.9858	35.7635
1.035	30.5749	31.3339	32.0971	32.8647	33.6364	34.4122	35.1922	35.9762
1.036	30.7428	31.5077	32.2770	33.0506	33.8284	34.6104	35.3966	36.1868
1.037	30.9092	31.6800	32.4552	33.2347	34.0186	34.8066	35.5988	36.3952
1.038	31.0743	31.8508	32.6318	33.4172	34.2069	35.0008	35.7990	36.6013
1.039	31.2379	32.0201	32.8068	33.5979	34.3933	35.1931	35.9970	36.8052
1.040	31.4000	32.1878	32.9800	33.7768	34.5779	35.3833	36.1930	37.0068

Mean Value Table to fit curve $y = ar^x$

r	n=49	n=50	n=51	n=52	n=53	n=54	n=55	n=56
1.040	31.4000	32.1878	32.9800	33.7768	34.5779	35.3833	36.1930	37.0068
1.041	31.5606	32.3539	33.1516	33.9539	34.7605	35.5715	36.3867	37.2062
1.042	31.7198	32.5184	33.3215	34.1292	34.9413	35.7577	36.5784	37.4033
1.043	31.8775	32.6813	33.4898	34.3027	35.1201	35.9418	36.7679	37.5981
1.044	32.0336	32.8427	33.6563	34.4744	35.2970	36.1239	36.9552	37.7906
1.045	32.1883	33.0024	33.8211	34.6443	35.4720	36.3040	37.1403	37.9809
1.046	32.3415	33.1605	33.9842	34.8124	35.6450	36.4820	37.3233	38.1688
1.047	32.4931	33.3170	34.1455	34.9786	35.8161	36.6580	37.5041	38.3545
1.048	32.6432	33.4719	34.3052	35.1430	35.9853	36.8319	37.6828	38.5378
1.049	32.7918	33.6251	34.4631	35.3055	36.1525	37.0037	37.8592	38.7189
1.050	32.9389	33.7767	34.6192	35.4663	36.3177	37.1735	38.0336	38.8977
1.051	33.0844	33.9267	34.7737	35.6252	36.4811	37.3413	38.2057	39.0742
1.052	33.2284	34.0751	34.9264	35.7822	36.6425	37.5070	38.3757	39.2485
1.053	33.3708	34.2218	35.0774	35.9375	36.8019	37.6706	38.5435	39.4204
1.054	33.5117	34.3669	35.2267	36.0909	36.9595	37.8323	38.7092	39.5902
1.055	33.6511	34.5104	35.3742	36.2425	37.1151	37.9919	38.8728	39.7576
1.056	33.7890	34.6523	35.5201	36.3923	37.2688	38.1494	39.0342	39.9229
1.057	33.9253	34.7925	35.6642	36.5402	37.4205	38.3050	39.1935	40.0859
1.058	34.0602	34.9311	35.8066	36.6864	37.5704	38.4586	39.3507	40.2467
1.059	34.1935	35.0682	35.9473	36.8308	37.7184	38.6102	39.5058	40.4054
1.060	34.3252	35.2036	36.0863	36.9733	37.8645	38.7598	39.6589	40.5618
1.061	34.4555	35.3374	36.2236	37.1142	38.0088	38.9074	39.8099	40.7161
1.062	34.5843	35.4696	36.3593	37.2532	38.1512	39.0531	39.9588	40.8682
1.063	34.7116	35.6003	36.4933	37.3905	38.2917	39.1968	40.1057	41.0182
1.064	34.8374	35.7294	36.6256	37.5260	38.4304	39.3386	40.2506	41.1661
1.065	34.9617	35.8569	36.7563	37.6598	38.5673	39.4785	40.3935	41.3120
1.066	35.0845	35.9828	36.8853	37.7919	38.7023	39.6165	40.5344	41.4557
1.067	35.2059	36.1072	37.0128	37.9223	38.8356	39.7527	40.6733	41.5974
1.068	35.3258	36.2301	37.1386	38.0509	38.9671	39.8870	40.8103	41.7371
1.069	35.4442	36.3515	37.2628	38.1780	39.0969	40.0194	40.9454	41.8747
1.070	35.5613	36.4713	37.3854	38.3033	39.2249	40.1500	41.0785	42.0104
1.071	35.6769	36.5897	37.5064	38.4269	39.3511	40.2788	41.2098	42.1441
1.072	35.7910	36.7065	37.6259	38.5490	39.4757	40.4058	41.3392	42.2758
1.073	35.9038	36.8219	37.7438	38.6694	39.5985	40.5310	41.4668	42.4057
1.074	36.0152	36.9358	37.8602	38.7882	39.7197	40.6545	41.5925	42.5336
1.075	36.1252	37.0483	37.9751	38.9054	39.8392	40.7763	41.7165	42.6597
1.076	36.2338	37.1593	38.0884	39.0211	39.9571	40.8963	41.8386	42.7839
1.077	36.3411	37.2689	38.2003	39.1352	40.0733	41.0147	41.9590	42.9063
1.078	36.4470	37.3771	38.3107	39.2477	40.1880	41.1313	42.0777	43.0268
1.079	36.5516	37.4839	38.4196	39.3587	40.3010	41.2464	42.1946	43.1456
1.080	36.6549	37.5893	38.5272	39.4682	40.4125	41.3597	42.3098	43.2627

Mean Value Table to fit curve $y = ar^x$

r	n=49	n=50	n=51	n=52	n=53	n=54	n=55	n=56
1.080	36.6549	37.5893	38.5272	39.4682	40.4125	41.3597	42.3098	43.2627
1.081	36.7569	37.6933	38.6332	39.5763	40.5224	41.4715	42.4234	43.3780
1.082	36.8575	37.7960	38.7378	39.6828	40.6308	41.5817	42.5353	43.4916
1.083	36.9569	37.8974	38.8411	39.7879	40.7377	41.6903	42.6456	43.6035
1.084	37.0550	37.9974	38.9429	39.8915	40.8431	41.7974	42.7543	43.7137
1.085	37.1519	38.0961	39.0434	39.9938	40.9470	41.9029	42.8614	43.8223
1.086	37.2475	38.1935	39.1426	40.0946	41.0494	42.0069	42.9669	43.9293
1.087	37.3419	38.2896	39.2404	40.1940	41.1505	42.1095	43.0709	44.0347
1.088	37.4351	38.3845	39.3369	40.2921	41.2500	42.2105	43.1734	44.1385
1.089	37.5271	38.4781	39.4321	40.3888	41.3482	42.3101	43.2744	44.2408
1.090	37.6179	38.5705	39.5260	40.4842	41.4451	42.4083	43.3739	44.3416
1.091	37.7075	38.6616	39.6186	40.5783	41.5405	42.5051	43.4719	44.4408
1.092	37.7959	38.7516	39.7100	40.6711	41.6346	42.6005	43.5685	44.5386
1.093	37.8832	38.8403	39.8002	40.7626	41.7274	42.6945	43.6637	44.6350
1.094	37.9694	38.9279	39.8891	40.8528	41.8189	42.7872	43.7575	44.7299
1.095	38.0545	39.0144	39.9769	40.9418	41.9091	42.8785	43.8500	44.8234
1.096	38.1384	39.0996	40.0634	41.0296	41.9980	42.9686	43.9411	44.9155
1.097	38.2213	39.1838	40.1488	41.1161	42.0857	43.0573	44.0309	45.0063
1.098	38.3031	39.2668	40.2330	41.2015	42.1721	43.1448	44.1193	45.0957
1.099	38.3838	39.3487	40.3161	41.2857	42.2574	43.2310	44.2065	45.1838
1.100	38.4635	39.4296	40.3980	41.3687	42.3414	43.3160	44.2925	45.2706

Mean Value Table to fit curve $y = ar^x$

r	n=57	n=58	n=59	n=60	n=61	n=62	n=63	n=64
1.000	28.0000	28.5000	29.0000	29.5000	30.0000	30.5000	31.0000	31.5000
1.001	28.2705	28.7801	29.2898	29.7997	30.3098	30.8201	31.3305	31.8410
1.002	28.5407	29.0599	29.5793	30.0991	30.6192	31.1397	31.6605	32.1816
1.003	28.8104	29.3391	29.8682	30.3979	30.9281	31.4588	31.9899	32.5216
1.004	29.0796	29.6178	30.1566	30.6961	31.2363	31.7771	32.3186	32.8608
1.005	29.3481	29.8958	30.4443	30.9936	31.5438	32.0947	32.6465	33.1991
1.006	29.6160	30.1731	30.7312	31.2903	31.8503	32.4114	32.9734	33.5364
1.007	29.8831	30.4496	31.0172	31.5860	32.1559	32.7270	33.2992	33.8726
1.008	30.1493	30.7252	31.3023	31.8807	32.4604	33.0415	33.6238	34.2074
1.009	30.4146	30.9997	31.5863	32.1743	32.7638	33.3547	33.9471	34.5409
1.010	30.6789	31.2732	31.8692	32.4667	33.0658	33.6666	34.2689	34.8728
1.011	30.9421	31.5455	32.1508	32.7577	33.3665	33.9769	34.5891	35.2031
1.012	31.2041	31.8166	32.4311	33.0474	33.6656	34.2857	34.9077	35.5316
1.013	31.4648	32.0864	32.7099	33.3356	33.9632	34.5929	35.2246	35.8583
1.014	31.7242	32.3547	32.9873	33.6221	34.2591	34.8982	35.5395	36.1829
1.015	31.9823	32.6216	33.2631	33.9070	34.5532	35.2017	35.8524	36.5055
1.016	32.2389	32.8869	33.5373	34.1902	34.8455	35.5032	36.1633	36.8258
1.017	32.4939	33.1506	33.8098	34.4716	35.1358	35.8027	36.4720	37.1439
1.018	32.7474	33.4126	34.0805	34.7510	35.4242	36.1000	36.7785	37.4595
1.019	32.9993	33.6729	34.3493	35.0285	35.7104	36.3951	37.0826	37.7727
1.020	33.2494	33.9313	34.6162	35.3039	35.9945	36.6879	37.3842	38.0834
1.021	33.4978	34.1879	34.8811	35.5772	36.2763	36.9784	37.6834	38.3913
1.022	33.7444	34.4426	35.1439	35.8484	36.5558	37.2664	37.9800	38.6966
1.023	33.9890	34.6953	35.4047	36.1173	36.8330	37.5519	38.2739	38.9990
1.024	34.2318	34.9459	35.6633	36.3839	37.1077	37.8348	38.5651	39.2985
1.025	34.4727	35.1945	35.9196	36.6482	37.3800	38.1151	38.8535	39.5952
1.026	34.7115	35.4409	36.1737	36.9100	37.6497	38.3927	39.1391	39.8888
1.027	34.9482	35.6851	36.4255	37.1694	37.9168	38.6676	39.4218	40.1794
1.028	35.1829	35.9271	36.6750	37.4264	38.1813	38.9397	39.7015	40.4668
1.029	35.4155	36.1669	36.9220	37.6808	38.4431	39.2089	39.9783	40.7511
1.030	35.6458	36.4044	37.1667	37.9326	38.7021	39.4753	40.2520	41.0322
1.031	35.8740	36.6395	37.4088	38.1818	38.9584	39.7387	40.5226	41.3101
1.032	36.1000	36.8723	37.6485	38.4284	39.2120	39.9993	40.7902	41.5847
1.033	36.3237	37.1027	37.8856	38.6723	39.4627	40.2568	41.0546	41.8559
1.034	36.5451	37.3307	38.1202	38.9135	39.7105	40.5113	41.3158	42.1239
1.035	36.7643	37.5563	38.3522	39.1519	39.9555	40.7628	41.5738	42.3885
1.036	36.9811	37.7794	38.5816	39.3876	40.1976	41.0112	41.8286	42.6497
1.037	37.1956	38.0000	38.8083	39.6206	40.4367	41.2566	42.0802	42.9075
1.038	37.4077	38.2181	39.0325	39.8508	40.6729	41.4989	42.3286	43.1619
1.039	37.6174	38.4337	39.2540	40.0782	40.9062	41.7381	42.5737	43.4129
1.040	37.8248	38.6468	39.4728	40.3028	41.1366	41.9742	42.8155	43.6605

Mean Value Table to fit curve $y = ar^x$

r	n=57	n=58	n=59	n=60	n=61	n=62	n=63	n=64
1.040	37.8248	38.6468	39.4728	40.3028	41.1366	41.9742	42.8155	43.6605
1.041	38.0297	38.8574	39.6890	40.5245	41.3639	42.2071	43.0540	43.9046
1.042	38.2323	39.0654	39.9025	40.7435	41.5884	42.4370	43.2893	44.1453
1.043	38.4325	39.2709	40.1133	40.9596	41.8098	42.6637	43.5214	44.3826
1.044	38.6302	39.4738	40.3214	41.17[illegible]	42.0283	42.8874	43.7501	44.6165
1.045	38.8255	39.6742	40.52[illegible]9	41.383[illegible]	42.2438	43.1079	43.9756	44.8470
1.046	39.0184	39.8721	40.7297	41.5911	42.4564	43.3253	44.1979	45.0740
1.047	39.2089	40.0674	40.9298	41.7960	42.6660	43.5397	44.4169	45.2977
1.048	39.3970	40.2601	41.1272	41.9981	42.8727	43.7509	44.6328	45.5181
1.049	39.5827	40.4504	41.3220	42.1973	43.0764	43.9591	44.8454	45.7350
1.050	39.7659	40.6381	41.5141	42.3938	43.2773	44.1643	45.0548	45.9487
1.051	39.9468	40.8232	41.7035	42.5875	43.4752	44.3664	45.2610	46.1590
1.052	40.1252	41.0059	41.8904	42.7785	43.6702	44.5655	45.4641	46.3661
1.053	40.3013	41.1861	42.0746	42.9667	43.8624	44.7616	45.6641	46.5699
1.054	40.4751	41.3638	42.2562	43.1522	44.0517	44.9547	45.8610	46.7705
1.055	40.6464	41.5390	42.4352	43.3350	44.2382	45.1449	46.0548	46.9678
1.056	40.8154	41.7117	42.6116	43.5151	44.4219	45.3321	46.2455	47.1621
1.057	40.9821	41.8820	42.7855	43.6925	44.6029	45.5165	46.4333	47.3531
1.058	41.1465	42.0499	42.9569	43.8673	44.7810	45.6980	46.6180	47.5411
1.059	41.3086	42.2154	43.1258	44.0395	44.9565	45.8766	46.7998	47.7260
1.060	41.4684	42.3785	43.2921	44.2091	45.1292	46.0525	46.9787	47.9079
1.061	41.6259	42.5393	43.4560	44.3761	45.2993	46.2255	47.1547	48.0868
1.062	41.7812	42.6977	43.6175	44.5405	45.4667	46.3958	47.3279	48.2627
1.063	41.9343	42.8538	43.7765	44.7025	45.6314	46.5634	47.4982	48.4357
1.064	42.0852	43.0076	43.9332	44.8619	45.7936	46.7283	47.6657	48.6058
1.065	42.2339	43.1591	44.0875	45.0189	45.9533	46.8905	47.8305	48.7731
1.066	42.3804	43.3083	44.2394	45.1735	46.1104	47.0501	47.9926	48.9375
1.067	42.5248	43.4554	44.3890	45.3256	46.2650	47.2072	48.1519	49.0992
1.068	42.6671	43.6002	44.5363	45.4754	46.4172	47.3617	48.3087	49.2582
1.069	42.8073	43.7429	44.6814	45.6228	46.5669	47.5137	48.4629	49.4145
1.070	42.9453	43.8833	44.8242	45.7679	46.7142	47.6631	48.6145	49.5681
1.071	43.0814	44.0217	44.9649	45.9107	46.8592	47.8102	48.7635	49.7192
1.072	43.2155	44.1580	45.1033	46.0513	47.0018	47.9548	48.9101	49.8676
1.073	43.3475	44.2922	45.2396	46.1897	47.1422	48.0971	49.0542	50.0136
1.074	43.4776	44.4244	45.3738	46.3258	47.2802	48.2370	49.1960	50.1570
1.075	43.6057	44.5545	45.5059	46.4598	47.4161	48.3746	49.3353	50.2981
1.076	43.7319	44.6827	45.6359	46.5917	47.5497	48.5100	49.4723	50.4367
1.077	43.8562	44.8088	45.7640	46.7214	47.6812	48.6431	49.6070	50.5729
1.078	43.9787	44.9331	45.89[illegible]	46.8491	47.8105	48.7740	49.7395	50.7069
1.079	44.0993	45.0554	46.0140	46.9748	47.9378	48.9028	49.8697	50.8386
1.080	44.2181	45.1759	46.1361	47.0985	48.0650	49.0294	49.9978	50.9680

Mean Value Table to fit curve $y = ar^x$

r	n=57	n=58	n=59	n=60	n=61	n=62	n=63	n=64
1.080	44.2181	45.1759	46.1361	47.0985	48.0630	49.0294	49.9978	50.9680
1.081	44.3350	45.2945	46.2562	47.2202	48.1861	49.1540	50.1237	51.0952
1.082	44.4502	45.4113	46.3745	47.3399	48.3073	49.2765	50.2475	51.2203
1.083	44.5637	45.5263	46.4910	47.4577	48.4264	49.3970	50.3693	51.3432
1.084	44.6755	45.6395	46.6056	47.5737	48.5437	49.5155	50.4890	51.4640
1.085	44.7855	45.7509	46.7184	47.6878	48.6591	49.6320	50.6067	51.5829
1.086	44.8939	45.8607	46.8294	47.8001	48.7725	49.7467	50.7224	51.6997
1.087	45.0007	45.9687	46.9387	47.9106	48.8842	49.8594	50.8362	51.8145
1.088	45.1058	46.0751	47.0463	48.0193	48.9941	49.9704	50.9482	51.9274
1.089	45.2094	46.1799	47.1522	48.1264	49.1021	50.0794	51.0582	52.0384
1.090	45.3113	46.2830	47.2565	48.2317	49.2085	50.1868	51.1665	52.1475
1.091	45.4118	46.3846	47.3591	48.3353	49.3131	50.2923	51.2729	52.2548
1.092	45.5107	46.4846	47.4601	48.4373	49.4160	50.3962	51.3776	52.3603
1.093	45.6061	46.5830	47.5596	48.5377	49.5173	50.4983	51.4806	52.4641
1.094	45.7041	46.6800	47.6575	48.6366	49.6170	50.5988	51.5819	52.5661
1.095	45.7986	46.7754	47.7539	48.7338	49.7151	50.6977	51.6815	52.6664
1.096	45.8917	46.8695	47.8488	48.8295	49.8116	50.7950	51.7795	52.7651
1.097	45.9834	46.9620	47.9422	48.9238	49.9066	50.8907	51.8759	52.8622
1.098	46.0737	47.0532	48.0342	49.0165	50.0001	50.9849	51.9707	52.9576
1.099	46.1626	47.1430	48.1247	49.1078	50.0921	51.0775	52.0640	53.0515
1.100	46.2503	47.2314	48.2139	49.1977	50.1827	51.1687	52.1558	53.1439

files: Niewiaroski

D. H. Niewiaroski, "The level of living of nations - meanings and measurements"

(IBRD 15 July 1964)

Request from Paris office 3/29/65 - something peculiar about this; was told to write Paris no copies; Mr. Niewiaroski on leave for a couple of years -- but Ruth Achcar under Mr. Goor has copies

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Magazine just rec'd in Library 8/66 (very slow)

8/25/70 - letter from him
Program Manager, International Development
Research Analysis Corporation
McLean, Va. 22101

BOOKS BY BANK STAFF

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PUBLICATIONS — PUBLICACIONES

THE LEVEL OF LIVING OF NATIONS: MEANING AND MEASUREMENT

Donald H. Niewiaroski, United States * **

RESUMEN

En los últimos años se ha desarrollado un nuevo enfoque para resolver el problema de la comparación internacional del bienestar económico relativo de los distintos países. Es éste el enfoque del "nivel de vida de las naciones" que parte del postulado fundamental de que las condiciones básicas de vida de la población de un país se reflejan en ciertas partidas clave de la economía nacional. En base a este principio, si se seleccionan en forma apropiada algunas de esas partidas y se atribuyen a cada una ponderaciones convenientes, es posible construir un índice que puede servir para medir directamente el bienestar alcanzado por una nación, es decir, la verdadera forma de vida y condiciones de la población. Además, como las partidas se expresan en cantidades físicas y no en moneda resulta innecesario el tener que usar, para hacer las conversiones correspondientes, las tasas de cambio corrientes.

En este trabajo se describen las distintas etapas que ha seguido el autor para construir un índice de ese tipo. Para seleccionar las variables ha usado un criterio objetivo tratando de que cada una sea, o un elemento en sí representativo de las condiciones de vida imperantes, o un factor contribuyente a tales condiciones. Las variables están clasificadas en dos categorías: económicas y sociales. En las primeras se incluyen: (a) la proporción, en la fuerza de trabajo, de persona ocupadas en actividades no agropecuarias; (b) el número de teléfonos per cápita; (c) el consumo de energía per cápita; (d) el consumo de acero per cápita; (e) el consumo de cemento per cápita; (f) el número de vehículos a motor per cápita; (g) el consumo de fibra per cápita; (h) la proporción de proteínas en la dieta alimenticia; e (i) las exportaciones per cápita. Entre las variables sociales: (a) el número de dentistas y médicos per cápita; (b) la circulación diaria de periódicos per cápita; (c) el número de aparatos de radio per cápita; (d) la inscripción escolar post primaria; y (e) el grado de alfabetismo.

Estas 14 variables aparecen relacionadas, en la fórmula que propone el autor, mediante una ecuación lineal. Para determinar las ponderaciones ha usado, como técnica matemática, la del análisis factorial.

La técnica expuesta, que incluye la construcción de un índice total combinado y un índice económico y otro social, que miden respectivamente los niveles económico y social, es aplicada aquí a datos obtenidos para 88 países.

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* International Bank for Reconstruction and Development, Washington, D.C., and United States Agency for International Development, Quito.

** Views or conclusions contained in this article should not be interpreted as representing the official opinion or policy of the World Bank or the Government of Ecuador. I wish to thank Philip Hayes, of the Organization for Economic Cooperation and Development, Frank Lowestein, of the United States Department of Agriculture, and Professor Irma Adelman, of John Hopkins University, for their constructive criticism. Professor P. T. Ellsworth, of the University of Wisconsin, gave unselfishly of time and effort to improve the paper in many respects. Any faults in this research, however, are owing to my own inadequacies.

INTRODUCTION

Among the many aspects of economic development which economists have explored during the past decade, one of central importance is the measurement of a country's relative position on the scale of economic attainment. The concept of what is to be measured has generally been taken to be relative well-being. The most widely used instrument of measurement has been per capita income expressed in a common currency.

Unfortunately, the national income statistics on which such a measure is based are frequently suspect, while conversion of different per capita incomes into a common currency (usually United States dollars) at current or official exchange rates makes no allowance for differences in purchasing power. As an alternative some economists have tried the method of valuing the product of different countries at United States prices or at an average of United States and other countries' prices, but the results vary depending upon the prices used.

In recent years a new approach to the problem of establishing international comparisons of well-being has been developed that avoids the purchasing power problem. This is the "level of living" approach. Its fundamental postulate is that well-being, or the basic living conditions of a people, is reflected by certain key items in a nation's economy. By selecting appropriate items and giving them suitable weights, an index can be constructed that provides a direct measure of well-being. Since the items chosen are expressed in physical quantities rather than money values, conversion at dubious exchange rates is unnecessary, and the troublesome purchasing power problem is avoided.

Clearly, the main problem confronting the level of living approach consists in the choice of appropriate variables and suitable weights. Bennett [1] was among the first to research the issue; although he defined his terms of reference as consumption levels, examination of the variables he used suggests that he was closer to a level of living concept. Bennett admitted to knowing of "no basis for assigning weights" to each of the 19 indicators he selected and used a crude means of aggregating his data. More recent studies by the United Nations [2] and Berry [3] have made no attempt to agregate the separate variables into a single index.

I. A SUGGESTED MEASURE OF THE LEVEL OF LIVING

The purpose of this paper is to construct an index of the level of living that will serve as a useful basis for international comparisons. To perform this function successfully, the index should present a composite picture that accurately reflects national well-being, defined as the actual living and conditions of the population. Although "living conditions" is

[1] "International Disposition in Consumption Levels", by M. K. Bennet. *The American Economic Review*, September 1951.

[2] *Report on International Definition and Measurement of Standards and Levels of Living. An Interim Guide*, United Nations, New York, 1961.

[3] *Atlas of Economic Development*, by Ginsberg and Berry, Chicago. 1961.

a broad and rather loose concept, its essential meaning seems clear. It refers to the economic and social characteristics of a nation that determine the quality of life of its people, including as qualitative element the relative abundance of goods and services they enjoy.

If our level of living index is to mirror accurately the quality and abundance of life in a country, each of its characteristics should represent one or more of the numerous elements that enter into or that determine living conditions. Since no index is all-inclusive, selection is essential. Judgment is unavoidable, but I have tried to make the selection as objective as possible by insisting that there be clear empirical evidence or theoretical expectation that each variable either itself constitutes an element of living conditions or acts as a determinant of those conditions. Although I have tried to justify each choice on these grounds, there is plenty of room for possible disagreement and therefore for attempts on the part of others to construct a better index.

Clearly, the number of characteristics chose must be sufficient to reflect all major aspects of the conditions of life. Yet these conditions or the factors that determine them constitute a fabric composed of many interwoven threads. To recognize the pattern, we need select only a relatively few of them. Especially for making international comparisons, it is undesirable to include items whose magnitude is roughly the same in all countries, for this merely reduces the sharpness of the comparison.

In selecting the specific variables, I have made use of United Nations documents [4], the work of Hagen [5], and that of Ginsberg and Berry [6]. The rationale of each choice is given in terms of supporting empirical evidence or theoretical expectation. The variables are classified into two categories: economic and social.

Economic Variables

(1) The first of the variables I have classified as economic, *the proportion of the labor force engaged in non-agricultural activities*, constitutes an indicator of the degree of industrialization. If a country is to attain a high level of development and thus a high level of living, it must have a competent and well qualified labor force, which varies with the extent of a nation's industrialization. (2) The number of *telephones per capita* is also a companion to industrial achievement, since the latter requires a sophisticated medium of communication. (3) The level of *energy consumption per capita* reflects the requirements of a society for the means with which to operate its farms, homes and complex industrial plant. (4) *Steel consumption per capita* and (5) *cement production per capita* reflect the production capabilities of the industrial sector. (6) *Motor vehicles per*

[4] *Demographic Yearbook*, 1961; *Report on International Definition and Measurement of Standards and Levels of Living*, 1954; *Report ... An Interim Guid*, *op. cit.*

[5] "On the Theory of Social Change", by Everett F. Hagen. *Development of the Emerging Countries: A Framework for Analyzing Economic and Political Change*, Brookings Institution, Washington, D. C., 1962.

[6] *Op. cit.*

capita suggest the size of the industrial sector as well as the need for a modern transportation system to satisfy the requirements of a developed society. (7) *Per capita fiber consumption* is indicative of the need for clothing and house furnishings which manifest themselves as or after development occurs. As a measure of nutrition (8), *the proportion of protein in the diet* was used. Wealthy societies, with a higher level of living, consume a larger proportion of the relatively expensive proteins. It can be argued that this factor might logically be included in the social or welfare section, as been done by the United Nations [7]. Strictly speaking, however, the "social" sector does not include the feeding of people, if one thinks of "social" as meaning welfare, recreation, education and cultural affairs. The last item in the economic sector is (9) *exports per capita*. This variable gives weight to a source from which some countries derive a substantial portion of their level of living. It could, of course, be argued that imports per capita would be a better measure, as a highly developed country can afford to import goods which it does not itself produce. But imports can be purchased either by export receipts or by foreig capital inflow; if the latter looms large, as it does in many developing countries today, imports reflect reliance upon outside sources rather than a nation's own attainment. Hence the choice of exports per capita.

Social Variables

The first is (1) *physicians and dentists per capita*, which measures the health aspect of welfare. However, this variable has its limitations as many less developed countries, through university emphasis on the medical profession, foster large medical faculties with a resultant proliferation of doctors. [8] This is particularly true in Latin America. A number of other related variables, such as hospital beds, morbidity index and life expectancy, could also logically be included, but because of lack of data these were omitted. (2) *Newspaper circulation per capita per day* and (3) *radios per capita* have been used as indicators of "cultural" achievement and interest. Lastly, (4) *post-primary school enrollment* was selected as the measure of the rate of human capital accumulation and (5) *literacy* as the indicator of the basic reading and writing attainments of a developing society.

I have used a total of 14 variables, and have gathered data for 88 countries. In an earlier stage of my research I had used 12 variables, omitting fiber consumption, diet and exports per capita but including railroad freight ton-miles. The railroad variable was dropped because the modes of transportation used in a country may not be entirely dependent upon level of economic development. Ideally, freight moved on all modes of transportation would be a better characteristic, but such data do not exist.

[7] *Report on the World Situation*, United Nations, New York, 1961

[8] *Hagen, op. cit.*

II. METHOD

My objective is to develop an index of the level of living of nations, that is, a single number that represents relative conditions of economic and social welfare. I have assumed a linear or additive relationship among the 14 variables of the form

$$I = b_1 X_1 + b_2 X_2 + \ldots + b_{14} X_{14} \qquad (1)$$

where:

I is the index of the level of living,
b is the weight applied to each variable,
X is the variable

The mathematical technique used to determine the weights and thus reduce the 14 variables to a single common component is a factor analysis. This technique posseses certain features germane to the concept of the level of living. The factor analysis:

1. Maximizes the variance of the index (I)
2. Maximizes the sum of the squared correlations between the index and the characteristics used.

Maximization of the variance of the index (I) ensures that it will discriminate effectively between countries that have high, medium and low levels of living. Maximizing the sum of the squared correlations ensures that the index explains a maximum proportion of the total variance of all the variables. Factor analysis, therefore, verifies statistically the aggregation of macroeconomic data and also provides the component which explains the highest proportion of variance among the variables used.

Factor analysis discloses relational patterns among a series of variables and enables one to extract the principal components reflected therein. For example, Olsen and Garb [9] have used this technique to discover regional differences that may affect economic growth. Forty-nine characteristics were selected for two regions in the United States, one of which was chosen for its relatively high level of development to serve as a basis of comparison. Applying factor analysis, the authors extracted six common factors or components which they believe explain the differences in level of development between the two regions. This approach, then, derives the principal components after the factor analysis is made based on the size and sign of the factor loadings for each characteristic.

Unlike this technique, my approach is to postulate the principal component first, namely, level of living and then deliberately select the variables I believe reflect this phenomenon. I have chosen Waugh's method

[9] *An Application of Factor Analysis to Economic Growth*, by Bernard M. Olsen Gerald Garb, paper presented at the 123rd Annual Meeting of the American Statistical Association, Cleveland, September 1963.

of factor analysis[10] because it is suited to my approach and has been used successfully by the United States Department of Agriculture[11] to determine level of living indices for counties in the United States. Moreover, his computational principles are simpler to use than those developed by others[12], and his technique is easily adaptable for programming for the computer. Finally, Waugh's procedure yields several additional tests, not obtained from the other factor analytic methods which aid in evaluating the results.

Like any index, the principal component or index (I) does not exist numerically before the factor analysis is applied. There are no dependent or independent variables as with regression analysis. Here all series are inter-related. Factor analysis solves for the weights (the "b's") and the index (I) is obtained as the weighted average of the selected variables. The simple correlations among all pairs of variables represent the raw data. Beside the factor loadings, several other measures are derived. A Lagrange multiplier is obtained, which measures the proportion of variation in the selected characteristics explained by the principal component—the level of living. The objective is to maximize this proportion, reaching, if possible, 100 per cent. The higher the proportion, the easier it is to claim that the characteristics chosen do measure the level of living. A low proportion suggests that the variables selected do not possess any homogeneity with regard to the principal component being measured. Such a result would suggest poor data, improper choice of variables, nonmeasurability of the principal component, or some combination of these. The important point to bear in mind is that a high proportion of variation (relative to the number of variables involved) indicates that the variables selected can be reduced to a common component.

The simple correlation of each variable with the index (I) indicates the usefulness of the characteristic as a measure of the level of living: a high correlation points to a strong (linear) relationship with the index (I), while a low correlation would suggest a weak relationship. With this statistic it is possible to gauge the merit of including a particular variable in the level of living exercise. I have added another statistic which measures for each country and group of countries the relative importance of each variable in the index. This statistic is for any given country, each variable multiplied by the corresponding weight divided by that country's index (I), i.e. b_1X_1/I, b_2X_2/I, ..., $b_{14}X_{14}/I$. And, if one is willing to accept averages of these percentages by index (I) decide, a useful pattern emerges, as will be explaned later on.

I have computed a combined index which measures overall economic and social well-being as well as an economic index and social index which

[10] "Factor Analysis: Some Basic Principles and an Application", by Frederick Waugh, *Agricultural Economic Research*, July 1962.

[11] *Agriculture and Economic Growth*, Agricultural Economics Report No. 28, Department of Agriculture, Washington, D.C., 1963.

[12] *Modern Factor Analysis*, by H. H. Harman, Chicago, 1960; "Analysis of a Complex of Statistical Variables into Principal Components", by H. Hotelling, *Journal of Educational Psychology*, No. 4, 1933; Econometrics, by Gerhard Tintner, New York, 1952.

measure, respectively, economic level of living and social level of living. These statistics plus the relative proportions referred to above provide much grist for the analytical mill. The indexes calculated for each country are strictly quantitative measures. No attempt has been made to adjust for differences in customs; the series chosen are basic to any economy in the course of economic development. Nor is any allowance made for differences in the importance or esthetic value attached to any variable although this does manifest itself to some degree in the relative importance statistic.

The mathematical details of the factor analysis method are explained in Appendix A.

III. THE ANALYSIS

For ease of reference all tables are assembled in Appendix B. Table B3 sets out the three indices, combined, economic and social, with the countries arranged in accordance to their combined index. Each index has been scaled to United States = 100 to facilitate comparison.

The combined index results: overall view

The range of the combined index (circa 1959) is striking but not surprising: from 100 for the United States to about six for Ethiopia, which means that the level of living in Ethiopia is about six per cent of that in the United States. Nearly half the countries (42) are at or below 20 per cent of the United States level of living, or in terms of the 1959 population of the 88 countries involved (Mainland China excluded) about 47 per cent of the world's population is existing at or below 20 per cent of the United States level of living. The 88 countries used accounted, in 1959, for 92 per cent of the world's population. As the countries not used were excluded for lack of data, it seems safe to assume that they too would be in the 20 per cent or below class, which would mean that over half the world's population is living at 1/5 or less of the level of the United States.

For the combined index range cero to 20, of the 42 countries 31 have an economic index exceeding their social index, in some cases by a rather wide margin. The difference equals or exceeds the value of the social index itself for Iraq, Rhodesia, Nyasaland, Morocco, Libya, Haiti, Nigeria, British East Africa, Sudan, Angola, Mozambique, Cameroon, Sierra Leone and Ethiopia. For the combined index range of 20 to 67, the economic index is below the social index in 29 out of 35 countries. Above a combined index of 67, the picture is mixed.

The implication is that at exceedingly low levels of living (20 or below) economic growth is easier to accomplish than is social development. On the other hand, above a combined index of 20 economic improvement is difficult to achieve while social growth comes easier. This suggests that the gap in development between the developed and less-developed countries is not automatically transferred into social terms above what I might call the "watershed" of a combined index of 20, where the pattern between economic and social development changes.

These findings are tentative (although the United Nations in its study[13] reached the same conclusion) because such important "social" series as life expectancy and morbidity have been omitted primarily for lack of data. However, I suspect that with the advent of miracle drugs and with the improvements in sanitation during the postwar era, the increase in life expectancy throughout the world has been achieved at relatively low cost and with a minimum of effort. The addition of a life expectancy variable to the social index might possibly increase it more for the less-developed than for the developed countries and thus disrupt the patterns described above. On the other hand, a morbidity index might offset this impact as most of the developing countries today suffer from debilitating diseases which have not been under massive attack by the medical authorities. Were it possible to include both of these variables, the same pattern noted above might emerge.

The reason for this sequence is that at relatively low levels of living (20 or below) any economic improvement looks big because the base is low, while social improvement must wait for the economic base to reach a level at which the former can be supported by the latter. Above the "watershed", economic change at the expense of social development becomes difficult for two reasons: first, an awareness of social needs and desires by the populace forces recognition of these needs and diverts resources into the social sphere; second, with a higher economic base, any improvement no longer looms as large.

The combined index results: by area

Table B6 in Appendix B contains a summary of the combined index by area. It shows that the non-communist world (excluding the United States) is living at about 25 per cent of the United States level, while the less-developed areas (excluding the communist bloc) are at 15 per cent, the developed areas (excluding the United States) at 58 per cent and the communist bloc at 46 per cent. A breakdown by area shows Western Europe at 61 per cent, Southern Europe at 24 per cent, Latin America at 26 per cent, the Middle East (excluding Israel) at 16 per cent, Asia (excluding Japan) and Africa each at 12 per cent and Oceania at 72 per cent.

Some interesting patterns emerge here. First, it seems surprising that Southern Europe and Latin America are at about the same index level. However, a close look at the individual country indices and the ranges by areas reveals that Latin America is a mixed or heterogenous area while Southern Europe appears to be homogenous. In other words, Latin America definitely is an unevenly developed rather than a completely underdeveloped region while Southern Europe appears to be a more uniformily underdeveloped area. (See table 1 below, combined index section).

[13] *Report of the World* , op. cit.

TABLE 1. COMBINED INDEX AND PER CAPITA INCOME: RANGE AND AVERAGE BY AREA

Statistic	Area							
	Latin America	Southern Europe	Western Europe	Asia (excl. Japan)	Africa: Total	Africa: (excl. S. Africa)	Middle East	Communist bloc
	COMBINED INDEX (U.S. 100)							
Range	10-41	16-31	40-79	8-20	6-32	6-20	14-30	53-65
[illegible]	31	15	39	12	26	14	16	32
Average	26	24	61	12	12	10	16	46
	PER CAPITA INCOME (INDEX U.S. 100)							
Range	4-24	9-12	22-59	3-14	2-17	2-10	4-15	15-29
[illegible]	20	3	37	11	15	8	11	14
Average	11	11	39	5	5	4	6	24

Average is weighted by population.
Source: Combined Index: Appendix B, Table B3.
Per Capita Index: International Bank for Reconstruction and Development estimate, Appendix B, Table B4.

Western Europe is unevenly developed but at a relatively high level, while Asia (excluding Japan) and the Middle East (excluding Israel) fall into the category of underdeveloped, with the Middle East at a slightly higher level. The communist bloc seems to be in a mid-range between the less developed and fully developed countries. Africa presents a unique situation in that its range is wider than that of Asia (excluding Japan) and the Middle East (excluding Israel). This results from the inclusion of South Africa in the African figures, which raises the upper limit of the range more than it does the average. Accordingly, there is an entry in Table 1 for Africa excluding South Africa. The range and the average then approach those for Asia (excluding Japan), and point to these areas as the most impoverished in the world, with the Middle East (excluding Israel) and Southern Europe next in line.

Essentially the same relationships can be observed using per capita income data as shown in the lower half of Table 1. There is one major difference: the income estimates suggest a much lower level of living relative to the United States. The next section discusses this in greater detail.

The combined index results: comparison with per capita income

Per capita income estimates have been widely used as indicators of the level of living during the postwar era; accordingly, a comparison of the combined index with these estimates needs to be made. Table B4 in Appendix B contains four per capita income estimates converted to an index basis with the United States = 100. The differences between the combined level of living index and the income indices are striking. The combined index shows every country substantially closer to the United States than is indicated by the income indices. For example, the combined index shows Japan and the USSR at about the same level of living (45 and 46 respectively), while the income estimates place Japan in the neighborhood of 12 to 18 per cent of the United States and Russia 23 to 25. The income indices show 40 countries at or below 10 per cent of the United States income, while the combined index indicates only 12. These observations reinforce the belief that current per capita income estimates converted to dollars at prevailing exchange rates generally understate the incomes of poor countries. Despite this difference in level a high correlation exists between the combined index and the per capita income estimates, implying a strong functional relationship.[14] Quite logically, one would expect as income rises the level of living improves more or less accordingly.

Use of adjusted exchange rates is unsatisfactory as one cannot be certain that the adjustment has brought one any closer to the equilibrium rate. Valuation of a country's aggregate of final goods and services is another method of estimating incomes in a common currency but one which requires prices. The question is whose prices: those of country A, of country B, or an average of both. Although this alternative is highly regarded, it has been applied to only a handful of countries "owing to the practical complexity of the procedure"[15], p. 318. The best known studies of this kind are those of Gilbert and Kravis.[16] The results emphasize that conventional exchange rates to obtain income estimates in a common currency generally underestimate the per capita income of other countries vis-a-vis the United States. Table 2 below summarizes these findings along with those obtained from the level of living analysis.

TABLE 2. COMPARISON BETWEEN LEVEL OF LIVING AND INCOME ESTIMATES BASED ON PURCHASING POWER
(U.S. = 100)

	Organization for European Economic Cooperation		Combined level of living index	U.N.
	Index of p/c GNP via purchasing power adjustments* 1955	Index of p/c GNP via exchange 1955	1959	Index of p/c GNP via exchange rates 1958
United States	100	100	100	100
United Kingdom	57 [1.36]	42	72 [1.53]	47
Denmark	51 [1.38]	37	68 [1.62]	42
France	49 [1.04]	47	58 [1.23]	47
Norway	55 [1.34]	41	62 [1.41]	44
Germany	51 [1.46]	35	68 [1.70]	40
Belgium	53 [1.20]	44	68 [1.45]	47
Italy	29 [1.53]	19	40 [1.90]	21
Netherlands	47 [1.52]	31	62 [1.88]	33

* Based on geometric average of European and United States prices.
Source: *Comparative National Products and Price Level*, by Milton Gilbert and Associates, Paris, 1958; Appendix B, Table B4.

[14] See Appendix B, Table B7.
[15] *Yearbook of National Accounts Statistics*, United Nations, New York, 1962, p. 318.
[16] *Comparative National Products and Price Levels*, by Milton Gilbert and Associates, Paris, 1958; *An International Comparison of National Products and the Purchasing Power of Currencies*, by Milton Gilbert and Irving B. Kravis, Organization for European Economic Cooperation (OEEC), Paris, 1954.

As can be seen, the 1958 Gilbert study demonstrates the downward bias in the estimates based on conventional exchange rates compared to the purchasing power technique. The purchasing power figures definitely show the other eight countries much closer to the United States than implied by the conventional exchange rates data. The same pattern evolves between the level of living and the United Nations per capita figures based on exchange rates. The numbers in brackets show the ratio of purchasing power to exchange rates, and level of living to exchange rates. The two sets of ratios present similar pattern although the level of living suggests a consistently higher correction factor. I conclude that the level of living approach provides a more realistic estimate of the comparative level of well-being that can be obtained from conventional exchange rate income figures.

Correlations of each of the indices (CI, EI, SI) with selected exogenous factors are summarized in table B7 in Appendix B. The correlations with birth rates, death rates and infant mortality are of the correct sign, negative, as the less developed countries have high birth, death and infant mortality rates while the opposite is true of the developed nations.[17] The lowest correlation is with the death rate variable. On the other hand, life expectancy correlations are positive, reflecting the high levels of life expectancy among the richer and the opposite among the poorer countries. These results suggest the possibility of adding infant mortality, birth rates and life expectancy series to further level of living investigations.

The combined index results: contribution of variables

Table B5 in Appendix B summarizes by combined index decile the proportion of the combined index accounted for by each of the 14 variables. The purpose here is to analyze the relative importance of each variable. The first observation worth noting is the almost totally consistent ratio of contribution by the economic and social sectors, namely, 2/3 and 1/3 respectively. There is an exception: that of the 0 to 10 group, which is 83 per cent and 17 per cent respectively. If diet were to be included in the social sector, the ratio of economic and social would change to 60 and 40 for the combined index range 50 to 100 and 40 and 60 for the combined index range 0 to 50. However, as noted earlier, I believe diet's logical place is in the economic sphere.

A close examination by decile and variable reveals the familiar demand pattern of household survey analyses. Diet constitutes the highest proportion in level of living at the low combined indexes and the smallest at the top. The same is generally true for fibers but to a lesser degree.

[17] *Demographic Yearbook*, op. cit.

Part of the explanation lies in the fact that all the countries in the 0 to 10 group are located in the extreme southern hemisphere with reduced fiber requirements. For the items in the industrial, transportation and communications segment the opposite is true: smaller proportions at the lower levels of living, larger proportions at the higher. Cement is an exception; it may be that in the advanced countries (from a combined index of about 70 upward), other products have proven capable substitutes for cement (brick, various metals, wood).

At first glance, the non-agricultural employment variable exhibits an odd sequence. There is almost a smooth progression from a high proportion at low levels of living to low proportions at high levels. This may seem strange, as countries with high levels of living are more advanced in education and technology. The reason for this anomaly rests with the relation of the data of different countries to the United States data. The table below shows each *actual* observation (by variable) as a percent of the United States data. The change in non-agricultural employment across countries is smaller than for telephones and some other variables. The second column for each country shows these percentages converted to Ethiopia + 1.0. For Uruguay, the non-agricultural employment figure is the second lowest while for Haiti it is fourth lowest; thus the rate of change for non-agricultural employment over this range of countries is one of the lowest for the 14 characteristics. Consequently, while at low levels of living non-agricultural employment is relatively important, as a country climbs up the level of living ladder other variables gain in importance.

TABLE 3. URUGUAY, HAITI AND ETHIOPIA: ACTUAL DATA AS PER CENT OF UNITED STATES AND RATIO TO ETHIOPIA

Variable	Uruguay		Haiti		Ethiopia	
	Per Cent	Ethiopia 1 0	Per Cent	Ethiopia 1 0	Per Cent	Ethiopia 1 0
Non-agricultural employment	71 6	6 3	19 3	1 7	11 4	1 0
Telephones	12 6	126 0	0 3	3 0	0 1	1 0
Energy	10 2	102 0	0 5	5 0	0 1	1 0
Steel	11 2	280 0	0 2	5 0	0 4	1 0
Cement	47 6	13 2	3 6	1 0	3 6	1 0
Vehicles	14 8	49 3	0 9	3 0	0 3	1 0
Fibers	38.7	8 6	9 7	2 2	4.5	1 0
Diet	80.0	1.8	60 0	1 4	44.0	1 0
Exports	42 9	14.8	9 5	3.3	2 9	1 0
Physicians and dentists	101.0	168.3	2 9	4 8	0 6	1 0
Newspapers	65 5	327 5	3 0	15.0	0 2	1 0
Schools	50 0	100.0	6 6	13 2	0.5	1 0
Literacy	85 5	27.6	2 6	0.8	3.1	1 0
Radios	6 3	21 0	0 6	2 0	0 3	1 0

Source: Based on data contained in Table B1, Appendix B.

So much for the statistical explanation. On the economic side, the role of non-agricultural employment in economic development is akin to that of agriculture. It has been said with regard to the U.S. economy that agriculture's contributions have been most dramatic in the past, "in the pre-take off stages of our development" and that agriculture now "will provide a persistent and firm underpinning to our economic growth and well-being rather than a major dynamic impetus".[18] Other research on United States experience indicates that, of the factors contributing to positive growth in the period 1929 – 1957, almost a third was accounted for by increased employment. While for the period 1909 – 1929 the contribution was closer to 40 per cent.[19] At lower stages of development, increases in non-agricultural employment constitute an important, even dynamic, impact upon economic growth. However, once a society has achieved a high level of technology the size of the industrial labor force becomes a firm underpinning rather than a dynamic impetus for growth. This serves to explain the relatively greater importance attached to the non-agricultural employment characteristic at the lower levels of living.

In the social sector the school enrollment and literacy series appear to resemble non-agricultural employment. These two variables show, generally, a declining importance with a rising level of living. The proportions for physicians and dentists are inconclusive for it would appear that the wealthier a society the more it can afford doctors and dentists. Part of the explanation is that the basic data are questionable as many countries report medical technicians, midwives and the like as physicians and dentists. Also, there is a tendency for some of the less developed nations to produce excessive numbers of doctors and dentists for a multitude of reasons.[20] Finally, the ratio proportions do conform to the demand pattern, that is, increasing contribution as the level of living raises. This variable along with motor vehicles reflects the degree of "affluency" in a society: the degree to which resources can be diverted from necessities to luxuries such as transportation, entertainment, recreation and leisure.

These patterns among countries parallel the experience with consumer survey data on evaluation of expenditures. In my own research on the Philippines, for seventeen expenditure categories (urban areas) the transportation category was second highest with an elasticity of 1.5 while recreation expenditure ranked fourth with 1.3 (the other two items were gifts and contributions, 1.6 and education, 1.4). Food and other necessities ranged from 0.5 to 0.8 elasticity.

There is a tendency for the proportions to become equal at the higher levels of living. In other words, at the lowest level the range of proportions

[18] *Index of Socio-Economic Status*, Bureau of the Census, Washington, D.C., 1963, p. 26.

[19] "United States Economic Growth", by Edward F. Denison, *The Journal of Business of the University of Chicago*, April 1962, p. 117.

[20] *Education, Manpower and Economic Growth: Strategies of Human Resource Development*, by Frederick Harbison and Charles A. Myers, New York, 1964

is 0.2 to 42.6 while in the highest decile the range is 4.1 to 12.2. This suggests a balancing of the degree of importance at the upper levels. Again it is tempting to ask what would happen to these proportions if it were possible to augment the social sector with series such as life expectancy, morbidity and recreation, and the economic sector with housing. I suspect that the overall sector allocation, 2/3 economic, 1/3 social, would remain unchanged but that there would be a redistribution within these two sectors. Moreover, the factor analysis might demonstrate that it was unnecessary to add, say housing, inasmuch as fiber, diet and possibly telephones resemble the same level of living associated with given levels of housing. In view of the strong interrelations among socio-economic variables, only a relatively few characteristics are required to measure a major component such as level of living.

IV. EVALUATION OF THE FACTOR ANALYSIS

As mentioned earlier, in addition to providing the weights, the factor analysis supplies several other statistics which can be used to evaluate the results of this research. Appendix C contains these measures. Table CI lists the factor analysis weights along with the reliability measures. The data are put in standardized form. There are 14 variables; the total variance of the variables is therefore 14.0, of which approximately 76 per cent is explained by the principal component derived from the factor analysis. In the two other indices, economic and social, the percentages are 77 and 81 respectively. This demonstrates that the principal component accounts for about three-quarters of the variation of the variables used and that these fourteen characteristics can be used as indicators of the level of living. Ideally we would like 100 per cent of the variation explained, but considering the fallibility of some of the data and possibly some missing series which should be part of the exercise, the proportion obtained is reassuring. Other variables amenable to the level of living concept that could be included in further research, assuming reliable data were available, are:

1. Housing measure
2. Demographic measure (life expectancy, birth rate, infant mortality)
3. Morbidity measure
4. Savings measure
5. Freight traffic measure
6. Social welfare measure

This list is by no means exhaustive. However, with additional research on these and other likely variables, it may be possible to reduce the sample to a relatively small number of reliable and pertinent indicators.

The coefficient of correlation between each variable and the indexes is shown in table C2. For the combined index only one variable has less than an 0.80 correlation (exports 0.78) while the highest correlation is for fibers (0.95). Ten of the variables have an "r" of 0.86 or higher. The results for the economic index and the social index are equally good. High correlation with the index demonstrate a meaningful linear relationship with the level of living. Put another way, a high correlation suggests that the level of living is a principal component of the variable involved. Hence the variable can be used to measure the level of living.

V. CONCLUSIONS

The test of a research project is its power to deepen understanding of the problem to which it is applied. There are two aspects of this issue. First, what have we learned? Second, how can we use our new knowledge? With respect to the first, (1) we have learned that not only is it possible to measure the level of living of nations, but we have the means for ascertaining the reliability of our estimates as well. Moreover, the level of living indices themselves support the view that per capita income estimates understate a country's income as shown by the favorable comparison with the Gilbert study. The same conclusion applies on an area basis. Nevertheless, the differences in the rankings between level of living and income are less severe, although for some countries (such as Japan, Venezuela, several other Latin American countries and some African nations) the differences are sharp. (2) The results suggest the possibility of two watersheds above and below which the relationship between economic and social development differs; one at a combined index of about 20 and another at about 67.[21] (3) The proportion contributed by each variable to the combined index demonstrates the existence of Engel curve demand patterns comparable to those found in household surveys. This suggests that whatever course of action a country wishes for its development, human desires and needs will force the satisfaction of necessities first, while luxury or luxury type elements of an economy must take second priority. (4) The tendency toward equality of these

[21] In this connection it should be mentioned that the watershed in 1959 probably would differ from the watershed five years later, inasmuch as the United States level of living would have increased. In other words, a 20 per cent level of living in 1964 (U.S. 100) would mean a higher absolute level than in 1959. The watershed in 1964 would correspond to a lower (than 20 per cent) proportionate level as this would equal the absolute level in 1959. The need is apparent for a level of living index over time for the United States which would facilitate measuring the rate of growth in the United States of living. We could then determine the growth of a given country by relating this rate to the combined index in the terminal year compared to the index in the original year. If the index were the same in both years, then the given country's growth was the same as that of the U.S.; if the index were higher in the terminal year then its growth would have been faster than that of the U.S. The difference could be measured by taking the U.S. growth (one plus the rate of change) and multiplying the apparent rate of change from the given country's level of living indices. Such a scheme would provide an alternative method of measuring the rate of change in economic development, as opposed to the income or product method.

proportions at the upper levels of living demonstrates the capacity of these countries to devote resources to affluent type activities and at the same time indicates to the developing nations that a firm economic foundation is required to achieve the higher level of living. On the other hand, this tendency limits the impact of any one sector upon the rate of growth of the level of living at the upper levels.

As to the second aspect of research, how we can make practical use of the findings, the following suggestions are made. (1) Having established that the level of living approach is a superior alternative to per capita income estimates, it follows that the combined index can be used in place of per capita income as a measure of economic development. (2) The next step is to develop the combined index over a period of time as to construct a series for each country and thus ascertain the rate of growth by country. A real problem is how often to change the weights. Periodic recalculation and evaluation against previous weights may provide the solution here. (3) Awareness of the watersheds can be helpful in designing economic plans (formal as in India, informal as in Japan) by indicating the possible directions in which the economy can move. For example, if a country is in the combined index range of 20 to 67, intensive efforts in the economic sphere will achieve favorable results but at the expense of the social sector—which may not coincide with the desires of the populace. And a persistent emphasis in the economic area, particularly in the industrial sector, will result in an exceedingly large proportion of the level of living being attributable to it. This process can continue but not for long; eventually the social sector needs to be recognized, as is indicated in Table B5, wherein is shown the 2/3 economic and 1/3 social contributions to the combined index. Thus, although an economy can deviate from this pattern, sooner or later it will return to it, primarily because of the needs and desires of the population. In the very lowest decile, 0 to 10, an economy must provide basic necessities for survival; above this level consideration can be given to satisfying other needs. A deviation from this sequence might lead to deprivation of basic necessities or a concentration on necessities only. I am not making a judgment as to the efficacy of this demand pattern; however, what seems apparent is that this sequence arises from the needs and desires of people, who are, in the final analysis, the ultimate consumers of all goods and services. To the extent that an economy is far removed from these patterns, return to them will satisfy the needs of the final consumers. This may quicken the development pace, as disequilibrium may be avoided—for example, excessive emphasis on steel or fertilizers or irrigation, without thought as to how these specific elements fit into the total picture. As the averaging process (used to obtain the contributions to the combined index) takes extremes into account, the resulting demand pattern can be viewed as anorm at which equilibrium exists. Equilibrium in the sense

that the needs and desires of a society are being satisfied and that each sector is complementary to the total. For example, Ethiopia's economic proportion is 95.4, while the average for its decile group is 86.9, which shows it is out of line, giving too much weight to diet (69.1) and too little to non-agricultural employment (15.1) as weel as to literacy (2.8) and schools (0.6). Haiti, also in the 0 to 10 decile, seems to be better balanced, with the economic proportion 85.6.

As noted earlier, substantially more research is required to verify or disprove these findings or perhaps to establish others. Additional experimentation with more and different combinations of variables is necessary to observe how the results are affected. One alternative is to subject the Bennett series to a factor analysis.

APPENDIX A

THE FACTOR ANALYSIS METHOD[22]

The method described here is that developed by Frederick V. Waugh of the United States Department of Agriculture and has been used in computing *Farm Operator Level of Living Indices for Counties of the United States*, 1950 and 1959.[23] A complete text of Waugh's method is contained in this publication.

The level of living index takes the form:

$$I = b_1 X_1 + b_2 X_2 + \dots + b_n X_n \tag{1}$$

were b_1, b_2, b_n are the weights to be determined. Transforming the data into standard form, equation (1) becomes, for a three variable problem,

$$i = w_1 z_1 + w_2 z_2 + w_3 z_3 \tag{2}$$

where w_1, w_2, w_3 are the weights for the standardized variables z. These weights should allow for substantial variation in the index so that it will discriminate between high, medium and low level of living countries. The variance of the index is

$$var\ i = \frac{1}{n} \Sigma\, i^2 = w_1^2 + 2r_{12}\, w_1 w_2 + 2r_{13}\, w_1 w_3 + w_2^2 + 2r_{23}\, w_2 w_3 + w_3^2 \tag{3}$$

where the r's represent the simple correlation. In its present form the variance has no maximum as it could be increased by multiplying all the w's by a constant greater than one. The variance of i can be maximized subject to the constraint that the sum of the squared weights is unity,

$$w_1^2 + w_2^2 + w_3^2 = 1 \tag{4}$$

This can be accomplished by using a Lagrange multiplier thus,

$$F = var\ i - g\,(w_1^2 + w_2^2 + w_3^2 - 1) \tag{5}$$

where g is the Lagrange multiplier.

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[22] The material presented in this appendix follows very closely the original article by Waugh. I have presented here the theoretical background and formulas necessary to use the Waugh factor analysis method. The reader is referred to his article for a numerical example.

[23] Statistical Bulletin 321, Department of Agriculture, Washington, D.C., 1962.

Differentiating (5) with respect to the w's and g we have:

$$(1-g)\,w_1 + r_{12}\,w_2 + r_{13}\,w_3 = 0$$
$$r_{12}\,w_1 + (1-g)\,w_2 + r_{23}\,w_3 = 0$$
$$r_{13}\,w_1 + r_{23}\,w_2 + (1-g)\,w_3 = 0 \qquad (6)$$
$$w_1^2 + w_2^2 + w_3^2 = 1$$

In order to solve for the true maximum, as equation (6) could be satisfied with some galues of g that result in a minimum F and some may result in neither a minimum nor a maximum, it is necessary to add that in the Hessian matrix.

$$H = \begin{matrix} 1-g & r_{12} & r_{13} \\ r_{12} & 1-g & r_{23} \\ r_{13} & r_{23} & 1-g \end{matrix} \qquad (7)$$

all diagonal elements must be negative, all principal 2-row determinants must be positive and so on. Accordingly, in order to arrive at a maximum solution g must be greater than one. We are concerned than with the maximum positive root which can be solved by changing equations (6) to:

$$w_1 - r_{12}\,w_2 + r_{13}\,w_3 = g w_1$$
$$r_{12}\,w_1 + w_2 + r_{23}\,w_3 = g w_2 \qquad (8)$$
$$r_{13}\,w_1 + r_{23}\,w_2 + w_3 = g w_3$$

Multiplying the first equation by w_1, these second by w_2 and the third by w_3, and adding, we obtain:

$$var\ i = g\,(w_1^2 + w_2^2 + w_3^2) \qquad (9)$$

invoking quotation (4) the constraint element, equation (9) indicates that the dominant root g equals the variance of the index. The Lagrange multiplier turns out to have a real meaning, as is often the case. Equation (8) can be solved by an iteration process, that is commencing with trial w's such as 1.0, and continuing until each new set of w's converge.

The correlation between the z's and the index are found by multiplying equations (2) successively by z_1, by z_2 and by z_3 and dividing each product by the standard deviation of the index (i.e., by$\sqrt{g}$). Then using (8),

$$r_{z_1 i} = \frac{w_1 + r_{12}\,w_2 + r_{13}\,w_3}{\sqrt{g}} = \sqrt{g}\cdot w_1$$
$$r_{z_2 i} = \frac{r_{12}\,w_1 + w_2 + r_{23}\,w_3}{\sqrt{g}} = \sqrt{g}\cdot w_2 \qquad (10)$$
$$r_{z_3 i} = \frac{r_{13}\,w_1 + r_{23}\,w_2 + w_3}{\sqrt{g}} = \sqrt{g}\cdot w_3$$

The sum of the squared correlation is:

$$r_{z_1 i}^2 + r_{z_2 i}^2 + r_{z_3 i}^2 = g\,(w_1^2 + w_2^2 + w_3^2) \qquad (11)$$

As the sum of the squared weights is 1, the Lagrange multiplier also equals the sum of the squared correlations between the index and its elements. Thus, in maximizing the variance of the index, we also maximize the sum of the squared correlations.

APPENDIX B

TABLE B1. INDICATORS OF LEVEL OF LIVING: ACTUAL DATA, CIRCA [illegible]

Country	Percent of active population in non-agriculture	Phones	Gross energy consumption	Steel consumption	Cement production	Vehicles	p/c Fiber consumption (Kg)	Quality of diet	p/c Exports (US$) average 1958-60	Physicians and dentists	Daily newspaper circulation	Post primary school enrollment	Percent of total population literate	Radios
Algeria	25	16.3	235	41.0	88	23.0	2.4	35	43	0.226	29	10	17.5	48.0
Angola	15	1.8	67	2.8	34	8.6	1.6	32	28	0.048	5	3	3.0	8.4
Argentina	75	60.3	1,033	97.0	115	35.0	7.3	56	50	1.744	155	32	87.5	164.0
Austrália	87	204.4	3,684	307.0	251	256.0	10.6	70	186	1.610	380	75	98.5	227.0
Austria	68	92.6	1,959	197.0	343	60.0	8.5	57	142	2.067	181	55	98.5	274.0
Belgium	88	118.5	3,853	276.0	488	93.0	9.2	58	357	1.360	275	79	96.5	272.0
Bolivia	28	6.1	142	5.8	8	12.0	2.1	40	16	0.364	34	10	32.5	61.0
Brasil	42	15.0	329	37.0	60	16.0	4.8	48	20	1.490	60	18	47.5	76.0
British E. Africa	20	3.0	77	8.1	16	6.0	1.4	28	16	0.051	5	4	17.5	6.4
Burma	32	0.6	50	4.0	2	2.0	1.8	24	11	0.054	12	17	57.5	1.3
Cameroon	15	1.9	75	4.9	16	8.6	1.9	28	33	0.041	3	4	7.5	3.3
Canada	87	294.7	5,606	355.0	326	283.0	11.1	74	323	1.379	222	64	97.5	574.0
Ceylon	47	3.7	101	7.1	10	11.0	2.0	39	39	0.200	20	25	62.5	27.0
Chile	70	24.6	787	59.0	112	16.0	3.8	43	61	1.32	128	34	77.5	96.0
China (Taiwan)	50	6.5	498	22.0	104	0.3	3.7	27	16	0.517	49	37	47.5	43.0
Colombia	46	19.2	465	19.0	98	12.0	4.2	58	34	0.454	54	16	52.5	62.0
Costa Rica	45	13.3	226	27.9	—	28.0	3.7	63	75	0.476	101	29	77.5	70.0
Cuba	58	28.9	777	17.0	100	34.0	5.4	50	104	1.329	129	19	77.5	155.0
Cyprus	68	30.5	785	36.0	149	56.0	3.9	45 e	93	0.104	112	42	62.5	152.0
Denmark	77	224.1	2,388	237.0	306	112.0	9.0	68	305	1.718	357	74	98.5	328.0
Dominican Republic	44	6.6	158	10.0	66	5.9	1.5	59	51	0.208	25	7	42.5	34.0
Ecuador	47	6.5	157	6.1	38	6.6	1.7	54	33	0.427	54	14	57.5	30.0
El Salvador	37	6.3	119	8.0	33	9.0	2.5	42	46	0.254	48	13	37.5	16.0
Ethiopia	10	0.5	8	0.2	12	1.1	0.7	33	3	0.011	0.5	0.4	3.0	2.5
Malaya Federation of	42	10.5	235	33.0	29	17.0	3.7	35	119	0.153	34	25	37.5	33.0
Finland	54	129.3	1,404	188.0	265	52.0	8.5	57	196	0.983	420	74	98.5	267.0
France	74	90.6	2,365	253.0	315	161.0	9.2	54	130	1.328	243	75	96.5	240.0
Germany Republic of	77	104.5	3,266	444.0	433	80.0	11.7	60	190	1.943	300	77	98.5	287.0
Ghana	30	5.1	87	7.6	-	5.4	3.9	31	57	0.045	42	26	22.5	22.0
Greece	47	17.9	400	33.0	175	11.0	5.8	42	26	1.515	125	35	72.5	90.0

TABLE BI. INDICATORS OF LEVEL OF LIVING: ACTUAL DATA. CIRCA 1959 (CONT.)

Country	Percent of active population in non-agriculture	Phones	Gross energy consumption	Steel consumption	Cement production	Vehicles	p/c Fiber consumption (Kg)	Quality of diet	p/c Exports (US$) average 1958–60	Physicians and dentists	Daily newspaper circulation	Post primary school enrollment	Percent of total population literate	Radios
Guatemala	32	5.4	161	10.7	31	9.1	2.2	40	27	0.195	22	7	27.5	11.0
Haiti	17	1.2	43	3.2	12	3.6	1.5	45	10	0.051	10	5	12.5	5.8
Honduras	34	3.2	142	5.5	6	6.5	2.3	33	36	0.242	25	8	37.5	18.0
Iceland	60	215.1	3,958	129.0	494	120.0	9.6	74	384	1.354	389	70	98.5	272.0
India	29	1.1	145	8.8	17	1.1	2.1	32	3	0.208	9	22	17.5	4.0
Indonesia	34	1.3	135	3.2	5	1.5	0.9	32	9	0.011	11	13	17.5	7.2
Iran	45	4.6	334	21.0	29	6.3	2.9	31	39	0.264	5	14	12.5	26.0
Iraq	19	6.6	336	38.0	321	7.0	3.4	31	88	0.198	10	21	12.5	15.0
Ireland	60	52.4	1,338	53.0	221	73.0	6.3	52	136	1.170	235	12	98.5	175.0
Israel	83	49.5	1,122	145.0	379	28.0	7.0	53	87	3.108	210	36	92.5	192.0
Italy	72	67.9	741	143.0	287	44.0	6.5	42	62	1.447	103	48	87.5	123.0
Jamaica	51	19.7	549	24.5	118	25.0	2.9	58	84	0.269	53	13	72.5	87.0
Japan	67	52.5	965	163.0	186	10.0	8.3	25	37	1.407	398	95	97.5	159.0
Jordan	46	12.8	156	33.6	67	7.2	3.2	33	7	0.173	19	35	17.5	37.0
Korea Republic of	20	3.9	214	[illegible]	15	1.2	2.9	15	1	0.320	57	32	37.5	17.0
Lebanon	50	27.1	567	116.0	327	30.0	5.5	32	25	1.073	97	28	47.5	52.0
Libya	20	12.8	292	17.0		20.0	2.8	46	10	0.135	7	10	7.5	22.0
Luxembourg	74	145.1	3,853	400.0	593	112.0	9.7	58	357	1.384	500	38	96.5	293.0
Mauritius	56	14.5	153	26.5		19.0	2.8	41	87	0.274	68	34	22.5	53.0
Mexico	42	14.8	817	37.0	81	23.0	4.0	45	23	0.526	79	10	62.5	[illegible]
Morocco	29	12.3	135	13.0	47	16.0	2.4	36	33	0.087	22	7	12.5	48.0
Mozambique	25	1.7	124	4.0	34	6.1	1.0	30 e	11	0.027	3	2	3.0	4.3
Netherlands	81	132.3	2,675	238.0	141	64.0	10.8	64	310	1.348	277	82	98.5	270.0
New Zealand	84	294.3	1,886	168.0	241	268.0	10.5	71	339	1.624	387	64	98.5	242.0
Nicaragua	32	5.6	180	9.6	25	9.7	2.3	53	44	0.403	66	4	37.5	26.0
Nigeria	41	1.1	42	2.4	4	1.6	1.4	23	13	0.025	8	5	12.5	2.1
Norway	74	195.2	2,477	250.0	318	85.0	8.3	65	228	1.681	368	51	98.5	279.0
Pakistan	35	0.8	56	2.8	12	0.9	2.2	26	4	0.068	9	16	17.5	2.7
Panama	50	25.4	478	23.0	95	23.0	3.0	47	31	0.360	104	35	67.5	131.0
[illegible]	46	5.8	71	5.4	8	4.6	1.0	45	18	0.691	37	13	67.5	63.0

TABLE BI. INDICATORS OF LEVEL OF LIVING: ACTUAL DATA CIRCA 1959 (CONT.)

Country	Percent of active population in non-agriculture	Phones	Gross energy consumption	Steel consumption	Cement production	Vehicles	p/c Fiber consumption (Kg)	Quality of diet	p/c Exports (US$) average 1958–60	Physicians and dentists	Daily newspaper circulation	Post primary school enrollment	Percent of total population literate	Radios
Peru	38	9.7	315	9.0	54	13.0	2.5	44	32	0.534	76	16	47.5	60.0
Philippines	41	3.5	149	15.0	30	6.5	1.9	34	21	0.120	19	25	62.5	13.2
Portugal	52	40.3	355	41.0	114	23.0	5.4	44	33	0.752	80	30	57.5	85.0
Rhodesia Á Nyasa	33	13.0	558	31.0	73	20.0	2.2	25	61	0.124	39	2	17.5	7.6
Sierra Leone	10	1.2	59	7.0		3.7	1.9	35 e	26	0.017	6	3	7.5	3.1
South Africa	53	62.6	2,496	134.0	180	70.0	4.9	40	83	0.613	61	25	42.5	63.0
Spain	51	54.5	827	70.0	175	13.0	5.5	46	19	1.036	70	30	82.5	84.0
Sudan	15	1.8	53	3.2	9	2.4	1.7	37	14	0.026	4	5	7.5	0.7
Sweden	87	338.9	2,995	453.0	383	172.0	11.1	67	307	1.542	462	65	98.5	357.0
Switzerland	88	298.1	1,686	254.0	512	98.0	10.7	62	324	1.852	296	39	98.5	264.0
Syria	30	10.8	286	27.0	81	6.5	4.7	35	26	0.272	19	17	27.5	57.0
Thailand	18	1.7	67	9.7	22	3.8	2.4	34	16	0.154	14	27	57.5	4.4
Trinidad Á Tobago	75	37.9	1,285	138.0	220	53.0	2.0	50	320	0.406	101	27	80.0	74.0
Tunisia	42	9.9	103	10.0	112	10.0	2.9	35	35	0.168	17	18	17.5	36.0
Turkey	23	2.0	246	14.0	64	7.5	5.1	28	11	0.508	31	18	37.5	44.0
U.A.R. — Egypt	35	8.6	235	8.6	70	3.6	4.0	31	20	0.389	20	19	22.5	41.0
United Kingdom	95	149.4	4,594	332.0	245	124.0	11.9	68	187	1.224	514	72	98.5	287.0
United States	88	398.5	7,834	491.0	336	393.0	15.5	75	105	1.762	328	76	96.5	941.0
Uruguay	63	50.3	800	55.0	160	58.0	6.0	60	45	1.779	215	38	82.5	261.0
Venezuela	59	27.6	2,512	105.0	287	55.0	4.3	58	365	0.704	97	22	52.5	122.0
Viet-Nam	20	1.1	56	5.9	28	2.1	0.8	30 e	5	0.048	28	2	17.5	6.8
Yugoslavia	33	12.8	794	78.0	120	4.3	4.1	34	27	0.761	56	22	72.5	71.0
U.S.S.R.	65	11.3	2.942	276.0	184	18.0	8.5	60 e	24	1.890	151	27	92.5	190.0
Czechoslovakia	62	69.0	4,590	438.0	350	18.0	10.1	60 e	127	1.800	189	21	97.5	253.0
Hungary	62	23.4	2,180	174.0	144	5.6	6.1	55 e	78	1.480	132	32	95.5	211.0
Poland	43	27.8	2,995	195.0	182	6.7	7.0	55 e	40	1.138	142	44	92.5	169.0
Rumania	30	9.6	1,253	124.0	156	1.8	4.9	70 e	31	1.250	132	32	77.5	101.0
East Germany	71	76.4	4,388	314.0	259	17.0	14.0	65 e	127	2.000	456	49	98.5	339.0

Note: All data are expressed on a per capita (or per thousand population) basis, with the exception of items 1, 8, 12 and 13 wich are on a percentage basis.

Source: *Statistical Yearbook*, United Nations, various issues; *Yearbook of International Trade Statistics, idem.*; *World Illiteracy at Mid-Century*, United Nations Educational, Scientific and Cultural Organization; *Basic Facts and Figures, idem.*, 1961, 1962; Automobile Manufacturer Association; World Motor Vehicle Registrations; World Railways; *Atlas of Economic Development*, by Ginsberg; *The Worldmark Encyclopedia of the Nations*, Benjamin A. Cohen (ed.), Worldmark Press Inc.; *Production Yearbook*, Food and Agriculture Organization of the United Nations, various issues; *Yearbook of Labor Statistics*, International Labour Office, 1962; *Compendium of Social Statistics*, United Nations, 1963, Table 61; *Monthly Bulletin of Agricultural Economics and Statistics*, Food and Agriculture Organization of the United Nations, Vol. 11, January 1962.

TABLE B2. CORRELATION MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1.0	0.73	0.73	0.77	0.74	0.67	0.92	0.73	0.71	0.80	0.80	0.80	0.85	0.76
2	0.73	1.0	0.77	0.78	0.71	0.90	0.81	0.70	0.74	0.60	0.78	0.72	0.63	0.86
3	0.73	0.77	1.0	0.91	0.74	0.77	0.87	0.73	0.66	0.66	0.76	0.69	0.69	0.90
4	0.77	0.78	0.91	1.0	0.81	0.70	0.90	0.70	0.66	0.74	0.82	0.73	0.73	0.85
5	0.74	0.71	0.74	0.81	1.0	0.58	0.80	0.63	0.77	0.75	0.78	0.69	0.72	0.70
6	0.67	0.90	0.77	0.70	0.58	1.0	0.72	0.65	0.63	0.49	0.65	0.66	0.53	0.83
7	0.82	0.81	0.87	0.90	0.80	0.72	1.0	0.73	0.65	0.83	0.89	0.84	0.80	0.88
8	0.73	0.70	0.73	0.70	0.63	0.65	0.73	1.0	0.67	0.69	0.72	0.59	0.77	0.74
9	0.71	0.74	0.66	0.66	0.77	0.63	0.65	0.67	1.0	0.50	0.73	0.64	0.61	0.60
0	0.80	0.60	0.66	0.71	0.75	0.49	0.83	0.69	0.50	1.0	0.75	0.68	0.84	0.71
1	0.80	0.78	0.76	0.82	0.78	0.65	0.89	0.72	0.73	0.75	1.0	0.84	0.80	0.77
2	0.80	0.72	0.69	0.73	0.69	0.66	0.84	0.59	0.64	0.68	0.84	1.0	0.79	0.74
3	0.85	0.63	0.69	0.73	0.72	0.53	0.80	0.77	0.61	0.84	0.80	0.79	1.0	0.72
4	0.76	0.86	0.90	0.85	0.70	0.83	0.88	0.74	0.60	0.71	0.77	0.74	0.72	1.0
	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Non-Agriculture Employment
Phones
Energy
Steel
Cement
Vehicles
Fiber
Diet
Exports
Physicians & Dentists
News-paper
Post-primary School Enrollment
Literacy
Radios

———ECONOMIC LEVEL———

———SOCIAL LEVEL———

———COMBINED ECONOMIC-SOCIAL LEVEL———

TABLE B3. SUMMARY OF LEVELS OF LIVING: COMBINED, ECONOMIC, SOCIAL RANKED ACCORDING TO COMBINED

(US = 100)

Country	Combined	Economic	Social	Rank Combined	Rank Economic	Rank Social
United States	100	100	100	1	1	1
Canada	83	87	76	2	2	7
Sweden	79	81	79	3	3	2
Australia	72	72	74	4	5	9
United Kingdom	72	69	78	5	9	4
Luxembourg	71	73	70	6	4	14
New Zealand	71	71	72	7	7	11
Iceland	70	69	72	8	8	10
Germany, Republic of	68	66	76	9	11	6
Switzerland	68	72	65	10	6	20
Belgium	68	68	70	11	10	16
Denmark	68	64	77	12	12	5
East Germany	65	59	78	13	14	3
Norway	62	60	70	14	13	15
Netherlands	62	58	70	15	15	13
France	58	55	66	16	17	19
Czechoslovakia	57	58	55	17	16	22
Finland	54	47	72	18	18	12
Israel	50	43	67	19	20	17
Austria	49	44	66	20	23	18
U.S.S.R.	46	43	52	21	21	24
Japan	45	32	75	22	31	8

TABLE B3. SUMMARY OF LEVELS OF LIVING: COMBINED, ECONOMIC, SOCIAL RANKED ACCORDING TO COMBINED (CONT.)

(US = 100)

Country	Combined	Economic	Social	Rank Combined	Rank Economic	Rank Social
[illegible]	42	38	54	23	24	23
[illegible]	41	35	57	24	30	21
[illegible]	41	36	50	25	28	25
[illegible]	41	37	49	26	26	27
[illegible]	40	37	50	27	26	26
[illegible]	40	38	48	28	25	28
Venezuela	40	46	29	29	19	42
Trinidad & Tobago	38	42	31	30	22	37
[illegible]	33	30	41	31	33	31
[illegible]	33	30	41	32	34	32
[illegible]	32	28	42	33	37	30
South Africa	32	36	24	33	29	47
Spain	31	29	37	35	36	33
Cyprus	30	30	33	36	32	35
Greece	30	25	42	37	41	29
Lebanon	29	29	30	38	35	40
Portugal	27	26	30	39	39	39
Costa Rica	26	25	32	40	42	36
Panama	26	23	33	41	43	34
Jamaica	25	27	24	42	38	46
Brazil	24	22	30	43	45	38
Colombia	24	25	22	44	40	49
Mexico	23	23	25	45	44	44
Yugoslavia	23	21	29	46	49	41
China (Taiwan)	21	18	25	47	52	43
Mauritius	20	21	20	48	47	53
Ecuador	20	20	20	49	51	51
Peru	19	18	22	50	53	48
Malaya	19	20	17	51	50	57
Paraguay	18	16	25	52	64	45
Dominican Republic	18	21	13	53	48	66
Ceylon	18	17	21	54	60	50
Iraq	17	21	10	55	46	71
Nicaragua	17	18	15	56	54	61
Jordan	17	18	15	57	56	58
Syria	17	18	14	58	55	63
Philippines	16	15	19	59	68	54
El Salvador	16	17	15	60	59	62
Turkey	16	15	18	61	70	55
U.A.R. — Egypt	15	16	14	62	63	64
Tunisia	15	17	11	63	57	69
Algeria	15	17	10	64	58	70
Ghana	14	15	13	65	69	65
Iran	14	17	9	65	61	73
Thailand	14	12	18	67	74	56
Guatemala	13	15	10	68	66	72
Honduras	13	14	12	69	71	67
Rhodesia & Nyasaland	13	16	7	70	62	77
Morocco	13	15	7	71	67	75
Libya	12	16	6	72	65	78
Korea	12	9	20	73	87	52
Burma	12	10	15	74	81	59
India	12	12	11	75	73	68
Bolivia	11	10	15	76	85	60
Pakistan	10	12	8	77	76	74
Indonesia	10	12	7	78	75	76
Haiti	10	12	5	79	72	81
Nigeria	9	11	4	80	77	82
B. E. Africa	8	10	5	81	83	80
Sudan	8	11	3	82	80	83
Viet-Nam	8	10	5	83	86	79
Angola	8	11	2	84	78	86
Mozambique	8	11	2	85	79	87
Cameroon	8	10	3	85	82	84
Sierra Leone	7	10	3	87	84	85
[illegible]	6	8	1	88	88	88

TABLE B4. PER CAPITA INCOME ESTIMATES AND LEVEL OF LIVING INDEX

Country	Level of living Combined	Per capita income estimates (US=100) MIT* 1957 GNP p/c	IBRD** 1958-60[1]	UN*** 1958[2] p/c	USAID**** 1961[3]	Combined	MIT	IBRD	UN	USAID
United States	100	100	100	100	100	1	1	1	1	1
Canada	83	76	72	76	66	2	2	2	2	2
Sweden	79	54	52	56	53	3	4	4	5	3
Australia	72	51	51	52	54	4	6	7	7	6
United Kingdom	72	46	45	47	50	4	8	9	8	9
Luxembourg	71	54	59	57	53	6	4	3	4	7
New Zealand	71	51	52	55	55	6	6	4	6	5
Iceland	70	22	33	41	30	8	22	16	13	17
Germany Republic of	68	36	44	40	52	9	13	10	14	8
Switzerland	68	55	52	58	61	9	3	4	3	4
Belgium	68	46	44	47	49	9	8	10	8	10
Denmark	68	41	46	42	49	9	11	8	12	10
East Germany	65	23	25	[a] n.a.	[a] n.a.	13	20	19	[a] n.a.	[a] n.a.
Norway	62	44	42	44	47	14	10	13	11	12
Netherlands	62	32	44	33	37	14	14	10	15	15
France	58	37	39	47	48	16	12	14	8	13
Czechoslovakia	57	26	29	[a] n.a.	[a] n.a.	17	17	18	[a] n.a.	[a] n.a.
Finland	54	31	36	32	39	18	15	15	16	14
Israel	50	28	24	25	29	19	16	21	19	18
Austria	49	26	31	28	31	20	17	17	18	16
U.S.S.R.	46	23	25	[a] n.a.	[a] n.a.	21	20	19	[a] n.a.	[a] n.a.
Japan	45	12	18	12	18	22	40	28	35	23
Ireland	42	21	22	20	25	23	23	24	23	19
Uruguay	41	19	17	19	16	24	25	31	24	24
Hungary	41	19	20	n.a.	n.a.	24	25	25	[a] n.a.	[a] n.a.
Poland	41	18	19	[a] n.a.	[a] n.a.	24	28	28	[a] n.a.	[a] n.a.
Argentina	40	19	20	21	13	27	25	25	21	33
Italy	40	20	24	21	25	27	24	21	21	19
Venezuela	40	25	24	31	25	27	19	21	17	19
Trinidad & Tobago	38	16	20	23	21	30	31	25	20	22
Rumania	33	14	15	[a] n.a.	[a] n.a.	31	34	33	[a] n.a.	[a] n.a.
Cuba	33	17	18	16	14	31	30	28	27	32
Chile	32	15	14	15	16	33	32	37	28	24
South Africa	31	15	17	17	15	33	32	31	26	26
Spain	31	11	12	14	13	35	42	42	32	33

TABLE B4. PER CAPITA INCOME ESTIMATES AND LEVEL OF LIVING INDEX (CONT.)

Country	Level of living Combined	Per capita income estimates (US=100) MIT* 1957 GNP p/c	IBRD** 1958-60[1]	UN*** 1958[2] p/c	USAID**** 1961[3]	Combined	MIT	IBRD	UN	USAID
Cyprus	30	18	15	18	15	36	28	33	25	26
Greece	30	13	11	13	15	36	38	45	33	26
Lebanon	29	14	14	11	15	38	34	37	36	26
Portugal	27	9	9	9	10	39	48	47	42	37
Costa Rica	26	14	15	15	12	40	34	33	28	35
Panama	26	13	14	15	15	40	38	37	28	26
Jamaica	25	12	15	15	42	40	33	28	28	26
Brazil	24	11	7	11	7	43	42	53	36	45
Colombia	24	10	12	13	10	43	44	42	33	37
Mexico	23	10	13	11	11	45	44	41	36	36
Yugoslavia	23	10	12	n.a.	10	45	44	42	n.a.	37
China (Taiwan)	21	6	5	5	5	47	61	61	61	58
Mauritius	20	10	10	10	[a] n.a.	48	44	46	39	[a] n.a
Ecuador	20	7	5	8	6	48	54	61	47	53
Peru	19	7	7	7	6	51	54	53	51	53
Malaya	20	14	14	9	10	48	34	37	42	37
Paraguay	18	4	4	5	5	52	71	70	61	58
Dominican Republic	18	9	9	9	8	52	48	47	42	42
Ceylon	18	5	4	5	5	52	67	70	61	58
Iraq	17	6	7	8	7	55	61	53	47	45
Nicaragua	17	6	8	10	8	55	61	51	39	42
Jordan	17	5	4	6	7	55	67	70	57	45
Syria	17	7	5	6	5	55	54	61	57	58
Philippines	16	9	5	8	4	59	48	61	47	63
El Salvador	16	9	9	9	8	59	48	47	42	42
Turkey	16	9	9	9	7	59	48	47	42	45
U.A.R. — Egypt	15	6	5	5	4	62	61	61	61	63
Tunisia	15	7	7	6	6	62	58	53	57	53
Algeria	15	7	7	10	10	62	54	53	39	37
Ghana	14	7	7	7	7	65	58	53	51	45

TABLE B4. PER CAPITA INCOME ESTIMATES AND LEVEL OF LIVING INDEX (CONT.)

Country	Level of living Combined	Per capita income estimates (US=100) MIT* 1957 GNP p/c	IBRD** 1958–60[1]	UN*** 1958[2] p/c	USAID**** 1961[3]	Combined	MIT	IBRD	UN	USAID
Iran	14	4	5	7	7	65	71	61	51	45
Thailand	14	4	5	3	3	65	71	61	74	66
Guatemala	13	7	8	7	6	68	54	51	51	53
Honduras	13	8	7	8	7	68	53	53	47	45
Rhodesia & Nyasaland	13	5	7	7	6	68	67	63	51	53
Morocco	13	6	5	7	5	68	61	61	51	58
Libya	12	2	4	6	7	72	82	70	57	45
Korea	12	6	3	5	3	72	61	77	61	66
Burma	12	2	4	2	2	72	82	70	80	76
India	12	3	3	3	3	72	76	77	74	66
Bolivia	11	4	4	4	4	76	71	70	68	63
Pakistan	10	3	3	2	3	77	76	77	78	66
Indonesia	10	5	3	3	3	77	67	72	74	66
Haiti	10	4	4	4	3	77	71	70	68	66
Nigeria	9	3	3	4	3	80	76	77	68	66
Br. East Africa	8	3	3	3	2	81	76	77	74	76
Sudan	8	2	3	4	3	81	82	77	68	66
Viet-Nam	8	3	3	4	3	81	76	77	68	66
Angola	8	2	3	4	[a] n.a.	81	82	77	68	[a] n.a.
Mozambique	8	3	3	3	[a] n.a.	81	76	77	74	[a] n.a.
Cameroon	8	2	3	5	3	81	82	77	61	66
Sierra Leone	7	2	5	3	2	87	82	61	74	76
Ethiopia	6	2	2	2	2	88	82	88	80	76

* Massachusetts Institute of Technology.
** International Bank for Reconstruction and Development.
*** United Nations.
**** United States Agency for International Development.
[1] Per capita at factor cost exchange rate adjustment.
[2] Domestic product at factor cost.
[3] GNP market prices.
[a] Not available.

Source: Per capita income estimates, World Income 1957, by Mikoto Usui and E.E. Hagen, MIT; IBRD, per capita GNP at factor cost, Economic Department estimates; *Yearbook National Accounts Statistics*, UN; *Estimates of Gross National Product (1961)*, USAID, Statistics and Reports Division, April 3L, 1963.

TABLE B5. LEVEL OF LIVING: COMBINED INDEX AND AVERAGE CONTRIBUTION OF EACH VARIABLE TO INDEX

Combined index decile	Number of countries	Percentages* Economic sector Total	Non-agric-employ-ment	Phones	Energy cons.	Steel cons.	Cement prod.	Vehicles	Exports	Fibers	Diet	Social sector Total	Phy-sicians	News-papers	Schools	Literacy	Radios
90.1-100.0	1	67.8	7.5	8.7	10.9	7.4	4.1	9.7	1.6	8.9	0.0	32.2	4.4	4.5	5.9	5.2	12.2
80.1- 90.0	1	70.7	9.0	7.8	9.5	6.5	4.9	8.4	6.1	7.8	10.7	29.3	4.2	8.3	6.0	6.4	9.0
70.1- 80.0	5	67.7	10.0	6.8	6.6	6.8	5.8	6.3	5.9	8.5	11.0	32.5	5.1	8.4	6.7	7.3	5.0
60.1- 70.0	8	65.5	10.0	5.6	6.5	6.1	6.8	3.1	6.6	9.1	11.7	34.7	6.3	7.0	7.7	8.0	5.7
50.1- 60.0	3	64.0	9.6	3.8	6.9	7.8	6.8	3.3	4.2	9.5	12.1	36.2	6.0	7.0	7.9	9.4	5.9
40.1- 50.0	[illegible]	59.2	13.0	2.5	5.2	5.1	5.4	1.7	2.5	9.5	14.3	41.1	9.8	6.0	8.1	11.5	5.7
30.1- 40.0	0	67.0	14.7	2.3	5.8	3.8	6.2	2.7	5.9	7.6	18.0	33.0	6.4	4.2	6.9	11.3	4.2
20.1- 30.0	12	63.9	16.4	1.6	2.7	2.4	5.1	1.7	2.6	9.6	21.8	35.9	7.0	4.2	8.1	12.6	4.0
10.1- 20.0	29	70.5	19.2	1.0	1.8	1.4	3.7	1.5	3.3	10.5	28.1	29.7	4.1	2.6	8.8	11.8	2.4
0 - 10.0	11	86.9	20.7	0.4	1.1	0.8	2.4	1.3	3.0	9.6	47.6	13.0	1.0	1.2	3.8	6.3	0.7
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* Obtained as simple arithmetic average of country percentages.

TABLE B6. LEVEL OF LIVING COMBINED INDEX BY AREA

(Circa 1959)

Area	Combined Index (US=100)
Free World	25
Developed countries	58
Less-developed countries	15
Soviet Bloc	46
Western Europe	61
Southern Europe	24
Latin America	26
Middle East	16
Africa (excl. South Africa)	12
Asia (excl. Japan)	12
Australia & New Zealand	72
Others:	
Canada	83
Israel	51
Japan	45
South Africa	32

Note: Developed countries includes Western Europe, Canada, Australia, New Zeland, Japan and Israel. Less-developed countries include Latin America, Africa, Asia, Middle East, Southern Europe and South Africa.

TABLE B7. SUMMARY OF CORRELATIONS WITH SELECTED EXOGENOUS FACTORS

Exogenous factors	Correlation with level of living index		
	Combined	Economic	Social
	r	r	r
Crude birth rate	—0.62	—0.58	—0.67
Crude death rate	—0.28	—0.26	—0.30
Infant mortality	—0.66	—0.64	—0.65
Life expectancy	0.76	0.72	0.79

Note: The number of countries used in each correlation varies inasmuch as data on the exogenous factors were not available for all countries.

TABLE C1. LEVEL OF LIVING: COMBINED, ECONOMIC, SOCIAL WEIGHTS AND RELIABILITY MEASURES

Variable	Weights		
	Combined	Economic	Social
1 Non-Agriculture employment	0.0855	0.1254	
2 Phones p/c	0.0219	0.0341	
3 Energy consumption p/c	0.0014	0.0017	
4 Steel consumption p/c	0.0150	0.0233	
5 Cement production p/c	0.0123	0.0191	Not Applicable
6 Vehicles p/c	0.0246	0.0390	
7 Fiber consumption p/c	0.5779	0.8488	
8 Quality of diet	0.1197	0.1802	
9 Exports p/c	0.0157	0.0241	
10 Physicians & Dentists p/c	2.4969		8.0845
11 Newspaper circulation p/c	0.0137		0.0423
12 Post-primary school enrollment	0.0780	Not Applicable	0.2472
13 Literacy	0.0540		0.1755
14 Radios p/c	0.0130		0.0384
		Reliability Measures	
Total variance of variables	14.0	9.0	5.0
Computed variance of variables	10.6049	6.9224	4.0580
Percentage of total variance explained by Level of Living concept	76	77	81

TABLE C2. LEVEL OF LIVING: CORRELATION BETWEEN EACH VARIABLE AND INDEX

Variable	Correlation coefficient		
	Combined	Economic	Social
1 Non-Agriculture employment	0.90	0.87	
2 Phones p/c	0.88	0.91	
3 Energy consumption p/c	0.90	0.91	
9 Steel consumption p/c	0.91	0.92	
5 Cement production p/c	0.86	0.86	Not Applicable
6 Vehicles p/c	0.80	0.84	
7 Fiber consumption p/c	0.95	0.93	
8 Quality of diet	0.82	0.83	
9 Exports p/c	0.78	0.82	
10 Physicians & Dentists p/c	0.83		0.88
11 Newspaper circulation p/c	0.91		0.92
12 Post-primary school enrollment	0.86	Not Applicable	0.90
13 Literacy	0.86		0.92
14 Radios p/c	0.91		0.87

* * *