Trade Models, Trade Elasticities, and the Gains from Trade

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Big Picture: Welfare Gains From Trade

How do different margins of trade affect estimates of the welfare gains?

Big Picture: Welfare Gains From Trade

How do different margins of trade affect estimates of the welfare gains?

One answer: Arkolakis, Costinot, and Rodríguez-Clare's (2011) result...

$$\text{Welfare cost of autarky} = \lambda_n^{-\frac{1}{\theta}}.$$

Holds for a wide class of models...

- Armington, Krugman (1980), Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), Melitz (2003) à la Chaney (2008).

Thus, given a common estimate of θ, the welfare gains are the same.

Big Picture: Welfare Gains From Trade

How do different margins of trade affect estimates of the welfare gains?

Our answer:

1. Trade models with various micro-level margins require lower trade elasticities (θs) to generate a given amount of price dispersion and trade.

   $\Rightarrow$ Richer models yield larger welfare gains.

2. Quantitatively, these differences are large.

3. Approaches that imply a common estimate of θ (Caliendo and Parro, '11) have no effect on our quantitative results.

Today's Talk

1. Outline three trade models:
   - Anderson (1979)—Armington,
   - Eaton and Kortum (2002)—EK,
   - Bernard, Eaton, Jensen, and Kortum (2003)—BEJK,
2. Theory (in pictures): Armington $<$ EK $<$ BEJK in welfare.
3. Estimation: Quantify the difference.
4. Extensions: (a) Macro estimation, (b) Endogenous variety models.

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Three Fixed Variety Trade Models

Anderson (1979)—Armington.

- Unit measure of tradable goods, subset differentiated by source $i = 1, \ldots, N$. Each country produces subset of measure $1/N$. CES preferences.
- Perfect competition; price reflects marginal cost.
- All goods traded/intensive margin only.

Eaton and Kortum (2002)—EK.

- Unit measure of tradable goods. Not differentiated by source. CES preferences.
- Producers differ in productivity; from Fréchet distribution w/ parameter θ.
- Perfect competition; price reflects marginal cost.
- Endogenous tradability/extensive margin.

Bernard, Eaton, Jensen, and Kortum (2002)—BEJK.

- Same as EK, but with Bertrand competition; price reflects next best competitor.
- Productivity for first and second best competitor in a country from a modified, Fréchet like distribution w/ parameter θ.
- Extensive margin + variable markups.

Trade Flows in All Models

Normalized trade flows of i goods to n,

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\Phi_i}{\Phi_n}\tau_{ni}^{-\theta}.$$

where $\Phi_n = \sum_{k=1}^{N} T_k(\tau_{nk} w_k)^{-\theta}$.

In all these models, θ controls ...

- love of variety in Armington; dispersion in micro-level productivity (previous slides),
- response of trade to changes in trade costs (above),
- welfare gains from trade (first slide).

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Theory Overview

Two symmetric countries, home and foreign. Trade flows are:

$$\frac{X_{h,f}/X_h}{X_{f,f}/X_f} = \tau_{h,f}^{-\theta}.$$

and we constrain trade to be the same across θs and models.

And then ask...

- How do inferences about θ using order statistics for relative prices rank across models?

Price Gaps

Definition

For any good, define the logged price gap between the home and the foreign country as $\log \tilde{P}_h \equiv \log P_h - \log P_f$.

Definition

Given a model $\mathcal{M}$, denote the cumulative density function of positive log price gaps as:

$$\forall \text{ models } \mathcal{M}, \quad G_{\mathcal{M}}(\log \tilde{p}_h, \theta) = \begin{cases} 0 & : \log \tilde{P}_h < 0 \\ \log \tilde{p}_h \in [0,1] & : 0 \leq \log \tilde{P}_h \leq \log \tau \\ 1 & : \log \tau < \log \tilde{P}_h. \end{cases}$$

Order Statistics

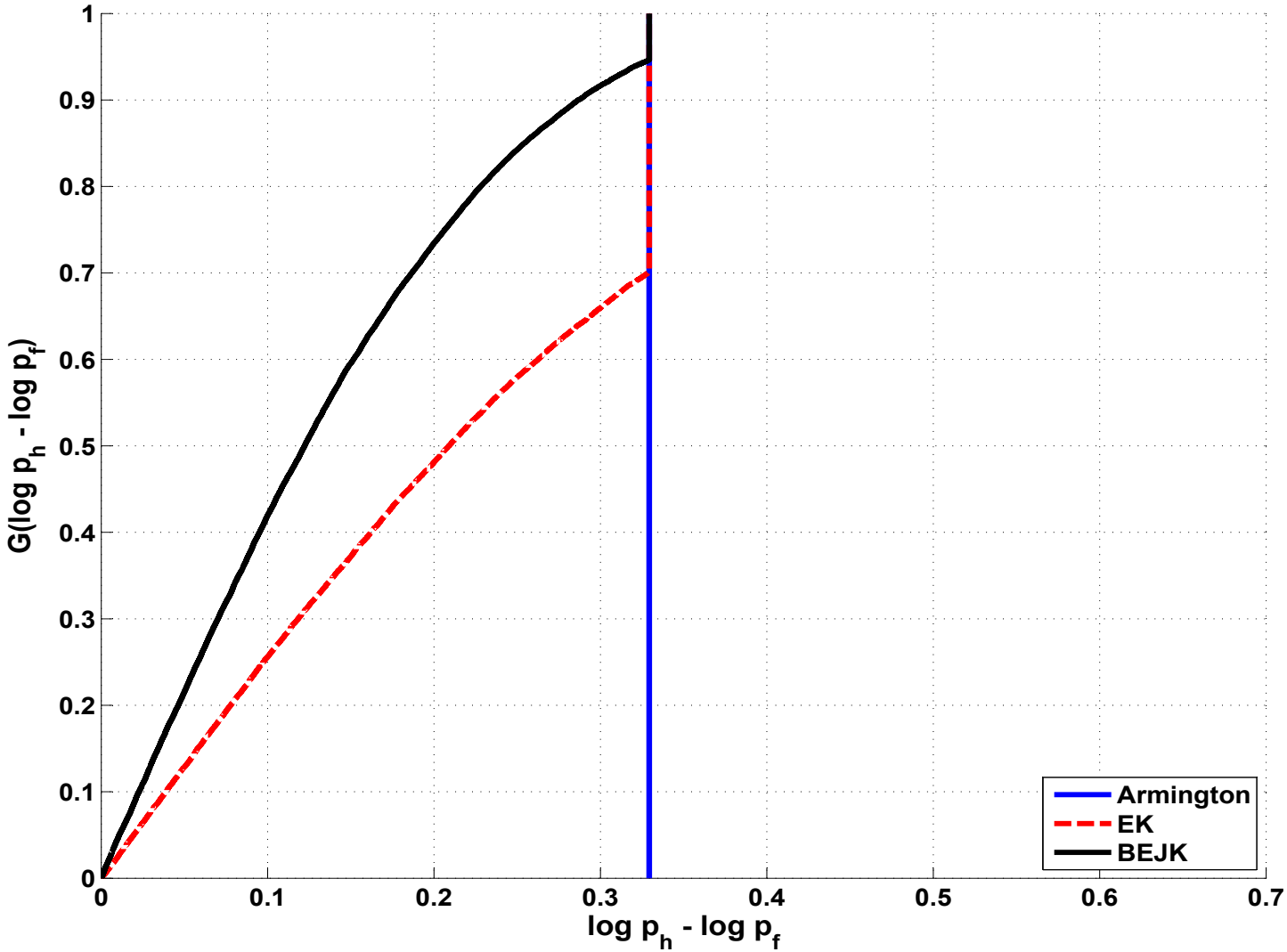
Definition

Consider a random sample of positive price gaps $\log \tilde{P}_{h,1}, \log \tilde{P}_{h,2}, \ldots \log \tilde{P}_{h,L}$ ordered from smallest to largest;

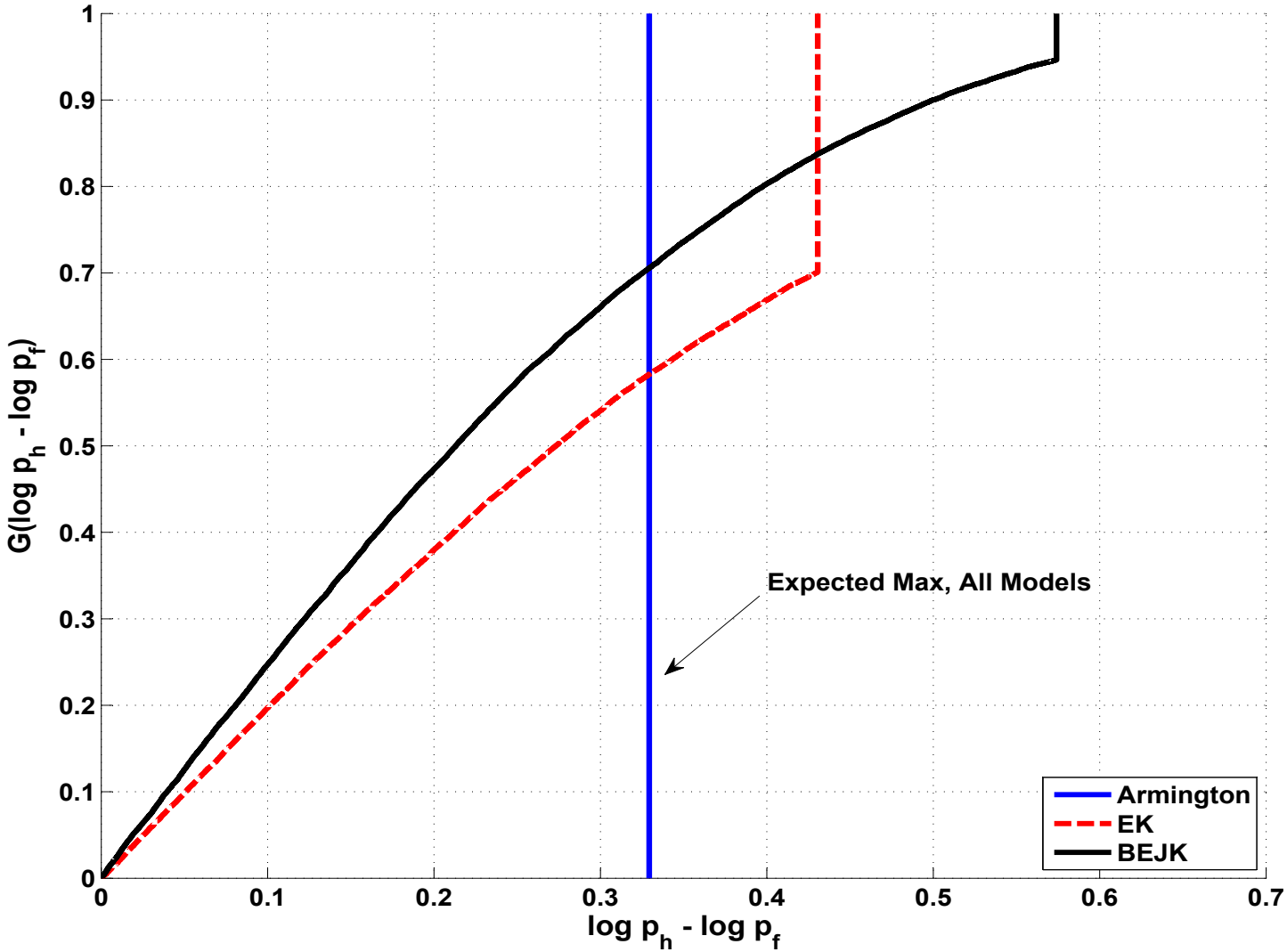
- The maximal price gap in the sample is $\log \tilde{P}_{h,L:L}$.
- Given a model $\mathcal{M}$, parameter θ, and sample of goods L, the expected maximal price gap is

$$\mathbb{E}\left(\log \tilde{P}_{h,L:L};\ \{\theta, \mathcal{M}\}\right).$$

Distribution of Price Gaps: Same θ



Distribution of Price Gaps: Same Largest Order Statistic



Estimates of θ Using Order Statistics

Proposition

Consider the following estimator of θ:

$$\hat{\theta}_{\mathcal{M}} = \arg\min_{\theta} h(\theta_{\mathcal{M}})' h(\theta_{\mathcal{M}})$$

$$\text{where} \quad h(\theta_{\mathcal{M}}) = \left(\log \tilde{P}_{h,L:L} - \mathbb{E}\left(\log \tilde{P}_{h,L:L};\ \{\theta, \mathcal{M}\} \right) \right).$$

Then:

1. $\hat{\theta}_{\mathcal{ARM}} > \hat{\theta}_{\mathcal{EK}} > \hat{\theta}_{\mathcal{BEJK}}$
2. Across models, the welfare costs of autarky are

$$\lambda_h^{-\frac{1}{\hat{\theta}_{\mathcal{BEJK}}}} > \lambda_h^{-\frac{1}{\hat{\theta}_{\mathcal{EK}}}} > \lambda_h^{-\frac{1}{\hat{\theta}_{\mathcal{ARM}}}}$$

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Our Estimation Approach: An Overview

1. Given micro-level prices, compute the statistic...

$$d_{ni} = \max_{\ell \in L} \{\log p_n(\ell) - \log p_i(\ell)\} + \log \bar{p}_i - \log \bar{p}_n,$$

$$\text{where} \quad \log \bar{p}_i = \frac{1}{L} \sum_{\ell=1}^{L} \log(p_i(\ell)),$$

2. Recover all parameters—up to an unknown scalar θ—from only trade data.

3. Given a θ and model $\mathcal{M}$, compute the theoretical moment $\mathbb{E}(d_{ni}; \{\theta, \mathcal{M}\})$

- Key computational issue: we approximate the theoretical moment $\mathbb{E}(d_{ni}; \{\theta, \mathcal{M}\})$ via simulation.

4. Update θ until the two moments are "close".

Price Gaps and Trade Determinants, 2004 ICP Data, 30 countries

Dependent Variable	Log Distance	Border	Mean Value
d_{ni}	0.13 [0.11, 0.15]	−0.11 [−0.18, −0.02]	2.5
d_{ni}^{85th}	0.055 [0.047, 0.063]	−0.035 [−0.060, −0.010]	1.4

Note: The first two columns report regression of dependent variables on distance, border indicator and country fixed effects. The last column reports the mean value, exponentiated.

Our Exercise...

Assume different data generating processes...

- Armington
- EK
- BEJK

and apply estimation approach described. Next couple of slides...

- Some details.
- Results using ICP price and trade data for 2004 from 30 largest countries.

In the paper...

- All details.
- Evidence that our approach works well.
- Treatment of measurement error in prices.

Exactly Identified Case

A zero function

$$h_1(\theta, d_{ni}, \mathcal{M}) = d_{ni} - \frac{1}{S}\sum_{s=1}^{S} d_{ni}(\theta, u_s, \mathcal{M}),$$

Our estimation is based on the moment condition

$$\mathbb{E}(h_1(\theta_o, d_{ni}, \mathcal{M})) = 0.$$

Thus our estimator is ...

$$\hat{\theta}(\mathcal{M}) = \arg\min_{\theta} g(\theta, \mathcal{M})' g(\theta, \mathcal{M}),$$

$$\text{where} \quad g(\theta, \mathcal{M}) = \left(\frac{1}{N^2 - N}\right) \sum_n \sum_i h_1(\theta, d_{ni}, \mathcal{M}).$$

Overidentified Case

Moments. . .

- Max price difference (same as in previous slide).
- Price difference in 85th percentile of sample.
- Covariance of max price difference and distance.

Our zero function is now a vector $h(\theta, \mathbf{d_{ni}}, \mathcal{M})$ and our estimator is

$$\hat{\theta}(\mathcal{M}) = \arg\min_{\theta} g(\theta, \mathcal{M})' W^{-1} g(\theta, \mathcal{M}),$$

$$\text{where} \quad g(\theta, \mathcal{M}) = \left(\frac{1}{N^2 - N} \right) \sum_n \sum_i h(\theta, \mathbf{d_{ni}}, \mathcal{M}).$$

Details: Standard Errors, Weighting Matrix, J-Statistic

Standard errors to be estimated using a parametric bootstrap procedure.

Weighting matrix W. Two specifications...

1. Identity matrix.
2. Optimal weighting matrix suggested by Gouriéroux and Monfort (1996).

   Depends on $\hat{\theta}(\mathcal{M})$, thus we continually update the weighting matrix within the minimization routine as in Hansen, Heaton, Yaron (1996).

J-statistic to test overidentifying restrictions...

- Test statistic is $(N^2 - N)g(\hat{\theta}, \mathcal{M})' W^{-1} g(\hat{\theta}, \mathcal{M})$.
- Use parametric bootstrap to construct the finite sample distribution of the test statistic under the null hypothesis that the DGP is correct.

Overview of Data

Data:

- Trade shares from data on bilateral trade and production in manufacturing among 30 largest (absorption) countries in 2004.

- Prices in 30 countries for 62 tradable products during 2003-2005.
  - Price data on goods with identical characteristics sampled across retail locations.

Interpretation within the model:

- A data set is a random sample of prices for 62 goods in all countries.

Estimation Results

	Exactly Identified	**Overidentified, Identity**	**Overidentified, Optimal**	
Model	Estimate of θ	Estimate of θ	Estimate of θ	J-stat P-value
Armington	**5.24** [5.02, 5.45]	**5.06** [4.84, 5.26]	**3.94** [3.73, 4.08]	< 0.01
EK	**4.17** [4.00, 4.34]	**4.13** [3.99, 4.29]	**4.05** [3.97, 4.14]	0.25
BEJK, $\rho = 2.5$	**3.32** [3.18, 3.48]	**3.28** [3.16, 3.43]	**3.15** [3.08, 2.96]	< 0.01
BEJK, est. $\hat{\rho}$	$- - -$	**3.21** [2.90, 3.64]	**2.74** [2.62, 2.89]	< 0.01

Note: Values within brackets report 90th and 10th confidence intervals. J-stat P-value reports probability of observing a J-statistic as large as observed. Estimates of ρ for BEJK model are 2.25 and 1.37.

Moments: Data and Model

Data and Model Moments

	Moment		
	Mean d_{ni}	Mean d_{ni}^{85}	Cov$(d_{ni}, \log m_{ni})$
Data	**0.93**	**0.37**	**0.14**
Armington	**1.23** [0.16]	**0.25** [0.02]	**0.21** [0.15]
EK	**0.95** [0.09]	**0.37** [0.01]	**0.10** [0.09]
BEJK, $\rho = 2.5$	**0.99** [0.09]	**0.35** [0.01]	**0.09** [0.09]
BEJK, est. $\hat{\rho}$	**0.95** [0.09]	**0.37** [0.01]	**0.07** [0.08]

Note: Reports model implied moments at estimated parameters values from the overidentified estimation with optimal weighting matrix. Values within brackets report the diagonal entry of W, the variance-covariance matrix. $\log m_{ni}$ is the log distance in miles between country n and i.

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Macro Estimation of θ

Cleanest/best approach is Caliendo and Parro (2011) and Parro (2013). We will follow them by denoting the

- Log of triple differenced trade flows by $\tilde{x}_{nih}$ for country triple n, i, h,
- Log of triple differenced tariffs by $\tilde{t}_{nih}$ for country triple n, i, h,
- N_t as the number of country triple observations.

All models $\mathcal{M}$ imply the same relationship:

$$\tilde{x}_{nih} = \theta \tilde{t}_{nih}.$$

Thus, given data on $\tilde{x}_{nih}$, $\tilde{t}_{nih}$, and the appropriate orthogonality condition, all models imply the **same** θ.

Next slides: combine this approach with ours and see what happens...

Macro Estimation of θ, Details #1

A zero function for tariff based approach

$$h_{nih}(\theta, \tilde{x}_{nih}, \tilde{t}_{nih}) = \tilde{t}_{nih}\left(\tilde{x}_{nih} - \theta\tilde{t}_{nih}\right).$$

Averaging across triples yields

$$g_t(\theta) = \left(\frac{1}{N_t}\right)\sum_n\sum_i\sum_h h_{nih}(\theta, \tilde{x}_{nih}, \tilde{t}_{nih}),$$

Note that

$$\hat{\theta}_t = \arg\min_\theta g_t(\theta)' g_t(\theta),$$

is the OLS estimate used in Caliendo and Parro (2011) and Parro (2013).

Macro Estimation of θ, Details #2

Stacking $g_t(\theta)$ with our moment conditions gives

$$\tilde{g}(\theta, \mathcal{M}) = \begin{bmatrix} g_t(\theta) \\ g(\theta, \mathcal{M}) \end{bmatrix}$$

Thus our estimator is ...

$$\hat{\theta}(\mathcal{M}) = \arg\min_{\theta} \tilde{g}(\theta, \mathcal{M})' \tilde{g}(\theta, \mathcal{M}).$$

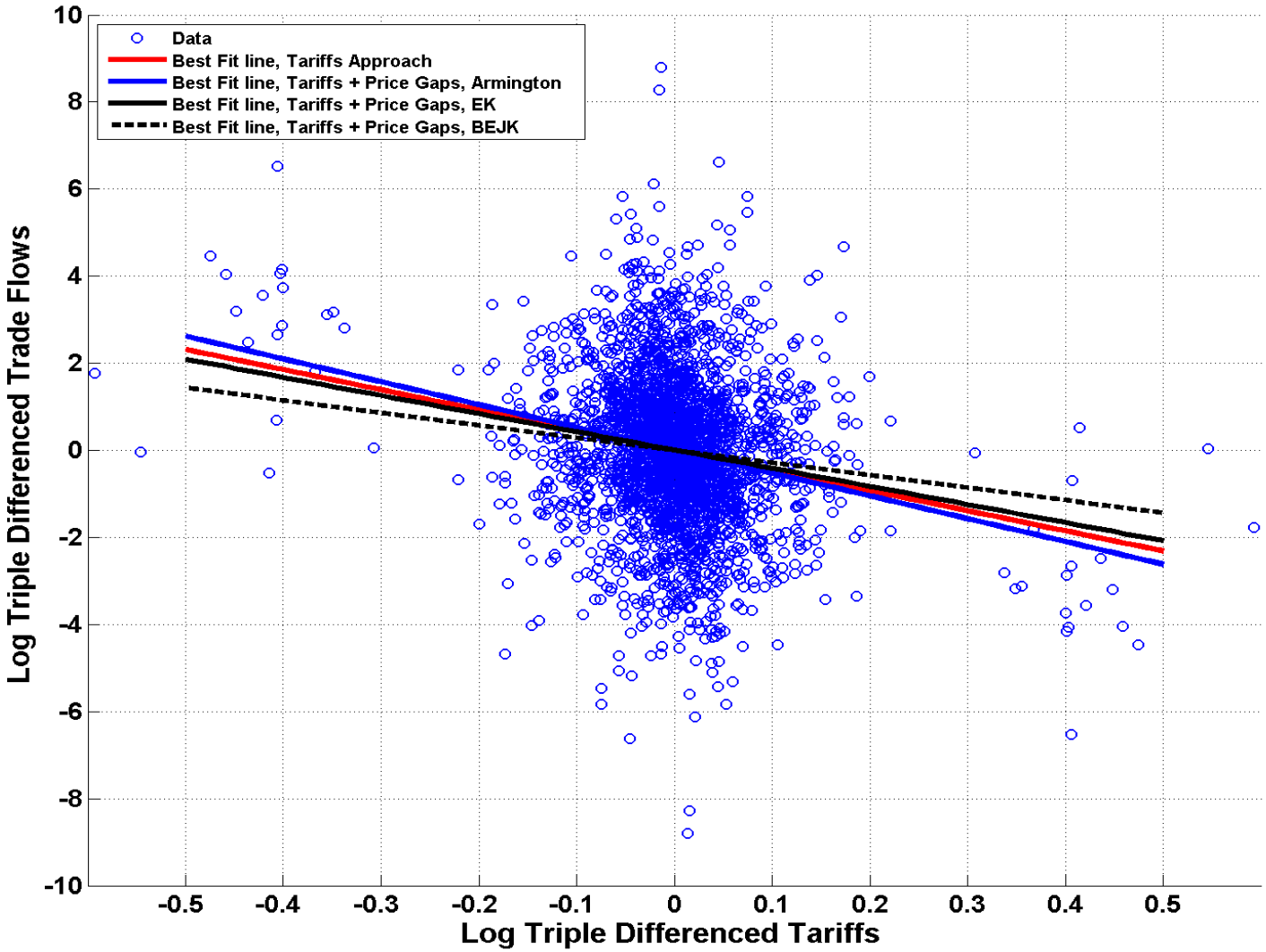
Results: Macro + Micro Estimates of θ

Macro and Micro Estimates of θ

	Estimation Method		
Model	Macro	Micro	Macro + Micro
Armington/Krugman	**4.63** [5.21, 4.09]	**5.24** [5.02, 5.45]	**5.24** [5.01, 5.45]
EK	**4.63** [5.21, 4.09]	**4.17** [4.00, 4.34]	**4.17** [4.03, 4.39]
BEJK, ρ=1.37	**4.63** [5.21, 4.09]	**2.74** [2.62, 2.89]	**2.74** [2.53, 2.80]

Note: Values within brackets report 90th and 10th confidence intervals. Macro data is from Parro (2013) for year 1990 for 21 countries, where the EU block represents a single observation, and tariffs are computed as a trade-weighted average of Parro's two sectors (capital goods and non-capital goods).

Macro Data and Best Fit Lines



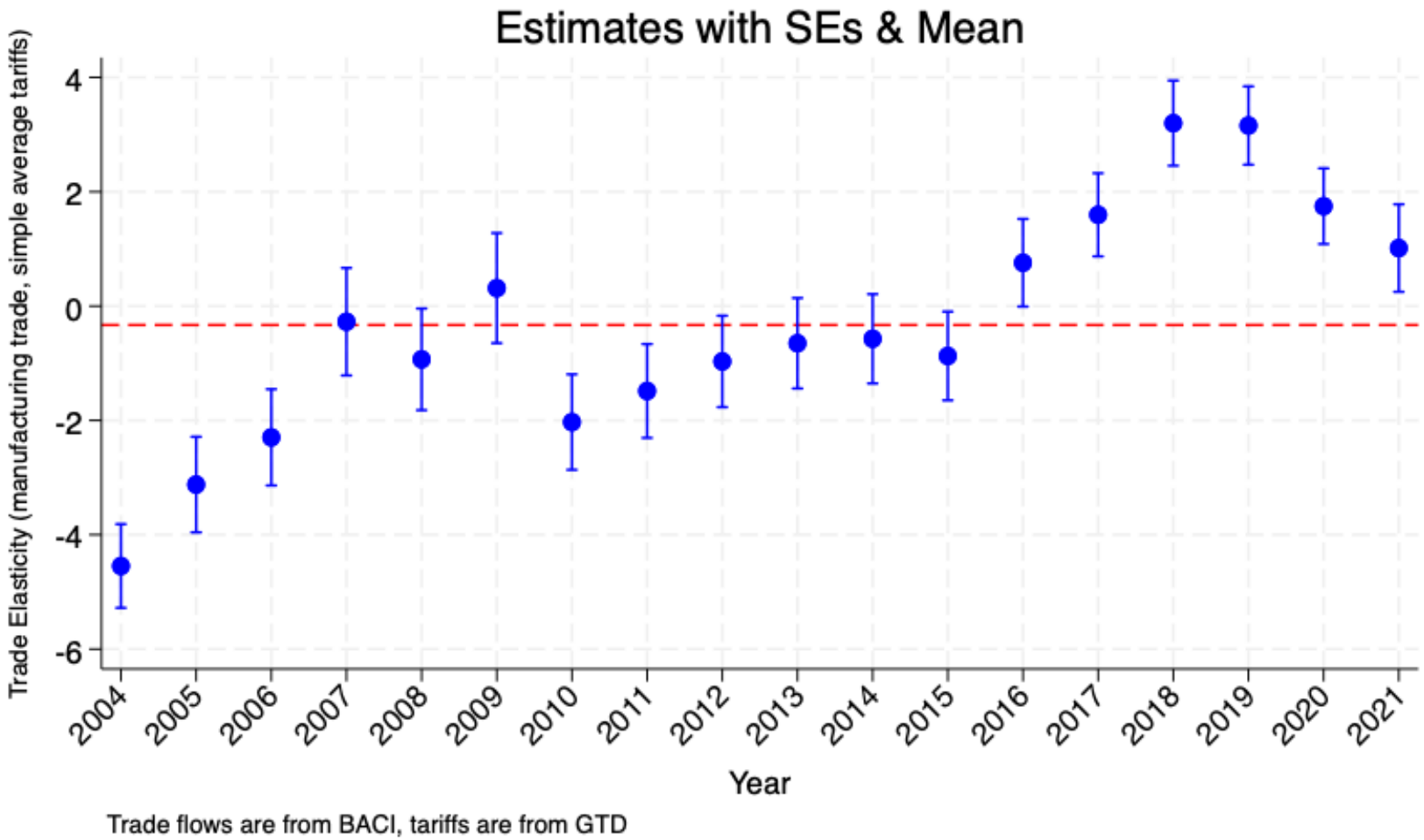
Estimation Results over Time

Estimation Results, Exactly Identified

	2004	2011	2017
Armington	**5.24** [5.02, 5.45]	**5.20**	**5.56**
EK	**4.17** [4.00, 4.34]	**3.91**	**4.21**
BEJK	**3.32** [3.18, 3.48]	**3.10**	**3.33**

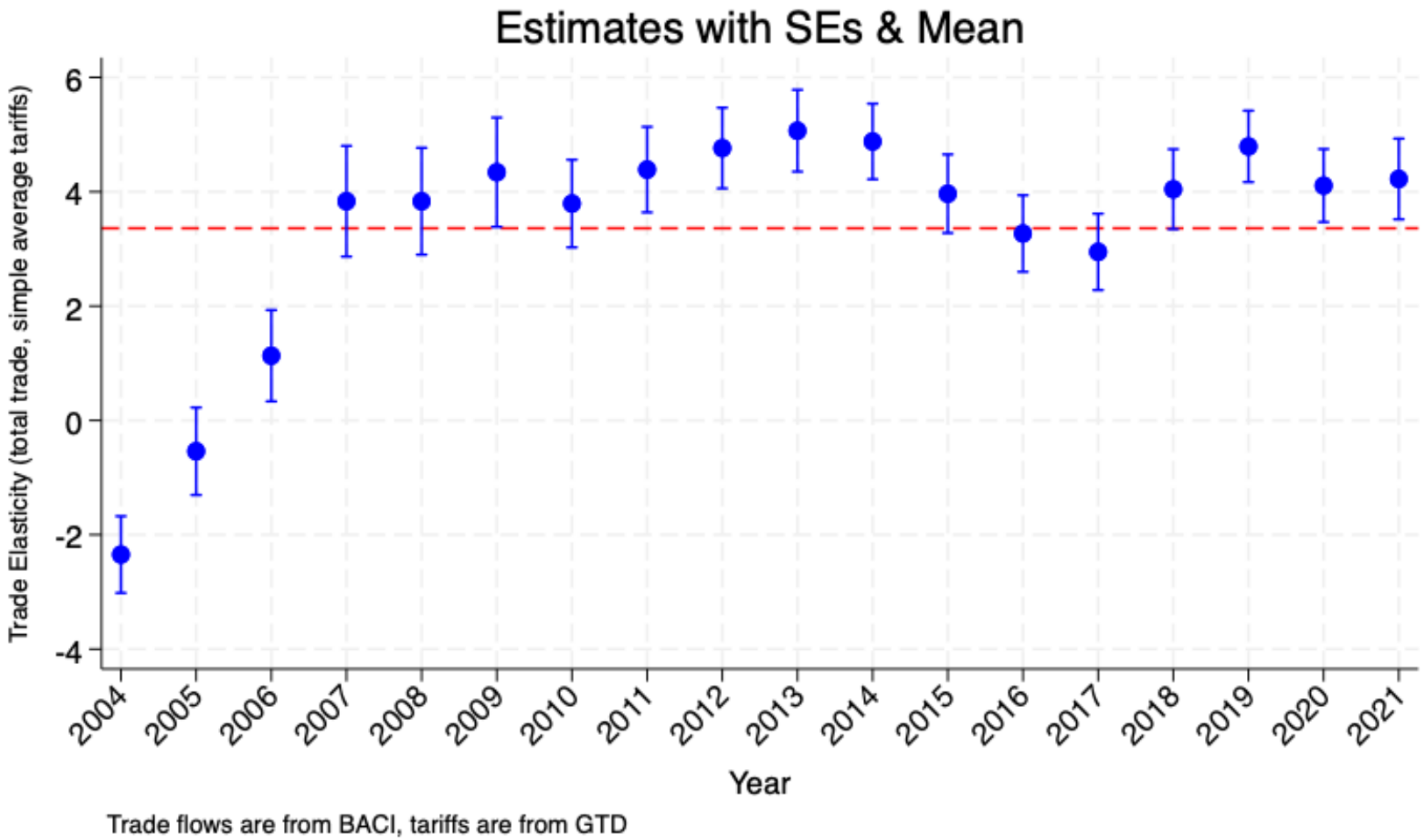
Data sources: Manufacturing trade shares (BACI 2017, Feenstra 2011, UN Comtrade 2017, 2011)

Macro Estimates, 30 countries



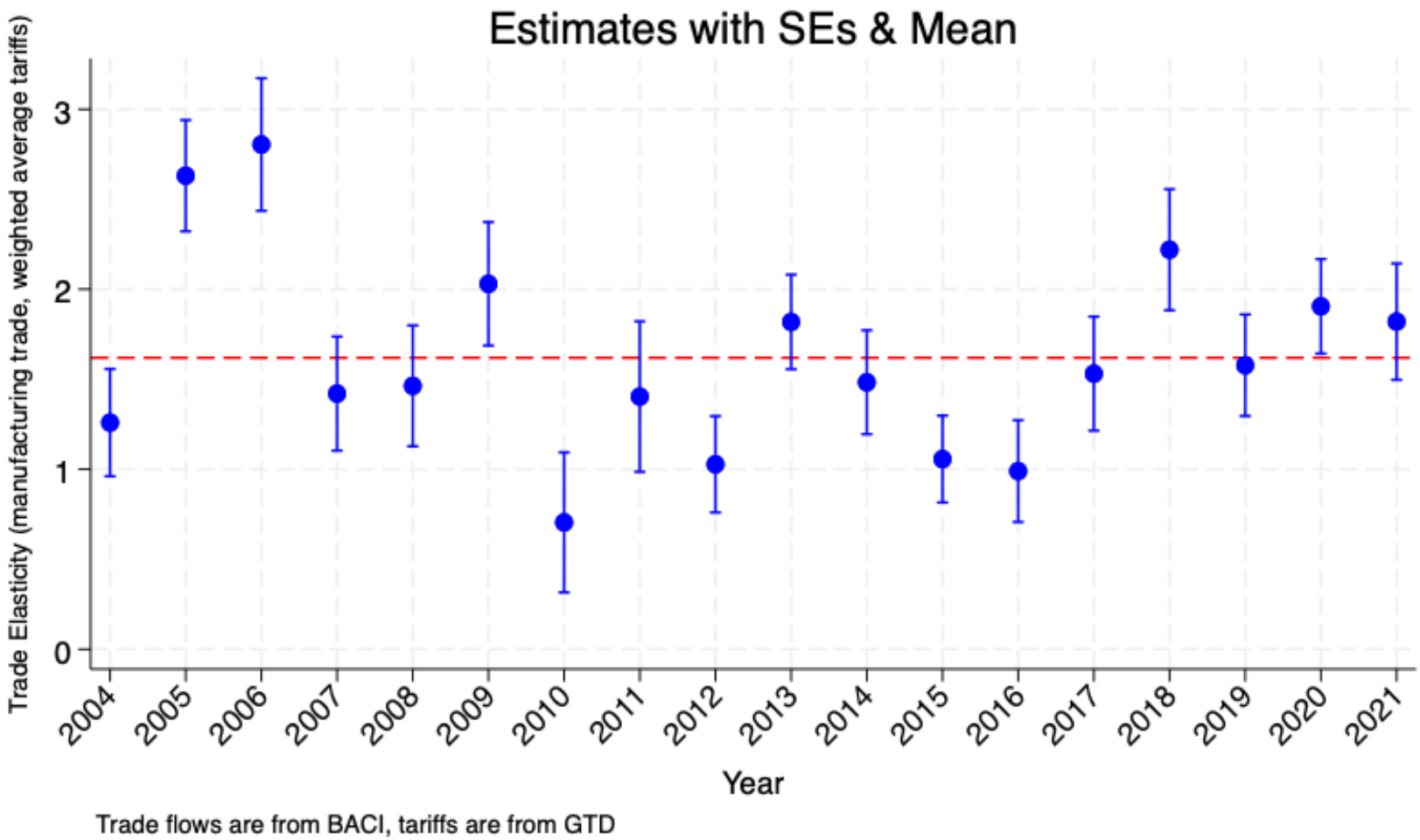
Data sources: BACI (manufacturing trade), Teti (average tariff)

Macro Estimates, 30 countries



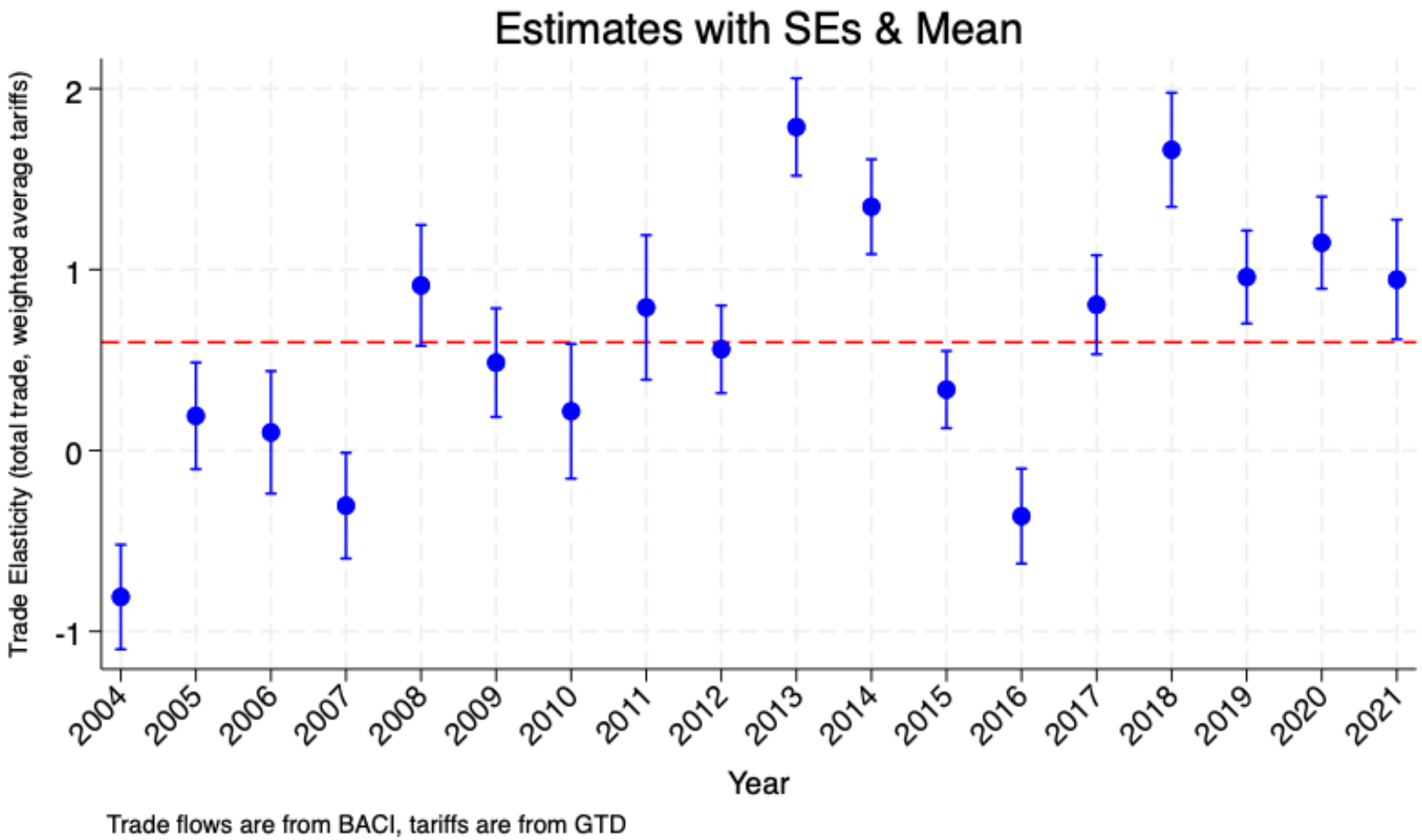
Data sources: BACI (all goods trade), Teti (average tariff)

Macro Estimates, 30 countries



Data sources: BACI (manufacturing trade), Teti (trade-weighted average tariff)

Macro Estimates, 30 countries



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Endogenous Variety Models (Krugman and Melitz)

These are more complicated for two reasons. . .

1. Require parameterizations/assumptions about fixed costs of production.
2. Require more assumptions/interpretations to match data with model.

   Example: In an asymmetric Melitz model, the set of goods differs across countries and thus for certain goods, no possible price comparison.

Solution? We forge ahead:

- Assume fixed costs are proportional to labor endowments.
- In a Krugman model, this solves issue number #2 above.
- To solve #2 Melitz, we assume data collectors only sample goods available in every market.

Results: Endogenous Variety Models

Estimation Results, Endogenous Variety Models

	Exactly Identified	**Over-Identified, Identity**	**Over Identified, Optimal**	
Model	Estimate of θ	Estimate of θ	Estimate of θ	J-stat P-value
Krugman	**5.24** [5.02, 5.45]	**5.06** [4.84, 5.26]	**3.94** [3.73, 4.08]	< 0.01
Melitz	**3.69** [3.32, 4.21]	**3.69** [3.37, 4.09]	**3.65** [3.36, 3.86]	0.35

Conclusion...

- Theoretically ordered fixed-variety models according to their implied trade elasticities and welfare gains from trade.
- Provided estimates of trade elasticity using price and trade-flow data for 30 countries.
- Results in welfare terms: Armington $<$ EK $<$ BEJK and Krugman $<$ Melitz
  - Ranking driven explicitly by different micro-level margins.
    Variable markups in EK vs. BEJK, Selection in Krugman vs. Meltiz.
- Incorporating information from estimators that imply a common trade elasticity has no effect on our results.

Appendix

Model: Armington With Continuum of Goods

N countries: i is source, n is destination. Set of tradable goods of unit measure, subset differentiated by source. Each i produces subset of measure $1/N$.

- CES consumer preferences over goods with elasticity of substitution ρ,

$$U_n = \left[\sum_{i=1}^{N} \int_0^{1/N} x_{ni}(j)^{\frac{\rho-1}{\rho}} dj \right]^{\frac{\rho}{\rho-1}}.$$

- P.C. producers from country i can supply good j to country n at price

$$p_{ni}(j) = \frac{\tau_{ni} w_i}{T_i^{1/\theta}}, \quad \text{where} \quad \theta = \rho - 1 \quad \text{and} \; T_i^{1/\theta} \text{ is productivity in } i.$$

- Trade: all countries buy all goods, CES determines quantities.

Model: Eaton and Kortum (2002)

N countries indexed by i. A continuum of tradable goods $x(j)$, $j \in [0,1]$.

- Consumers preferences over goods

$$U_i = \left[\int_0^1 x_i(j)^{\frac{\rho-1}{\rho}} dj \right]^{\frac{\rho}{\rho-1}}.$$

- P.C. producers from country i can supply $x(j)$ to country n at price

$$p_{ni}(j) = \frac{\tau_{ni} w_i}{z_i(j)}.$$

- Productivity, $z_i(j)$, is drawn from a Fréchet distribution

$$F_i(z) = \exp\left(-T_i z^{-\theta}\right).$$

- Trade: n buys j from lowest-cost producer, CES determines quantities.

Model: BEJK (2003)

N countries indexed by i. A continuum of tradable goods $x(j)$, $j \in [0,1]$. Preferences are the same as in EK.

- Costs are $c_{kni}(j) = \frac{\tau_{ni} w_i}{z_{ki}(j)}$ for the cost of k-th most efficient producer.
- Bertrand competition with prices based on next best competitor,

$$p_n(j) = \min \left\{ \min \left\{ c_{2ni^*}(j) \ , \ \min_{i \neq i^*} \{ c_{1ni}(j) \} \right\} \ , \ \bar{m} c_{1ni^*}(j) \right\},$$

  where $\bar{m} = \rho/(\rho - 1)$ and $i^* =$ location of lowest cost producer.

- Productivity, $z_k(j)$ for $k = 1, 2$ drawn from

$$F_i(z_1, z_2) = \left[1 + T_i (z_2^{-\theta} - z_1^{-\theta}) \right] \exp\left(-T_i z_2^{-\theta} \right).$$

- Trade: n buys j from lowest-cost producer, CES determines quantities.

Details: Simulation Approach

For any model $\mathcal{M}$:

- Fix θ.
- Recover marginal cost parameters $S_i \equiv \log(w_i^{-\theta} T_i)$ and $\log(\tau_{ni})$—up to an unknown scalar θ—from only trade data.
- Simulate micro-level prices from the model, $\mathcal{M}$.
- Compute $\frac{1}{S}\sum_{s=1}^{S} d_{ni}(\theta, u_s, \mathcal{M})$ to approximate $\mathbb{E}(d_{ni}, \{\theta, \mathcal{M}\})$; compare to d_{ni} from real data.
- Update θ to minimize the distance between moments.

Detail: ρ matters in BEJK. Two approaches: fix $\rho = 2.5$; estimate ρ and θ.

Does Our Approach Work? Yes.

Estimation With Artificial Data, True $\theta = 4$

	Model	Estimate of θ	Mean d_{ni}	Cov$(d_{ni}, \log m_{ni})$	Mean d_{ni}^{85th}
Exactly Identified	Armington	**3.99** [3.83, 4.26]	1.27	—	—
	EK	**3.99** [3.80, 4.18]	0.96	—	—
	BEJK	**3.99** [3.84, 4.15]	0.63	—	—
Over Identified	Armington	**3.97** [3.82, 4.19]	1.21	0.20	0.24
	EK	**3.99** [3.90, 4.10]	0.96	0.10	0.37
	BEJK	**4.00** [3.88, 4.10]	0.63	0.04	0.25

Note: Value is the median estimate across simulations. In each simulation there are 30 countries with 62 prices sampled per country; 100 simulations are performed. Values within brackets report 90th and 10th percentiles.

Why is θ for Melitz $<$ Krugman? Selection.

Focus on the max price difference for intuition...

Common to both models:

For a pair n and i, $\frac{p_n}{p_i} = \frac{\tau_{nk}}{\tau_{ik}}$, where k = good's origin.

- If $k = i$, trade barrier is identified, otherwise it is underestimated.

Different across models: the common set of goods.

1. Krugman model: the fraction of common goods from country i is $1/N$.
2. Melitz model: only the lowest-cost firms select into all markets $\Rightarrow$
   - Distribution of price gaps only reflects low trade cost pairs, misses high trade costs pairs.

Macro Estimation of θ, Details #3

Use tariff and trade data from Parro (2013)...

- 24 countries (20 of them are in our data set) for the year 1990.
- Aggregate tariffs are a trade-weighted average of Parro's two sectors (capital goods and non-capital goods).
- Simple averaging gives similar results.

Model: Krugman (1980)

N countries indexed by i. CES preferences over goods, with the measure of goods being endogenous.

- M.C. firm from country i alone can supply $x(j)$ to country n at price

$$p_{ni}(j) = \bar{m}\left(\frac{\tau_{ni} w_i}{T_i^{\frac{1}{\theta}}}\right), \quad \text{where} \quad \bar{m} = \rho/(\rho-1), \quad \text{and} \quad \theta = \rho - 1$$

- Fixed costs of production f_i assumed to be proportional to mass of workers at home L_i.

- Trade: Firms enter only if average profits are positive. Conditional on entry, all producers serve all destinations, CES determines quantities.

- Note: No new margin here, identical predictions to Armington.

Model: Melitz(2003) / Chaney (2008) / EKK (2011)

N countries indexed by i. CES preferences over varieties, with the measure of goods being endogenous.

- M.C. firms from country i can supply variety $x(j)$ to country n at price

$$p_{ni}(j) = \bar{m}\left(\frac{\tau_{ni} w_i}{z_i(j)}\right), \quad \text{where} \quad \bar{m} = \rho/(\rho - 1)$$

- Fixed market access costs f_n assumed to be proportional to mass of workers in a market L_n.

- Fixed measure of firms with efficiency of at least z for any j,

$$\mu_i^z(z) = T_i z^{-\theta}.$$

- Trade: Firms sell to market n only if profits are positive, CES determines quantities.

Results: Endogenous Variety Models

Estimation Results, Endogenous Variety Models

	Exactly Identified	**Over-Identified, Identity**	**Over Identified, Optimal**	
Model	Estimate of θ	Estimate of θ	Estimate of θ	J-stat P-value
Krugman	**5.24** [5.02, 5.45]	**5.06** [4.84, 5.26]	**3.94** [3.73, 4.08]	< 0.01
Melitz	**3.69** [3.32, 4.21]	**3.69** [3.37, 4.09]	**3.65** [3.36, 3.86]	0.35

Estimation of Parameters Needed For Simulation

Derive the following gravity equation from the models:

$$\log\left(\frac{X_{ni}/X_n}{X_{nn}/X_n}\right) = S_i - S_n - \theta \log \tau_{ni} + \nu_{ni}$$

$$\log(\tau_{ni}) = \alpha + \gamma \log d_{ni} + b_{ni} + ex_i.$$

$S_i = \log\left[w_i^{-\theta} T_i\right]$:

- Recovered as the coefficients on country-specific dummy variables.

Trade cost function:

- log distance with elasticity γ.
- shared border (b_{ni})— 1 coefficient.
- exporter fixed effect (ex_i) — N coefficients.

Simulation of EK and BEJK Models

Assuming a value for θ parameterizes:

- The level of trade costs.
- The marginal cost distribution in each model.

EK Model

- Inverse of marginal costs $u_i(j) \equiv z_i(j)/w_i \sim G_i(u_i) = \exp(-\exp(S_i)u_i^{-\theta})$.

BEJK Model

- Transformed marginal costs $v_{ki}(j) \equiv (z_{ki}(j)/w_i)^{-\theta}$, $k = 1, 2$, where

$$Pr[v_{1i} \leq v_1] = 1 - \exp(-\exp(S_i)v_1)$$

$$Pr[v_{2i} \leq v_2 | v_{1i} = v_1] = 1 - \exp(-\exp(S_i)v_2 + \exp(S_i)v_1)$$

- Draw second lowest cost, conditional on lowest cost draw.

Finally, compute prices and trade flows from models and generate β's.

Simulation of Melitz Model

Assuming a value for θ parameterizes:

- The level of trade costs.
- The marginal cost distribution, under proportional fixed-cost assumption.

Simulation Steps:

1. Discretize continuum to set of potentially-produced goods, 30,000*N
2. Determine measure of goods produced domestically in each country.
   - Define standardized unit cost $u_i(j) \equiv T_i z^{-\theta}$ (depends on gravity objects).
   - Using Pareto distribution, $\mu_z^i(\cdot)$, determine measure of firms that serve domestic market, $\bar{u}_{ii}(j)$.
   - Normalization: Compute $\bar{u}_{i^*} = \max_i \bar{u}_{ii}$ and assign $30,000$ to i^*.
   - Let measure of domestic producers in i be $\bar{u}_{ii}/\bar{u}_{i^*} \cdot 30,000$.
   - Procedure selects set of domestically-produced goods in all countries.

Simulation of Melitz Model... Continued

3. Simulate inverse marginal costs for each good in each country

   Define $v(j) \equiv u_i(j)/\bar{u}_{ii}(j) \sim U[0,1]$.

   Simulate $v(j)$, compute $u_i(j)$, and obtain $c_{ni}(j) \equiv \tau_{ni} w_i / z(j)$.

4. Determine set of exported varieties for all country-pairs

   Compare $c_{ni}(j)$ to cost cutoff to sell j in n, $\bar{c}_n(j)$, given by

$$\bar{c}_n = \left[\frac{f_n}{L_n} \left(\frac{1-\rho+\theta\rho}{1-\rho+\theta} \right) \sum_k \exp(S_k) \tau_{nk}^{-\theta} \right]^{-\frac{1}{\theta}}$$

1-4 **only** use objects estimated from gravity because of fixed-cost assumption.

From Pareto productivity distribution, $\bar{u}_{ii}(j) = \exp(S_i)\bar{c}_i^{\theta}$.

If $f_n \propto L_n$, then $\bar{c}_n/\bar{c}_i \propto \left(\sum_k \exp(S_k)\tau_{nk}^{-\theta} / \sum_k \exp(S_k)\tau_{ik}^{-\theta} \right)^{-1/\theta}$.

Hence, all objects depend on gravity parameters, up to a normalization.

Characterization of Krugman Model

Assuming a value for θ parameterizes:

- The level of trade costs.
- The common marginal cost, $\exp(S_i)$, under proportionality assumption.

Common goods

- If $f_i \propto L_i$ and expensed in w_i, then mass of entrants from i is $\propto \frac{1}{\theta+1}$
- All entrants serve all markets $\Rightarrow$ share of i-goods on common list is $1/N$

Common Goods in Melitz Model

Since a good is identified by the productivity and origin of the monopolist, the good appears on the common list only if the firm sells to all destinations.

Hence, any firm from i with marginal cost $\frac{w_i}{z(j)} \equiv \frac{c_{ni}(j)}{\tau_{ni}} \leq \bar{c}_n$ serves market n.

Then, the cutoff marginal cost to serve all N markets from i is

$$\bar{u}_i^{min} = \min_{l=1,\ldots,N} \frac{1}{\tau_{li}} \left[\frac{f_l}{L_l} \left(\frac{1-\rho+\theta\rho}{1-\rho+\theta} \right) \sum_k \exp(S_k) \tau_{lk}^{-\theta} \right]^{-\frac{1}{\theta}}.$$

From Pareto distribution, the measure of firms from i that serve all markets is

$$M_i = \exp(S_i) \min_{l=1,\ldots,N} \tau_{li}{}^{-\theta} \left[\frac{f_l}{L_l} \left(\frac{1-\rho+\theta\rho}{1-\rho+\theta} \right) \sum_k \exp(S_k) \tau_{lk}^{-\theta} \right]^{-1}.$$

The share of i-goods on the common list is $M_i / \sum_m M_m$ is

$$\frac{M_i}{M} = \frac{\min_l \exp(S_i) \tau_{li}{}^{-\theta} \left[\sum_k \exp(S_k) \tau_{lk}^{-\theta} \right]^{-1}}{\sum_m \min_l \exp(S_m) \tau_{lm}{}^{-\theta} \left[\sum_k \exp(S_k) \tau_{lk}^{-\theta} \right]^{-1}}$$

- Even if $f_l \propto L_l$, it is country-specific, unlike Krugman.

Fixed Cost Assumption

CES exact price index in model is

$$\bar{P}_n = \left\{ B \left[\frac{f_n}{L_n} \right]^{\frac{1-\rho+\theta}{1-\rho}} \Phi_n \right\}^{-\frac{1}{\theta}},$$

where $\Phi_n = \sum_{k=1}^{N} T_k (\tau_{nk} w_k)^{-\theta}$, B is constant.

If $f_n \propto L_n$, then $\bar{P}_n \propto \Phi_n^{-\frac{1}{\theta}}$, as in EK/BEJK.

Ordering θs Across Models

Proposition

Given models $G_{\mathcal{ARM}}(\log \tilde{p}_h, \theta_{\mathcal{ARM}})$, $G_{\mathcal{EK}}(\log \tilde{p}_h, \theta_{\mathcal{EK}})$, $G_{\mathcal{BJEK}}(\log \tilde{p}_h, \theta_{\mathcal{BJEK}})$, if the expected maximal price gaps are the same,

$$\mathbb{E}\left(\log \tilde{P}_{h,L:L};\ \{\theta, \mathcal{M}\}\right) = \mathbb{E}\left(\log \tilde{P}_{h,L:L};\ \{\theta', \mathcal{M}'\}\right), \quad \forall \text{ models } \mathcal{M},$$

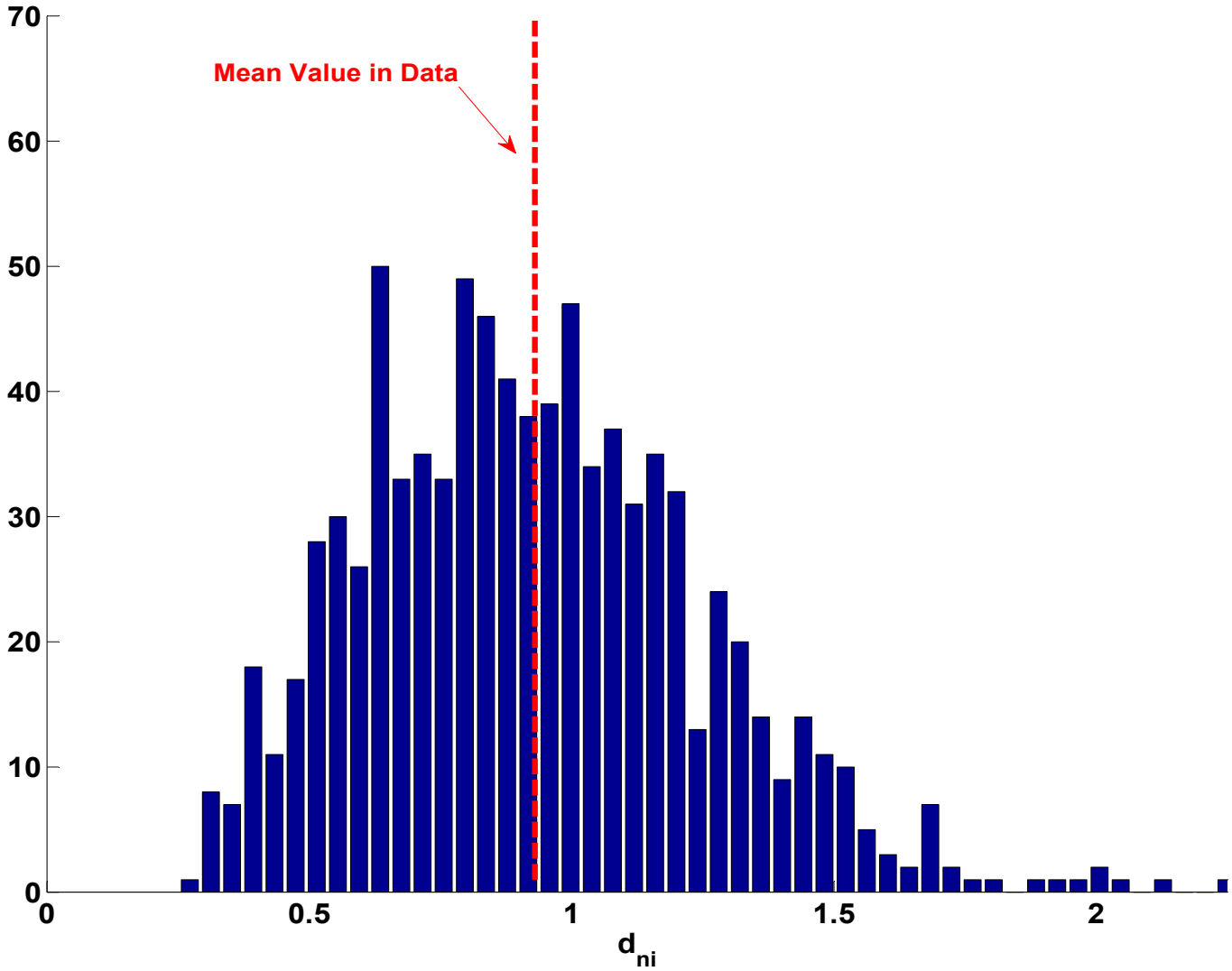
then

$$\theta_{\mathcal{ARM}} > \theta_{\mathcal{EK}} \quad \text{and} \quad \theta_{\mathcal{ARM}} > \theta_{\mathcal{BEJK}}.$$

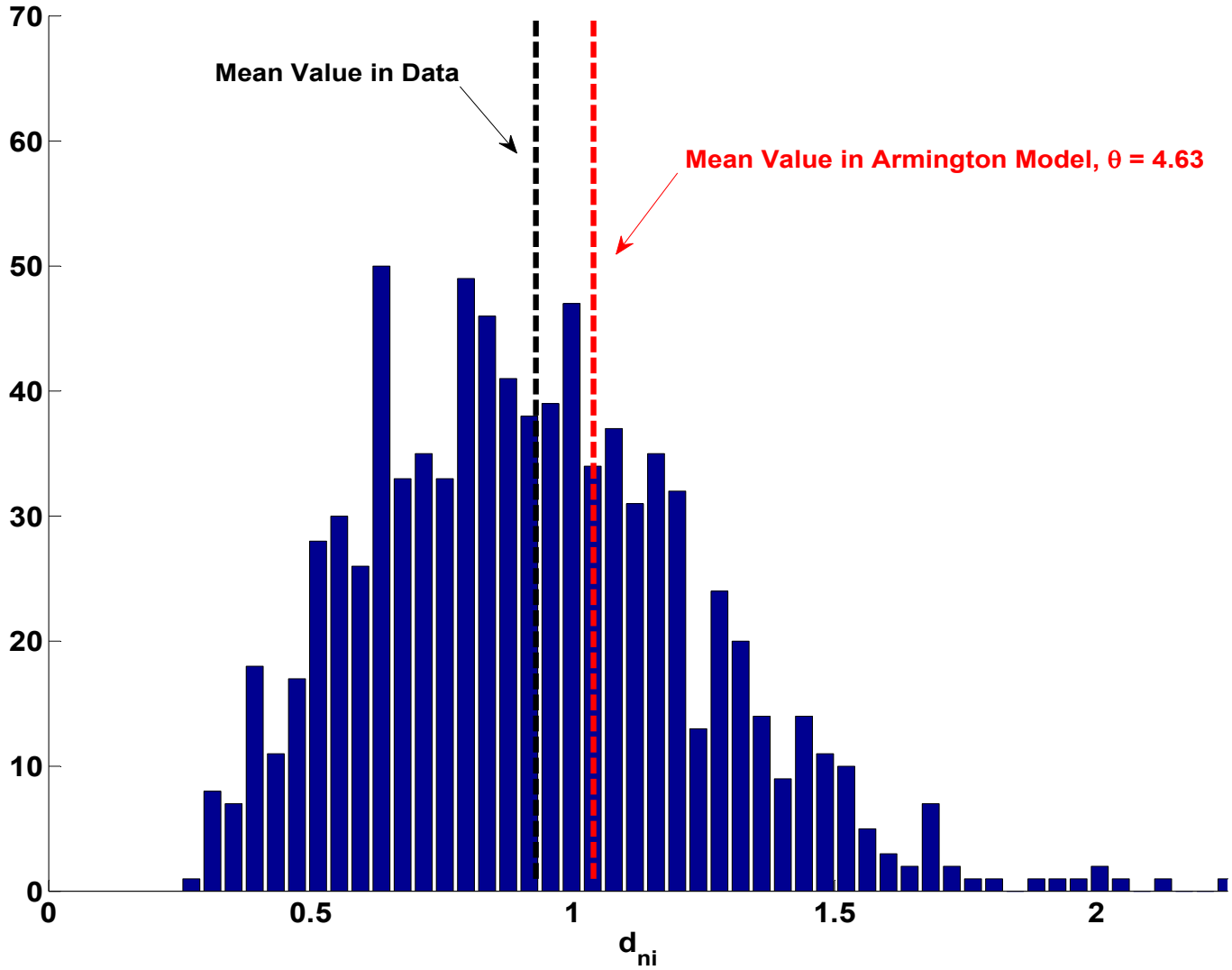
Moreover, if $G_{\mathcal{EK}}(\log \tilde{p}_h, \theta) >_{fosd} G_{\mathcal{BJEK}}(\log \tilde{p}_h, \theta)$, then

$$\theta_{\mathcal{ARM}} > \theta_{\mathcal{EK}} > \theta_{\mathcal{BEJK}}.$$

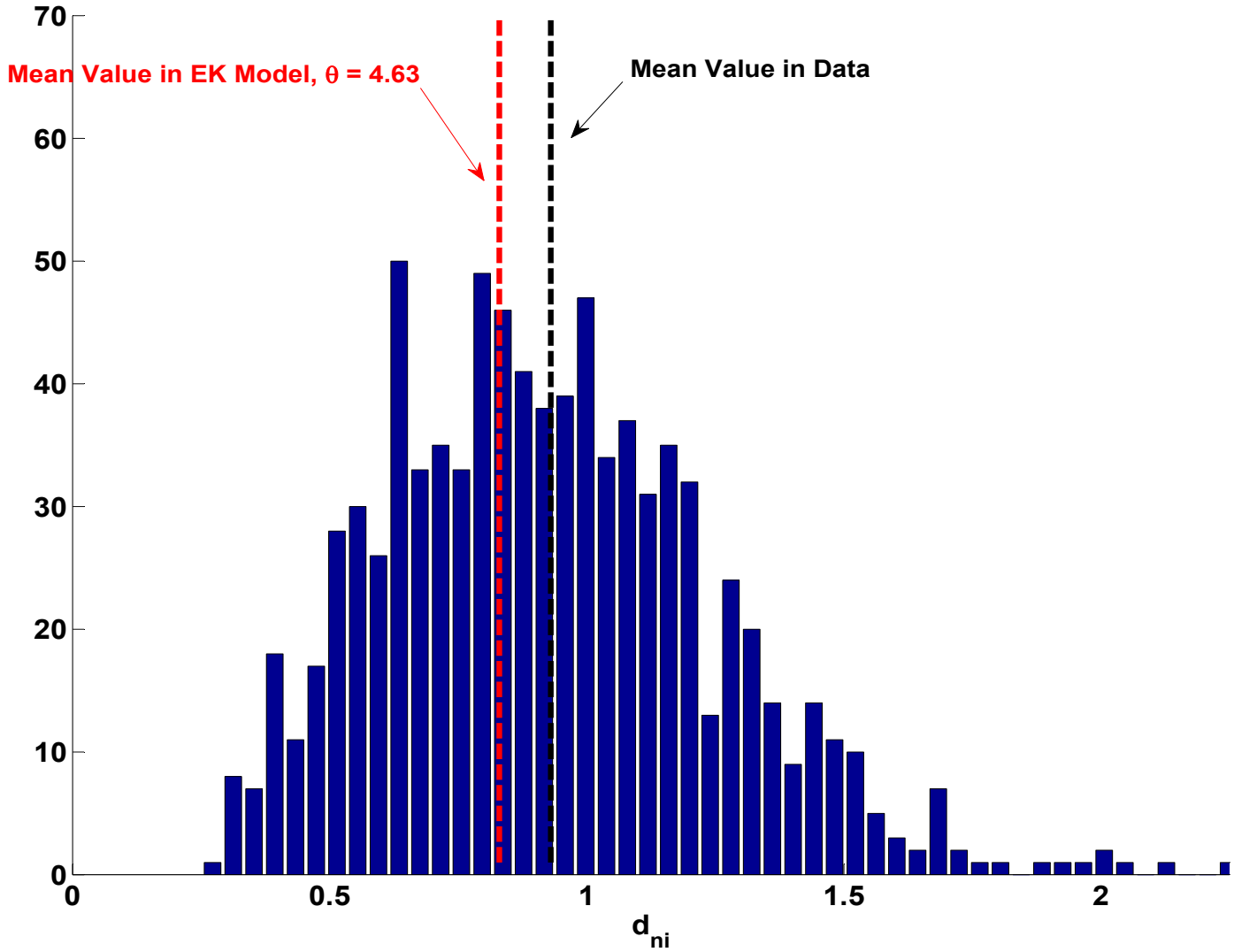
Micro Data: Histogram of d_{ni}s



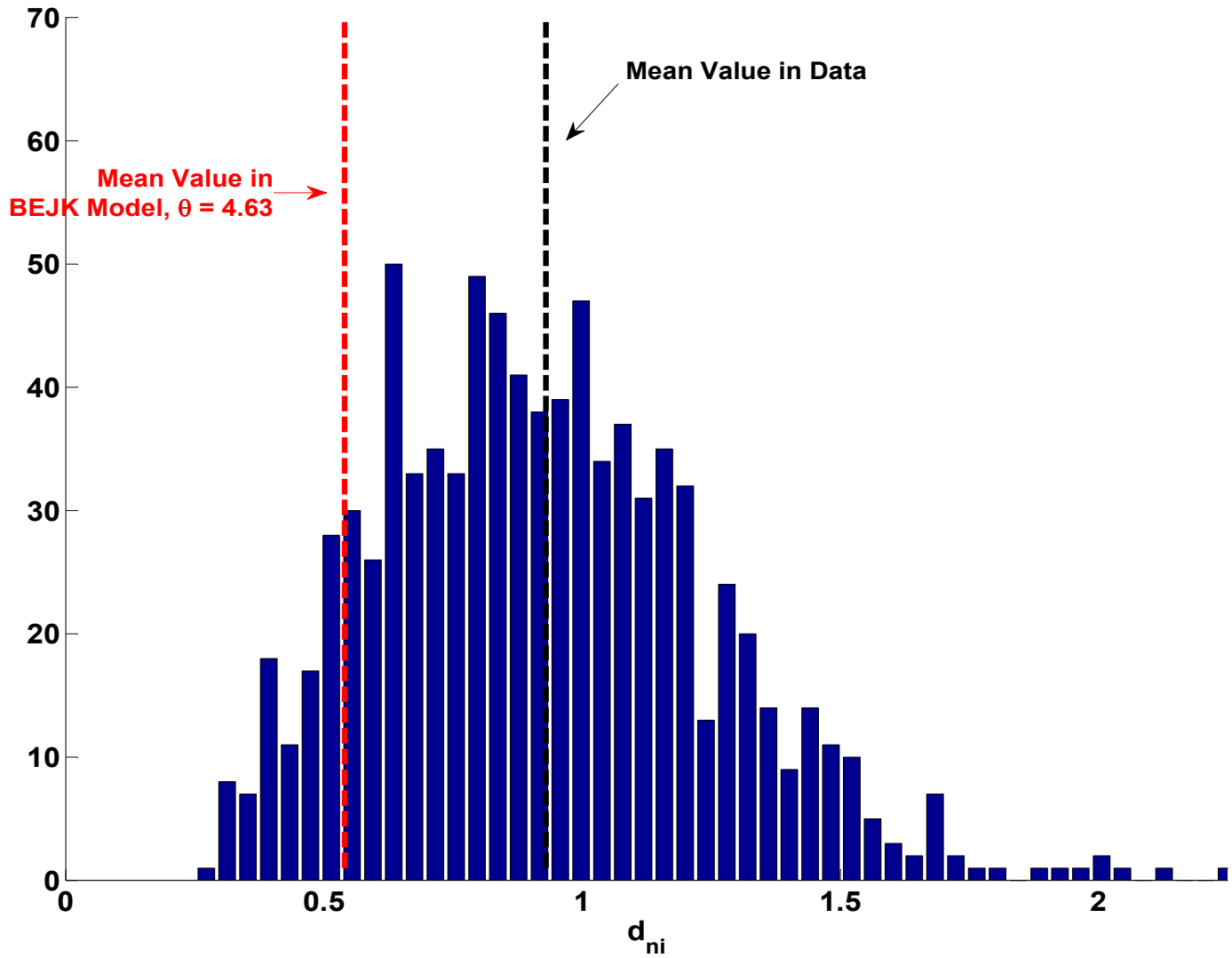
Micro Data and Armington with Macro Elasticity



Micro Data and EK with Macro Elasticity



Micro Data and BEJK with Macro Elasticity



Measurement Error

Measurement error is a serious concern. . .

- Our estimates may be biased downwards.
- The bias may be model specific, i.e. the differences in θ across models could be smaller (or larger).

To address this concern, we assume prices are measured with error,

$$\log p_n(\ell) = \log p_n^*(\ell) + \epsilon, \quad \text{where} \quad \epsilon \sim \mathsf{N}(0, \sigma).$$

and jointly estimate θ and σ using the same moments in the overidentified estimation.

Results: θ and Measurement Error

Estimation Results: θ and σ

	Overidentified, Optimal		
Model	Estimate of θ	Estimate of σ	J-stat P-value
Armington	**6.79** [6.70, 7.27]	**0.186** [0.175, 0.194]	< 0.01
EK	**4.05** [3.96, 4.29]	**0.000** [0, 0.074]	0.20
BEJK, $\rho = 2.5$	**3.60** [3.37, 3.91]	**0.143** [0.106, 0.167]	< 0.01

Note: σ is the standard deviation of measurment error. Values within brackets report 90th and 10th confidence intervals. J-stat P-value reports probability of observing a J-statistic as large as observed.