

# The Economics of Kenneth J. Arrow

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- study of markets for all goods *together*
  - e.g., can all markets clear simultaneously?  
(supply = demand in each market)



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- how does *uncertainty* about outcome affect economic behavior

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- In 18<sup>th</sup> century, Condorcet studied *simple majority rule* as method for aggregating preferences over social alternatives (social welfare function)
  - alternative  $x$  chosen over  $y$  if more individuals prefer  $x$  to  $y$  than  $y$  to  $x$



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*independence of irrelevant alternatives*

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  - dictatorship not satisfactory

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# Arrow –Debreu model

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- each firm described by its *technology* (relationship of inputs to outputs)
- each firm chooses *production plan*, choice of inputs and outputs (feasible for its technology) to maximize *profit* (revenue from selling outputs minus cost of inputs, given prices)

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budget = income from endowment + shares of firms' profits

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- many models in applications are special cases of Arrow-Debreu

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  - but patent holder is monopolist, so dissemination suboptimal

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- foundation of endogenous growth literature

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