Trade Liberalization, Wage Rigidity, and Labor Market Dynamics with Heterogeneous Firms

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International Seminar on Trade
Washington, DC, May 2025

Motivation

- Trade liberalization shocks entail **significant labor market responses**
  - labor income loss, temporary unemployment, job loss (Autor, Dorn, Hanson 2013)
  - firms exit and heterogeneous adjustment strategies (Verhoogen 2008)
  - firm employment history is at least as important as industry, skill and occupation (Amiti and Davis 2012; Helpman et al. 2017)

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- **This paper:** theoretical and quantitative analysis of **labor market transitions** in response to a reduction in trade costs
  - positive aggregate shock with heterogeneous firms outcomes
  - heterogeneous workers' outcomes in frictional LM (search frictions + wage rigidity)
  - dissipated gains from trade, winners & losers (good vs bad jobs)

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  - Sectoral reallocation: Kambourov (2009); Artuç et al. (2010); Coşar (2013); Dix-Carneiro (2014); Traiberman (2019)
  - Steady-state comparisons with heterogen. firms: Cacciatore (2014); Coşar et al. (2016)
  - Regional mobility: Caliendo et al. (2019); Kim and Vogel (2018); Rodríguez-Clare et al. (2022)

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- **The role of LM frictions → firms' adjustment after the trade shock**
  - Firms heterogeneity and search frictions: Elsby and Michaels (2013); Kircher and Kaas (2013); Acemoglu and Hawkins (2014); Fajgelbaum (2020); Bilal et al. (2022)
  - Wage rigidities: Flinn (2006); Engbom and Moser (2022); Costinot et al. (2022)

Results Preview

- Dynamic adjustment to a trade shock features **slow transition**
  - delayed productive entry due to slowly shrinking unproductive incumbents
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- **Instantaneous consumer gains from trade** due to free entry
  - do not depend on labor market frictions
  - focus on short-run misallocation of employment and delayed exit
- LM frictions result in **profit loss, job destruction and depressed wages**
  - 'good jobs' vs 'bad jobs'
  - search and matching frictions shield workers from unemployment
  - labor markets frictions redistribute losses from workers to firms

Model

Environment

- Two symmetric countries
- Two sectors (w/ free entry):
  - homogeneous non-traded good (numeraire)
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- Permanent unanticipated **reduction in bilateral trade cost**
  - discrete time with short time intervals, $\Delta \approx 0$

Consumer Demand

- Representative household with flow utility $\mathcal{U}(q_{0t}, Q_t)$ and discount rate r
- CES aggregator of differentiated traded varieties

$$Q = \left(\int_{\omega \in \Omega} q(\omega)^{\beta} d\omega \right)^{1/\beta}, \quad 0 < \beta < 1$$

- The utility function is quasi-linear

$$\mathcal{U}(q_0, Q) = q_0 + \frac{1}{\zeta} Q^{\zeta}, \quad 0 < \zeta < \beta$$

$\rightarrow$ period-t utility is then: $\mathcal{U}_t = I_t + \frac{1-\zeta}{\zeta} Q_t^{\zeta}$ with income I_t.

Households Labor Supply

- Unit-continuum of households with L units of labor:
  - N workers are assigned to differentiated traded sector
  - $N_0 = L - N$ workers are assigned to non-traded sector
- Workers can be employed H or unemployed (searching) U with
  - exogenous separation rate s
  - equilibrium job-finding rate x
- Unemployed workers are mobile across sectors

Non-traded Sector

- CRS Cobb-Douglas matching function $\rightarrow$ the cost of filling a vacancy is details

$$b_0 = \frac{\text{Vacancy-posting cost}}{\text{Job-filling rate}} = a_0 x_0^{\alpha}$$

- Single-worker firms, each worker produces 1 unit of output
- Wages are Nash bargained without commitment

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- Wages are Nash bargained without commitment
- **Assumption:** *L is large enough that along the equilibrium path $U_{0t} > 0, \forall t$.*
- **Lemma:** *(i) (x_0, b_0) are constant; (ii) The value of unemployed is constant and given by*

$$rJ_0^U = b_u + x_0 b_0$$

proof

Traded Sector: Setup

- Entrant draws productivity θ from Pareto distribution, w/ fixed cost of entry f_e
- Production technology $y = \theta h$, where h is employment level
- Export decision $\iota \in \{0, 1\}$ at fixed cost f_x and iceberg cost $\tau > 1$
- The optimal revenue in each period is

$$R(h, \iota; \theta) = \Theta(\iota; \theta)^{1-\beta} h^{\beta}, \qquad \Theta(\iota; \theta) = \left[1 + \iota \tau^{-\frac{\beta}{1-\beta}}\right] Q^{-\frac{\beta-\zeta}{1-\beta}} \theta^{\frac{\beta}{1-\beta}}$$

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- Nash bargaining w/o commitment: $J^E(h) - J^U = J_h^F(h) \quad \Rightarrow w(h)$ details
- Firms die at rate δ and existing matches are destroyed at rate $\sigma \rightarrow$ exogenous separation rate $s \equiv \delta + \sigma$

Traded Sector: Problem of a Firm

- The Bellman equation of a firm θ with export status $\iota \in \{0, 1\}$

$$J^F(h) = \max_{h'} \left\{ \varphi(h) - b[h' - (1-\sigma)h]^+ + \frac{1-\delta}{1+r} J_+^F(h') \right\}$$

Per-period operating profit is given by $\varphi(h) = R(h) - w(h)h - f_d - \iota f_x$ FOC + ET

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$$J_h^F(h') = b \qquad \Rightarrow \qquad \varphi'(h) = (r+s)b$$

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- **Lemma:** *(a)* (x, b) *are constant and satisfy* $xb = x_0 b_0$. *(b) Equilibrium employment conditional on hiring is*

$$h = \Phi^{1/\beta} \Theta(\iota; \theta), \qquad \text{where} \quad \Phi \equiv \left[\frac{\beta}{1+\beta} \frac{2}{b_u + [2(r+s)+x]b} \right]^{\frac{\beta}{1-\beta}}.$$

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- Full analytical characterization for DNWR $w \geq w_{\min}$ ⏩ details ⏩ value of entry

Steady State

Long-run Gains from Trade

- **Total welfare** is given by

$$W = \underbrace{\frac{x}{x+s} wL}_{\text{wage payments}} + \underbrace{\frac{1-\zeta}{\zeta} Q^{\zeta}}_{\text{consumer surplus}}$$

Long-run Gains from Trade

- **Total welfare** is given by

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- **Proposition:** *Comparing steady states after trade costs reduction, we find:*
  - (i) Q^{ζ}, H, *and* M *increase proportionally.*
  - (ii) *Aggregate employment and labor income don't change.*
  - (iii) *Welfare rises due to an increase in CS only.* details

Transition Dynamics

Dynamic Gains

- Consider a one-time unanticipated and permanent reduction in trade cost, $\tau' < \tau$
- **Proposition:** *Along the transition path, $Q_t \geq Q'$. If there is entry in every period, then $Q_t \equiv Q'$.* proof

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- **Corollary:** Dynamic gains in consumer surplus are instantaneous and do not depend on labor market frictions!
- Income I_t **changes throughout the transition:**
  - decrease in the aggregate value of firms
  - decrease in the value of employed at shrinking firms
  - endogenous separation into unemployment (firing)

Adjustment Strategies

- With entry, gains in CS (reduction in price level) do not depend on $\mathcal{G}_t(h, \theta)$
- Agg. product., trade, employment and income depend on $\mathcal{G}_t(h, \theta)$ and LM frictions

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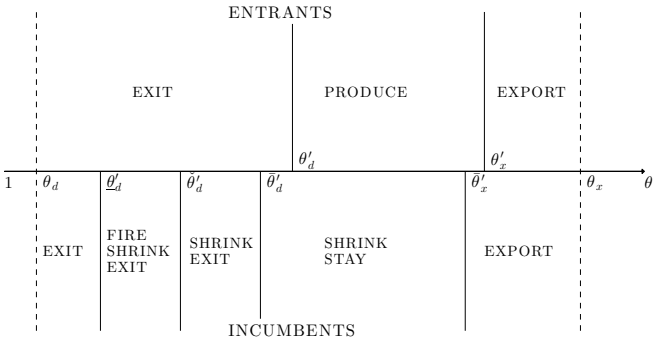

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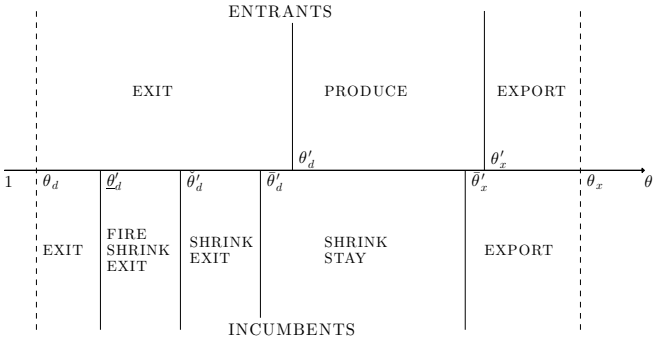
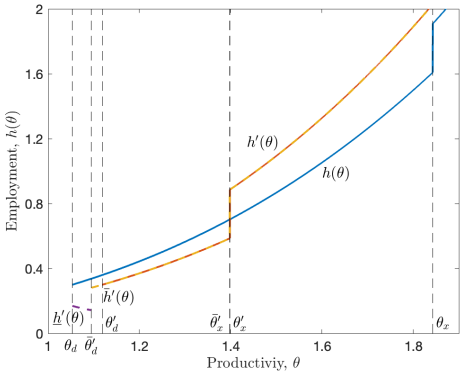
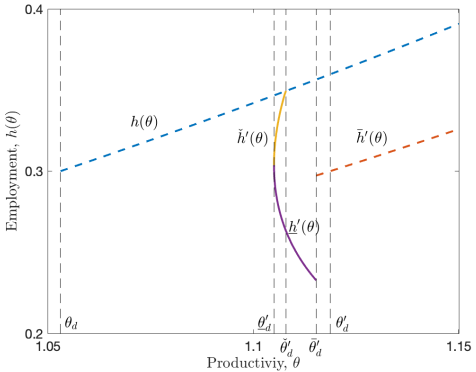


- full analytical characterization in the paper, now numeral simulations ⏩ parameters

Employment Adjustment

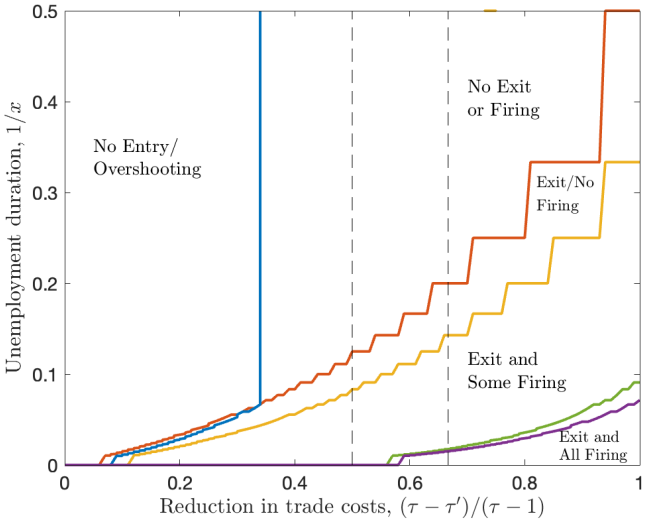


(a) $x = 2$, $\tau' = 1.375$

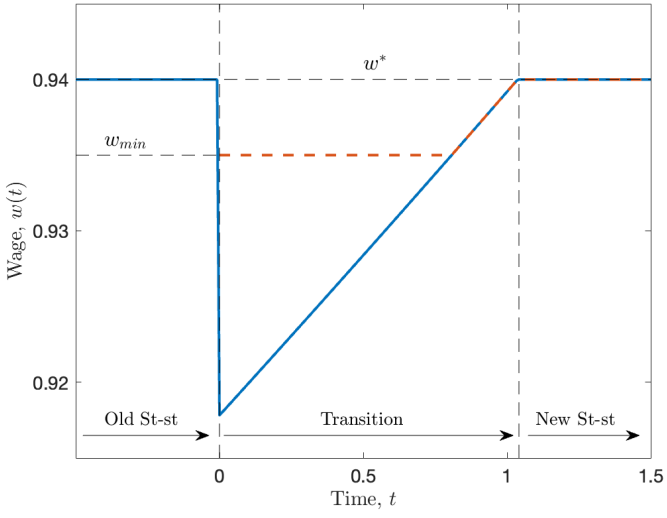


(b) $x = 24$, $\tau' = 1.25$

Adjustment Patterns: Regions of the Parameter Space

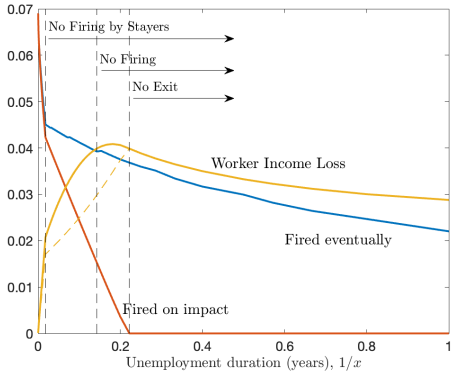


Nominal Wage Dynamics

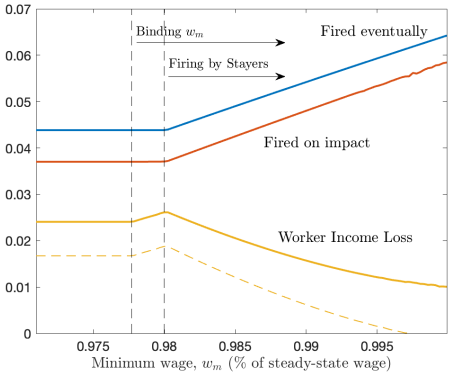


details

Displacement Rates and Income Losses

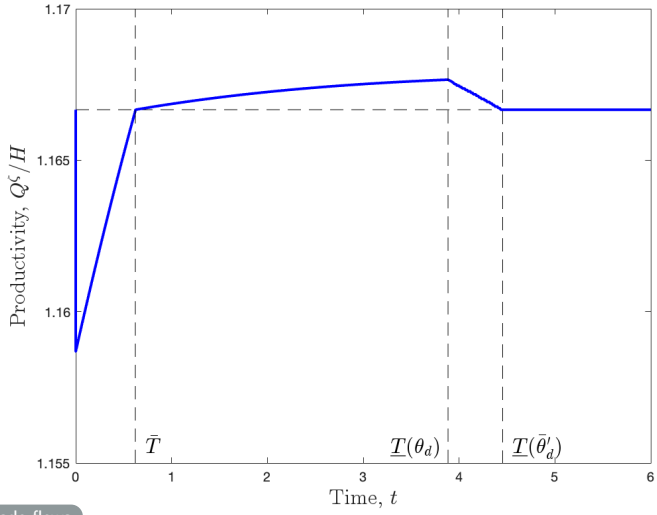


(a) unemployment duration, $1/x$ (no w_m)



(b) minimum wage, w_m ($x = 24$)

Aggregate Productivity Dynamics



rigid wages | trade flows

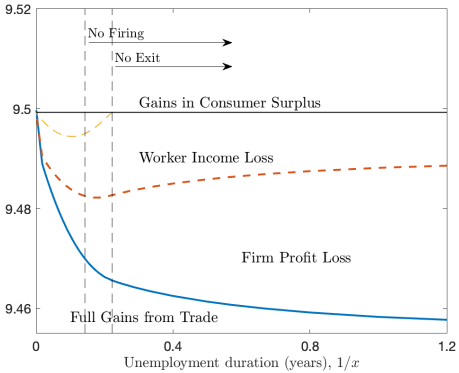
Decomposition of Dynamic Gains from Trade

- The annualized change in consumer surplus is $\frac{1-\zeta}{\zeta}(Q'^{\zeta} - Q^{\zeta})$
- The annualized flow change of wage income is $r(W' - W)$
- The annualized flow change of incumbents' value is $r(J^{F'} - J^{F})$

Dynamic gains from trade are equal to

$$GT = \underbrace{\frac{(1-\zeta)(Q'^{\zeta} - Q^{\zeta})}{\zeta Q^{\zeta}}}_{\Delta CS} + \underbrace{\frac{r(W' - W)}{Q^{\zeta}}}_{\Delta\text{Workers' Income}} + \underbrace{\frac{r(J^{F'} - J^{F})}{Q^{\zeta}}}_{\Delta\text{Firms' Profit}}$$

LM Frictions and Dynamic Gains from Trade



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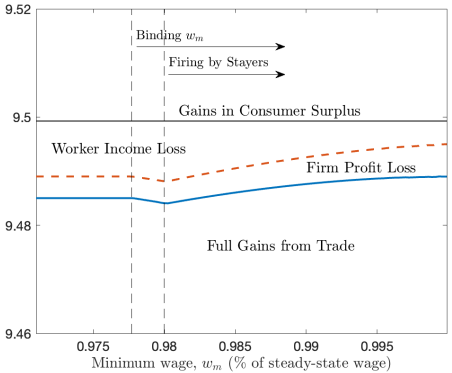

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Conclusion

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  - misallocation of labor and reduced productivity
  - decline in value of exports
  - bad jobs and good jobs
  - instantaneous gains in CS independent from the extent of frictions

- Quantitatively modest disruptions in the labor market

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  - misallocation of labor and reduced productivity
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  - bad jobs and good jobs
  - instantaneous gains in CS independent from the extent of frictions

- Quantitatively modest disruptions in the labor market

- Strong assumptions to relax in future work:
  - Linear hiring costs
  - Frictionless free entry
  - Perfect labor mobility across sectors
  - Symmetric countries/single traded sector

Thank you!

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Matching Technology

Matching function with constant returns to scale

$$m(U, V) = \frac{1}{\tilde{a}} U^{\chi} V^{1-\chi}$$

where U - the stock of unemployed workers, V - the stock of open vacancies, and $\tilde{a}$ is the inverse of productivity of matching. The job finding rate is

$$x = \frac{m}{U} = \frac{1}{\tilde{a}} \left(\frac{V}{U} \right)^{1-\chi}$$

and the vacancy filling rate is

$$g = \frac{m}{V} = \frac{1}{\tilde{a}} \left(\frac{V}{U} \right)^{-\chi}$$

Cost of Filling Vacancy

Suppose the flow cost of posting a vacancy as $\gamma\Delta$ (where Δ is sufficiently small), then the cost of filling a vacancy is equal to

$$b = \frac{\gamma\Delta}{g\Delta} = \frac{\gamma}{g} = ax^{\alpha}$$

where $\alpha = \frac{\chi}{1-\chi}$ and $a = \gamma\tilde{a}^{1+\alpha}$.

back to slides

Constant Job-Finding Rate

1. $U_0 > 0$ ensures entry of firms (vacancy posting) $\rightarrow \quad J_0^V \equiv 0 \quad$ and $\quad J_0^F = b_0$
2. Then Nash bargaining results in $J_0^E - J_0^U = J_0^F = b_0$
3. Surplus from employment satisfies:

$$(r + s_0)J_0^F = (1 - w_0) + \dot{J}_0^F,$$
$$(r + s_0 + x_0)(J_0^E - J_0^U) = (w_0 - b_u) + (\dot{J}_0^E - \dot{J}_0^U)$$

   Has unique stationary solution (x_0, b_0) with $\dot{b}_0 = 0$

4. Finally, the value of unemployed and equilibrium wage are:

$$rJ_0^U = b_u + x_0 b_0,$$
$$w_0 = b_u + (r + s_0 + x_0)b_0.$$

back to slides

Firm Optimization

- FOC wrt h' is given by

$$\frac{1-\delta\Delta}{1+r\Delta}J^F_{h,+}(h') = \begin{cases} b, & \text{if } h' > (1-\sigma\Delta)h, \\ \in [0,b], & \text{if } h' = (1-\sigma\Delta)h, \\ 0, & \text{if } h' < (1-\sigma\Delta)h \end{cases}$$

- Envelope Theorem

$$J^F_h(h) = \varphi'(h)\Delta + \frac{1-s\Delta}{1+r\Delta}J^F_{h,+}(h')$$

back to slides

Nash Bargaining

- Nash bargaining with commitment:

$$J^E(h) - J^U = J_h^F(h)$$

- from which

$$\varphi'(h) = w(h) - \Delta^U, \quad \text{where } \Delta^U = -\frac{1}{\Delta}\left(\frac{1}{1+r\Delta}J_+^U - J^U\right)$$

- Using the definition of $\varphi(h)$,

$$R'(h) - w'(h)h - w(h) = w(h) - \Delta^U$$

- After integrating,

$$w(h) = \frac{\beta}{1+\beta}\frac{R(h)}{h} + \frac{1}{2}\Delta^U$$

back to slides

Value of the Entrant

1. The value of a hiring entrant ($bh' > (1-\sigma\Delta)h$)

$$J^F(h) = \varphi(h)\Delta + (1-\sigma\Delta)h - bh' + \frac{1-\delta\Delta}{1+r\Delta}J^F_+(h')$$

2. FOC together with the Envelope Theorem give us

$$\varphi'(h)\Delta + (1-\sigma\Delta)b = \frac{1+r\Delta}{1-\delta\Delta}b_-$$

3. Combining (1) and (2)

$$\underbrace{J^F(h) - \frac{1+r\Delta}{1-\delta\Delta}b_- h}_{\frac{1-\delta\Delta}{1+r\Delta}J^F_{-1}(0)} = \underbrace{(\varphi(h) - \varphi'(h)h)\Delta}_{\frac{1-\beta}{1+\beta}\Phi\Theta(\iota;\theta) - f_d - \iota f_x} + \underbrace{\frac{1-\delta\Delta}{1+r\Delta}J^F_+ - bh'(h')}_{J^F(0)}$$

$$(r+\delta)J^V(\theta) - j^V(\theta) = \max_{\iota\in\{0,1\}}\left\{\frac{1-\beta}{1+\beta}\Phi\Theta(\iota;\theta) - f_d - \iota f_x\right\}.$$

back to slides

Traded Sector: Downward Wage Rigidity

- Minimum wage or downward wage rigidity: $w \geq w_{\min}$

- **Lemma 5:** *(a) Wages of a non-hiring firm with h workers:*

$$\bar{w}(h, \iota; \theta) = \max\{w(h, \iota; \theta), w_{\min}\}.$$

(b) Hiring firm pays $\bar{w} = \max\{w, w_{\min}\}$ *and hires:*

$$\bar{h}(\theta) = \bar{\Phi}^{\frac{1}{\beta}} \Theta(\iota; \theta), \qquad \bar{\Phi} \equiv \left(\max\left\{ \frac{2\beta}{1+\beta} \frac{1}{2\bar{w} - rJ^U}, \frac{\beta}{\bar{w} + (r+s)b} \right\} \right)^{\frac{\beta}{1-\beta}},$$

and $rJ^U = b_u + \frac{x}{r+s+x}(\bar{w} - b_u)$.

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Downward Wage Rigidity and Optimal Employment

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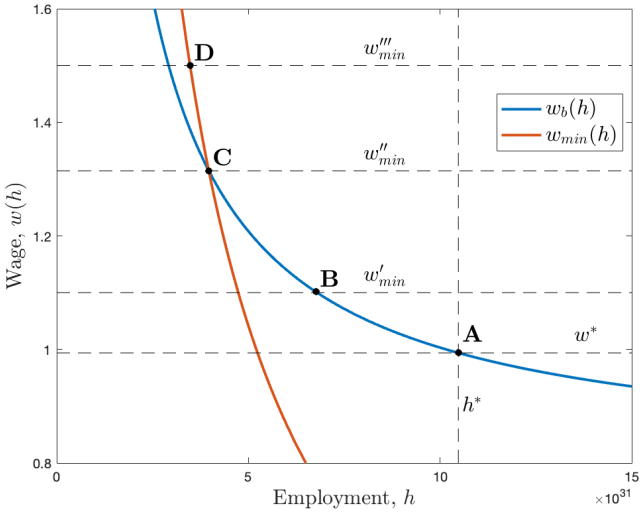

Steady State Aggregates

- The number of firms evolves as

$$M_{t+1} = (1 - \delta\Delta)M_t + m_t^e$$

  where m_t^e is the number of entrants.
- The aggregate employment in differentiated sector is

$$H = M \int h d\mathcal{G}(h, \theta)$$

- In steady state,

$$H = \Phi^{\frac{1-\beta}{\beta}} Q^{\zeta}$$

$$H_0 = \frac{xL}{x+s} - \Phi^{\frac{1-\beta}{\beta}} Q^{\zeta}$$

$$M = \frac{1}{k(r+\delta)f_e} \frac{\beta}{1+\beta} Q^{\zeta}$$

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Productivity Cutoffs

- Without minimum wage in steady state

$$\theta_d = \left[\frac{1 + \tau^{-k} (f_x/f_d)^{1-k\frac{1-\beta}{\beta}}}{(\frac{1-\beta}{\beta} k - 1)(r+\delta)/(1-\delta\Delta)} \frac{f_d}{f_e} \right]^{1/k}$$

$$\theta_x = \tau \left(\frac{f_x}{f_d} \right)^{\frac{1-\beta}{\beta}} \left[\frac{1 + \tau^{-k} (f_x/f_d)^{1-k\frac{1-\beta}{\beta}}}{(\frac{1-\beta}{\beta} k - 1)(r+\delta)/(1-\delta\Delta)} \frac{f_d}{f_e} \right]^{1/k}$$

$$Q = \left[\frac{1-\beta}{1+\beta} \Phi \frac{1}{f_d} \right]^{\frac{1-\beta}{\beta-\zeta}} \left[\frac{1 + \tau^{-k} (f_x/f_d)^{1-k\frac{1-\beta}{\beta}}}{(\frac{1-\beta}{\beta} k - 1)(r+\delta)/(1-\delta\Delta)} \frac{f_d}{f_e} \right]^{\frac{\beta}{k(\beta-\zeta)}}$$

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Proof of Proposition 2

- **Proposition 2:** *Along the transition path,* $Q_t \geq Q'$. *If there is entry in every period, then* $Q_t \equiv Q'$.
- **Proof:** $\tau_t = \tau'$ and $b_t = b$. Two cases:
  (i) Continuous entry, free entry implies $\int J^V(\theta) dG_\theta = (r + \delta) f_e \;\Rightarrow\; Q_t = Q'$.
  (ii) No entry in $t \in [0, T_e)$. Then $\dot{Q}_t < 0$ and $Q_{T_e} = Q'$ (short-run overshooting).

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Parameter Values

Moment	Parameter	Value	Comment
Discount rate	r	0.05	
Exogenous separation rate	s	0.2	$s_0 = s$
— Labor force attrition rate	σ	0.175	
— Firm death rate	δ	0.025	
Job-finding rate	x	2	$a_0 = a = 0.12$
Relative elasticity of matching	α	1	
Unemployment benefit	b_u	0.4	
Pareto shape parameter	k	4	
CES within sector	ε	4	$\beta = 3/4$
Semi-elasticity across sectors		2	$\zeta = 1/2$
Rigid wage	$w_{\min}$	0.97	$(w - w_{\min})/w = 2\%$
Employment share in the traded sector		14%	$L = 10$, $f_d = 0.05$
Fraction of exitors		25%	$(r + \delta)f_e/f_d = 2.7$
Fraction of exporters		11%	$f_x/f_d = 1$
Fraction of output exported		16%	$\tau = 1.75$
Trade liberalization	τ'	1.375	
— Fraction of exporters		28%	

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Dynamic Wages

- Firms that are gradually shrinking

$$w_t = \frac{\beta}{1+\beta}\frac{R(h_t)}{h_t} + \frac{1}{2}\Delta^U$$

- Fire, shrink, exit. Time until exit

$$\check{T}_e(\theta) = -\frac{1}{\sigma}\log\left(\frac{\underline{h}'(\theta)}{\check{h}'_e(\theta)}\right)$$

and dynamic wage is equal to

$$w_t(\theta) = \frac{\beta}{1+\beta}Q'^{-(\beta-\zeta)}\theta^{\beta}e^{-\sigma(\beta-1)(t-\check{T}_e(\theta))}\left(\underline{h}'(\theta)\right)^{\beta-1} + \frac{1}{2}\Delta^U$$

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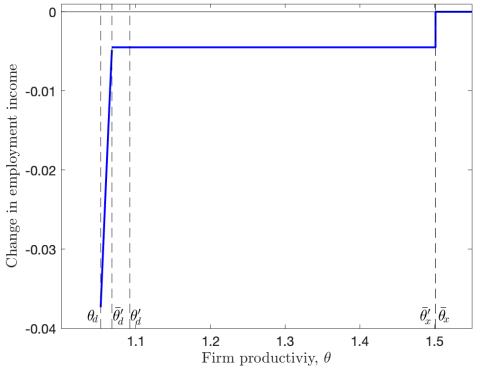
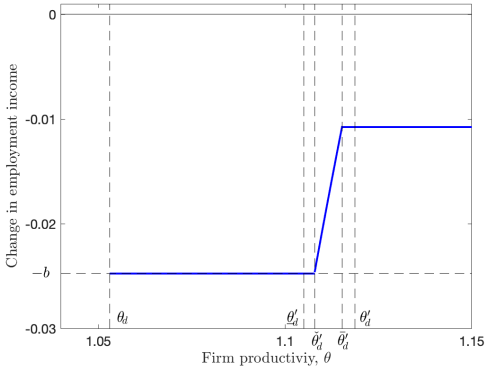
Loss in Value of Employment



(a) no wage rigidity, $x = 2$



(b) no wage rigidity, $x = 24$

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Loss in Value of Employment: Rigid Wage

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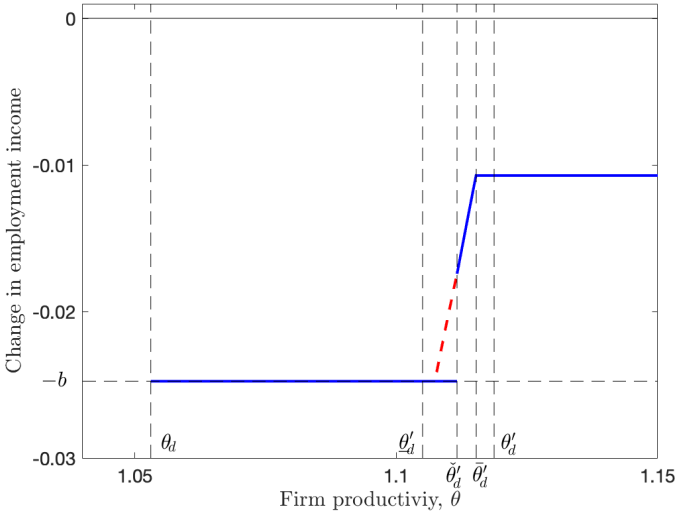
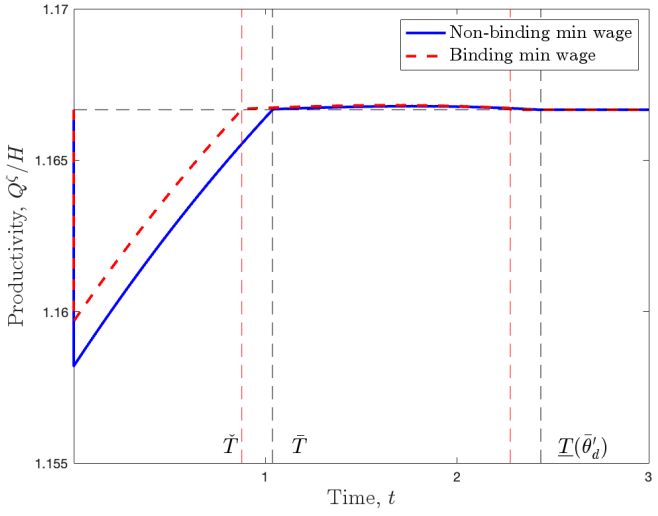


Aggregate Productivity Dynamics



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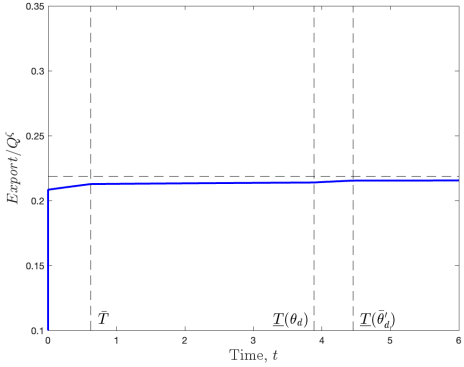
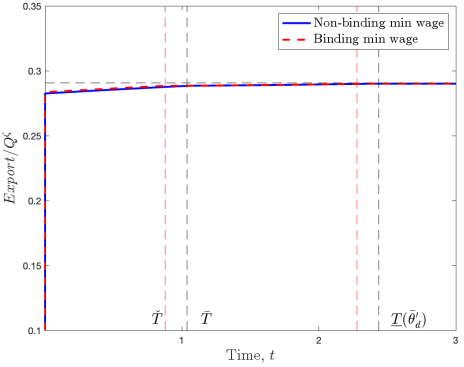
Trade Flows Dynamics



(a) $x = 2$, $\tau' = 1.375$



(b) $x = 24$, $\tau' = 1.25$, w/ and w/o min wage

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