Fisher versus Törnqvist when Relative Price Changes are Large

(*Multiplier effect of price variations [uncertainties] on the Fisher index*[1])

Yuri Dikhanov

World Bank

ydikhanov@worldbank.org

Improving ICP methodology in the upcoming ICP round is an urgent topic on the ICP Technical Advisory Group (TAG) agenda. Among other things, we think the basic aggregation method used in ICP (the Fisher-based GEKS) should be improved as it can be quite unstable in the face of price and expenditure volatility (esp. due to measurement errors). In addition, the GEKS computation can collapse if some weights become negative. The note discusses the sources of GEKS instability, introduces a model to explain the effects of price shocks, and proposes a remedy for the situation, - the Törnqvist-based CCD (Caves, Christensen and Diewert) index. The results of this paper will be relevant to both spatial and temporal indices.

1. Case of temporal price indices

As it is quite difficult to come up with a counterfactual in the international comparisons setting, we start with temporal indices to demonstrate the extent of the problem.

For temporal analyses, we utilize the 1990-2023 NIPA PCE data from the US BEA website (bea.gov). In Table A below three types of indices are considered: (1) multilateral, or GEKS (multilateral Fisher) and CCD (multilateral Törnqvist), (2) direct, or 2023/1990 direct estimates using only two years, and (3) chained indices. For reference, the Paasche and Laspeyres indices (both the regular and geometric, direct and chained) are provided as well. The chained indices, both based on the Törnqvist and Fisher, are a good approximation of the Divisia index and can serve as our approximation of the "true" price index (for more on that see below, also see Figures 1 and 2, as well as Tables 1 and 2).

We consider two aggregation scopes: (1) over all items, and (2) over all items without technology items. The fast-moving tech items will be our proxies for volatile items present in the ICP (of course, in the temporal setting those items are "volatile" due to the fast pace of the technical change, not due to measurement errors).

[1] The author extends sincere gratitude to Erwin Diewert for his generous provision of insightful suggestions, comments, and paper reviews.

Table A. Price index, 2023 to 1990, NIPA Personal Consumption Expenditures, annual accounts

	2023 indices (1990 = 1.0)		Törnqvist (chained) = 1.0	
	All items [a]	All w/o tech items[1] [b]	All items [c] = [a] / [a.6]	All w/o tech items[1] [d] = [b] / [b.6]
1. Fisher (multilateral), GEKS	1.7547	2.1710	0.8656	0.9927
2. Törnqvist (multilateral), CCD	2.0229	2.1799	0.9979	0.9968
3. Fisher (direct), two-year	1.5324	2.1557	0.7559	0.9857
4. Törnqvist (direct), two-year	2.0162	2.1746	0.9946	0.9943
5. Fisher (chained)	2.0269	2.1872	0.9999	1.0001
6. Törnqvist (chained)	2.0272	2.1869	1.0000	1.0000
reference				
7. Paasche (direct)	1.0117	1.9717	0.4990	0.9016
8. Laspeyres (direct)	2.3213	2.3570	1.1451	1.0777
9. Geometric Paasche (direct)	2.0061	2.2025	0.9896	1.0071
10. Geometric Laspeyres (direct)	2.0264	2.1470	0.9996	0.9817
11. Paasche (chained)	2.0042	2.1738	0.9887	0.9940
12. Laspeyres (chained)	2.0499	2.2006	1.0112	1.0063
13. Geometric Paasche (chained)	2.0566	2.2205	1.0145	1.0153
14. Geometric Laspeyres (chained)	1.9982	2.1539	0.9857	0.9849

Note: Net purchases abroad and NPISH are excluded

(1) - tech items are:

Video, audio, photographic, and information processing equipment and media (75, 76, and part of 93);

Telephone and related communication equipment.

Source of data: bea.gov

[Note that the indexes for Figures 1-4 for all years are listed in an Appendix]

Table A shows the 2023/1990 price indices with 1990 = 1.0. As we can see from Table A, both Fisher and Törnqvist chained indices are almost identical. This is the case both in aggregation over all items and in that over all items w/o technology items [a.5 vs. a.6, and b.5 vs. b.6].

We observe that the Törnqvist (multilateral) index – or CCD - is very close to the Törnqvist (chained), but the Fisher (multilateral) – or GEKS – is not that close for all items case [a.1 vs. a.5]. This changes if we remove the tech items, - then the GEKS becomes very close to the CCD [b.1 vs. b.2].

This is important, as it shows that using the Törnqvist chained index does not lead to chain drift since it is close to CCD. This justifies the use of the Törnqvist (or Fisher) chained index as a good target index.

So, what is happening is that the volatile (i.e., technology) items exhibit tremendous price declines over time and so using fixed base indexes, their prices in the later sample period are very low and this is what leads to outliers or "shocks" in relative prices, and the Fisher formula does not respond well to these large changes in relative prices.

In the All-items case, the Törnqvist (direct, or using only 1990 and 2023 data) is quite close to the Törnqvist (chained), but, again, this is not the case for the Fisher (direct) [compare a.3 to a.4]. However, if we remove the tech items, the Fisher (direct) comes much closer to our reference index, though still with some gap (a 1.4% difference, see b.3).

Important to note that even geometric Paasche and Laspeyres are not too far from our reference chained indices.

We see that the regular Paasche 2023/1990 index is distorted more than the Laspeyres, which creates a problem for the Fisher, especially in the All-items case.

Tables 1 and 2 show in detail that the chained Törnqvist (or Fisher) index is approaching the "truth" for this example, so comparisons of GEKS and CCD with this index are justified.

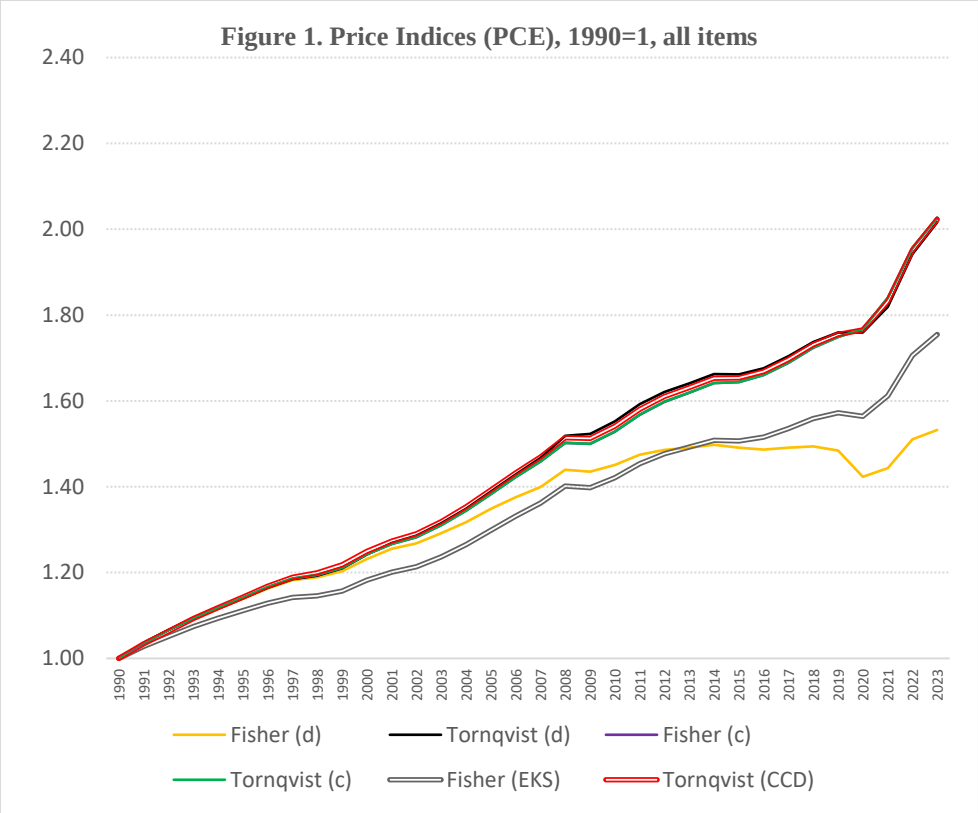


Figure 1. Price Indices (PCE), 1990=1, all items
2.40
2.20
2.00
1.80
1.60
1.40
1.20
1.00
1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023
Fisher (d)
Tornqvist (d)
Fisher (c)
Tornqvist (c)
Fisher (EKS)
Tornqvist (CCD)

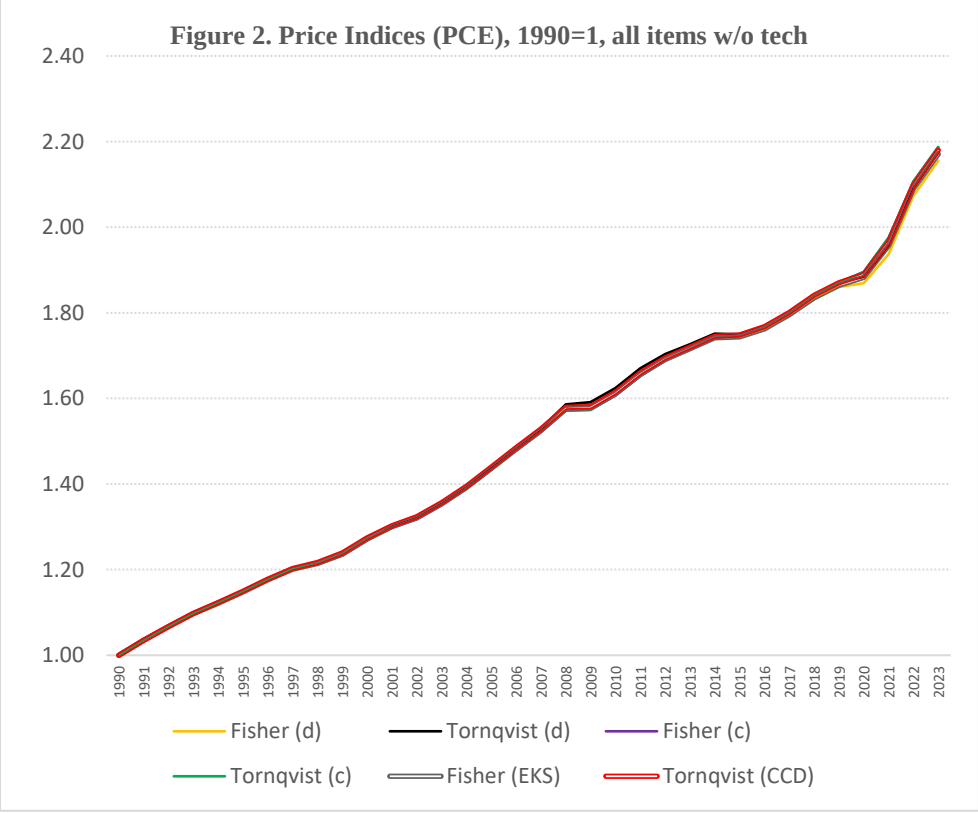


Figure 2. Price Indices (PCE), 1990=1, all items w/o tech
2.40
2.20
2.00
1.80
1.60
1.40
1.20
1.00
1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023
Fisher (d)
Tornqvist (d)
Fisher (c)
Tornqvist (c)
Fisher (EKS)
Tornqvist (CCD)

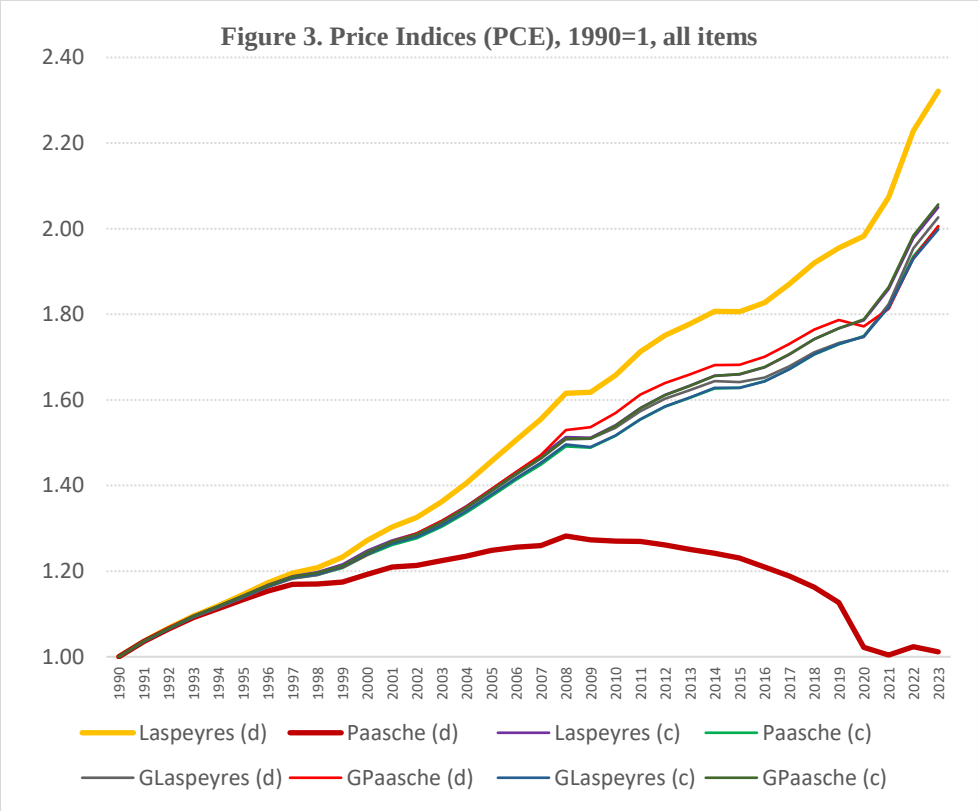


Figure 3. Price Indices (PCE), 1990=1, all items
2.40
2.20
2.00
1.80
1.60
1.40
1.20
1.00
1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023
Laspeyres (d)
Paasche (d)
Laspeyres (c)
Paasche (c)
GLaspeyres (d)
GPaasche (d)
GLaspeyres (c)
GPaasche (c)

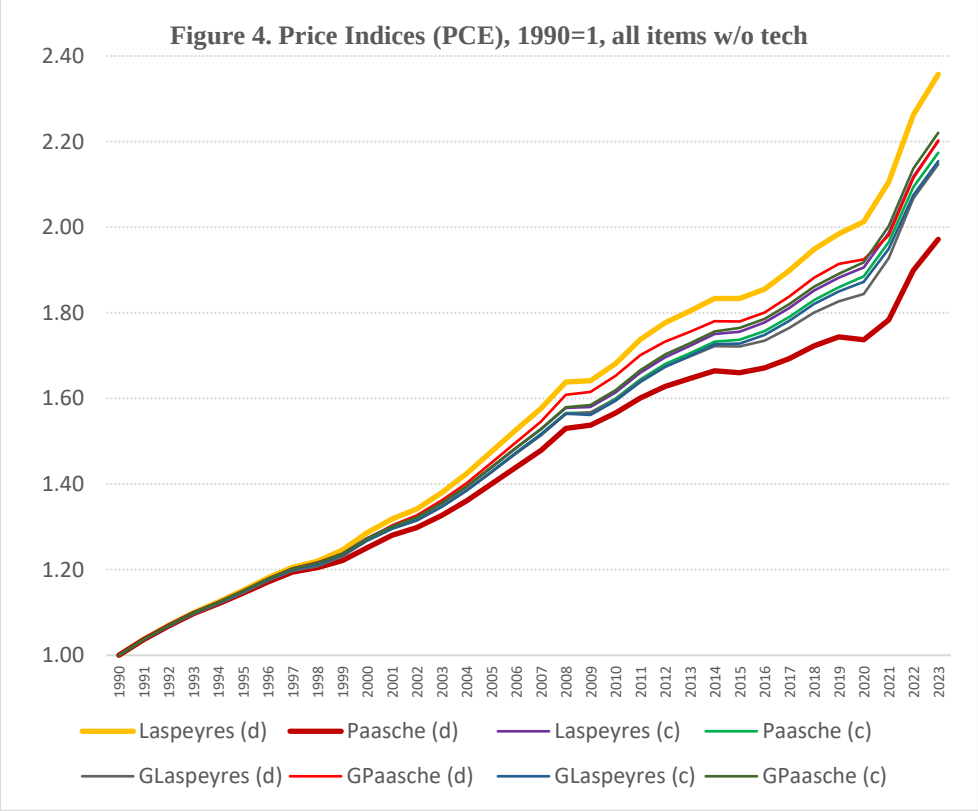


Figure 4. Price Indices (PCE), 1990=1, all items w/o tech
2.40
2.20
2.00
1.80
1.60
1.40
1.20
1.00
1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023
Laspeyres (d)
Paasche (d)
Laspeyres (c)
Paasche (c)
GLaspeyres (d)
GPaasche (d)
GLaspeyres (c)
GPaasche (c)

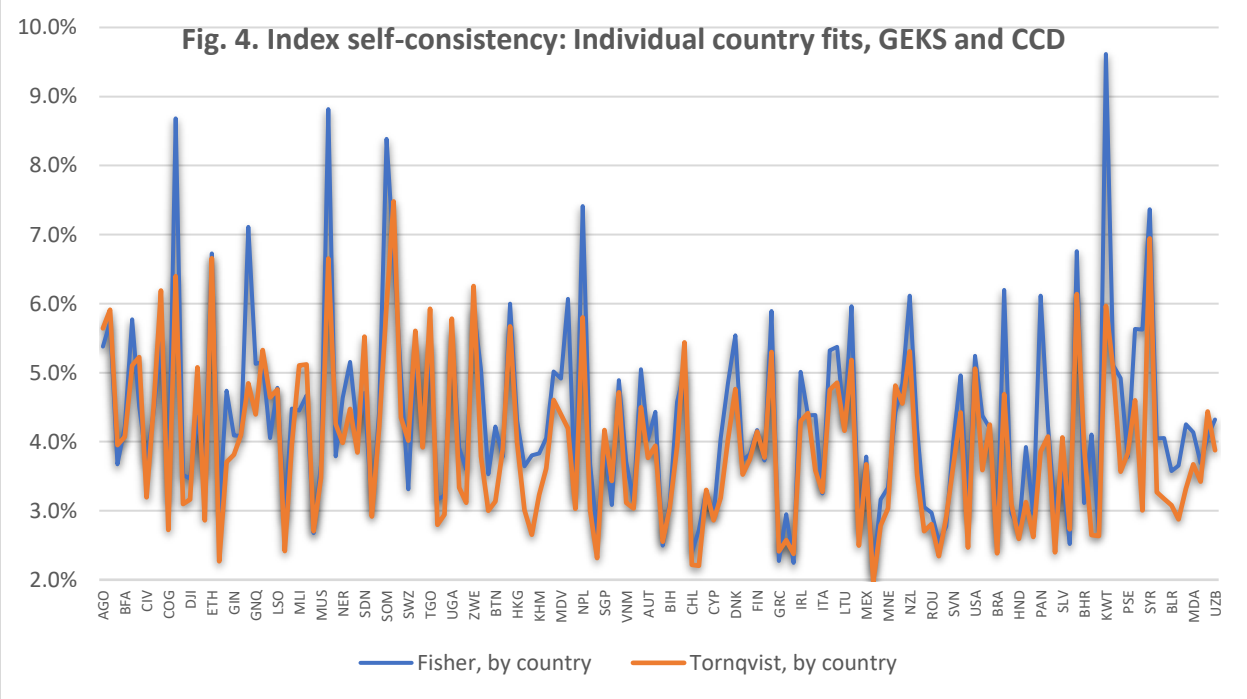
2. Case of spatial price indices

As we observe significant differences between the GEKS and CCD indices in the temporal context, the question arises what would be the extent of the differences in the ICP context?

For the spatial, the situation is a bit different. We are not so sure what the "true" index is in this case. We know that Fisher gives a very different answer than Törnqvist if there are huge fluctuations in prices. But in the International Comparisons context, we cannot yet say right away that one of these indexes is the "truth" and the other is "distorted" because it is different. But what we could say is: that one of the two multilateral systems is likely to give more volatile price indexes (or PPPs) than the other.

Consider a test based on the self-consistency of aggregation methods. If we look at the EKS methods [and both the GEKS and CCD are EKS methods, meaning that they reproduce the closest fit that goes via all their constituent binaries] via the regression perspective, we can assess how well the binaries that these methods are built on fit the EKS regression line. I.e., we can estimate the fitness parameter (*s.d.* of the differences between binary price index Bij for countries i and j and EKS index for those countries – EKSij) for all 154 countries of the 2021 ICP that were part of the regional linking. Estimated in logs, those values are **4.5%** for **GEKS** and **4.1%** for **CCD** (relative to the world average). If we look at individual countries – for **KWT** it is **9.6%** and **6%**, respectively. Thus, the CCD exhibits a significantly better consistency than the GEKS.

The individual country consistency is shown in the chart below:



Thus, the CCD index built on the binary Törnqvist would be preferable to the EKS index built on the binary Fishers. In the sections below we will discuss the mechanism of those inconsistencies.

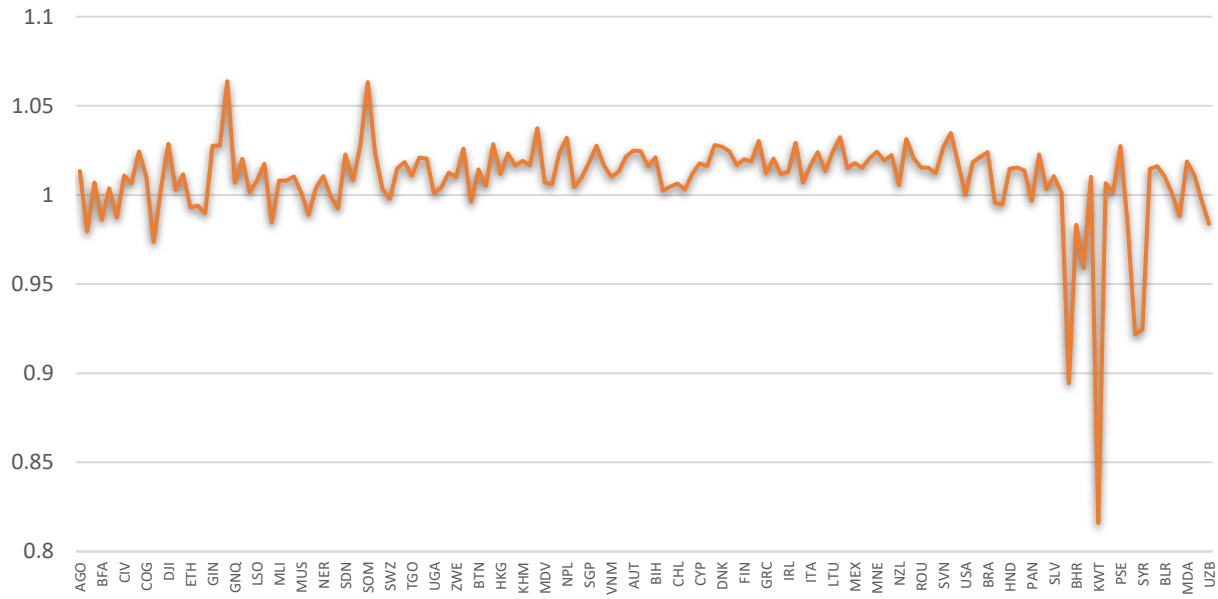
We can also look at the matrix of binary Fisher–Törnqvist differences in the global context. We see that the Fisher-Törnqvist ratio reaches 194% in the Kuwait-Somalia case. In general, we see that the intra-region consistency is better than the consistency for countries in different regions. Among the regions, WAS shows the highest inconsistencies. The heatmap of the Fisher-Törnqvist differences for 154 countries of the world by country, grouped by the ICP region is provided below:

Fig. 5. Fisher-Törnqvist differences (in logs, absolute value), matrix, by region, by country, 154 ICP countries, 2021 PCE w/o Net Expenditure Abroad

legend for colors:
black > 0.40; **red** > 0.20, **orange** > 0.10, grey > 0.05, no color < 0.05

The binary inconsistencies result in a significant effect on the GEKS/CCD ratio. Below is the graph showing this ratio for the 154 countries of the 2021 ICP comparison:

Fig. 6. GEKS/CCD PPP ratios, ICP 2021 global unrestricted results, US = 1, by country



Thus, as we can see as the Fisher is seriously affected by asymmetric price volatility due to its arithmetic-harmonic nature, the GEKS is affected too to a significant extent. So, from this perspective, the Törnqvist-based CCD index is preferable to the Fisher-based GEKS.

3. Model to simulate effects of price shocks on indices

Consider the following model to simulate price shocks, whether due to data errors or legitimate price deviations such as price declines of certain products associated with technical change (information processing, electronics, etc.):

1) Expenditure shares are kept constant between the two periods for *all* items.
2) One product [let's call it product 1] with expenditure share *s* experiences a price shock R [i.e., the price of product 1 changes from 1 to R], and prices for all other products are kept constant and equal to one[2].

In particular, we consider the effects of price jolts [shocks] on the Fisher index relative to the Törnqvist index. We want to compare the behavior of the Fisher and Törnqvist indexes as the price of product 1 increases or decreases substantially so that we can ignore other items. We will show that numerically there will be a tendency for the Fisher index to show bigger increases if the price of product 1 increases substantially and to show lower index levels if the price of product 1 decreases substantially, i.e., with large movements in relative prices, the Fisher index will show a greater level of volatility than the corresponding Törnqvist index [multiplier effect of the price volatility on the Fisher index relative to the Törnqvist].

This leads to the following expressions for the indices corresponding to our model (Laspeyres, Paasche, and Törnqvist price indices):

$$La(R,s) = 1 - s + sR$$

$$Pa(R,s) = 1/(1 - s + s/R)$$

$$T(R,s) = e^{s \log(R)}$$

Those are simple formulae that can be approximated using Taylor expansions.

3.1 Linear presentation

For simplicity purposes we start with the linear formulation. Using the expansions for logarithms and exponentials as well as for the inverse function, and discarding higher than third-order terms[3], we obtain the following Taylor approximations around (R=1):

$$La(R,s) = 1 + s(R-1)$$

$$Pa(R,s) = 1 + s(R-1) + (s-1)s(R-1)^2 + s(s-1)^2(R-1)^3 + O((R-1)^4)$$

[2] The idea of this model was developed with inputs from E. Diewert.

[3] Note that the fact that the first and second-order Taylor series expansions for the Fisher and Törnqvist indices coincide has been known since Diewert (1978).

$$T(R,s) = 1 + s(R-1) + \frac{1}{2}(s-1)s(R-1)^2 + \frac{1}{6}s(s-1)(s-2)(R-1)^3 + O((R-1)^4)$$

Or the linear approximation of the Fisher index ($F(R,s) \approx (La(R,s) + Pa(R,s))/2$) can be written as[4]:

$$F(R,s) = 1 + s(R-1) + \frac{1}{2}(s-1)s(R-1)^2 + \frac{1}{2}s(s-1)^2(R-1)^3 + O((R-1)^4)$$

Finally,

$$F(R,s) - T(R,s) = s\left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)(R-1)^3 + O((R-1)^4)$$

However, the approximation in terms of *linear* price deviations does not account for the symmetrical[5] and highly nonlinear nature of the Fisher/Törnqvist ratio and becomes quite distortive for any R deviating from 1 in a significant manner, making it unfit for the purpose of studying the effects of *large price shocks* in our model (though this approximation will be OK for smaller differences). Thus, we will have to move on to non-linear approximations.

3.2 First logarithmic presentation

Fortunately, there is a more efficient way to approximate the indices. From Dikhanov (2024), p.12, [Dikhanov-A-New-Elementary-Index.pdf (unece.org)] we know that in the general case, the log difference between the indices is described by expression (10) of the paper:

$$log\ F(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_1, \beta)\ -\ log\ T(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_1, \beta) \approx (\frac{1}{6} - \frac{\beta}{4})\langle \mathbf{d}^3 \rangle \qquad (1)$$

Where $x_i = \frac{p_i^1}{p_i^0} / \prod \left(\frac{p_i^1}{p_i^0}\right)^{\mathbf{s}_{0i}}$, and $\prod \left(\frac{p_i^1}{p_i^0}\right)^{\mathbf{s}_{0i}}$ is the Geometric Laspeyres index (GL). $\langle \mathbf{d}^3 \rangle$, or the third moment, using the notation $d_i = log\ x_i$ and $\langle \mathbf{d}^n \rangle = \sum \mathrm{s}_{0i} {d_i}^n$, is the measure of *skewness* in price changes. It is assumed here that the item shares **s** change with prices with elasticity β: $\mathrm{s}_{1i} = \mathrm{s}_{0i} e^{\beta d_i} / \sum \mathrm{s}_{0i} e^{\beta d_i}$.

The expenditure share elasticity β equals zero when all expenditure shares stay the same between the two periods not reacting to price changes, this also means that quantities are inversely proportional to prices; and β equals one when item quantities do not respond to price changes and stay proportional between the two periods.

Setting the share elasticity β to 0 keeps all shares **s** constant between the two periods, and in this case, we can write:

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[4] Note that the Fisher index can be approximated in this manner only for small values of R.

[5] Meaning that the effect of price shock R will be equivalent to the effect of 1/R with an opposite sign.

$$log\ F(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_0, 0)\ -\ log\ T(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_0, 0) \approx \frac{1}{6}\langle \mathbf{d}^3 \rangle$$

Noting that log(GL) = sr and r = log(R), with all other prices kept constant, we get

$$\langle \mathbf{d}^3 \rangle = (1-s)(0-sr)^3 + s(r-sr)^3 = s(1-3s+2s^2)r^3$$

From which we immediately obtain:

$$log\ F(r,s) -\ log\ T(r,s) \approx s\left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)r^3 \qquad (2)$$

This expression can be re-written as the multiplier effect on the Fisher relative to the Törnqvist as r changes:

$$log\ F(r,s) \approx log\ T(r,s)\left[1 + \left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)r^2\right] \qquad (3)$$

The multiplier effect $1 + \left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)r^2$ is always greater than one if $s < 0.5$. Thus, an error in price r would always create a bigger jolt to the Fisher than to the Törnqvist index. This would also work for other price hikes or drops, such as fast-declining tech prices over time. The multiplier effect reaches the maximum at s being around 20% (this is $argmax\left(s(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3})\right)$, 0≤s≤1, also see Figure A).

3.3 Second logarithmic presentation

There is another way to arrive at formulae (2) and (3). Writing the indices directly in log form, we get:

$$log\ T(r,s) = log\ e^{s\ \log(R)} = s\ \log(R) = sr$$

$$log\ La(r,s) = log\ (1 - s + s\ R) = log\ (1 - s + se^r\ )$$

$$log\ Pa(r,s) = -log\ (1 - s + s\ /R) = -log\ (1 - s + se^{-r}\ )$$

Using the expansions for logarithms and exponentials, and discarding all terms higher than the fourth order, we obtain the following:

$$log\ La(r,s) = sr + \frac{s}{2}(1-s)r^2 + \frac{s}{6}(2s^2 - 3s + 1)r^3 + \frac{s}{24}(1 - 7s + 12s^2 - 6s^3)r^4 + O(r^5)$$

$$log\ Pa(r,s) = sr - \frac{s}{2}(1-s)r^2 + \frac{s}{6}(2s^2 - 3s + 1)r^3 - \frac{s}{24}(1 - 7s + 12s^2 - 6s^3)r^4 + O(r^5)$$

Thus, the Fisher index is approximated as:

$$log\ F(r,s) = sr + s\left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)r^3 + O(r^5)$$

Or,

$$log\ F(r,s) = sr + \frac{1}{6}sr^3(1-2s)(1-s) + O(r^5)$$

Note that the fourth-order terms get canceled out.

Finally, the distance between the Fisher and Törnqvist indices can be expressed as our already familiar expression (2), except that now it is correct up to the fourth order:

$$log\ F(r,s) - log\ T(r,s) = s\left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)r^3 + O(r^5) \qquad (3a)$$

Here is the simple intuition behind the algebra[6]: The Törnqvist price index starts off by taking logarithms of the price ratios in scope and then averages these log price ratios. Taking logarithms compresses the amount of variation in these price ratios. The Paasche, Laspeyres, and Fisher indexes average the raw price ratios so a big outlier price ratio will have a bigger effect on the resulting index than averaging the log price ratios and then exponentiating the average.

4. Explaining the NIPA PCE index discrepancies with the model of the tech items' price shocks

First, it will be useful to investigate higher-level approximations of the model as in our case the price "shocks" are extremely large over the 1990-2023 period. From Dikhanov (2024b, *Range of possible results for certain classes of superlative price indices, ANNEX II,* forthcoming) we know that when β = 0, the distance between the Fisher and Törnqvist indices can be approximated as:

$$log\ F(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_1, 0) - log\ T(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_1, 0) \approx \frac{1}{6}\langle \mathbf{d}^3 \rangle + \frac{1}{120}(\langle \mathbf{d}^5 \rangle + 10\langle \mathbf{d}^2 \rangle \langle \mathbf{d}^3 \rangle) \quad (4)$$

Thus, using the same derivations as in the case above for the third-order approximation, the distance between the Törnqvist and Fisher indices using the fifth-order approximation becomes:

$$log\ F(r,s) - log\ T(r,s) = s\left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)r^3 + s\left(\frac{1}{120} - \frac{s}{8} + 5\frac{s^2}{12} - \frac{s^3}{2} + \frac{s^4}{5}\right)r^5 + O(r^7) \qquad (5)$$

Or, if **s** is small, the above expression simplifies to:

$$log\ F(r,s) - log\ T(r,s) = \frac{s}{6}r^3 + \frac{s}{120}r^5 + O(r^7) \qquad (6)$$

The same results can be obtained by applying Taylor expansions directly to $log\ La(r,s) = log\ (1 - s + se^r)$ and $log\ Pa(r,s) = -log\ (1 - s + se^{-r})$ as in Section 3.3 of Chapter 3.

[6] This paragraph is due to E. Diewert (from private correspondence).

It is worthwhile to note that the fifth order term becomes commensurate to the third term at the value of price shock of approximately R ≈ Exp(Sqr(120/6)) = 88. In our case, the relative decline of tech prices versus non-tech over the 1990 - 2023 period is 42.97 times (!), so it is quite high. Thus, it becomes beneficial to use the fifth order approximation. Average weight of the tech products is 2.01%, so it's not negligeable. This justifies using expression (5).

Thus, we estimate the effect of the third term to be 0.168 (in logs), the effect of the fifth term – 0.090 (in logs) or taken together those two terms amounts to 0.258 which practically explains away the entire extent of our Fisher-Törnqvist discrepancy (0.274 = log (2.16/1.53)), see Table A for the direct Fisher and Törnqvist indices. If we were to explain the Fisher-Törnqvist discrepancy over a shorter period, say of 10 or 15 years, the third order approximation (expression (2)) would be entirely sufficient.

This confirms that our model is adequate for describing real effects of price shocks such as technical change. As the GEKS is built upon binary Fisher indices, and the CCD is constructed of binary Törnqvist indices, our model can be used to explain the discrepancies between the GEKS and CCD indices.

Let us apply our model to explain the multiplier effect on the GEKS system. The 2023/1990 GEKS index can be written using its constituent binary Fisher indices as:

$$log\ GEKS_{1990}^{2023} = \frac{1}{34}\sum_{1990}^{2023} log\ F_{1990}^{t} + log\ F_{t}^{2023} \qquad (7)$$

This expression can be rewritten in the integral form. Thus, assuming that the average rate of relative price declines for the tech products has been constant over the period, and using expression (6) to approximate the Fisher – Törnqvist discrepancies, we obtain the following for $log\ \Delta GEKS_0^T = log\ GEKS_0^T - log\ CCD_0^T$:

$$log\ \Delta GEKS_0^T \approx s/T \int_0^T \left(\frac{1}{6}\left(r\frac{t}{T}\right)^3 + \frac{1}{120}\left(r\frac{t}{T}\right)^5 + \frac{1}{6}\left(r\frac{T-t}{T}\right)^3 + \frac{1}{120}\left(r\frac{T-t}{T}\right)^5\right) dt \qquad (8)$$

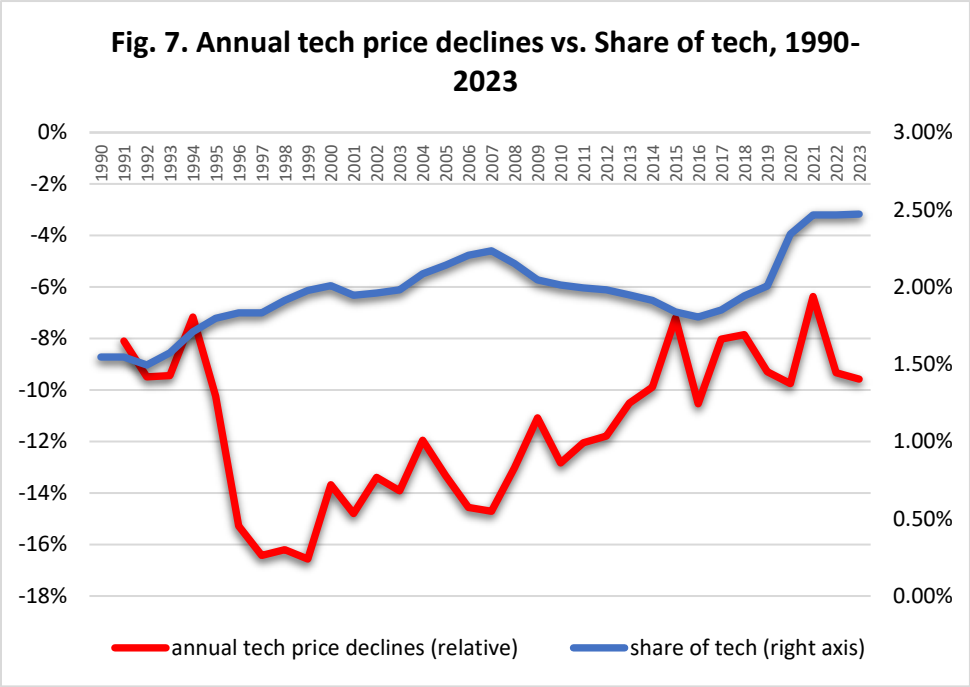
Here we scaled the time period without loss of generality to [0; T]. Then the tech price shock becomes $r\frac{t}{T}$ for time t, assuming a constant rate of relative price decline for tech items over the period. Taking the definite integral we arrive at (note that T gets canceled out):

$$log\ \Delta GEKS_0^T \approx {}^{s}/_{360}\ r^3(r^2 + 30) \qquad (9)$$

Now we can find out the ratio of expression (9) to expression (6) under the assumption that s is small:

$$log\ \Delta GEKS_0^T\ /(log\ F_0^T - log\ T_0^T) \approx \frac{1}{3} + \frac{10}{3(r^2 + 20)} \qquad (10)$$

Thus, $log\ \Delta GEKS_0^T$ at its minimum does not drop below 1/3 of $(log\ F_0^T - log\ T_0^T)$. [Note that if we use the third order approximations (expression (2)), expression (8) becomes equal to ½.] And in our case, where r = log (42.97) = 3.76, the shock of tech prices on GEKS should be around 43.1% of the shock on the Fisher index for the end of the period. In actuality, from Table A we find the ratio to be equal to about 50% (compare the logs of ratios of the direct and multilateral indices). The difference between the predicted and actual values can be attributed to the uneven pace of the tech relative price declines slowing down towards the second part of the period (see the chart below):



In general, our model describing the effects of a tech price shock adequately explains its multiplier effect on the Fisher index as well as the distortion of the GEKS index in this situation. Thus, it is important to keep in mind that the GEKS index, being built from the binary Fisher indices, is subject to the distortive effects of a price shock to the tune of anywhere from 1/3 to ½ of the maximum distortion for its constituent binary Fishers.

5. Explaining the ICP discrepancies with the model of price shocks (errors)

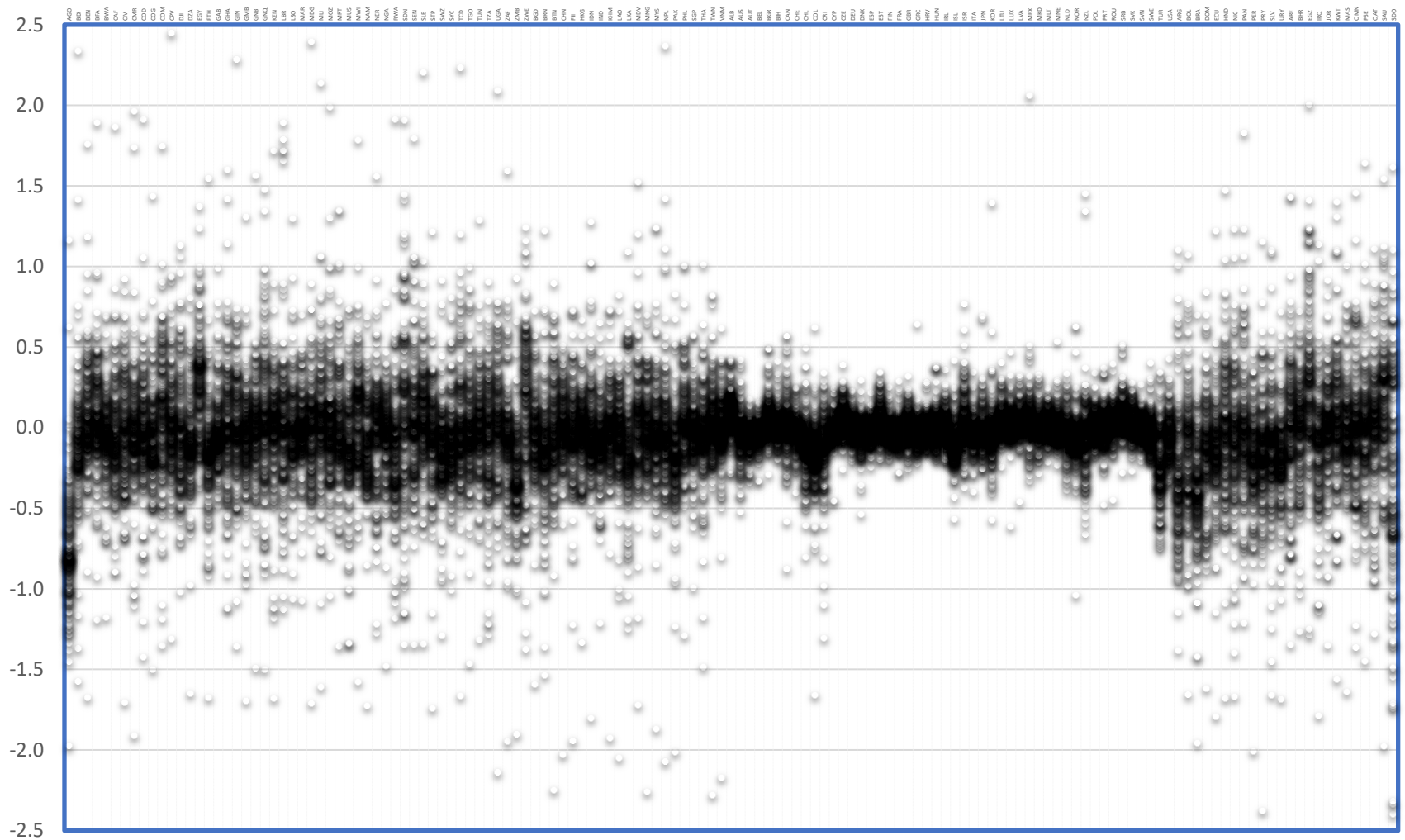
Now we will apply the same model to the ICP (spatial comparison). First, we notice that we cannot know the "true" index, and all we can do is to look at the GEKS/CCD and Fisher/Törnqvist differences. But what we do know is that the GEKS and Fisher do amplify distortions due to shocks. The task at hand is immensely complicated by an extremely high uncertainty of the prices relatives – the PPPs (as well as an equally distortive uncertainty in expenditure shares, but this is a story for another paper). Indirectly, the PPP volatility can be inferred from the temporal volatility of the BH PPPs which we can compare against our expectations.

The US NIPA PCE data we work with contain 72 components which is not too far off from the ICP BH count for the same aggregate – 109. The 2017-2021 PCE component volatility in the US was less than 10% for those 72 components. In the international setting we expect many components (BHs) to behave in a correlated manner across countries, esp. those traded internationally such as cars, electronics, fuel, etc. Thus, if we assume that within, say, the OECD the markets are functioning reasonably well, we can expect temporal volatility of US prices relative to the OCED average (or, PPP(US, 2017)/PPP(OECD, 2017) vs. PPP(US, 2021)/PPP(OECD, 2021)) to be even lower than 10% (probably significantly lower). In reality, it is 13.9% (in logs) which is still not too bad, and this probably means that the average BH PPP error (uncertainty) for the US in the OECD comparison is about 10% or so.

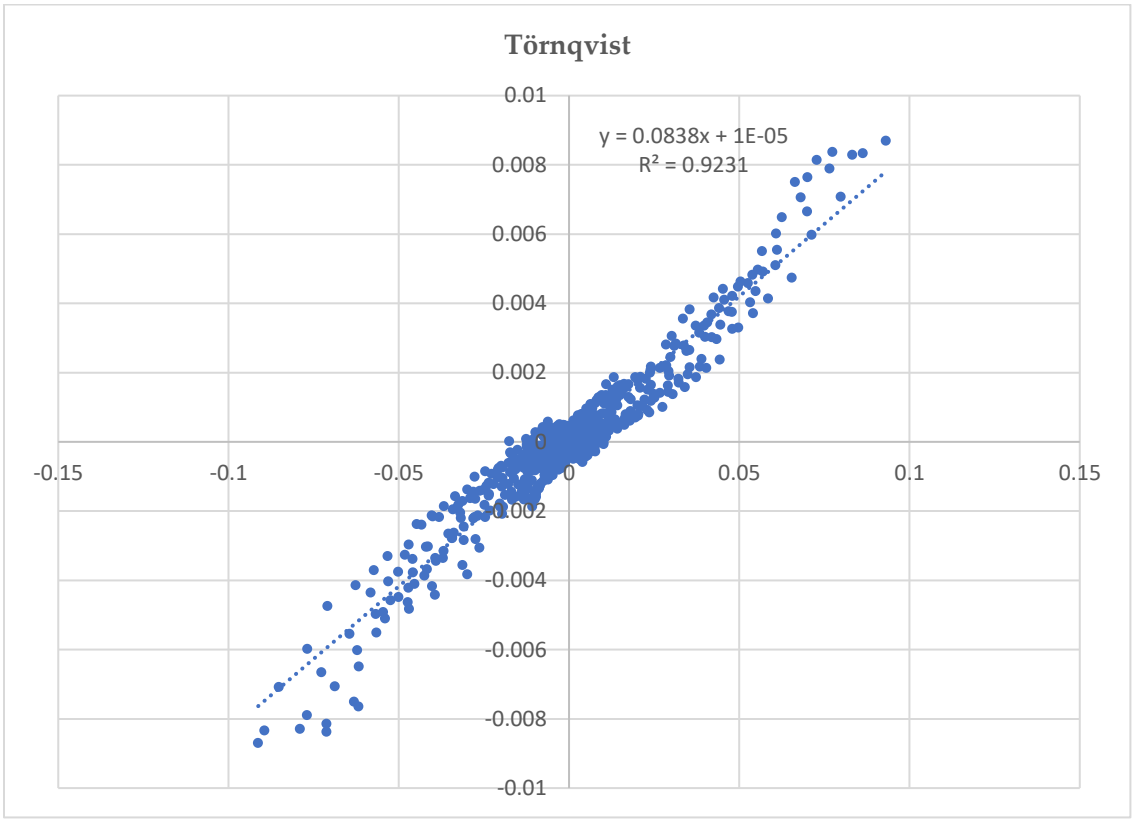
Unfortunately, it is worse for many other countries: the chart below shows the 2021/2017 PPP changes by country (shown in PLI form in order to exclude local inflation). The chart below is presented in logs, thus the scale spans from -2.5 to +2.5, or 148 times (148 = Exp(5)). This is quite significant as a price jolt of that magnitude can seriously distort the aggregate PPPs.

It would be fair to say that the data points beyond the 20% band around the midpoint on this graph probably indicate price measurement errors. Which are significant.

Fig. 8. PLI changes by BH, by country, 2021 to 2017, OECD=0.0, ICP 2021, in logs

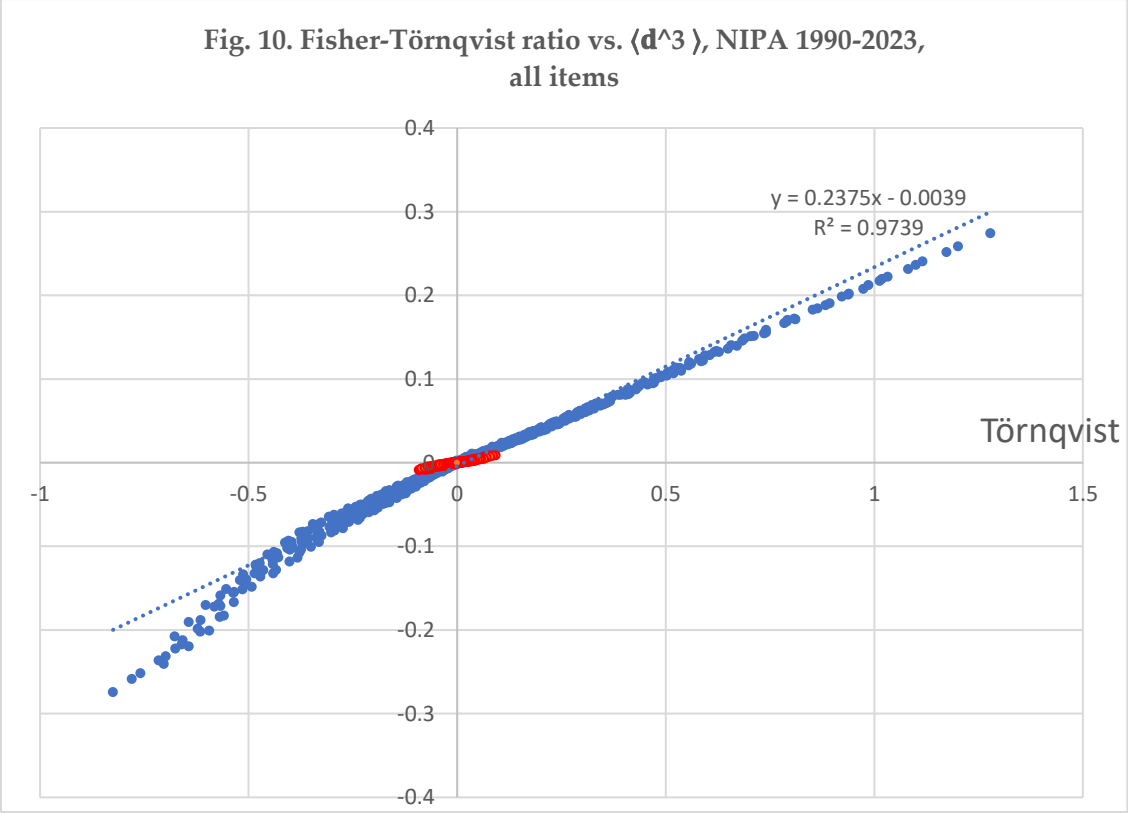


Let's go back to the NIPA data to consider the relationship between the binary Fisher – Törnqvist differences and price skewness $\langle \mathbf{d}^3 \rangle$. First, for all items without tech, for all pairs in the 1990-23 period, we get:

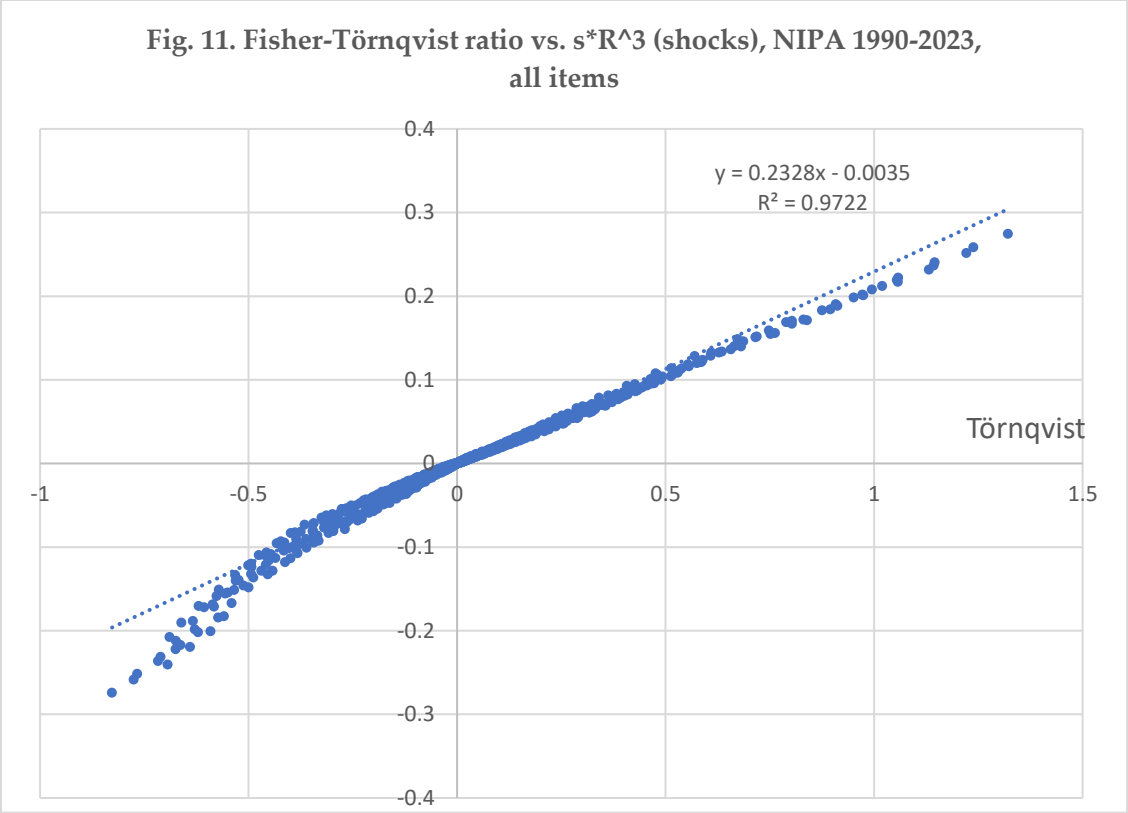


Note that the magnitude of the binary differences is quite small, it does not exceed 0.8 percent in both directions. The correlation between $log\ F\ -\ log\ T$ and $\langle \mathbf{d}^3 \rangle$ is quite high (r^2=0.92). Using expression (1) we find elasticity β being equal to 0.3315.

Now consider the same graph for all items, including tech:



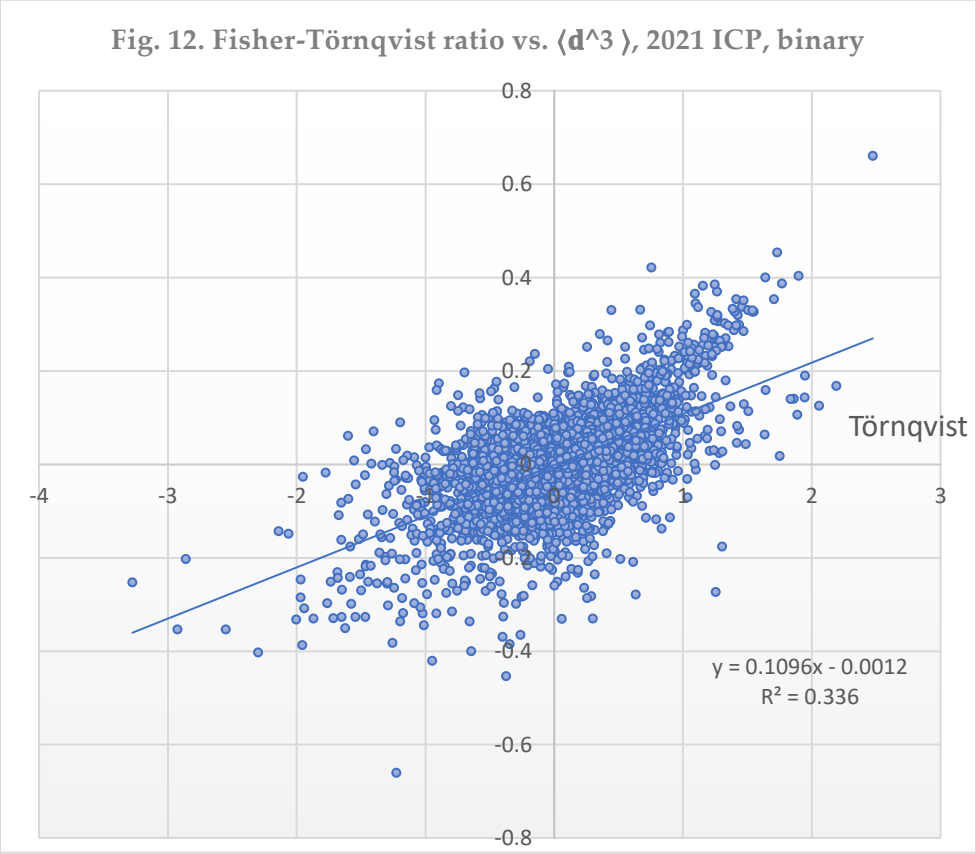
For comparison, the previous chart is shown in red. We can appreciate the extent of the effect of the tech items [also note that due to the high extent of the tech price declines, the fifth order expansion terms now become significant, and, as a result, the slope coefficient is well above 1/6, - compare to expression (1)]. We are now in the realm of shocks. Instead of being described by expression (1) graph (10) is now described by expression (6) for price shocks. The graph below shows precisely that:



The graphs (10) and (11) above are practically indistinguishable and thus the F-T differences are almost perfectly described by the model with shocks from tech items (expression (6)), *ignoring all other items*.

The tech items have such an extraordinary influence on the index numbers that we can rightfully ponder if we measure the utility of the tech correctly and/or if our index numbers used need a correction.

Let's go now over to the spatial comparisons. The same graph of the Fisher – Törnqvist ratios vs. $\langle \mathbf{d}^3 \rangle$ for the 154 ICP countries [all country pairs] is plotted below:



The first thing we learn from the graph is that the relationship becomes much fuzzier (r^2=0.34). The fuzziness is attributed not only to the uncertainty of prices (price shocks of different directions) but also to the fact that there is also no such single dominant item as tech in the NIPA temporal series. Besides, tech price declines apply to all countries simultaneously and thus are not relevant for the spatial case. Thus, we should compare this chart to the NIPA temporal chart w/o tech items – Fig.9. We see that $\langle \mathbf{d}^3 \rangle$ in the ICP case is extremely high – up to 3 and more, compared to 0.09 in the temporal NIPA case (Figure 9). Thus, we can conclude that the ICP case should be almost completely described with the shock equations.

Even though the slope coefficient looks plausible (0.11), it is probably dominated by price shocks and, thus, it can't be used to estimate the "true" share elasticity in the ICP context.

We know that the effects of price shocks are symmetric (shocks of opposite directions of the same magnitude and weight get canceled out), hence we can conclude that these price shocks in ICP are non-symmetric, which points to the possibility of serious biases due to data errors. Thus, the

resulting effect of multiple price shocks can be described by the sum of models described by expression (2) (when **s** and **r** are small).

$$log\ F(r,s) - log\ T(r,s) \approx \sum \frac{s_k}{6} {r_k}^3 \qquad (11)$$

It is clear, that the uncertainty in expenditure weights has substantial influence as well.

Kuwait-Germany case

The applicability of expression (10) for explaining the results of the ICP can be tested. Consider Kuwait and Germany, both high-income countries. The Törnqvist – Fisher ratio for the pair is 149.5% (0.402 in log terms), with the Fisher and Törnqvist being 54.24% and 81.07%, respectively. Two items stand out: Water supply and Electricity.

	PLI (US=100)		**Nominal share (PCE=100%)**		**Nominal share/PLI**	
	DEU	KWT	DEU	KWT	DEU	KWT
Water supply	197%	1%	0.5%	0.8%	0.25%	61.7%
Electricity	260%	4%	2.5%	1.4%	0.97%	33.3%

An approximation for the real impact of an item would be the ratio of nominal expenditure to price (due to non-additivity of both indices, the sum of ratios of nominal expenditures to PLIs will not add up to 100%). We can see from the last two columns of the table above that those two items pretty much determine a good part of the whole PCE in real terms, with Kuwait having dozens of times higher per capita real expenditures on those two items than Germany. This also illustrates the extreme importance of national accounts along with prices.

Without those two items, the Fisher and Törnqvist price indices are quite close: 90.30% and 90.46%, respectively. <d^3>/6 in this case is -6.0%, and, using formula (1) we obtain elasticity $\beta = 0.69$ which is plausible.

The value of <d^3>/6 for all items is -35.4% which is pretty much in accordance with the estimated Fisher – Törnqvist ratio (-40.2%). Thus, those two items add 29.4% to the overall value of <d^3>/6.

Treating those two items as being subject to shocks, we can use formula (11) to determine their impact on <d^3>/6, and hence on the Fisher – Törnqvist ratio. Correspondingly, the impact of the two items in log terms is found to be -13.6% and -22.9% (treating all other items as constant), respectively, which explains the bulk of the whole Fisher – Törnqvist ratio for the pair (-40.2%). The remainder is due to the fifth-order terms (due to significant values of the price shocks) and since the shares are not that close to zero (i.e., formula (5) should be used), as well as due to treating all other items as constant.

Another takeaway from this result is the fact that even under these conditions the Törnqvist index is significantly more stable than the Fisher index as it is less impacted by outliers/ data errors.

6. Model to simulate uncertainties in expenditure shares

The same principles can be applied to model shocks on expenditure shares. Obviously, we cannot assume that all prices stay the same but only expenditures experience a shock, because in that case all indices will stay at one. Instead, we assume that product 1, simultaneously with a price shock e^r, also experiences an expenditure shock se^{br}, or b = ln(s_1/s_0)/r, where s_1 is expenditure share in period 1, and s_0 – in period 0.

Thus, writing the indices directly in log form, we get:

$$log\ T(r,s,b) = s(1+\frac{e^{br}}{2})r$$

$$log\ La(r,s,b) = log\ (1-s+se^r)$$

$$log\ Pa(r,s,b) = -log\left(1-se^{br}+se^{-r+br}\right)$$

Using the expansions for logarithms and exponentials, and discarding all terms higher than the third order, we obtain the following expression:

$$log\ T(r,s,b) \approx sr+\frac{s}{2}br^2+\frac{s}{4}b^2r^3$$

$$log\ La(r,s,b) \approx sr+\frac{s}{2}(1-s)r^2+\frac{s}{6}(2s^2-3s+1)r^3$$

$$log\ Pa(r,s,b) \approx sr-\frac{s}{2}(1-s-2b)r^2+1/6\ sr^3\ (3b^2+\ \mathrm{b}\ (6\ \mathrm{s}\ -\ 3)+2\ \mathrm{s}^2-\ 3\ \mathrm{s}+\ 1)$$

Thus, the Fisher index is approximated as:

$$log\ F(r,s,b) \approx sr+\frac{s}{2}br^2+\frac{s}{4}b^2r^3+\frac{1}{6}sr^3(1-3s+2s^2+b(3s-3/2))$$

Finally, the distance between the Fisher and Törnqvist indices can be expressed as follows:

$$log\ F(r,s,b)-\ log\ T(r,s,b) \approx s\left(\frac{1}{6}-\frac{s}{2}+\frac{s^2}{3}\right)r^3+bsr^3\left(\frac{s}{2}-\frac{1}{4}\right) \qquad (12)$$

Or,

$$log\ F(r,s,b)-\ log\ T(r,s,b) \approx \frac{1}{6}sr^3(1-2s)\left(1-s-\frac{3}{2}b\right) \qquad (13)$$

Comparing expression (12) to expression (3) we see an additional term due to the shocks on expenditure shares: $bsr^3(s/2-1/4)$. As expected, we observe that the Törnqvist and Fisher indices are approximating each other to the second order.

If b=0 then expression (12) becomes expression (3) as expected. If b = 1 (i.e., the quantity of product 1 stays the same when s is small, $log\ F(r,s,b)-\ log\ T(r,s,b) \approx \frac{1}{6}sr^3(1-2s)(-0.5-s)$, and the difference between the two models (the one with price shocks only and the one with both price

and expenditure shocks) becomes $sr^3(s/2 - 1/4)$. I.e., if s is small, a positive value of b creates a dampening effect, reducing the multiplier effect. It will be the opposite under a negative value of b.

Correspondingly, expression (11) will be transformed into:

$$log\ F(r,s) - log\ T(r,s) \approx \sum \frac{s_k}{6} {r_k}^3 (1 - \frac{3}{2} b_k) \qquad (14)$$

Where $(1 - \frac{3}{2} b_k)$ represent the correction due to expenditure shocks. It follows that if $b_k = \frac{2}{3}$ (i.e., if log changes of nominal shares are 2/3 of log changes of prices), the shocks on prices will be compensated by the shocks on expenditure shares (when s are small), eliminating the Fisher-Törnqvist differences. It also follows that the most detrimental cases will be large magnitudes of price shocks with expenditure shocks moving in the opposite directions. Finally, this means that the equality of the Törnqvist and Fisher indices does not guarantee the absence of distortions/ other problems: the shocks might just compensate each other, but the underlying data can still be wrong. Compare also expression (14) to expression (15) from Dikhanov (2024b): $log\ F(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_1, \beta) - log\ T(\mathbf{x}, \mathbf{s}_0, \mathbf{s}_1, \beta) \approx \left(\frac{1}{6} - \frac{\beta}{4}\right) \langle \mathbf{d}^3 \rangle = \frac{1}{6} \langle \mathbf{d}^3 \rangle (1 - \frac{3}{2}\beta)$, from where we can see that in the general case the Fisher index approaches Törnqvist when share elasticity β approaches 2/3.

7. Multi-stage aggregation model

In previous section we considered only the one-stage [direct] aggregation as it is habitually implemented in both the ICP and CPI [i.e., the weighted aggregation with item elementary indices and item weights]. Formulae (1) and (12) show the effects of price (and expenditure) shocks on the results. However, consider the two-stage process where we do a partial aggregation at the first stage and then combine its results with the rest of items at the second stage[7].

The model will be set up as follows:

1) Expenditure shares are kept constant between the two periods for *all* items.
2) One product [let's call it product 1] with expenditure share *s* experiences a price shock R [i.e., the price of product 1 changes from 1 to R], and prices for all other products are kept constant and equal to one.
3) Product 2 with expenditure share z is to be combined with product 1 into a first stage aggregate to be used at the second stage with the rest of products.

Direct (one-stage) estimate is written as:

[7] Note that the Törnqvist index is decomposable when expenditure shares kept constant and, thus, allows for hierarchical computation. Hence, the one-stage Törnqvist will be the same as the two-stage one.

$$\Delta f_{1S} = log\ F(r,s) - log\ T(r,s) \approx s\left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right)r^3$$

Now, consider the two-stage aggregation where, at the first stage we combine products 1 and 2:

$$r_{BH} = log\ F_{BH}(r,s) \approx r\sigma + \sigma\left(\frac{1}{6} - \frac{\sigma}{2} + \frac{\sigma^2}{3}\right)r^3$$

Where $\sigma = \frac{s}{s+z}$

Then the two-stage aggregation will be described as:

$$\Delta f_{2S} \approx (s+z)\left(\frac{1}{6} - \frac{s+z}{2} + \frac{(s+z)^2}{3}\right)r_{BH}^3$$

$$\Delta f_{2S} \approx (s+z)\left(\frac{1}{6} - \frac{s+z}{2} + \frac{(s+z)^2}{3}\right)r^3\sigma^3\left(1 + r^2(\frac{1}{6} - \frac{\sigma}{2} + \frac{\sigma^2}{3})\right)^3$$

Collecting only the terms up to the third order, we get:

$$\Delta f_{2S} \approx (s+z)\left(\frac{1}{6} - \frac{s+z}{2} + \frac{(s+z)^2}{3}\right)r^3\left(\frac{s}{s+z}\right)^3 = sr^3\left(\frac{1}{6} - \frac{s+z}{2} + \frac{(s+z)^2}{3}\right)\left(\frac{s}{s+z}\right)^2 \qquad (15)$$

Define compression as the reduction of the Fisher-Törnqvist ratio due to an extra stage in aggregation. Then the compression coefficient $C = \frac{\Delta f_{2S}}{\Delta f_{1S}}$ equals to:

$$C = \left(\frac{1}{6} - \frac{s+z}{2} + \frac{(s+z)^2}{3}\right)\left(\frac{s}{s+z}\right)^2 \Big/ \left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right) \qquad (16)$$

Assuming that s = z[8]:

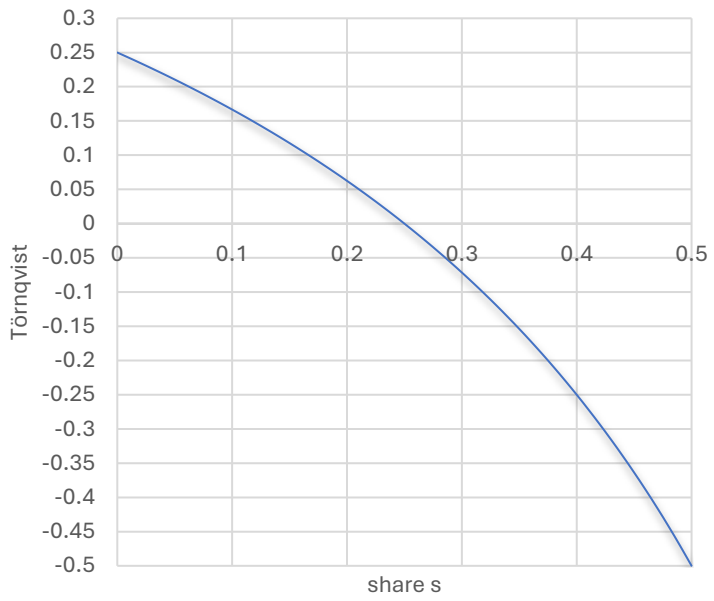
$$\Delta f_{2S} \approx \frac{1}{4}sr^3\left(\frac{1}{6} - s + \frac{4s^2}{3}\right) \qquad (17)$$

$$C \approx \frac{1}{4}\left(\frac{1}{6} - s + \frac{4s^2}{3}\right) \Big/ \left(\frac{1}{6} - \frac{s}{2} + \frac{s^2}{3}\right) \qquad (18)$$

Figure (13) below plots possible results from expression (18) when 0<s<0.5.

[8] From expression (16) we see that some z ≠ s may deliver a somewhat better compression ratio, but s=z delivers a close to the maximum compression ratio.

Fig. 13. Compression of Fisher-Törnqvist distance due to second stage of aggregation



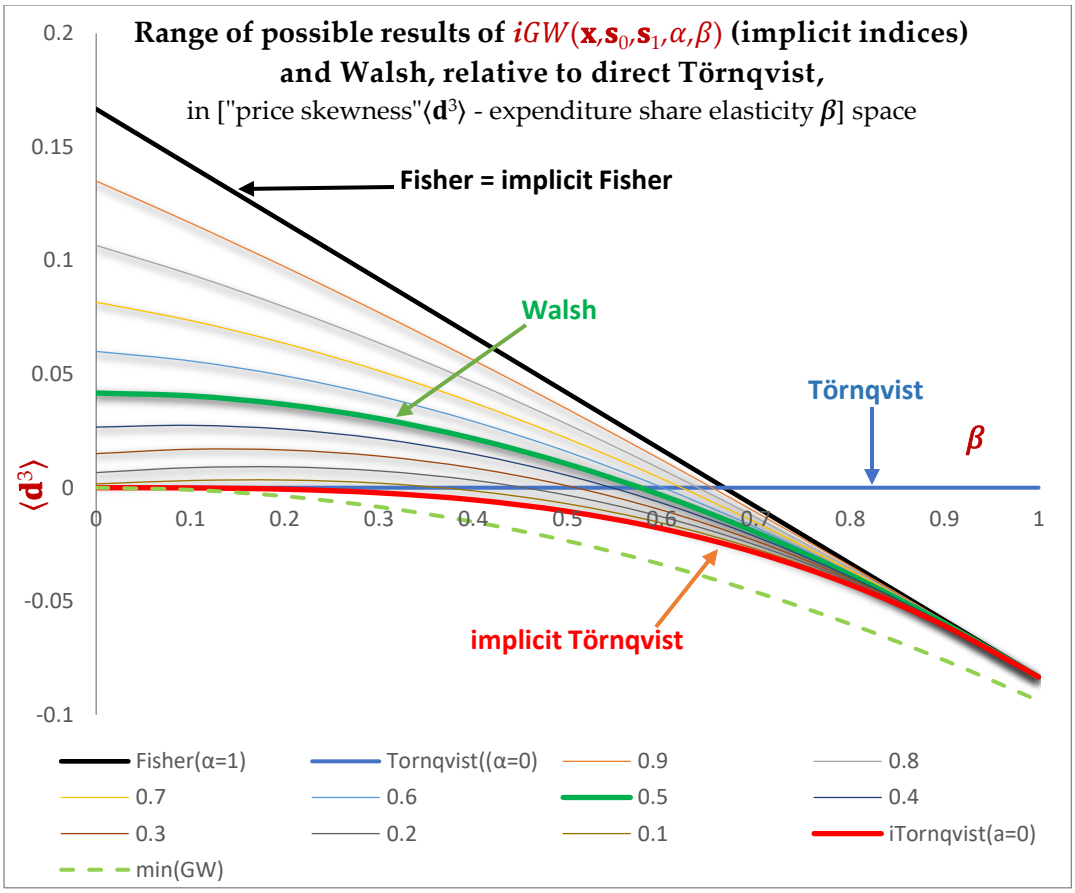
Thus, when s=0.25 then the Fisher two-stage aggregation will be the same as Törnqvist's [i.e., C=0]. When s=0.1, the compression ratio C (the reduction of the Fisher-Törnqvist ratio) is about 0.167, or the Fisher-Törnqvist ratio decreases six times. When s approaches 0, the ratio approaches 0.25 (a decrease of four times). On the other hand, when s>0.25, the compression ratio changes its sign.

Lastly, discarding the terms above the third order, we get as a simpler but still relatively good approximation of expression (18) (with some overshooting for s > 0):

$$C \approx \frac{1 - 3s - 3s^2 - 3s^3}{4}$$

8. Conclusions

This paper finds that the Törnqvist-based CCD index is vastly preferrable to the Fisher-based GEKS index under the condition of strong price and expenditure shocks [uncertainties]. So how does it correspond to the results of Dikhanov (2024b) that finds the Walsh index "ideal"? In short, that paper finds that the range of the best superlative indices lies between the Fisher and *implicit* Törnqvist, with the Walsh being lodged comfortably in the middle [see Figure 6 of Dikhanov (2024b) reproduced below, the Walsh index is in green].



Source: Dikhanov (2024b)

First, the Walsh was found to be the best for the whole interval of $\beta \subset [0,1]$, and until β=2/3 the regular Törnqvist was within that range of the best superlative indices, thus for most reasonable cases of elasticity β the Törnqvist is still quite good. Another consideration is the following: that paper assumes measurement errors to be small. However, when shocks [measurement errors] start contributing significantly (and, especially, overwhelmingly) to $\langle \mathbf{d}^3 \rangle$ (i.e., to the "skewness" of price variations), the situation changes dramatically, and the index stability considerations become extremely important. Perhaps, when the ICP price uncertainties diminish in the future ICP rounds we could consider the Walsh-based EKS index [we can call it W-EKS] for PPP calculations. Until then, the CCD index will be the best index we can get for the ICP under the circumstances.

References

Caves, D. W., L. R. Christensen, and W. E. Diewert (1982), "Multilateral Comparisons of Output, Input, and Productivity Using Superlative Index Numbers," *Economic Journal*, 92, 73–86.

Diewert, W.E. (1978), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900.

Dikhanov, Y. (2024), "A New Elementary Index Number", 18th Ottawa Group Meeting, Ottawa, May 13, [Dikhanov-A-New-Elementary-Index.pdf (unece.org)]

Dikhanov, Y. (2024b), "Range of possible results for certain classes of superlative price indices", forthcoming.

ADDENDUM

The following figure is a graphic presentation of the multiplier effect of a price shock on the Fisher-Törnqvist log differences:

Figure A. Effect of price shock R on the Törnqvist-Fisher log differences

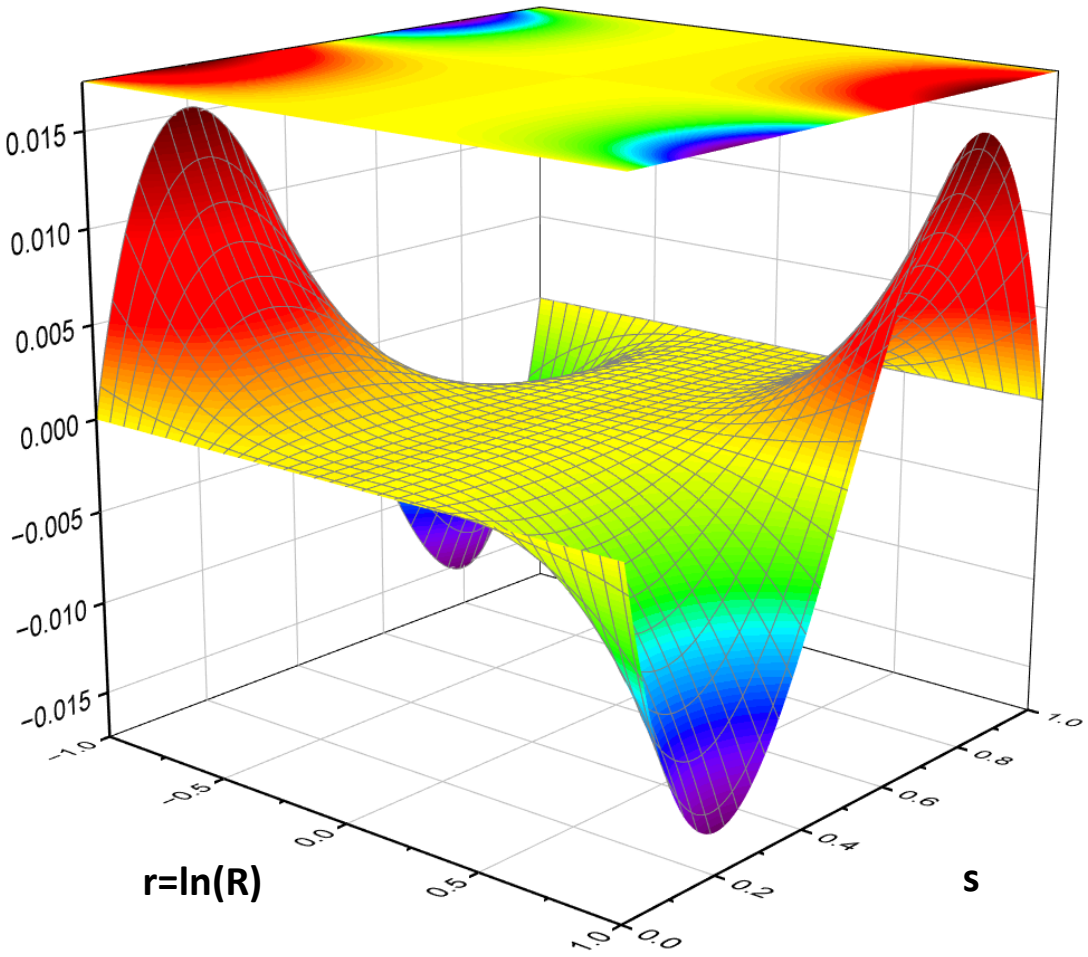


Table 1. Price Indexes for Personal Consumption Expenditures, all items

[Index numbers,
1990=1.00]

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Direct																	
Fisher (d)	1.0000	1.0359	1.0657	1.0935	1.1158	1.1391	1.1631	1.1822	1.1889	1.2032	1.2316	1.2555	1.2680	1.2917	1.3176	1.3483	1.3754
Törnqvist (d)	1.0000	1.0359	1.0657	1.0936	1.1160	1.1395	1.1642	1.1844	1.1928	1.2099	1.2416	1.2691	1.2858	1.3152	1.3487	1.3888	1.4284
Chained																	
Fisher (c)	1.0000	1.0359	1.0659	1.0944	1.1174	1.1412	1.1658	1.1860	1.1945	1.2116	1.2424	1.2662	1.2825	1.3106	1.3441	1.3830	1.4224
Törnqvist (c)	1.0000	1.0359	1.0659	1.0944	1.1174	1.1413	1.1659	1.1860	1.1946	1.2117	1.2425	1.2664	1.2827	1.3107	1.3442	1.3832	1.4226
Multilateral																	
Fisher (EKS)	1.0000	1.0281	1.0512	1.0740	1.0935	1.1115	1.1286	1.1419	1.1456	1.1570	1.1822	1.2012	1.2135	1.2366	1.2652	1.2986	1.3318
Törnqvist (CCD)	1.0000	1.0338	1.0633	1.0930	1.1183	1.1428	1.1682	1.1888	1.1993	1.2178	1.2490	1.2734	1.2903	1.3189	1.3531	1.3925	1.4322

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Direct																	
Fisher (d)	1.3993	1.4392	1.4354	1.4506	1.4745	1.4858	1.4910	1.4980	1.4911	1.4866	1.4913	1.4940	1.4841	1.4232	1.4429	1.5102	1.5324
Törnqvist (d)	1.4671	1.5187	1.5228	1.5519	1.5928	1.6211	1.6412	1.6625	1.6618	1.6761	1.7044	1.7373	1.7597	1.7595	1.8188	1.9423	2.0162
Chained																	
Fisher (c)	1.4589	1.5023	1.5000	1.5281	1.5676	1.5978	1.6190	1.6416	1.6438	1.6601	1.6889	1.7240	1.7487	1.7676	1.8396	1.9565	2.0269
Törnqvist (c)	1.4591	1.5025	1.5002	1.5282	1.5678	1.5980	1.6193	1.6419	1.6440	1.6602	1.6890	1.7242	1.7488	1.7677	1.8398	1.9568	2.0272
Multilateral																	
Fisher (EKS)	1.3622	1.4012	1.3979	1.4208	1.4539	1.4769	1.4922	1.5082	1.5064	1.5155	1.5354	1.5592	1.5720	1.5638	1.6121	1.7056	1.7547
Törnqvist (CCD)	1.4692	1.5142	1.5128	1.5411	1.5809	1.6101	1.6308	1.6526	1.6533	1.6686	1.6971	1.7311	1.7548	1.7653	1.8320	1.9514	2.0229

Note: Net purchases abroad and NPISH are excluded
source of data: Bureau of Economic Analysis

Table 2. Price Indexes for Personal Consumption Expenditures, all items w/o tech items

[Index numbers, 1990=1.00]

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Direct																	
Fisher (d)	1.0000	1.0372	1.0686	1.0983	1.1223	1.1482	1.1761	1.1999	1.2123	1.2334	1.2685	1.2993	1.3196	1.3531	1.3915	1.4371	1.4826
Törnqvist (d)	1.0000	1.0372	1.0686	1.0983	1.1223	1.1482	1.1762	1.2000	1.2123	1.2337	1.2693	1.3003	1.3207	1.3544	1.3930	1.4387	1.4847
Chained																	
Fisher (c)	1.0000	1.0372	1.0688	1.0990	1.1235	1.1496	1.1777	1.2017	1.2142	1.2355	1.2704	1.2986	1.3188	1.3514	1.3893	1.4335	1.4791
Törnqvist (c)	1.0000	1.0372	1.0688	1.0990	1.1235	1.1496	1.1777	1.2017	1.2142	1.2355	1.2704	1.2986	1.3188	1.3514	1.3893	1.4335	1.4791
Multilateral																	
Fisher (EKS)	1.0000	1.0351	1.0664	1.0969	1.1219	1.1478	1.1762	1.2007	1.2145	1.2366	1.2715	1.3004	1.3210	1.3539	1.3920	1.4363	1.4816
Törnqvist (CCD)	1.0000	1.0353	1.0670	1.0979	1.1235	1.1496	1.1781	1.2027	1.2166	1.2389	1.2741	1.3030	1.3237	1.3567	1.3950	1.4395	1.4851

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Direct																	
Fisher (d)	1.5267	1.5834	1.5885	1.6223	1.6679	1.7005	1.7237	1.7472	1.7448	1.7606	1.7930	1.8322	1.8604	1.8697	1.9376	2.0733	2.1557
Törnqvist (d)	1.5295	1.5858	1.5911	1.6245	1.6704	1.7032	1.7266	1.7509	1.7501	1.7674	1.8009	1.8410	1.8699	1.8840	1.9539	2.0906	2.1746
Chained																	
Fisher (c)	1.5220	1.5718	1.5732	1.6068	1.6523	1.6882	1.7143	1.7417	1.7465	1.7673	1.8007	1.8409	1.8708	1.8954	1.9757	2.1062	2.1872
Törnqvist (c)	1.5221	1.5718	1.5731	1.6067	1.6523	1.6882	1.7142	1.7416	1.7463	1.7671	1.8005	1.8407	1.8706	1.8951	1.9755	2.1060	2.1869
Multilateral																	
Fisher (EKS)	1.5244	1.5754	1.5775	1.6110	1.6563	1.6908	1.7158	1.7416	1.7436	1.7626	1.7956	1.8352	1.8644	1.8825	1.9576	2.0899	2.1710
Törnqvist (CCD)	1.5281	1.5790	1.5809	1.6144	1.6599	1.6945	1.7196	1.7456	1.7480	1.7674	1.8008	1.8408	1.8702	1.8899	1.9658	2.0983	2.1799

Note: Net purchases abroad and NPISH are excluded

tech items are: Video, audio, photographic, and information processing equipment and media (75, 76, and part of 93)
Telephone and related communication equipment

source of data: Bureau of Economic Analysis

Table 3. Price Indexes for Personal Consumption Expenditures, all items

[Index numbers,
1990=1.00]

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Direct																	
Laspeyres (d)	1.0000	1.0361	1.0665	1.0955	1.1194	1.1451	1.1728	1.1956	1.2084	1.2329	1.2719	1.3031	1.3252	1.3624	1.4053	1.4560	1.5061
Paasche (d)	1.0000	1.0356	1.0648	1.0916	1.1122	1.1332	1.1534	1.1690	1.1697	1.1742	1.1925	1.2097	1.2132	1.2246	1.2353	1.2486	1.2561
Chained																	
Laspeyres (c)	1.0000	1.0361	1.0664	1.0951	1.1184	1.1426	1.1678	1.1883	1.1976	1.2156	1.2475	1.2717	1.2885	1.3172	1.3514	1.3912	1.4315
Paasche (c)	1.0000	1.0356	1.0654	1.0936	1.1164	1.1399	1.1639	1.1836	1.1915	1.2075	1.2372	1.2609	1.2766	1.3040	1.3368	1.3748	1.4135
Direct																	
GLaspeyres (d)	1.0000	1.0356	1.0647	1.0921	1.1146	1.1379	1.1630	1.1830	1.1907	1.2099	1.2444	1.2693	1.2839	1.3129	1.3470	1.3862	1.4254
GPaasche (d)	1.0000	1.0362	1.0667	1.0950	1.1174	1.1412	1.1654	1.1858	1.1948	1.2098	1.2388	1.2689	1.2878	1.3176	1.3503	1.3914	1.4315
Chained																	
GLaspeyres (c)	1.0000	1.0356	1.0654	1.0937	1.1167	1.1404	1.1650	1.1850	1.1933	1.2104	1.2406	1.2638	1.2797	1.3070	1.3401	1.3783	1.4173
GPaasche (c)	1.0000	1.0362	1.0664	1.0950	1.1182	1.1421	1.1668	1.1870	1.1959	1.2129	1.2444	1.2689	1.2856	1.3145	1.3484	1.3881	1.4280

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Direct																	
Laspeyres (d)	1.5548	1.6156	1.6179	1.6571	1.7126	1.7505	1.7778	1.8066	1.8063	1.8274	1.8706	1.9195	1.9549	1.9824	2.0737	2.2287	2.3213
Paasche (d)	1.2594	1.2821	1.2735	1.2699	1.2696	1.2612	1.2504	1.2421	1.2309	1.2094	1.1890	1.1628	1.1266	1.0217	1.0041	1.0233	1.0117
Chained																	
Laspeyres (c)	1.4688	1.5129	1.5116	1.5405	1.5808	1.6116	1.6334	1.6568	1.6595	1.6764	1.7062	1.7422	1.7675	1.7862	1.8590	1.9780	2.0499
Paasche (c)	1.4490	1.4917	1.4885	1.5158	1.5544	1.5842	1.6048	1.6266	1.6283	1.6438	1.6717	1.7060	1.7301	1.7493	1.8204	1.9353	2.0042
Direct																	
GLaspeyres (d)	1.4628	1.5078	1.5090	1.5344	1.5734	1.6030	1.6233	1.6439	1.6417	1.6520	1.6785	1.7109	1.7330	1.7470	1.8244	1.9538	2.0264
GPaasche (d)	1.4714	1.5297	1.5366	1.5695	1.6124	1.6393	1.6593	1.6814	1.6821	1.7006	1.7307	1.7641	1.7868	1.7721	1.8131	1.9309	2.0061
Chained																	
GLaspeyres (c)	1.4536	1.4959	1.4899	1.5171	1.5552	1.5850	1.6060	1.6283	1.6279	1.6435	1.6718	1.7064	1.7307	1.7476	1.8166	1.9298	1.9982
GPaasche (c)	1.4647	1.5092	1.5105	1.5395	1.5805	1.6111	1.6327	1.6555	1.6602	1.6771	1.7065	1.7421	1.7672	1.7881	1.8633	1.9841	2.0566

Note: Net purchases abroad and NPISH are excluded
source of data: Bureau of Economic Analysis

Table 4. Price Indexes for Personal Consumption Expenditures, all items w/o tech items

[Index numbers,
1990=1.00]

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Direct																	
Laspeyres (d)	1.0000	1.0374	1.0692	1.0995	1.1245	1.1515	1.1810	1.2056	1.2198	1.2457	1.2861	1.3184	1.3414	1.3797	1.4237	1.4755	1.5268
Paasche (d)	1.0000	1.0370	1.0679	1.0970	1.1201	1.1448	1.1712	1.1943	1.2049	1.2213	1.2513	1.2806	1.2982	1.3269	1.3599	1.3997	1.4398
Chained																	
Laspeyres (c)	1.0000	1.0374	1.0692	1.0995	1.1242	1.1504	1.1788	1.2030	1.2157	1.2377	1.2734	1.3016	1.3221	1.3551	1.3933	1.4381	1.4840
Paasche (c)	1.0000	1.0370	1.0685	1.0985	1.1228	1.1487	1.1766	1.2005	1.2126	1.2333	1.2674	1.2956	1.3155	1.3477	1.3852	1.4290	1.4742
Direct																	
GLaspeyres (d)	1.0000	1.0369	1.0677	1.0968	1.1206	1.1459	1.1740	1.1973	1.2082	1.2308	1.2687	1.2970	1.3146	1.3472	1.3848	1.4279	1.4716
GPaasche (d)	1.0000	1.0375	1.0695	1.0998	1.1241	1.1505	1.1784	1.2027	1.2165	1.2365	1.2700	1.3036	1.3268	1.3616	1.4012	1.4496	1.4979
Chained																	
GLaspeyres (c)	1.0000	1.0369	1.0683	1.0983	1.1227	1.1486	1.1767	1.2005	1.2126	1.2339	1.2680	1.2957	1.3153	1.3471	1.3845	1.4279	1.4729
GPaasche (c)	1.0000	1.0375	1.0693	1.0997	1.1244	1.1506	1.1788	1.2029	1.2157	1.2371	1.2727	1.3016	1.3222	1.3556	1.3941	1.4392	1.4854

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Direct																	
Laspeyres (d)	1.5766	1.6386	1.6412	1.6812	1.7377	1.7764	1.8042	1.8337	1.8334	1.8549	1.8989	1.9486	1.9847	2.0126	2.1054	2.2629	2.3570
Paasche (d)	1.4783	1.5300	1.5376	1.5655	1.6010	1.6278	1.6468	1.6649	1.6604	1.6711	1.6930	1.7227	1.7440	1.7369	1.7831	1.8996	1.9717
Chained																	
Laspeyres (c)	1.5274	1.5776	1.5798	1.6140	1.6601	1.6962	1.7226	1.7505	1.7558	1.7771	1.8113	1.8522	1.8824	1.9061	1.9867	2.1186	2.2006
Paasche (c)	1.5167	1.5661	1.5666	1.5996	1.6446	1.6803	1.7060	1.7329	1.7372	1.7574	1.7901	1.8298	1.8593	1.8849	1.9647	2.0938	2.1738
Direct																	
GLaspeyres (d)	1.5136	1.5633	1.5673	1.5968	1.6404	1.6744	1.6981	1.7220	1.7213	1.7346	1.7645	1.8007	1.8265	1.8440	1.9274	2.0672	2.1470
GPaasche (d)	1.5456	1.6085	1.6153	1.6527	1.7010	1.7325	1.7556	1.7803	1.7794	1.8008	1.8382	1.8822	1.9144	1.9250	1.9807	2.1143	2.2025
Chained																	
GLaspeyres (c)	1.5155	1.5642	1.5616	1.5943	1.6383	1.6737	1.6994	1.7265	1.7284	1.7485	1.7813	1.8209	1.8503	1.8722	1.9490	2.0753	2.1539
GPaasche (c)	1.5286	1.5795	1.5846	1.6192	1.6664	1.7028	1.7292	1.7569	1.7644	1.7858	1.8199	1.8608	1.8912	1.9183	2.0023	2.1371	2.2205

Note: Net purchases abroad and NPISH are excluded

tech items are: Video, audio, photographic, and information processing equipment and media (75, 76, and part of 93)
Telephone and related communication equipment

source of data: Bureau of Economic Analysis