Sensitivity of PPP-Based Income Estimates to Choice of Aggregation Procedures

by

Yuri Dikhanov

Development Data Group
International Economics Department
The World Bank
Washington, D.C.

January, 1997

Yuri Dikhanov
The World Bank

08/17/99

Sensitivity of PPP-Based Income Estimates to Choice of Aggregation Procedures[1]

The choice of aggregation method significantly influences the results of international comparisons (both real incomes and rankings). The two most widely used methods of aggregating detailed data to get GDP in international prices -- the EKS and Geary-Khamis (GK) -- are discussed and contrasted here with the other additive and non-additive indexes (altogether 11 indexes discussed). The additivity issue, Paasche-Laspeyres spread and Gerschenkron effect are discussed in more detail. Special attention is paid to the Ikle index which while being additive minimizes the Gerschenkron effect (in contrast to the EKS and GK, the first of which is not additive and the second manifests significant Gerschenkron effect), and which is recommended for use in analytical work involving comparisons of the GDP structures as well as levels. Some of the results of this investigation include the following: (i) the system of international prices corresponding to the Iklé system is developed; (ii) a generalized Geary-Khamis system (GGK) is introduced; (iii) iteration procedures for solving the Iklé, Geary-Khamis and other systems were developed and implemented in a Microsoft Excel environment; (iv) existence and uniqueness of the Ikle solution is shown. The indexes were used to aggregate the 1985 ICP results, which are discussed as well.

1 I would like to thank Erwin Diewert (University of British Columbia) who kindly agreed to be the discussant of this paper at the Twenty-Third General Conference of the International Association for Research in Income and Wealth held at St. Andrews, New Brunswick, Canada, August 21-27, 1994, and Jitendra Borpujari (World Bank) who helped to launch this project and provided helpful suggestions. I am especially thankful to Prasada Rao (University of New South Wales) for numerous and productive discussions. I would like to thank Michael Ward (World Bank) for extensive reviewing of this paper. I have benefitted also from Kumaraswamy Velupillai (UCLA), Doris Iklé and B.J.Stone (Conservation Management Corporation), Alan Heston (University of Pennsylvania), Sultan Ahmad (World Bank), and Nancy Wagner (IMF), who commented on earlier versions. The responsibility for any remaining errors is of course entirely my own. The author alone is responsible for the paper's findings, interpretations, and conclusions, which should not be attributed to the World Bank, its board of Directors, its management, or any of its member countries.

Sensitivity of PPP-Based Income Estimates to Choice of Aggregation Procedures

> The main use of PPPs is to extend the domain of usefulness of national accounts data by making it possible to compare or combine data for different countries in an economically meaningful way. (P.Hill, *Multilateral Measurements of Purchasing Power and Real GDP*, Eurostat, 1982, p. 8)

> No one in normal practice bothers with theory, and quite rightly (S.N.Afriat, *The Price Index,* Cambridge University Press, 1977, p. 27)

> We cannot hope for one ideal formula for the index number (P.A.Samuelson and S.Swamy, *Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis*, The American Economic Review, Vol. 64, N. 4, 1974, p.592)

I. Introduction

The United Nations International Comparison Programme (ICP) was launched in 1968 as a worldwide effort to compare country income levels on a purchasing power adjusted basis. Built on the earlier work of Clark (1940) and Gilbert and Kravis (1954), the initiative has been developed under the guidance primarily of a group of scholars from the University of Pennsylvannia. It also represents a cooperative effort of the international agencies, including especially the United Nations Statistical Office (UNSTAT), the Statistical Office of the European Communities (EUROSTAT), the Organisation of Economic Cooperation and Development (OECD) and the World Bank.[2]

The ICP comparisons collect detailed statistics on prices and on expenditures on gross domestic product (GDP) as basis for making PPP-based international comparisons of income levels. While lags in basic data development constitute a major challenge to the prospects for this initiative, the focus of this paper is on the methodological issues posed by sensitivity of ICP comparisons to the choice of a procedure for aggregating the basic survey data. Mitigation of the data as well as methodological concerns remains a matter of priority in view of ICP's importance as the only available worldwide basis for a comparisons of country incomes on a purchasing power adjusted basis.

The ICP methodology can be briefly described as follows. First, observed prices and expenditures in local currency for individual commodity and service components of GDP are grouped into separate basic headings. Unweighted PPPs or price relatives are then obtained for each of these groupings on a *comparable* and *representative* basis, with adjustments made as needed for quality differences.[3] Finally, these unweighted basic heading parities are aggregated to arrive at the PPPs--and hence price-adjusted *real quantities*--for each expenditure category up to the level of GDP. This paper's focus is on the choice of aggregation procedures in the final step of ICP computations rather than on the generation of basic heading PPPs.

Section II below assesses the relative merits of Geary-Khamis (GK) and Elteto-Köves-Szulc (EKS) systems in the context of the various index number properties considered desirable in such aggregation procedures. Section III provides a formulation of Iklé system. Section IV describes a generalized Generalized Geary-Khamis system and analyzes alternative weighting schemes for the system. It is shown that the Iklé system is essentially an equal-weighted Geary-Khamis system. Section V discusses the *Gerschenkron effect* in additive procedures in the *Paasche-Laspeyres*

---

2 For more details on ICP history, methodology and data base, see, for instance, Kravis *et al* (1982) and Kurabayashi *et al* (1990) and World Bank (1993).

3 One can note that the idea of achieving both comparability and representativity in the PPP context is not unlike the Heisenberg Uncertainty Principle in nuclear physics on determining location and speed of an elementary particle: it is impossible to determine both simultaneously.

Spread (PLS) framework. Section VI discusses sensitivity of diffrerent indexes to stochastic errors in estimation. Section VII discusses uniqueness of the solution of the Ikle system. Section VIII assesses results from implementation of the Generalized Geary-Khamis framework and a number of other indexes to make PPP-adjusted GDP comparisons for 57 of the countries that participated in the 1985 ICP surveys. This section discusses the stability and correlation of PPP results (both real incomes and rankings) with respect to different aggregation procedures. The concluding Section IX points out that the Iklé system, which minimizes the *Gerschenkron effect* without loss of the *additivity* of ICP results can be used simultaneously for comparisons of economic structures as well as income levels. It is also stressed that the choice of aggregation procedure does influence the comparisons, especially for the lower income countries.

II. Desired properties of aggregation procedure and Geary-Khamis (G-K) and Eltetö-Köves-Szulc (EKS) approaches

The aggregation procedures are to satisfy certain properties. There is no universal agreement, however, on what properties should be satisfied. Listed below are a number of important properties for aggregation procedures (see, Kravis et al, (1982) *pp.71-74*), which is presented more formally in, for instance, Diewert (1987), *p. 767*).

Base-country invariance The choice of a base country does not affect the relative income or price levels of individual countries, i.e., the country selected as the base serves as a numéraire only. (In intertemporal comparisons this is equal to the time-reversal test.)[4]

Matrix consistency This property is sometimes called additivity. Quantities obtained through applying the index should satisfy the two requirements: the values for any category should be directly comparable between countries as well as between categories.

Factor-reversal test The product of the price and quantity ratios equals the nominal expenditure ratio.

Transitivity Any pairwise comparisons between the indexes are transitive in the sense that $I_k^j = I_l^j / I_l^k$.

Transactions equality This property, which requires that the relative importance of each transaction be dependent only on its magnitude.

World representativeness This property implies that the international price structure reflects the price and quantity structures of the world.[5]

Statistical efficiency The results should be minimally sensitive with respect to the sampling errors in the original data on prices and expenditures.

However, it is theoretically impossible to develop the perfect index for generating internationally comparable data that meet all the conditions above. In real life, one thus has to sacrifice some properties. In this paper we pursue a mixture of the so-called axiomatic (statistical) and functional approaches to the construction of index number[6].

Geary-Khamis and EKS aggregation procedures

The most widely used methods of aggregation used in ICP's international comparisons are the EKS and G-K.

An aggregation procedure that delivers additivity of the real expenditures on the level of GDP provides a set of common *international prices* (price vector to be applied to notional quantities to generate GDP in real terms), and is called an additive procedure. Some aggregation procedures are non-additive so that the resulting PPP-adjusted GDP components do not sum up to total GDP. An example of additive procedure is the GK system, an example of non-additive procedure is the EKS method.

---

4 It can be shown that the base-country invariance requires homothetic preferences.

5 In KHS [Kravis-Heston-Summers, see Kravis et al. (1982)] version of Geary-Khamis this property is satisfied through introducing super-country weights. This allows treating the countries-participants in the ICP as the representatives of their respective income groups. In practice, this procedure assumes that all the other countries not present in the comparison have price and quantity structures identical to those in their respective income groups. In our view, this is not fully compatible with the transaction equality principle.

6 As stated in *Kravis et al, (1982), p. 74* the *statistical* approach "compares two situations by a summary number that simply reflects in some sense the average difference between the statistics describing each of the situations. The *functional* approach, on the other hand, compares two situations on the basis of a theoretical structure derived from economic considerations... "

EKS (Fisher) procedure

The simplest procedure used in international comparisons is the binary Fisher index, which is a geometric mean of country A-based and country B-based indexes. This index was used in the very first international comparisons (see, Gilbert and Kravis (1954)). The Fisher index is hard to justify theoretically and it lacks some important properties, including transitivity and matrix consistency (additivity). A logical extension of the Fisher index is the EKS index, which can be seen as a multilateral Fisher. In the EKS procedure, countries are treated as a set of independent entities and each country is assigned an equal weight. The EKS method does not produce a set of *international prices* for the aggregation, although for each category of expenditures, it produces a set of price relatives. The EKS can be seen as a procedure that minimizes the differences between multilateral binary PPPs and bilateral binary PPPs. Or, it can be presented as:

$$F_{j,k} = (\prod_{l} F_{j,l} / F_{k,l})^{1/m} \qquad \textbf{(1)}$$

where $F_{j,k}$ - Fisher index for country j and country k
m - the number of all countries

The EKS provides:

- base country invariance;
- transitivity;
- direct information for two countries, including real prices;
- less vulnerability to stochastic errors (in tests, EKS has shown less sensitivity to stochastic errors in price and quantity data than GK);
- reduction[7] of the Gerschenkron effect (which is because the EKS is an unweighted geometric mean of the Fishers, and the Fishers do not have this effect); and
- compliance with the factor-reversal test.

The EKS fails to provide:

- matrix consistency; and
- transactions equality (i.e., the index provides neither invariance to changes in political subdivisions nor equal treatment of individual transactions in different countries).

It should be noted that it is possible to introduce a weighting scheme into the EKS to allow for larger countries to play a greater role in determining PPPs (i.e., to allow for some transactions equality). Thus, this modified EKS can be represented as a weighted geometric mean of Fisher indexes, where the weights are the shares of individual countries' GDP in world GDP. The modified EKS is, however, close to the original EKS in terms of results (see Table 1 of the Appendix).

G-K system

Another widely used index is the classic Geary-Khamis index (G-K), in which countries are treated as elements of a set rather than independent entities. The G-K method yields a vector of international prices and a vector of PPPs such that the international price for an individual good is a weighted average of relative prices in individual countries. Each country has a weight corresponding to its share in the GDP of the group /for each of the basic headings/. Thus, the larger countries have more influence in this procedure than the smaller ones. Hence, a change in the composition of the group can change the average prices as well as the relationships between countries. Providing a set of international prices makes the G-K an additive procedure.
The system can be written as follows:

$$\Pi_i = \sum_{j=1}^{n} \frac{p_{ij}}{PPP_j} * \{ \frac{q_{ij}}{\sum_{j=1}^{n} q_{ij}} \}$$

$$PPP_j = \frac{\sum_{i=1}^{m} p_{ij} q_{ij}}{\sum_{i=1}^{m} \Pi_i q_{ij}} \qquad \textbf{(2)}$$

where

Π_i - international price of commodity i;
p_{ij} - price of commodity i in country j;
q_{ij} -quantity of commodity i in country j;
PPP_j - overall PPP for country j;

The GK satisfies:

- base-country invariance;
- transitivity;
- matrix consistency; and

[7] Some researchers would probably argue that I should have written "elimination" rather than "reduction", but as it turns out the EKS (Fisher) might become relatively more biased due to the measurement errors in basic data than some of other indexes, such as Törnqvist. This behavior is based upon the following fact: in the two-country case the EKS (Fisher) is equal to Van-Yzeren balanced method (see Annex I). Because the Van-Yzeren balanced method generates a common price vector equal to an unweighted arithmetic mean of relative prices in two countries, the errors in prices will distort the outcome more than in the Törnqvist case. Nevertheless, the EKS (Fisher) was chosen as the benchmark in this study because the 1985 ICP database was or rather high quality and other biases become more manifested.

transactions equality, or "world representativeness".

The GK does not satisfy:

- neutrality with respect to the Gerschenkron effect (GK is biased towards countries with large GDPs).

The GK method results in the situation where a country with an unusual price structure will be shown as having higher volume levels than it would have if a set of prices closer to this country's price structure had been used instead. In this sense the GK "rewards" the countries that deviate in terms of structure from a “norm”. This so called *Gerschenkron effect* occurs because the GK international price structure becomes skewed to the price structure of large (in terms of total GDP) countries. In general, the developing countries are especially, but not exclusively, affected by this. The developed countries with significant deviations in their price structures from the GK international also demonstrate this effect.

Before 1990, the UNSTAT, the OECD and Eurostat all used the GK method in their comparisons. The GK had been widely criticized by experts in the ICP, but then some sort of compromise was reached. As the authors of OECD (1992) note:

> "nevertheless, the method [the GK] never gained general acceptance, being criticized by experts, countries and, on different occasions, by the international organizations themselves. Eurostat, in particular, has always had reservations about the GK method... The experts [of Eurostat] recognized that the results of such calculations are used for many different purposes and that there is no one method of aggregation which can be considered satisfactory for all these purposes. They recommended the calculation and dissemination of two sets of results: one set to be aggregated using the EKS method, the other to be aggregated using the GK method".

Additional statistical tests

Diewert (1987) thoroughly summarized statistical, axiomatic, and micro-economic properties of the index number. Analyzing different indexes he stated that given the imperfections of the real world Generalized EKS (GEKS) seems to be the least biased index. The GEKS can be used with the Fisher indexes (regular EKS) or with indirect translog indexes (with Törnqvist indexes for price index, we use this version of GEKS in our calculations, it is marked as Törnqvist). He noted:

> “... how are we to discriminate between P_F, P_W and P_T [Fisher, Walsh and Törnqvist]? Fortunately, it does not matter very much which of these formulae we chose to use in applications: they will all give the same answer to a reasonably high degree of approximation.” (Diewert, 1987)[8]

Because the GEKS is not additive, and additivity is highly desirable property, we can explicitly specify the proximity of an index to the GEKS as a requirement. Adding here imperfections of the real world, where our data are subject to errors, we can add to the usual set of tests two more:

1) distance (in some sense) from GEKS;
2) sensitivity of the index to data errors;

In defining the test of proximity of the results to the GEKS, we shall keep in mind that the GEKS itself is an approximation (though a “reasonably” good one) to the “ideal” index.

III. Formulation of the Iklé system

The Iklé system was first introduced in 1972 in a paper published in *The Quarterly Journal of Economics* (Iklé, 1972). In what follows, the system of *international prices* corresponding to the Iklé system is developed based on a proposed system of notation. The relation of the Iklé system to the G-K is shown as well.

To obtain an explicit expression for an Iklé price vector in the two country case, we will use equation (2) from Iklé's paper :

$$R = \frac{\sum_i q_I^i \left(\frac{e_I^i + REe_{II}^i}{q_I^i + Rq_{II}^i} \right)}{\sum_i q_{II}^i \left(\frac{e_I^i + REe_{II}^i}{q_I^i + Rq_{II}^i} \right)} \qquad \textbf{(3)}$$

where p=prices, q=quantities, e=p x q, R - ratio of quantities, E - ratio of prices

Hence, the price vector can be written as follows:

---

[8] We should like to note, however, that the degree of approximation, although of the same order, might vary due to different responses of the indexes to basic data deficiencies (see also previous note).

$$\Pi_i = \frac{e_I^i + REe_{II}^i}{q_I^i + Rq_{II}^i} \qquad \textbf{(4)}$$

Or, it can be rewritten as:

$$\Pi_i = \frac{\frac{e_I^i}{E_I}E_I + \frac{e_{II}^i}{E_{II}}E_{II}\frac{E_I}{E_{II}}}{\frac{e_I^i}{p_I^i} + \frac{e_{II}^i}{p_{II}^i}\frac{E_I}{E_{II}}\frac{PPP_{II}}{PPP_I}} \qquad \textbf{(5)}$$

where

$$E_j = \sum_i p_j^i q_j^i, \quad RE = \frac{E_I}{E_{II}}, \quad R = \frac{E_I}{E_{II}}\frac{PPP_{II}}{PPP_I}$$

Let $\boldsymbol{\delta_j^i = e_j^i / E_j}$ be the share of commodity i in the total expenditures of country j.

Thus, (5) can be rewritten as follows:

$$\Pi_i = \frac{\delta_I^i E_I + \delta_{II}^i E_I}{\frac{\delta_I^i}{p_I^i}E_I + \frac{\delta_{II}^i}{p_{II}^i}E_I\frac{PPP_{II}}{PPP_I}} = PPP_I \frac{\delta_I^i + \delta_{II}^i}{\delta_I^i\frac{PPP_I}{p_I^i} + \delta_{II}^i\frac{PPP_{II}}{p_{II}^i}} \qquad \textbf{(6)}$$

Or, knowing that prices are defined with accuracy up to a scalar, we can write:

$$\Pi_i = \frac{\delta_I^i + \delta_{II}^i}{\delta_I^i\frac{PPP_I}{p_I^i} + \delta_{II}^i\frac{PPP_{II}}{p_{II}^i}} \qquad \textbf{(7)}$$

In the multilateral case, formula (7) can be generalized as follows:

$$\Pi_i = \frac{\sum_j \delta_j^i}{\sum_j \delta_j^i \frac{PPP_j}{p_j^i}} \qquad \textbf{(8)}$$

Let us now introduce a system of notation that allows us to represent such indexes in a more obvious way:

A and **H** - operators of the arithmetic and harmonic means, respectively. They are determined as:

$$H_i(f_j^i, \delta_j^i) = \frac{\sum_j \delta_j^i}{\sum_j \frac{1}{f_j^i}\delta_j^i} \qquad \textbf{(9)}$$

$$A_i(f_j^i, \delta_j^i) = \frac{\sum_j f_j^i \delta_j^i}{\sum_j \delta_j^i}$$

Thus, (6) can be rewritten as $\mathbf{H_i(p_j^i / PPP_j\,,\ \delta_j^i)}$. This is just a different way of saying that (6) is a harmonic mean of the relative commodity prices $\mathbf{p_j^i\ /\ PPP_j}$ weighted by shares δ_j^i. Let's discuss some properties of the index operator:

I. $\mathbf{H_i(\ f_j^i\ ,\ \delta_j^i) = A_i(\ f_j^i\ ,\ \delta_j^i / f_j^i)}$[9]

II. $\mathbf{A_i(\ a\ f_j^i\ ,\ \delta_j^i) = a\ A_i(\ f_j^i\ ,\ \delta_j^i)}$

III. $\mathbf{A_i(\ f_j^i\ ,\ a\ \delta_j^i) = A_i(\ f_j^i\ ,\ \delta_j^i)}$

IV. $\mathbf{A_i(\ f_j^i + y_j^i,\ \delta_j^i) = A_i(\ f_j^i\ ,\ \delta_j^i) + A_i(y_j^i,\ \delta_j^i)}$

V. $\mathbf{A_i(\ 1,\ \delta_j^i) = 1}$

VI. $\mathbf{A_i(\ 0,\ \delta_j^i) = 0}$

Now we can rewrite (8) using respectively properties I and II in the following form[10]:

$$\Pi_i = H_i\left(\frac{p_j^i}{PPP_j}, \frac{p_j^i q_j^i}{\sum_i p_j^i q_j^i}\right) = A_i\left(\frac{p_j^i}{PPP_j}, \frac{p_j^i q_j^i}{\sum_i p_j^i q_j^i}\frac{PPP_j}{p_j^i}\right) =$$

$$= A_i\left(\frac{p_j^i}{PPP_j}, \frac{p_j^i \Pi_i q_j^i / \Pi_i}{PPP_j \sum_i \Pi_i q_j^i}\frac{PPP_j}{p_j^i}\right) = A_i\left(\frac{p_j^i}{PPP_j}, \frac{\Pi_i q_j^i}{\sum_i \Pi_i q_j^i}\frac{1}{\Pi_i}\right) =$$

$$= A_i\left(\frac{p_j^i}{PPP_j}, \frac{\Pi_i q_j^i}{\sum_i \Pi_i q_j^i}\right) = A_i\left(\frac{p_j^i}{PPP_j}, w_j^i\right)$$

where

$$w_j^i = \frac{\Pi_i q_j^i}{\sum_i \Pi_i q_j^i}$$

w_j^i can be seen as the real share of commodity i in the total expenditures of country j measured in *international prices.*

---

[9] Proof can be seen from the following:

$$\frac{\sum_j \delta_j}{\sum_j \delta_j \frac{1}{f_j^i}} = \frac{\sum_j (\delta_j / f_j^i)\ f_j^i}{\sum_j (\delta_j / f_j^i)}$$

[10] Note that this notation allows $\sum_j \delta_j^i \neq 1$,

because: $A_i(\varphi_j^i, \delta_j^i) = \dfrac{\sum_j \varphi_j^i \delta_j^i}{\sum_j \delta_j^i}$

On the other hand, to close the system, we need to add an expression for PPP_j.

By definition:

$$\mathbf{PPP_j = H_j(p_j^i} / P_i , \ q_j^i \mathbf{)}$$

Or, as it can be easily shown:

$$\mathbf{PPP_j = A_j(p_j^i} / P_i , \ q_j^i P_i \mathbf{/p_j^i) = \ A_j(p_j^i} / P_i , w_j^i \mathbf{)}$$

Thus, the Iklé system can be written as follows:

$$P_i \mathbf{= A_i(p_j^i /PPP_j ,} w_j^i \mathbf{)}$$

$$\mathbf{PPP_j = \ A_j(p_j^i} / P_i , w_j^i \mathbf{)} \qquad \textbf{(10)}$$

We can also represent Iklé and Geary-Khamis prices as:

$$\Pi_i \ (Ikle) \ = \ A_i(\frac{p_j^i}{PPP_j} \ , \ w_j^i)$$

$$\Pi_i \ (G - K) \ = \ A_i(\frac{p_j^i}{PPP_j} \ , \ q_j^i) \qquad \textbf{(11)}$$

In general, based on representation (11) one can introduce the Generalized Generalized Geary-Khamis System (GGK) as:

$$P_i \mathbf{= A_i(p_j^i /PPP_j ,} \ x_j^i \mathbf{(\ p_j^i \ ,q_j^i ,} P_i \mathbf{)})$$

$$\mathbf{PPP_j = \ A_j(p_j^i} / P_i , w_j^i \mathbf{)}$$

where $x_j^i(\ p_j^i \ ,q_j^i ,P_i)$ is a weight function of local prices, quantities and international prices.

As we can see from (10) and (11), the Iklé and GK systems have some symmetry with both PPPs and Πs expressed as arithmetic means weighted by the same real shares. For the Iklé index, the *international prices* on commodities are determined by the respective relative prices in individual countries, weighted by the shares of these commodities in total expenditures in the respective countries, expressed in real terms.

Comparison of the Geary-Khamis and Iklé indexes

The difference between the Geary-Khamis and Iklé systems can be understood as the difference between weights: in the Geary-Khamis system, the weights are the elements of the matrix of real quantities, whereas in the Iklé system, the weights are the elements of the matrix of the same real quantities normalized to set the sums of the entries into the matrix columns equal to unity. The Iklé system can be represented as *an* equal-weighted Geary-Khamis as well, so that we have:

$$\Pi_i \ (Ikle) \ = \ A_i(\frac{p_j^i}{PPP_j} \ , \ w_j^i)$$

$$\Pi_i \ (G - K) \ = \ A_i(\frac{p_j^i}{PPP_j} \ , \ w_j^i \ * \ Y_j) \qquad \textbf{(12)}$$

where

$\mathbf{Y_j} \ ^{o} \ \mathbf{E_j/PPP_j}$ is the total GDP of country j in *international prices.*

Thus, the Iklé system can be seen as *a* “democratic” Geary-Khamis system, in which each country exercises the same influence on the *international price* structure. The Geary-Khamis system becomes the Iklé system when all the economies are of the same size in terms of real total GDP. It should also be noted that such a "democratization" entails loss of some information in the transaction equality framework.

The Geary-Khamis index displays an explicit bias towards high-income (which happen to be large in GDP terms) countries, the so called *Gerschenkron effect*. To increase the representativity of the GK and to reduce the *Gerschenkron effect*, a procedure based on introducing "super-country weights" has been developed by Kravis, Heston and Summers see Footnote 5). However, it should be noted that the procedure (i) does not eliminate the Gerschenkron effect completely, and (ii) contains a very strong assumption that the countries that are represented by a country-representative have the same price and quantity structures as the country-representative does. The Geary-Khamis price structure might change significantly when we bring a new large economy into the analysis, and is not practically influenced by adding a number of small countries. In its turn, the Iklé index has a selection bias, i.e., a number of small countries with unusual price and quantity structures might influence the index. Although, in practice, the selection bias is significantly smaller than the *Gerschenkron effect*, for example.

IV. Alternative weighting schemes in Generalized Geary-Khamis Systems

Let us discuss some possible arithmetic weighting schemes for the price vector. We can use weights based on either (a) nominal quantities (in national currency), or (b) real (notional) quantities (in *international prices*). On the other hand, we can implement either (1) quantities in absolute value, or (2) normalized ones. Normalization can be only "vertical" (i.e. the sum of commodity weights for each country equals unity), since "horizontal" normalization is already embodied into the calculation of *international prices*. Finally, the last possibility is with no weights at all (0). We consider the latter as the reference point for studying the effects of introducing a weight into our weighting scheme.

These weighting schemes are represented in the following table:

Weighting schemes for Generalized Geary-Khamis Systems

	Quantities expressed in national prices	Quantities expressed in international prices
Unweighted scheme	weights$_j^i$ =1 GK-I B (Van-Yzeren) (0)	
Non-normalized weights	$f^i_j = p^i_j q^i_j$ (1a)	$\Pi_i q^i_j$ Geary-Khamis (1b)
Weights, normalized by columns	$d^i_j = \dfrac{p^i_j q^i_j}{\sum_i p^i_j q^i_j}$ GK-I A (Own Weights) (2a)	$w^i_j = \dfrac{\Pi_i q^i_j}{\sum_i \Pi_i q^i_j}$ Iklé (2b)

Thus, we can summarize the following possibilities:

no weights *(0)*

weights are defined as: $f^i_j = p^i_j q^i_j$

This option *(1a)* makes little sense from an economic point of view.

weights are $\delta_j^i = e_j^i / E_j$ - shares of commodity i in total expenditures in country j in local currency *(2a)*:

$$d^i_j = \frac{p^i_j q^i_j}{\sum_i p_i q^i_j} = \frac{p^i_j \;\; \Pi_i q^i_j}{PPP_j \;\; \Pi_i \;\; \sum_i \Pi_i q^i_j} = \frac{p^i_j}{PPP_j \;\; \Pi_i} w^i_j$$

weights are $w^i_j = \dfrac{\Pi_i q^i_j}{\sum_i \Pi_i q^i_j}$

This is the Iklé index (*2b*)

weights are q_j^i. This is the Geary-Khamis index (*1b*).

Hence, one can introduce two modifications of the Generalized Geary-Khamis indexes:

(*2a*) GK-I A: with the shares δ_j^i as weights (Own Weights);

(*0*) GK-I B: unweighted (Van-Yzeren)[11]

These indexes are shown in expression (13):

$$\begin{aligned} \Pi_i \;\; (Own\ Weights) &= A_i\left(\frac{p^i_j}{PPP_j}\,,\; d^i_j\right) \\ \Pi_i \;\; (Van-Yzeren) &= A_i\left(\frac{p^i_j}{PPP_j}\,,\; 1\right) \end{aligned} \qquad \textbf{(13)}$$

V. The Gerschenkron effect and Paashe-Laspeyres spread (PLS) in international comparisons

The *Gerschenkron effect* can be defined as an overvaluation of a country's real GDP due to the deviation of the country's price structure from the base price structure. The base price structure can be another country's price structure or, under the aggregation method that produces a set of international prices, the international price structure. Thus, the Gerschenkron

[11] It can be shown that the Van-Yzeren balanced index is equivalent to our GK-I B (referred to as Van-Yzeren throughout this paper). The Van-Yzeren index is equal to the GK when q^i_j =1. In the two-country case, the Van-Yzeren is equivalent to the EKS (Fisher), i.e. the Van-Yzeren yields prices that, when applied to the quantities, produce solution equal to that of EKS (Fisher) [see Annex I].

effect translates into an overvaluation of the poorer country, while utilizing for the comparison the richer country's price structure, and, correspondingly, an overvaluation of the richer country, while using for the comparison the poorer country's price structure. To determine the Gerschenkron effect, we have to reference this "overvaluation" to some "true" index and establish boundaries between which the effect can potentially lie.

Let's consider the Paasche and Laspeyres indexes in the two-country case. It is known that in bilateral comparisons the Laspeyres index is an upper bound on a "true" index, corresponding to the preference ordering and indifference curve attained in the base-country situation, and the Paasche index is a lower bound on the other "true" index corresponding to the preference ordering and indifference curve attained in the comparison-country situation. However, it is not generally true that the Paasche and Laspeyres indexes are the lower and upper bounds of one "true" index. But, if the preference ordering is homothetic, then the "true" index lies between its Paasche and Laspeyres bounds (Pollak (1990), *p. 30*). Thus,

$$Q_1^{\,2}(Laspeyres) \equiv \frac{\vec{p}_1\vec{q}_2}{\vec{p}_1\vec{q}_1} \geq Q_1^{\,2} \geq \frac{\vec{p}_2\vec{q}_2}{\vec{p}_2\vec{q}_1} \equiv Q_1^{\,2}(Paasche) \Rightarrow \quad \textbf{(14)}$$
$$\Rightarrow \frac{\vec{p}_2\vec{q}_1}{\vec{p}_2\vec{q}_2} \geq Q_2^{\,1} \wedge \frac{\vec{p}_1\vec{q}_2}{\vec{p}_1\vec{q}_1} \geq Q_1^{\,2}$$

if preference ordering is homothetic.

That is, the fixed-country-based Laspeyres price (and quantity) index is, under homothetic preferences, always higher than the "true" index, and the Gerschenkron effect is bounded by the Paasche-Laspeyres spread.

As Samuelson and Swamy (1974) note, "Empirical experience is abundant that the Santa Claus hypothesis of homotheticity in tastes and in technical change is quite unrealistic." However, Konüs (1924) showed that bounds similar to (15) hold in the general nonhomothetic case as well, provided we choose a reference vector $\mathbf{q} = \lambda\mathbf{q}^1 + (1-\lambda)\mathbf{q}^2$, which is a weighted average of the two observed quantity points. The following description of the Konüs index is adapted from W. Diewert (1987). Konüs introduced an index that can be approached as follows: Given a reference utility level, $\underline{u}$ ° F(**q**), the Konüs index $P_K(\mathbf{p}^1, \mathbf{p}^2, \mathbf{q})$ is the ratio of the minimum cost of achieving the utility level $\underline{u}$ when facing prices $\mathbf{p}^2$ relative to the minimum cost of achieving the same $\underline{u}$ when facing prices $\mathbf{p}^1$. Or we can write $P_K(p^1, p^2, q) = \dfrac{C(F(q), p^1)}{C(F(q), p^2)}$, where $C(u,p) = \min\limits_{q}\{pq : F(q) \geq u\}$. The Laspeyres-Konüs price index is defined as $P_K(\mathbf{p}^1, \mathbf{p}^2, \mathbf{q}^1)$ and, correspondingly, the Paasche-Konüs price index is defined as $P_K(\mathbf{p}^1, \mathbf{p}^2, \mathbf{q}^2)$. Specifically, Konüs showed that there exists a λ between 0 and 1 such that if $P_P \leq P_L$, then

$$P_P \leq P_K(\mathbf{p}^1, \mathbf{p}^2, \lambda\mathbf{q}^1 + (1-\lambda)\mathbf{q}^2) \leq P_L \quad \textbf{(15)}$$

W. Diewert (1987) noted that "the bounds given by (15) are the best bounds that we can obtain without making further assumptions on the utility function."

In addition, Diewert (1981) has shown that the Allen quantity index $Q_A(q^1, q^2, p) = \dfrac{C(F(q^1), p)}{C(F(q^2), p)}$ is also bounded by the Paasche and Laspeyres indexes in the general nonhomothetic case[12]:

$$Q_P \leq Q_A(\mathbf{q}^1, \mathbf{q}^2, \lambda'\mathbf{p}^1 + (1-\lambda')\mathbf{p}^2) \leq Q_L \quad \textbf{(16)}$$

Or, we can say that Paasche-Laspeyres boundaries are valid for quite a wide range of underlying utility functions.

The Paasche-Laspeyres spread

Thus, we can deduce that the Gerschenkron effect is intrinsically related to that of the Paasche-Laspeyres spread[13]. Let us define $Q_L = Q_1^{\,2}$ (Laspeyres), $Q_P = Q_1^{\,2}$ (Paasche) for quantity indexes, and, correspondingly, $P_L = P_1^{\,2}$ (Laspeyres), $P_P = P_1^{\,2}$ (Paasche) for price indexes. Then, following von Bortkiewicz (1922, 1924), we obtain:

[12] It should be noted, however, that $Q_A{*}P_K = \mathbf{p}^1\mathbf{q}^1/\,\mathbf{p}^2\mathbf{q}^2$ only in the homothetic case.

[13] In the two-country case the Gerschenkron effect *is* equal to the Paasche-Laspeyres spread.

$$P_L = \Sigma \frac{p_1}{p_0} w_0\ ,\ Q_L = \Sigma \frac{q_1}{q_0} w_0$$

$$s_p^2 = \Sigma\left(\frac{p_1}{p_0} - P_L\right)^2 w_0\ ,\ s_q^2 = \Sigma\left(\frac{q_1}{q_0} - Q_L\right)^2 w_0$$

hence,

$$r_{p,q} = \frac{\Sigma\left(\frac{p_1}{p_0} - P_L\right)\left(\frac{q_1}{q_0} - Q_L\right) w_0}{s_p s_q} = \frac{\Sigma\left(\frac{p_1 q_1}{p_0 q_0} - P_L Q_L\right) w_0}{s_p s_q} =$$

$$= \frac{P_P Q_L - P_L Q_L}{s_p s_q}$$

thus,

$$\frac{P_P}{P_L} = \frac{Q_P}{Q_L} = 1 + r_{p,q} \frac{s_p}{P_L} \frac{s_q}{Q_L} \quad \text{(von Bortkiewicz formula)} \qquad \textbf{(17)}$$

where $r_{p,q}$ -- weighted coefficient of correlation between price and quantities relatives.
σ_p , σ_q -- weighted variances in prices and quantities.
weights ω_0 are base-country weights.

Or, we can write this formula as:

$$PLS = \frac{Q_P}{Q_L} = 1 + r_{p,q}\, s_{\frac{p}{P_L}}\, s_{\frac{q}{Q_L}} \qquad \textbf{(18)}$$

where $\sigma_{p/P}$, $\sigma_{q/Q}$ -- weighted variances in relative prices and quantities.

The above expression 18 simply states that the Laspeyres index exceeds the Paasche index if the correlation between relative price and quantity changes is negative, i.e., the direction of movements in price ratios en masse is opposite to those in quantity ratios [or most of the product mix are normal goods]. This intuitively looks right: if technical change (or something else) makes products cheaper then their consumption increases; or, when some goods experience a price shock, their consumption decreases. Moreover, it can be observed that the hypothesis of normality of goods is validated by the whole experience of the International Comparison Programme: examining a table of the PLS compiled from the ICP results for different years, one can find that the PLS is always less than unity for any pair of countries.

Given actual ICP or growth rates detailed data, it is difficult to separate components of the PLS. In some countries, for example, expenditures for certain commodities are reported to be equal to zero, which reflects inadequate information, rather than the actual expenditure pattern.

Simulating the Gerschenkron effect in a multilateral case

In this exercise the Gerschenkron effect was simulated for a multilateral case. The assumptions used in this simulation were the following:

1. income among the countries is distributed log-normally;
2. the utility function has a Cobb-Douglas form;
3. estimation errors in measuring prices and expenditures are independently and normally distributed, with a standard deviation of 25 percent;
4. expenditure shares of commodities are exponentially related to income per capita.

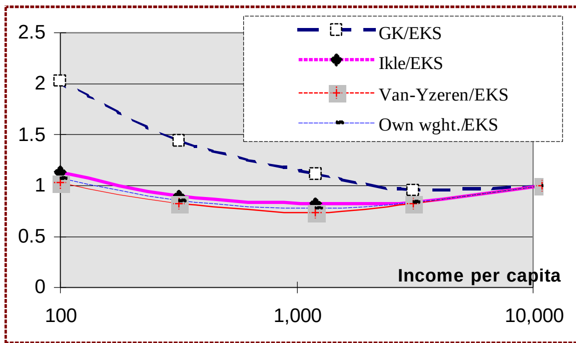

Figure 1. Gerschenkron effect of different indexes

Figure 1 shows that the Iklé, Van-Yzeren and Own weights indexes do indeed closely approximate each other. (Note that in a 2-country case, the Van-Yzeren index is equivalent to the EKS). The deviation of the classic Geary-Khamis from the EKS predictably gears upwards as income per capita (and total GDP of the country) decreases. In this example, judging by the standard deviation of these indexes, the Iklé has the lowest value of the deviation, closely followed by the Van-Yzeren and Own weights. Again, all these indexes produce similar results for "high-income" countries from our simulation, and the deviation from the EKS increases as the income decreases. Note that simulations with other utility functions produce similar results.

One can point out that in the real world measurement errors are obviously not just of a stochastic nature. There exist numerous biases that modify the relationship between the PLS and income differential. And the residual Gerschenkron effect is not readily distinguishable in the “democratic” Generalized Geary-Khamis case, i.e. for the Iklé, Van-Yzeren and Own weights (see Appendix, Figure 1), because of the randomly [and systematically] distributed measurement errors. On the other hand, in the Geary-Khamis case, the Gerschenkron effect is clearly visible.

The PLS and income per capita

Let's consider the relationship between the PLS and per capita income. It turns out that, using 1985 ICP data, the correlation between the two tends not to be uniform. The PLS and per capita income show the strongest correlation for high-income countries (R^2 = 0.73); they have a lower correlation (R^2 = 0.461) for all 57 countries. Figure 2 shows the distribution of the PLS by pairs of countries, arranged by income differential for high-income countries. Here one can see that there is a less distinctive pattern to the relationship between the PLS and income per capita in the middle and in the "southeast" section of the graph, which displays a decrease in correlation.

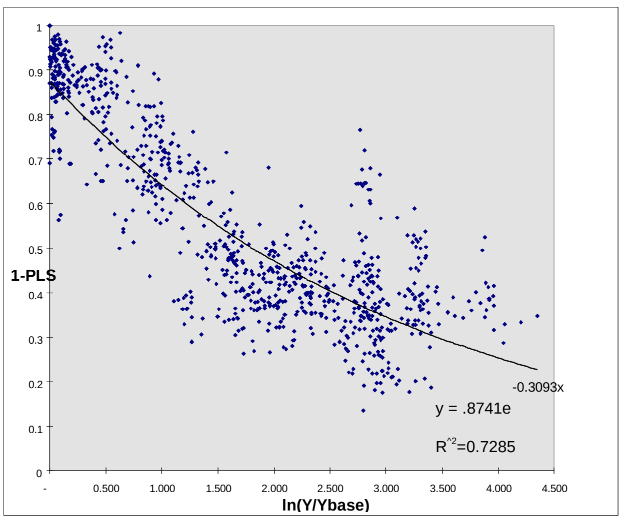

Figure 2. Relationship between PLS and per capita income differential (high-income countries)

As one can see from Figure 2, the regression shows a 12.6 percent discrepancy between unity and PLS that corresponds to $Y=Y_{base}$ case. This discrepancy can probably be treated as some measure of discrepancies in consumer preference sets over the countries, as well as of systematic errors of estimation (one can easily see that stochastic errors of estimation *practically* do not influence PLS).

VI. Sensitivity of different indexes to stochastic errors in estimation

A sensitivity analysis of different GGK indexes to normally distributed errors in prices and quantities yields the following results:

a. 1 percent distortion:

	Relative to original results for resp. indexes		Relative to new (distorted) EKS results	
	Diff.	STD	Diff.	STD
GK	-0.19%	0.29%	20.73%	21.97%
Ikle	-0.05%	0.21%	-0.79%	7.26%
Van-Yzeren	0.03%	0.22%	-0.77%	8.03%
Own weights	-0.01%	0.24%	2.00%	9.58%
EKS	-0.14%	0.17%	N/A	N/A

b. 10 percent distortion:

	Relative to original results for resp. indexes		Relative to new (distorted) EKS results	
	Diff.	STD	Diff.	STD
GK	-0.36%	2.94%	19.70%	21.77%
Ikle	-0.40%	2.15%	-1.45%	7.68%
Van-Yzeren	-0.35%	2.44%	-1.42%	8.04%
Own weights	-0.21%	2.34%	1.16%	9.79%
EKS	-1.19%	1.80%	N/A	N/A

As we can see for both 1- and 10-percent distortions, the EKS is the least sensitive index (with a standard deviation of 0.17% and 1.80%, respectively), whereas the Iklé index is the most consistent of the GGK indexes (0.21% and 2.15%, respectively). The second portion of these tables is provided for reference. It indicates new (distorted) results for the GGK indexes in relation to those for the new (distorted) EKS index. This portion shows if our conclusion about the deviations of different indexes from the EKS sustain this distortion in the original data. As one can see the indexes (both GGK and EKS) are positioned relative to each other after the distortion almost exactly before any distortion in data.

A 10-percent error in measurements should probably be viewed as realistic. It is unwise to expect smaller measurement errors from price and expenditure data.

The above could be expanded to include the errors in expenditure data that are not independent, in which case the expenditure categories can be misrepresented due to inconsistencies in specifications across countries. As for prices, one can note that the systematic errors (like those in prices of services in low-income countries) influence the outcome more than the stochastic ones.

VII. Existence and uniqueness for the Iklé procedure

Proofs of existence and uniqueness for the Ikle system in this chapter will use some properties of the system discussed earlier in this paper. We will show that the existence and uniqueness of the solution of the Iklé system follows immediately from the relation of the Iklé to the Geary-Khamis system, and from the fact that the Geary-Khamis procedure is a continuous transformation Y with respect to prices ***p*** and quantities ***q*** (with respect to quantities it is a continuous monotonously increasing transformation limited from above as we think of ***Q*** as a share in "world" output), and it has a unique solution[14]. This proof can be seen also as a computational algorithm for solving the Iklé system. In fact, it is useful to use this algorithm to check both other Iklé and Geary-Khamis algorithmae simultaneously.

Let us start with some sets of prices p_j^i and quantities q_j^i. We know that there exists a unique Geary-Khamis solution for these sets. We will show that certain transformation of q_j^i will yield a Geary-Khamis prices that is equal to the Iklé prices for the original q_j^i. Let us write:

$$Q_j = \Psi_j(q_j^i,(\cdot)) = \sum_i \Pi_i(q_j^i,(\cdot))\, q_j^i \qquad \textbf{(19)}$$

is the Geary-Khamis solution for quantities q_j^i, prices are assumed to be constant; $\Pi_i(q_j^i,(.))$ - are Geary-Khamis prices (throughout this chapter).

Let's introduce the following transformation of quantities q_j^i:

$$q_j^{i\,N} = q_j^{i\,N-1} \,/\, Q_j^{N-1} \qquad \textbf{(20)}$$

Iterations to arrive at the Ikle solution will consist of estimating consecutive GK solutions for each $q_j^{i\,N}$:

$$Q_j^{\,N} = \sum_i \Pi_i\,(q_j^{i\,N},(\cdot))\, q_j^{i\,N} = \sum_i \Pi_i\left(\frac{q_j^{i\,N-1}}{Q_j^{\,N-1}},(\cdot)\right)\frac{q_j^{i\,N-1}}{Q_j^{\,N-1}}$$

Without restricting generality we can assume that $\sum_{j=1}^{M} Q_j^N = M$. We will prove that $\lim_{N\to\infty} q_j^{i\,N} = q_j^{i\,N-1}$, or, equivalently, $\lim_{N\to\infty} Q_j^N = 1$.

Let us rewrite (20) as: $q_j^{i\,N} = q_j^{i\,(0)} \,/\, \prod_{L=1}^{N-1} Q_j^L$

Then, substituting it in expression (19), we obtain $\prod_{L=0}^{N} Q_j^{\,L} = \sum_i \Pi_i(q_j^{i\,N},(\cdot))\, q_j^{i\,(0)}$, which is limited from above and below.

We will rewrite two consecutive Q_j^{L+1} and Q_j^L as follows:

$$1 = \sum_i \Pi_i\,(q_j^{i\,L},(\cdot))\,\frac{q_j^{i\,L}}{Q_j^L}$$

$$Q_j^{L+1} = \sum_i \Pi_i\,(\frac{q_j^{i\,L}}{Q_j^L},(\cdot))\,\frac{q_j^{i\,L}}{Q_j^L} \qquad \textbf{(21)}$$

It follows from here that $\left|Ln(Q_j^{L+1})\right| < \left|Ln(Q_j^{\,L})\right|$, because $e^{Q_j}_{\Pi_i(q_j^i)}$ - the elasticity of Q_j on international prices $\Pi_i(q_j^i\ ,(\cdot))$ due to changes in q_j^i - is less than unity in absolute value[15]. This means that each iteration produces smaller changes in $\prod_{L=0}^{N} Q_j^{\,L}$ than the previous one, and because

---

[14] See, *Khamis (1970), Rao (1971)* for proofs of existence and uniqueness for the solution to the Geary-Khamis system.

[15] One can easily show that

$$\left|e^{Q_j}_{\Pi_i(q_j^i)}\right| = \left|\sum_i w_j^i e^{\Pi_i}_{q_j^i}\right| < 1 \qquad \textbf{(21a),}$$

because $|e^{\Pi_i}_{q_j^i}| < 1$. Here $e^{\Pi_i}_{q_j^i}$ stands for the elasticity of $\Pi_i(q_j^i\ ,(\cdot))$ on quantities $q_j^i = q_j^{i(0)} z_j$, where z_j is a scalar.

$Ln\left(\prod_{L=0}^{N} Q_j^{\,L}\right) = \sum_{L=0}^{N} Ln(Q_j^L)$ has a limit[16] we get $\lim_{N\to\infty} Q_j^{\,N} = 1$.

Thus, we can write $\lim_{L\to\infty} Q_j^{\,L} = \lim_{L\to\infty} \sum_i \Pi_i(q_j^{i\,L},(\cdot))\, q_j^{i\,L} = 1$, which is the solution for a Geary-Khamis system with $\tilde{q}_j^i = \lim_{L\to\infty} q_j^{i(0)} / \prod_{N=1}^{L} Q_j^N$. Then, using expression (12) we can say that international prices $\Pi_i(\tilde{q}_j^i,(\cdot))$ are the Iklé international prices both for the original set q_j^i and for the transformed set $\tilde{q}_j^i$ (because under the Iklé system, international prices are determined by expenditure structures $w_j^i = \frac{\Pi_i q_j^i}{\sum_i \Pi_i q_j^i}$) and, in turn, correspond to the Geary-Khamis international prices for the transformed set $\tilde{q}_j^i = \lim_{L\to\infty} q_j^{i(0)} / \prod_{N=1}^{L} Q_j^N$. Weights in the Iklé system in this case are $w_j^i = \frac{\tilde{q}_j^i}{\lim_{L\to\infty} Q_j^L} = \frac{\tilde{q}_j^i}{1}$. Thus, we can write that

$$Q_j^{Ikle} = \lim_{L\to\infty} \prod_{N=0}^{L} Q_j^{\,N} = \sum_i \Pi_i(\tilde{q}_j^i,(\cdot))\, q_j^{i\,(0)} =$$

$$= \sum_i \Pi_i\left(\lim_{L\to\infty}\left(\frac{q_j^{i\,(0)}}{\prod_{N=0}^{L} Q_j^{\,N}}\right),(\cdot)\right) q_j^{i\,(0)} = \qquad \textbf{(22)}$$

$$= \sum_i \Pi_i(q_j^{i\,(0)} / Q_j^{Ikle},(\cdot))\, q_j^{i\,(0)}$$

and, thus, the iterations converge to a unique solution, which is the Ikle solution. **QED**

The above discourse provides a proof of existence and uniqueness, as well as an algorithm to solve the Ikle system. Uniqueness and existence alone can be shown easier:

PROPOSITION. *Solution for the Ikle system defined in (10) exists and is unique.*

Existence: Consider the following transformation $\Omega(Q_j^I) = \sum_i \Pi_i(q_j^i / Q_j^I,(\cdot))\, q_j^i$, again $\Pi_i(q_j^i,(.))$ - are Geary-Khamis prices, $\frac{1}{M}\sum_{j=1}^{M} Q_j^I = 1$. The solution obviously exists and is unique. $\Omega(Q_j^I)$ is a continuous mapping of M-dimensional simplex $\{Q_j^I\}$ onto itself, henceforth, applying Brouwer's Fixed Point Theorem, we have at least one point where $Q_j^* = \Omega(Q_j^*)$. This is the Ikle solution[17].

Uniqueness: Assume, we have two Ikle solutions: Q_j^*, Q_j^{**}. Then, denoting $x_j^i Q_j^* = y_j^i Q_j^{**} = q_j^i$, we can write:

$$1 = \sum_i \Pi_i(x_j^i,(\cdot))\, x_j^i = \sum_i \Pi_i(y_j^i,(\cdot))\, y_j^i$$

and

$$Q_j^* \big/ Q_j^{**} = \sum_i \Pi_i(y_j^i\, Q_j^{**} \big/ Q_j^*,(\cdot))\, y_j^i$$

$$Q_j^{**} \big/ Q_j^* = \sum_i \Pi_i(x_j^i\, Q_j^* \big/ Q_j^{**},(\cdot))\, x_j^i$$

Q_j^* / Q_j^{**} and 1 are Ikle solutions for quantities $\{y_j^i\}$, Q_j^{**} / Q_j^* and 1 are Ikle solutions for quantities $\{x_j^i\}$. This would involve $\sum_j Q_j^{**} \big/ Q_j^* = \sum_j Q_j^* \big/ Q_j^{**} = M$, which, for positive Q_j^*, Q_j^{**}, is possible only when $Q_j^* \equiv Q_j^{**}$[18]. **QED**

VIII. Implementation of alternative aggregation procedures

---

[16] According to D'Alembert's Test $\sum_{L=0}^{N} \left|x^L\right|$ is convergent if $\left|\frac{x^{L+1}}{x^L}\right| \le q < 1$. On the other hand, if a series is absolutely convergent (i.e. convergent for moduli) then the series is convergent. I.e., ∃ finite $\lim_{L\to\infty} \sum_{L=0}^{\infty} x^L$.

[17] In a way, previous proof can be seen as a computational realization of Brouwer's FPT for this system (compare it with equation (22)).

[18] In fact, the elasticity consideration of expression (21a) can be applied here as well: one can consider moving between Ikle solutions 1 to Q_j^* / Q_j^{**} for quantities $\{y_j^i\}$, which would require $\left|e^{Q_j}_{\Pi_i(q_j^i)}\right| = 1$: a contradiction.

In this section, the effect of individual weighting schemes in the Generalized Geary-Khamis framework first discussed. Algorithms solving the Generalized Geary-Khamis systems were implemented using environment of Microsoft Excel Visual Basic for Applications. The results of the GGK and some other indexes (11 of them are presented in Figures 1 and 2 and Tables 1and 2 of the Appendix) have been used in this analysis. The PPPs for 56 countries that took part in the 1985 ICP Phase have been estimated for this exercise (seven Caribbean countries were excluded from consideration because of the unreliability of their results).

As could be expected from formula (11), the results shown in Figures 1 and 2 indicate that the Iklé index approximated both the Van-Yzeren and Own Weights (the "democratically" modified indexes from (13)). Thus, in our case, the difference between using weights q_j^i and w_j^i for our aggregation is marginal. Moreover, even unweighted, the arithmetic mean produces results approximating both weighted arithmetic means. The GK in its turn deviates significantly from these three indexes as could be predicted. All four Generalized Geary-Khamis indexes are arithmetic mean operators applied to the vector of relative prices. The only difference among the four is the weighting scheme, which leads to a case where the normalization of weights by columns yields results close to the unweighted index (Van-Yzeren). Put in other terms, the variation in normalized weights across countries is considerably less than that of total real expenditures on the category level. One can see from Figures 1 and 2 of the Appendix that the Iklé index is not biased towards the price structure of the high-income countries as the original Geary-Khamis is.

In this investigation, the Geary-Khamis method was applied directly to 139 basic headings, and no supercountry weighting was used; thus, the GK results became closer in value to those obtained using the bilateral USA-based Laspeyres index (Table 1 of Appendix). It should be noted that introducing the "super-country weighting" would shift the GK index closer to the Iklé and EKS. The introduction of the "fixity principle" would cause a similar shift, but would bring about a loss of additivity.

The considerable discrepancies one can see between the Iklé and Geary-Khamis indexes stem from the fact that in the GK system the high-income countries influence the international prices more than the low-income countries do. In the Iklé world, it is only the relative expenditure shares of individual countries expressed in international currency that matter, irrespective of the total GDP in those countries.

To illustrate this point, Figure 3 of the Appendix plots international prices according to various GGK indexes. We observe that the GK prices are predictably geared towards the US price structure (on this graph all international prices are plotted against the US price background).

Table 2 of the Appendix also explicitly shows the results of one of the statistical tests for additive indexes that we introduced (*closeness to the EKS*). The GK system predictably displays the highest deviation of GGK indexes, 20.3 percent; the Ikle displays the smallest magnitude, 6.3 percent. The other two - Van-Yzeren and Own weights - generate a 7.4 and 8.5 percent deviation, respectively. Thus, the test shows that compared to other GGK indexes the Ikle minimizes the Gerschenkron effect.

A correlation analysis for the indexes[19] demonstrates the following (see Table 2 of Appendix):

- the GEKS indexes aggregated from bilateral Fishers and Törnqvist indexes display the strongest correlation (r = 0.9999);
- the Iklé and Own Weights, and the Iklé and Rao show the same high degree of correlation (r = 0.9999);
- in general, equal-weighted GGK indexes (Iklé, Van-Yzeren and Own Weights) and the Rao index are a highly correlated group of indexes (r = 0.9993 - 0.9999);
- of all equal-weighted GGK indexes (Iklé, Van-Yzeren and Own Weights) and the Rao index, the Iklé index displays the <u>highest</u> correlation with the GEKS (aggregated from both Fishers and Törnqvists) and the EKS (weighted) indexes;

The income levels according to different indexes are presented in Figures 1 and 2. In general, one can see that all aggregation procedures, with the notable exception of the Paasche, Laspeyres and Geary-Khamis indexes, yield per capita incomes similar to

[19] Logarithms are used here in order to emphasize relative differences as opposed to absolute ones.

those of the EKS (Fisher) index. The EKS index has been chosen as the reference point, because being a multilateral Fisher, it is practically devoid of the *Gerschenkron effect.* The GEKS index aggregated from bilateral Törnqvist indexes is shown as well. As was mentioned above, the EKS index could be modified by introducing weights corresponding to the GDP size of the countries (weighted EKS index or EKS(w))[20]. Correlation analysis shows a very tight correlation between the EKS (w) and the Fisher, as well as with the original EKS (see Table 2 of the Appendix). The EKS (w) does not have the selection bias that the EKS has, thereby reflecting some of the transaction equality property.

Finally, one has to mention a data problem that is generated by the ICP itself: the expenditures and price data collection are not related to each other. This generates significant incompatibilities not only between different countries but between different benchmark years for the same country. Moreover, introducing quality adjustment coefficients might create an additional distortion. This produces significant errors in the data. We can see this from the example of India and Bangladesh: these countries seem not to follow the general pattern in the Paasche-Laspeyres spread. Indian prices for goods were adjusted by coefficients that were too large (or services, by ones too low), thereby making the Indian price structure closer to that of the high-income countries than it apparently is. With Bangladesh, the reverse occurred. This results in significantly different rankings for the countries depending on what aggregation procedure is used.

Rankings

One of the most important issues to deal with in international comparisons is that of rankings among the countries. All the aggregation procedures discussed here produce rankings that are substantially different from exchange rate rankings. Yet, there exists a much stronger internal relationship among all the indexes constructed on the basis of PPP aggregation procedures than between any of the PPP indexes and the exchange-rate-converted index. This is related to the fact that all the aggregation procedures are applied to the same set of national prices and quantities, but they are aggregated in different ways.

In this analysis, we used standardized logarithmic rankings. One can see that a significant drawback to using discrete rankings (simple ranks) is that the same difference in ranks for very disparate differences in income level can be assigned. The normalization of rankings can allow for this drawback[21].

Next, to elaborate the rankings they were converted to logarithms and standardized. Standardization[22] allows us to compare the beginning as well as the end of the list (the discrete rankings and normalized nominal rankings have restricted comparability at the beginning and the end of the list). For example, the USA is ranked number one according to all indexes, which does not mean that the relative and absolute position of the country with respect to other countries is not affected by the choice of aggregation procedure. Again, logarithms emphasize relative differences instead of absolute ones. In other words this intends to emphasize the situation when 10 % deviation at \$10,000 level would be comparable with the 10 % deviation at \$100 level.

Using these calculations we can now address the issue of sensitivity from a different prospective: namely, <u>how different countries react to the application of various indexes</u>. The sensitivity of the results for different countries due to the aggregation procedure used is presented in Figure 4. Here, the rankings based on different aggregation procedures are compared on

---

[20] The EKS (w) was realized as an iteration procedure, with the weights being the GDP of individual countries in EKS (w) terms:

$$EKS_j(w) = \prod_k (F_k^j)^{EKS_j(w) / \sum_k EKS_k(w)}$$

Thus defined "plutocratic" version of the EKS opposes the "democratic" EKS, where all the countries have the same weight in the formula. However, as it can be seen from our calculations, the EKS (w) produces results practically identical to those obtained using the regular EKS. From the theoretical point of view, the EKS (w) betters the EKS because the former satisfies the *insignificance of small country* property. Being aggregated from the same Fisher indexes, EKS (w) and EKS share all other major properties.

[21] The normalization procedure used here ascribes 100 to the highest income level and 0 to the lowest.

[22] The standardization procedure involves the division of nominal values over their standard deviation.

the basis of standard deviation in standardized logarithmic rankings (standard deviation is estimated across indexes). As one can see, the aggregation procedures under discussion produce the most coherent results for OECD and Group II countries. A number of countries such as Congo, India, Bangladesh, Nepal, Sierra-Leone and Tanzania display rather unstable results. All these countries have standard deviations in rankings of 10 or more percent. This possibly tells us that these countries have problems with their basic category data. However, in part, the variation is accounted for by the differences in aggregation procedures themselves. If the price structure of a country is distorted (e.g., the relative commodity prices are too high), the different aggregation procedures will yield results incorporating this distortion to varying extent.

We can deduce from these results that although the Ikle, Van-Yzeren and Own weights indexes all exibit a smaller Gerschenkron effect than the Geary-Khamis does, the Ikle index is superior to other GGK indexes in *all* conducted statistical tests (namely, standard deviation from the EKS for the simulated and actual 1985 ICP details, the sensitivity to data errors, and the correlation with the GEKS procedures aggregated from both Fisher and Törnqvist pairwise indexes). One can conclude that *though the Iklé index does not eliminate the Gerschenkron effect completely, as some residual effect is intrinsically embodied in any additive aggregation procedure, it does minimize the influence of this effect.*

IX. Conclusion

The Geary-Khamis index provides *additivity* (matrix consistency), but it also displays a significant *Gerschenkron effect*. The EKS index is free from the *Gerschenkron effect*, but it does not deliver *additivity*. On one hand, *additivity* is important in comparing price and expenditure structures across different countries. This property is crucial in comparing, for instance, poverty levels, which is important for operational purposes in international organizations engaged in development issues. On the other hand, the *Gerschenkron effect* might significantly distort the income levels in the developing countries, which are more sensitive to the results of international comparisons. Thus, from the axiomatic (statistical) point of view, the Iklé index is seen as the most suitable index for specific purposes of the World Bank and other international organizations that prefer making use of a single index in their analytical work. The Iklé index (like G-K and EKS) is simple for the algorithmization, it minimizes the *Gerschenkron effect* (being essentially the equal-weighted Geary-Khamis index), and it maintains additivity (being an additive procedure with a set of *international prices).*

Annex I

Some theory of the Fisher Index

Lemma 1. *If utility function f(x) is linear homogeneous, and the constraint is linear, then* $\frac{\nabla f(\mathbf{x})}{f(\mathbf{x})}=\frac{\mathbf{p}}{y}$.

Proof: Assume that $\mathbf{x}^*$ is a solution to the utility maximization problem
$\max_x\{f(\mathbf{x}): \mathbf{px}\le y\}$, then
$\frac{\nabla f(\mathbf{x}^*)}{\mathbf{p}}= l$, and $f(\mathbf{x}^*)=\nabla f(\mathbf{x}^*)\cdot\mathbf{x}^*$ because $f(\mathbf{x})$ is linear homogeneous. Thus, we obtain
$f(\mathbf{x}^*)= l\mathbf{px}^*= ly$, and $\frac{\nabla f(\mathbf{x}^*)}{f(\mathbf{x}^*)}=\frac{\mathbf{p}}{y}$.

Lemma 2. *Fisher index corresponds to quadratic utility function.*

Proof: Consider the following optimization problem - $\nu(\mathbf{p},y)=\max_x\{f(\mathbf{x})=(\mathbf{x}'\mathbf{A}\mathbf{x})^{1/2}: \mathbf{p}\times\mathbf{x}\le y\}$. Then,
$\frac{\nabla f(\mathbf{x})}{f(\mathbf{x})}=\frac{\mathbf{p}}{y}$ (see **Lemma** 1).

And, consequently, $\frac{\mathbf{p}}{y}=\frac{\mathbf{x}'\mathbf{A}}{\mathbf{x}'\mathbf{A}\mathbf{x}}$.

Selecting two points $\{\mathbf{x}^1,\mathbf{p}^1\}$ and $\{\mathbf{x}^0,\mathbf{p}^0\}$, we obtain
$\frac{\mathbf{p}^1\mathbf{x}^0}{y^1}=\frac{\mathbf{x}^{1\prime}\mathbf{A}\mathbf{x}^0}{\mathbf{x}^{1\prime}\mathbf{A}\mathbf{x}^1}$, and $\frac{\mathbf{p}^0\mathbf{x}^1}{y^0}=\frac{\mathbf{x}^{0\prime}\mathbf{A}\mathbf{x}^1}{\mathbf{x}^{0\prime}\mathbf{A}\mathbf{x}^0}$.

Finally, taking the ratio of the two and noting that $\mathbf{A}=\mathbf{A}'$, we arrive at

$$Q^2(\mathbf{x}^0,\mathbf{x}^1)=\frac{\mathbf{p}^0\mathbf{x}^1}{y^0}\Big/\frac{\mathbf{p}^1\mathbf{x}^0}{y^1}=\frac{\mathbf{x}^{0\prime}\mathbf{A}\mathbf{x}^1}{\mathbf{x}^{0\prime}\mathbf{A}\mathbf{x}^0}\Big/\frac{\mathbf{x}^{1\prime}\mathbf{A}\mathbf{x}^0}{\mathbf{x}^{1\prime}\mathbf{A}\mathbf{x}^1}=\frac{\mathbf{x}^{1\prime}\mathbf{A}\mathbf{x}^1}{\mathbf{x}^{0\prime}\mathbf{A}\mathbf{x}^0}=\frac{f^2(\mathbf{x}^1)}{f^2(\mathbf{x}^0)}$$

. I.e. Q(Fisher)=f.

Lemma 3. *The Fisher index can be represented as an additive index with common prices* $\mathbf{p}=\mathbf{p}_1/\Pi+\mathbf{p}_0$.

Proof: Consider the following optimization problem - $\max_x\{f(\mathbf{x}): \mathbf{p}\times\mathbf{x}\le y\}$, where $f(\mathbf{x})$ is linear homogeneous. Thus, we can write $f(\mathbf{x})=\nabla f\,\mathbf{x}$ (*Euler's Theorem*). On the other hand,
$\nabla f = f\,\frac{\mathbf{p}}{\mathbf{px}}$ (see **Lemma** 1).

We, thus, can write:

$$f_1-f_0=\nabla f_1\mathbf{x}^1-\nabla f_0\mathbf{x}^0=\nabla f_1\mathbf{x}^1-\nabla f_0\mathbf{x}^0+\nabla f_0\mathbf{x}^1-\nabla f_0\mathbf{x}^1=$$

$$\nabla f_1\mathbf{x}^1-\nabla f_0\mathbf{x}^0+\nabla f_0\mathbf{x}^1-\nabla f_1\mathbf{x}^0+(1-\frac{f_1}{f_0})\nabla f_1\mathbf{x}^0=$$

$$(\nabla f_1+\nabla f_0)(\mathbf{x}^1-\mathbf{x}^0)-(\frac{f_1-f_0}{f_0})\nabla f_1\mathbf{x}^0,$$

because $\nabla f_1\mathbf{x}^0=f_1\frac{\mathbf{p}^1\mathbf{x}^0}{\mathbf{p}^1\mathbf{x}^1}$, and $\nabla f_0\mathbf{x}^1=f_0\frac{\mathbf{p}^0\mathbf{x}^1}{\mathbf{p}^0\mathbf{x}^0}$, and therefore $\frac{\nabla f_0\mathbf{x}^1}{\nabla f_1\mathbf{x}^0}=\frac{f_1}{f_0}$.

Or, $(f_1-f_0)(1+\frac{\nabla f_1\mathbf{x}^0}{f_0})=(\nabla f_1+\nabla f_0)(\mathbf{x}^1-\mathbf{x}^0)$, and
$f_1-f_0=\frac{\nabla f_1+\nabla f_0}{1+PLS^{-1/2}}(\mathbf{x}^1-\mathbf{x}^0)$, where

$$PLS\,(\text{Paasche}-\text{Laspeyres spread})=\frac{\mathbf{p}^1\mathbf{x}^1}{\mathbf{p}^1\mathbf{x}^0}\Big/\frac{\mathbf{p}^0\mathbf{x}^1}{\mathbf{p}^0\mathbf{x}^0}.$$

Which translates into $\boxed{f_1-f_0=\frac{\mathbf{p}_1/\Pi+\mathbf{p}_0}{1+PLS^{-1/2}}(\mathbf{x}^1-\mathbf{x}^0)}$.

Or, written differently,
$f_1-f_0=(l_0^*\mathbf{p}_0+l_1^*\mathbf{p}_1)(\mathbf{x}_1-\mathbf{x}_0)(1+PLS^{-1/2})^{-1}$,
where λ^* is marginal utility of income.

Lemma 4. *Contributions of individual components to the growth expressed by the Fisher index is*

$$C_i=\frac{\mathbf{p}^i(\mathbf{x}_1^i-\mathbf{x}_0^i)}{\mathbf{px}_0}=\frac{(\mathbf{p}_1/\Pi+\mathbf{p}_0)(\mathbf{x}_1^i-\mathbf{x}_0^i)}{(\mathbf{p}_1/\Pi+\mathbf{p}_0)\mathbf{x}_0}.$$

Proof: We can express growth as follows:
$f_1/f_0-1=\frac{\mathbf{p}(\mathbf{x}_1-\mathbf{x}_0)}{\mathbf{px}_0}=\frac{(\mathbf{p}_1/\Pi+\mathbf{p}_0)(\mathbf{x}_1-\mathbf{x}_0)}{(\mathbf{p}_1/\Pi+\mathbf{p}_0)\mathbf{x}_0}$ (see **Lemma** 3). Thus, contributions of individual components to the total growth will be written as

$$C_i=\frac{\mathbf{p}^i(\mathbf{x}_1^i-\mathbf{x}_0^i)}{\mathbf{px}_0}=\frac{(\mathbf{p}_1^i/\Pi+\mathbf{p}_0^i)(\mathbf{x}_1^i-\mathbf{x}_0^i)}{(\mathbf{p}_1/\Pi+\mathbf{p}_0)\mathbf{x}_0}.$$

References

S.N.Afriat (1977), *The Price Index*, Cambridge University Press

L. von Bortkiewicz (1922, 1924), *Zweck und Structur einer Preisindexzahl*, Nordisk Statistisk Tidskrift, **1** and **3**

C. Clark (1940), *The Conditions of Economic Progress*, London: Macmillan

W.E. Diewert (1981), *The Economic Theory of Index Numbers: a Survey*, in *Essays in the Theory and Measurements of Consumer Behavior in Honor of Sir Richard Stone*, ed. A. Deaton, Cambridge University Press, London

W.E. Diewert (1987), *Index Numbers*, in *The New Palgrave: Dictionary of Economics*, W.W. Norton & Company, New York

M. Gilbert and I. Kravis (1954), *An International Comparison of National Products and the Purchasing Power of Currencies*, Paris: OEEC

P.Hill (1982), *Multilateral Measurements of Purchasing Power and Real GDP*, Eurostat

Doris Iklé (1972), *A New Approach to the Index Number Problem*, Quarterly Journal of Economics

Salem H. Khamis (1970), *Properties and Conditions for the Existence of a New Type of Index Numbers*, Sankhya: The Indian Journal of Statistics, Series B.

A.A. Konüs (1924), *Ê Âîïðîñó î íàñòîÿùåì Èíäåêñå Ñòîèìîñòè Æèçíè*, Moscow, [*The Problem of the True Index of the Cost of living*, Trans. in Econometrica, 7 (1939)]

I. Kravis, A. Heston, R. Summers (1982), *World Product and Income*, The Johns Hopkins University Press, Baltimore and London

Y. Kurabayashi and I. Sakuma (1990), *Studies in International Comparisons of Real Product and Prices*, Kinokuniya Company Ltd., Tokyo

OECD (1992), *Purchasing Power Parities and Real Expenditures, 1990,* Paris

R.A. Pollak (1990), *The Theory of the Cost-of-living Index*, in *Price Level Measurement* (ed. W.E. Diewert), North-Holland

D.S. Prasada Rao (1971), *On the Existence and Uniqueness of a New Class of Index Numbers*, Sankhya: The Indian Journal of Statistics, Series B.

P.A. Samuelson and S. Swamy (1974), *Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis*, The American Economic Review, Vol. 64, N. 4

World Bank (1993), *Purchasing* Power *of* Currencies*: Comparing National Incomes Using ICP Data*, Washington, D.C.

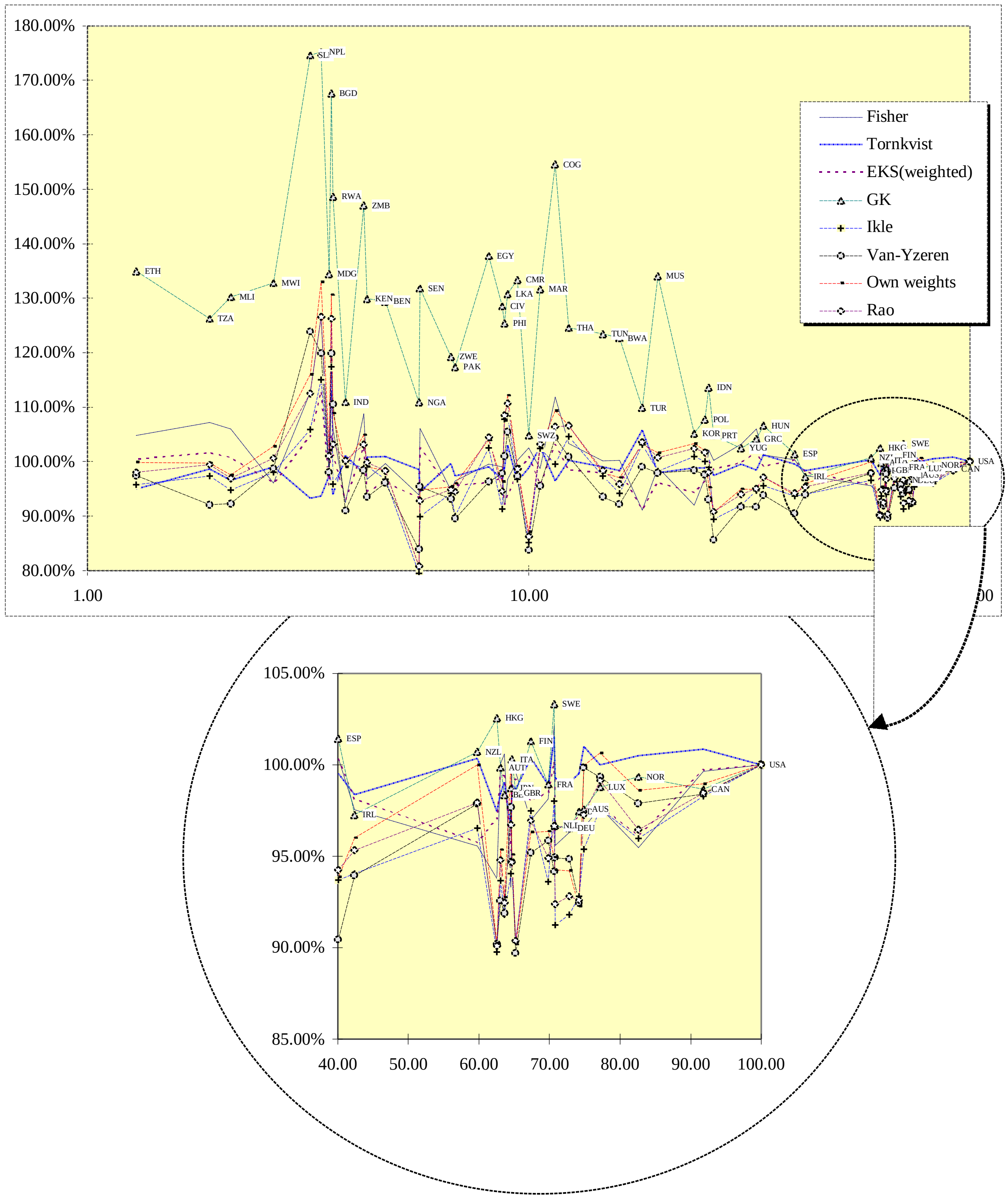

Figure 8. Deviation from EKS of per capita income according to various indexes, 1985

(Horizontal axis shows countries arranged according to their per capita income in EKS terms, US=100)

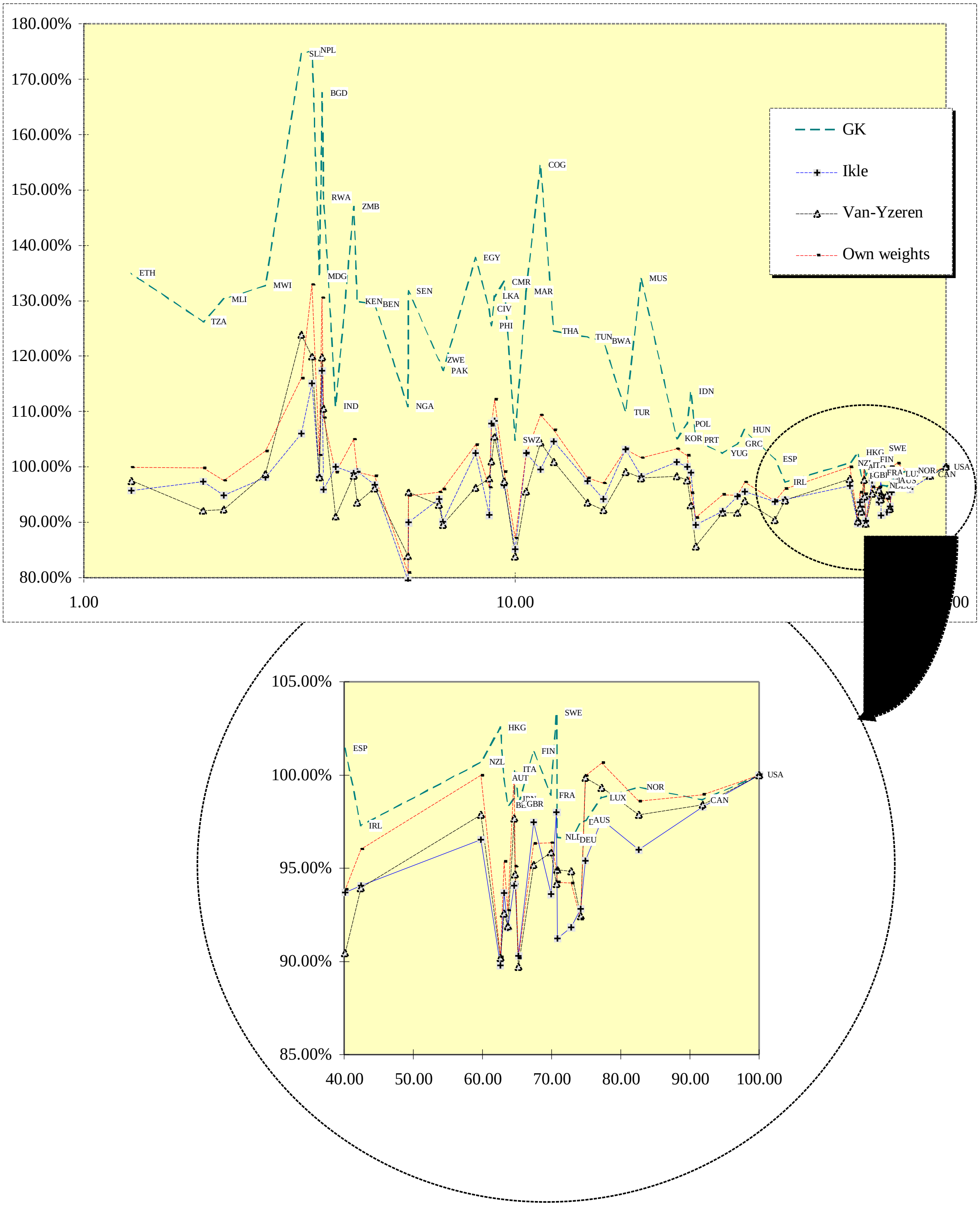

Figure 9. Deviation from EKS of per capita income according to various *additive* indexes (GGK), 1985

(Horizontal axis shows countries arranged according to their per capita income in EKS terms, US=100)

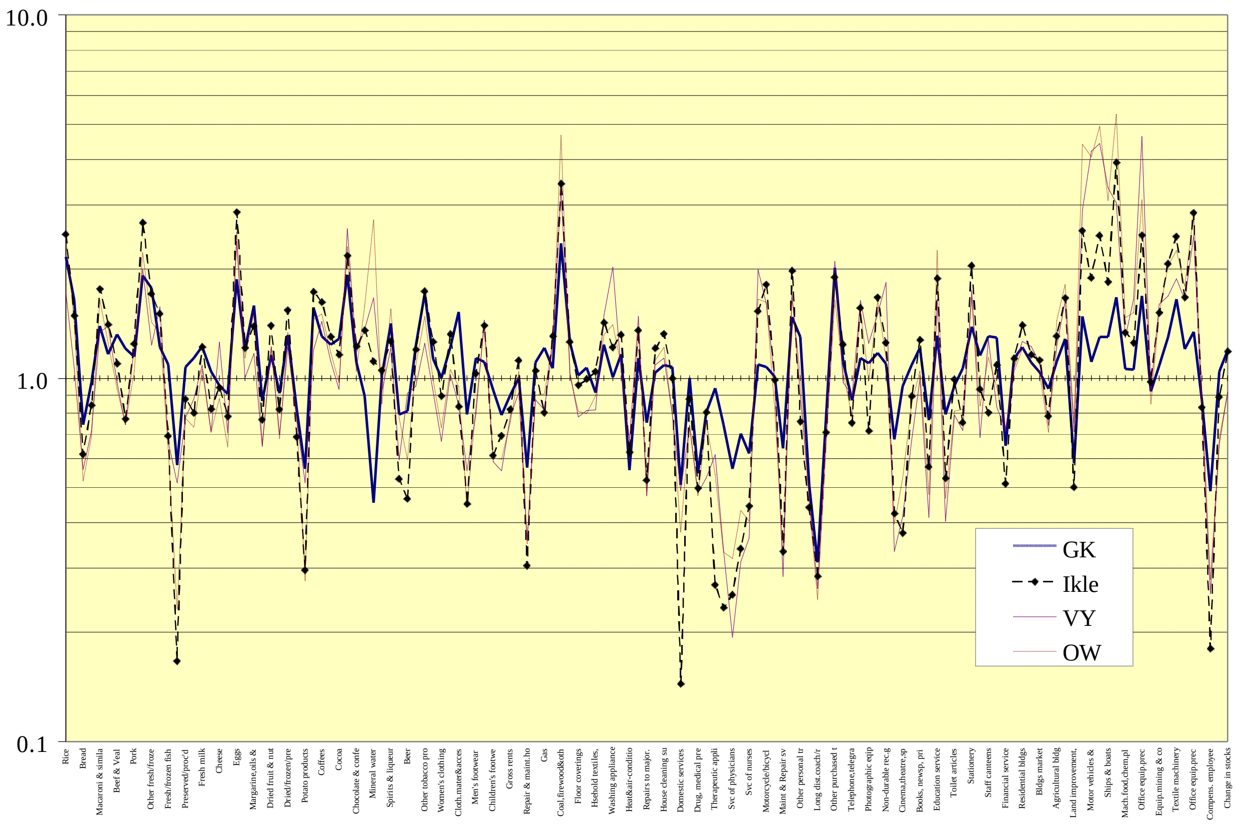

Figure 10. International prices according to various Geary-Khamis-Ikle indexes, 1985. (US prices = 1.00)

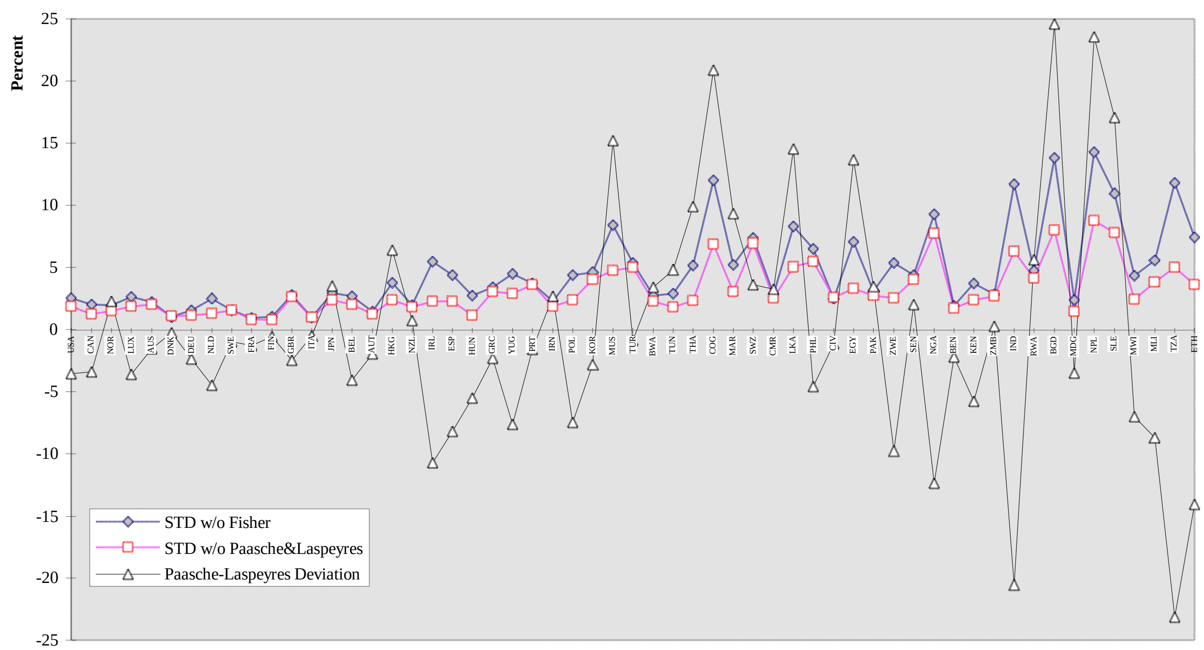

Figure 11. Sensitivity of country rankings to various aggregation indexes, by country, 1985

(standard deviation in standardized log-rankings across 11 indexes from Table 1)

Table 1. GDP per capita in International Dollars, 1985

Country	EKS	GK	Ikle	Van-Yzeren	Own weights	Rao	Fisher	Paasche	Laspeyres	Tornkvist	EKS(weighted)	Exchange Rate
United States	16,786	16,786	16,786	16,786	16,786	16,786	16,786	16,786	16,786	16,786	16,786	16,786
Canada	15,408	15,200	15,143	15,161	15,244	15,170	15,352	15,599	15,109	15,537	15,363	13,805
Norway	13,859	13,769	13,300	13,564	13,663	13,369	13,232	14,736	11,882	13,930	13,324	14,010
Luxembourg	12,953	12,794	12,641	12,866	13,036	12,842	12,650	13,217	12,107	12,953	12,660	9,416
Australia	12,562	12,256	11,983	12,541	12,556	12,217	12,127	13,070	11,252	12,687	12,205	10,648
Denmark	12,448	12,131	11,555	11,507	11,484	11,529	12,030	13,188	10,972	12,386	12,098	11,347
Germany	12,226	11,801	11,225	11,595	11,516	11,348	11,775	12,622	10,985	12,104	11,834	10,147
Netherlands	11,881	11,484	10,840	11,276	11,197	10,977	11,354	11,931	10,806	11,791	11,456	8,901
Sweden	11,861	12,254	11,626	11,168	11,260	11,465	12,111	13,148	11,157	12,041	11,878	12,051
France	11,724	11,597	10,973	11,237	11,296	11,126	11,503	12,540	10,552	11,597	11,550	9,482
Finland	11,314	11,461	11,028	10,769	10,897	10,968	10,971	12,155	9,903	11,357	11,090	11,026
United Kingdom	10,941	10,773	9,880	9,814	9,865	9,888	10,893	11,809	10,049	10,796	10,851	8,072
Italy	10,855	10,886	10,220	10,277	10,321	10,277	10,862	12,056	9,786	10,733	10,735	7,431
Japan	10,839	10,701	10,196	10,586	10,849	10,483	10,113	11,918	8,581	10,462	10,332	11,121
Belgium	10,687	10,509	9,809	9,818	9,910	9,880	10,748	11,450	10,088	10,584	10,619	8,099
Austria	10,584	10,568	9,913	9,798	10,092	10,032	10,592	11,600	9,671	10,422	10,526	8,624
Hong Kong	10,500	10,767	9,426	9,471	9,475	9,462	9,848	12,064	8,039	10,232	10,191	6,142
New Zealand	10,031	10,102	9,684	9,816	10,029	9,825	9,586	11,014	8,342	10,064	9,605	6,865
Ireland	7,111	6,917	6,689	6,681	6,827	6,779	6,935	7,305	6,584	6,996	6,979	5,315
Spain	6,730	6,826	6,306	6,086	6,315	6,342	6,754	7,364	6,194	6,697	6,751	4,301
Hungary	5,720	6,100	5,466	5,369	5,560	5,554	5,790	6,682	5,018	5,794	5,669	1,936
Greece	5,504	5,735	5,210	5,048	5,221	5,232	5,833	6,986	4,870	5,417	5,608	3,366
Yugoslavia	5,086	5,209	4,676	4,665	4,830	4,783	5,249	5,998	4,593	5,062	5,076	2,024
Portugal	4,412	4,624	3,947	3,779	4,005	4,006	4,425	5,583	3,507	4,297	4,351	2,042
Iran	4,292	4,877	4,247	3,993	4,091	4,209	4,385	5,831	3,298	4,307	4,288	3,880
Poland	4,210	4,536	4,211	4,109	4,296	4,282	4,084	4,860	3,433	4,161	4,056	1,908
Korea	3,984	4,189	4,020	3,918	4,114	4,079	3,666	4,691	2,865	3,936	3,758	2,277
Mauritius	3,296	4,418	3,238	3,227	3,348	3,321	3,261	5,284	2,012	3,231	3,169	1,055
Turkey	3,033	3,332	3,127	3,005	3,129	3,142	2,764	3,785	2,019	3,208	2,770	1,049
Botswana	2,696	3,307	2,539	2,486	2,617	2,586	2,704	3,914	1,868	2,650	2,649	1,058
Tunisia	2,472	3,051	2,407	2,311	2,422	2,432	2,475	3,694	1,658	2,449	2,421	1,140
Thailand	2,067	2,575	2,162	2,086	2,204	2,205	2,136	3,470	1,315	2,073	2,036	722
Congo	1,925	2,975	1,916	2,010	2,105	2,048	2,152	3,982	1,163	1,856	1,983	1,115
Morocco	1,781	2,345	1,824	1,701	1,825	1,838	1,758	2,924	1,057	1,829	1,706	584
Swaziland	1,679	1,760	1,430	1,407	1,462	1,449	1,722	2,681	1,106	1,660	1,694	548
Cameroon	1,584	2,112	1,532	1,542	1,569	1,563	1,588	2,494	1,011	1,526	1,549	817
Sri Lanka	1,504	1,968	1,630	1,586	1,687	1,665	1,456	2,654	798	1,550	1,411	384
Philippines	1,480	1,856	1,595	1,494	1,591	1,606	1,348	1,976	919	1,463	1,365	562
Cote d'Ivoire	1,461	1,878	1,334	1,430	1,407	1,382	1,448	2,188	958	1,414	1,418	716
Egypt	1,362	1,876	1,396	1,311	1,416	1,423	1,356	2,473	744	1,350	1,309	746
Pakistan	1,144	1,343	1,031	1,025	1,098	1,081	1,104	1,841	663	1,114	1,095	324
Zimbabwe	1,119	1,334	1,053	1,042	1,067	1,061	1,061	1,517	741	1,114	1,060	538
Senegal	951	1,254	856	908	902	883	1,010	1,676	608	899	974	404
Nigeria	947	1,051	753	795	766	766	890	1,268	624	934	884	973
Benin	794	1,027	768	763	780	780	791	1,297	482	801	764	259
Kenya	722	938	716	675	715	721	699	1,119	436	728	682	303
Zambia	709	1,043	701	698	744	732	771	1,308	455	694	728	334
India	644	715	644	587	638	646	596	819	433	652	599	280
Rwanda	604	898	580	668	658	624	641	1,194	344	568	611	288
Bangladesh	600	1,005	704	719	783	757	678	1,572	292	576	629	135
Madagascar	592	795	581	580	604	598	576	977	339	576	567	286
Nepal	568	996	654	682	755	719	720	1,633	317	533	642	150
Sierra Leone	536	936	568	664	622	604	604	1,304	280	500	561	360
Malawi	443	588	434	437	455	446	425	726	249	439	426	157
Mali	355	462	336	327	346	344	376	640	221	343	357	157
Tanzania	317	401	309	292	317	316	340	495	234	313	323	284
Ethiopia	217	292	207	211	216	212	227	392	131	206	218	110
St. deviation from EKS, %		*20.3*	*6.3*	*7.4*	*8.5*	*7.6*	*6.1*	*40.9*	*14.6*	*2.3*	*3.5*	*23.4*

Table 2. Correlation matrix of various aggregation procedures

	EKS	***GK***	***Ikle***	***Van-Yzeren***	***Own weights***	***Rao***	***Fisher***	***Paasche***	***Laspeyres***	***Tornkvist***	***EKS(weighted)***	***Exchange Rate***
EKS	1.0000											
GK	0.9966	1.0000										
Ikle	0.9987	0.9979	1.0000									
Van-Yzeren	0.9983	0.9986	0.9993	1.0000								
Own weights	0.9981	0.9985	0.9997	0.9997	1.0000							
Rao	0.9984	0.9984	0.9999	0.9996	0.9999	1.0000						
Fisher	0.9990	0.9978	0.9983	0.9985	0.9983	0.9984	1.0000					
Paasche	0.9908	0.9973	0.9929	0.9942	0.9944	0.9939	0.9941	1.0000				
Laspeyres	0.9976	0.9907	0.9950	0.9942	0.9938	0.9943	0.9969	0.9825	1.0000			
Tornkvist	0.9999	0.9960	0.9987	0.9979	0.9979	0.9983	0.9985	0.9898	0.9974	1.0000		
EKS(weighted)	0.9996	0.9973	0.9986	0.9986	0.9984	0.9985	0.9998	0.9928	0.9976	0.9992	1.0000	
Exchange Rate	0.9760	0.9688	0.9715	0.9730	0.9702	0.9706	0.9750	0.9572	0.9808	0.9751	0.9761	1.0000