Folder Title: Goreux, L. M. - Articles and Speeches (1971 - 1972)

Folder ID: 1651772

Fonds: Records of Office of External Affairs (WB IBRD/IDA EXT)

Digitized: October 04, 2013

GOREUX, L.M. - Articles and Speeches (1971-1972)

DECLASSIFIED
WBG Archives

GOREUX

INTERNATIONAL BANK FOR RECONSTRUCTION AND DEVELOPMENT

A B S T R A C T

THE CONFLICT BETWEEN NATIONAL AND INTERNATIONAL RETURNS; AN APPLICATION TO INTERNATIONAL LENDING FOR COMMODITY PROJECTS

To be presented at the
IBRD Economic Research Seminar
on November 5, 1971, at 4:00 p.m., Room C-1006

The reader interested in a more qualified presentation of the arguments, in the algebra and in numerical examples should refer to the non-abbreviated version dated September 30, 1971.

October 27, 1971

L. M. Goreux
Development Research Center

The objective of this paper is to investigate how some of the welfare concepts developed for the allocation of public funds within a country could be applied to the allocation of international funds by an international project lending agency. By analogy with the distinction between private and social return within a country, we shall draw here a distinction between national and international returns.

The national return to country j from the project p financed with a foreign exchange loan X_{jp} extended by the agency is measured by the increment Y_{jp} of the national objective function of country j. The impact on country j's objective function of all the loans extended by the agency to all countries is measured by Y_j. Therefore, Y_j accounts not only for the direct impact of the loans to country j, but also for the indirect impact on country j of loans extended by the agency to countries other than j.

The international return of the projects financed by the agency is measured by their contribution to the value of the objective function of the international lending agency. This objective function is defined by: $I = \sum_j w_j Y_j + \sum_i w_i Y_i$, where the w's are weights reflecting the agency's valuation of different countries' gains (or losses) due to its lending operations. The first term refers to the developing countries j and the second to the developed countries i.

Developed countries transfer to the agency a given amount of capital on given terms, which defines the resource endowment of the agency. Taking this exogenous resource endowment, the agency maximizes I (value of the international objective function), subject to politico-institutional constraints defined by the minimum gains $\overline{Y}_j$ required (or maximum losses $\overline{Y}_i$ permitted) for individual countries on account of the lending operations of the agency.[1/]

1/ A description of the model together with a picture of the matrix are given in "Conflict between National and International Returns; An Application to International Lending for Commodity Projects", L.M. Goreux, IBRD, Sept. 30, 1971, pp. 25-30.

The agency can lend only to developing countries j. Therefore, the variable Y_i entering into the objective function I measures only the indirect impact on developed countries i of loans made to developing countries j. The welfare loss of developed countries i on account of their capital transfer to the agency does not enter into Y_i, since this capital transfer constitutes the agency's resource endowment taken as an exogenous datum for the agency's lending model.

Investment allocation problems are dynamic by nature. They are analyzed here within a static model by transforming flows of benefits and costs into their present discounted values. Following usual practice in project evaluation, the gain Y_{jp} of country j on account of project p is expressed by the present value of the stream of benefits minus costs measured in relation to a given discount rate. This gain is measured in relation to the rate of national return r_{jp} and the rate of interest on the loan r by $Y_{jp} = \partial_{jp} (r_{jp} - r) X_{jp}$, where X_{jp} is the amount of the loan and ∂_{jp} is a coefficient converting flows into present discounted values.

After having defined the model, we shall analyze the implications on the rates of national reutrns r_{jp} of the projects selected in the optimal solution. Before considering the general formulation of the model, we shall start with two simplified formulations.

1. Efficiency Only ($w = 1$), No Externalities

We assume that (i) a project implemented in country j has no welfare impact on any country other than country j; (ii) the benefits from loans of the agency in all developing countries can be measured in monetary terms and; (iii) the agency values a dollar gain from all countries identically. Consequently, the value of the international objective function can be written

$I = \sum_j Y_j$ $(w_j \equiv 1$ and $Y_i \equiv 0)$. If the national objective functions are measured by the values of the GDP, the agency maximizes its contribution to the GDP of the LDC as a group, irrespective of the distribution of the increment among countries.

The function of the agency is to eliminate discrimination in the capital market against poor nations. Let us assume that the world can be stratified between two internally perfect capital markets, a large one for the rich and a small one for the poor. The equilibrium rate of interest is 8% for the rich and 15% for the poor, because commercial banks take a 7% premium when they lend to the poor. But the international agency, which is considered as eminently creditworthy among the rich, need not pay any premium. Assuming the administrative expenses of the agency to be negligible, the agency lends to the poor at the rate at which it borrows from the rich. If the limit in the capital absorptive capacity of the poor is such that the total demand for loans from the agency remains small in relation to the capital market of the rich, the agency can be treated as a price-taker when it borrows on that market.

Under those conditions, the agency does not need any capital budgeting model[1/] to maximize its objective function. The agency should[2/] finance any project with a rate of national return r_{jp} higher than the rate $\bar{r}$ at which it can borrow. The agency should not extend any loan to countries which have no project with a rate of return higher than $\bar{r}$.

1/ All constraints would either become redundant or would lead to infeasibility.

2/ If externalities attached to the impact of projects on world prices were recognized, this rule would lead to maximizing the agency's contribution to world GDP irrespective of the distribution of the increment among countries. However, if other types of externalities, such as those attached to the diffusion of research findings among countries were recognized, the conclusion would not apply any more.

2. Mix between Efficiency and Equity, No Externalities

We still assume that a project implemented in country j has no impact on any country other than country j. But we now assume that the weight w_j applied to the gain Y_j of country j is negatively correlated with the average per caput income $\bar{y}_j$ which that country would have reached in the absence of the agency's lending. Since the agency's lending has no impact on the developed countries $(Y_i \equiv 0)$, the value of the international objective function can be written $I = \sum_j w_j Y_j$.

To maximize I, the agency needs to differentiate its terms of lending with w_j and, therefore, $\bar{y}_j$. For this purpose, the agency may draw a distinction between hard and soft loans. We shall assume there that the agency receives a subsidy from the rich countries and lends at 7% on hard terms and at 1% on soft terms. By using a mix between soft and hard loans, the agency can effectively lend at any rate between 1 and 7%.

The optimal solution for the agency does not consist in drawing a cut-off point for the rate of return r_j which would be identical for all countries and lending on soft terms to the poorest countries up to the exhaustion of soft-term loans.[1/] The optimal solution is to differentiate among countries the cut-off point r_j specific to country j, reflecting the trade-off between efficiency (r_j) and equity (w_j). If countries j were able to borrow in unlimited quantities at 15% on the commercial market, the optimal solution would consist of a mix of projects with rates of return which could vary from 1 to 15%.

1/ With $w_A = 3\, w_B$, if X_A and X_B are the demand of A and B at a common rate of return $r_A = r_B = 8\%$, the value of I can be raised by increasing X_B by ΔX (which implies a reduction of r_A) and reducing X_B by the same amount ΔX (which implies an increase of r_B) up to the point where the marginal dollar lent to A and B has the same impact on the objective function I $(w_A dY_A = w_B dY_B)$.

In the first alternative (efficiency only), the cut-off point was a single number $\bar{r}$ equal for all countries. In the second alternative (mix between efficiency and equity), the single number $\bar{r}$ is replaced by a vector r_j obtained from the solution of the optimizing model.

3. Mix between Efficiency and Equity with Externalities

This alternative differs from the previous one only by the introduction of externalities in the optimizing model. The research on improved wheat and rice varieties conducted respectively in Mexico and the Philippines provides a classical example of external economies. The impact of the additional commodity export generated by a project located in country j on world prices and, consequently, on countries other than j trading that commodity provides another example of externalities. The latter example is used below to illustrate how the introduction of externalities may affect the project selection by the agency.

Let us consider two types of projects which produce commodities C and P respectively. Let us assume further that the world import demand and the world export supply are less than infinitely price-elastic for these two commodities. By financing project C or P in country j, the agency contributes to increasing the world's supply, thereby reducing the world price. It, therefore, induces a reduction in the producers' surplus of the other exporting countries and an increase in the consumers' surplus of the importing countries. But for commodity C (cocoa), poor countries have a large export surplus, while for commodity P (pulp and paper) they have a large and growing import surplus.

If, as in the case of alternative one (efficiency only), the agency attaches the same value to one dollar gained by a rich or by a poor country, the agency should treat projects C and P in the same way. If, as in alternative three, the agency attaches a much greater weight to one dollar gained by a poor country than to one dollar gained by a rich country, the international rate of return on the cocoa project (r_C) is much lower than the national rate of return (r_{jC}) to country j where the project is located, while the international rate of return on the pulp and paper project (r_P) is higher than the national rate of return (r_{jP}) to country j where the project is located.

We have to exclude the case of a cocoa project with an ex ante[1/] rate of national return (r_{jC}) higher than 15%, since country j would implement this project anyway, whether or not the agency finances it. Let us then assume that, for projects C and P, the rates of national and international returns can be ranked as follows:

$$r_C < r_{jP} < r_{jC} < r_P \quad \text{with} \quad r_{jP} > 1\%$$

Country j cannot implement project C nor project P without receiving a loan from the agency, because country j cannot borrow commercially below 15%. If country j is to receive a loan from the agency with the same terms of lending for projects C and P, country j will obviously choose project C. But, the agency prefers project P to project C. This conflict of interests between country j and the agency can be resolved if the agency adjusts the mix between soft and hard loans to the difference between the rates of international and national returns. Country j can be made better off by receiving soft loans for project P than hard loans for project C.

1/ Due to the technical expertise brought in by the agency, the rate of national return of a project financed by the agency could, but only ex post, exceed 15%.

To optimize I, while fulfilling the country's income constraints, the agency should allocate its lending (and especially its soft lending) not only in relation to efficiency (r_{jp}) and equity (w_j), but also in relation to the difference between international and national rates of returns ($r_p - r_{jp}$). As a result, the vector r_j obtained from the optimal solution in alternative two is replaced by the matrix r_{jp} in alternative three. The cut-off point for the rate of national return which was the single number $\bar{r}$ in alternative one has become the matrix r_{jp} differentiated by countries and by types of projects in alternative three.

This two-way classification by income groups and by types of projects is consistent with the allocation of public funds within a welfare State. On the one hand, subsidies are given to low income groups while high income groups are taxed. On the other, subsidies are given to sectors such as education and health where the social return exceeds the private return, while taxation is imposed on polluting industries where the social cost exceeds the private cost. Short of taking into account the interaction between these two types of criteria and short of building up the full matrix r_{jp}, the welfare State can use the two criteria independently. It can extend soft loans to individuals or regions falling below a given income level and to sectors for which the social return substantially exceeds the private return.

A similar simplification could be made by the international lending agency. In alternative two, we have seen that the optimal solution is to take into account the trade-off between efficiency (r_j) and equity (w_j) in order to establish the cut-off rate of return vector r_j. Short of measuring this vector, it is better, nevertheless, to use a common cut off point for all countries and to allocate soft loans by priority to the poorest countries than to ignore the distinction between poor and rich countries.

For the third alternative, we have outlined a somewhat complex optimizing model reflecting the trade-offs between efficiency (r_{jp}), equity (w_j), political considerations ($\overline{Y}_j$) and externalities ($r_p - r_{jp}$). Short of solving this model, a simpler method is proposed to compute the rate of national return r_{jp} below which the project would bring a decline in the value of the international objective function I, even if hard loans were not scarce. Short of computing r_{jp}, the agency could draw a black list[1/] of the types of projects not to be financed even with hard loans and a white list of the types of projects to be financed with soft loans. Such a list should not be used as the bible but as "a strong presumption." The charge of the proof should be given to the one who wants to waive the presumption.

The differentiation made above by types of projects is not trivial for an agency lending to all developing countries and only to developing countries. Most countries at an early stage of development have fairly similar trade patterns and, for many of the export products common to those countries, the price-elasticities of the import demand of the developed countries is low. This clearly applies to tropical non-competing commodities, such as coffee, cocoa and tea. It may also apply to a number of competing manufactured products with a high labor content, such as cotton textiles and clothing, because developed countries may unfortunately impose quota restrictions to protect their "depressed domestic sectors", once the level of imports exceeds a critical mass.

1/ The basic formula for drawing the list is given in Annex.

Even if the international lending agency has no clearly specified objective function, it cannot ignore the problem of its borrowers' repayment capacity. The agency can project the developing countries' trade balance which would be likely to prevail in the absence of the agency's operations. The impact of the agency's lending on the trade balance thus projected depends on the sectors in which the agency's loans will be invested. Clearly, if those were to be invested only in sectors where the international return is substantially lower than the national return,[1/] the trade balance of the developing countries would deteriorate as a result of the agency's operations. The only solution would be an ever increasing volume of lending with an ever growing debt from the poor to the rich countries. We argue here that the types of projects for which the agency has to lend should be treated as one of the key endogenous variables in the agency lending model. The agency needs not only to have a clear trade strategy. It needs to translate this strategy in its project selection criteria. A black and white list by types of projects would be better than to ignore externalities.

1/ Sectors with a positive β_g coefficient, as defined in the Annex.

ANNEX: THE BLACK AND WHITE COMMODITY LIST

Let us assume that the agency maximizes the returns to the developing countries as a whole (referred to below by subscript g) without taking into account the impact on the developed countries nor the distribution of the gain among developing countries ($w_j \equiv 1$, $w_i \equiv 0$). Let us call C_p the cost (inclusive of borrowing) per unit of commodity produced from the project, P the world price and β_g a coefficient which will characterize the ranking of the commodity concerned in our list. The contribution ΔI to the objective function I, per unit of the project output, can be written:

$$\Delta I = P\,(1-\beta_g) - C_p$$

(marginal contribution) = (marginal return) - (marginal cost)

The project brings a positive contribution ($\Delta I > 0$) if:

$$C_p < (1 - \beta_g)\,P$$

If the impact of the project on world prices is small, the coefficient β_g can be written:[1]

$$\beta_g = \frac{\alpha_g}{-\eta_m + \eta_x}\left[1 + \frac{\alpha_{xg}}{\alpha_g}(1 - \gamma_g)\,\eta_{xg}\right]$$

with: α_g = Developing countries net exports (+) or net imports (-) over world exports (+)

α_{xg} = Developing countries gross exports (+) over world exports (+)

η_m = Price elasticity of world import demand (-)

η_x = Price elasticity of world export (+)

η_{xg} = Price elasticity of world export from developing countries (+)

γ_g = Opportunity cost of the resources released per unit of production displaced in the developing countries as a result of the project divided by the world price

---

1/ "The Conflict ...", pp. 7-16, op. cit.

Since γ_g is generally lower than unity, the term between brackets is higher than unity. Since, under normal conditions, $-\eta_m + \eta_x$ is positive, the coefficient β_g is positive if the developing countries have a net export surplus, and negative if they have a net import surplus. The value of β_g depends critically on the developing countries net trade balance (α_g) and the price elasticities of world import demand and export supplies ($-\eta_m + \eta_x$).

If the project was sufficiently large to induce a substantial price decline, the size of the project should be included as an additional parameter[1/] in the formula of β_g. The simplified formula shown above measures the limit towards which tends β_g for infinitesimally small projects. The important fact is that, for most of the traditional LDC exports, this limit is positive and differs significantly from zero. The value of β_g is generally not very sensitive to the size of the project. Consequently, for most practical cases, the approximation given for small projects is satisfactory. The price decline induced by the project does not, therefore, even need to be computed.[2/]

Similarly, the national return to country j may be defined by $P(1 - \beta_j) - C_p$, where β_j is defined in the same way as β_g was. The coefficients α_g, α_{xg}, γ_g and η_{xg}, which referred to the LDC as a group in the above formula, are simply replaced by the coefficients α_j, α_{xj}, γ_j, and η_{xj}, which refer to the particular country concerned. Since $\alpha_g = \sum_j \alpha_j$, the coefficient α_j for individual countries may greatly differ from the

---

1/ The absolute value of β_g would then be higher than shown in the above formula.

2/ It is sometimes argued in this context that, even if the agency wants to maximize LDC returns, it can safely ignore the price effect of the project. The reasoning runs as follows: The output generated by the project is small in relation to the volume of world trade. Consequently, its impact on world prices is bound to be small and well within the margin of the projection errors. Following the same reasoning, one can argue that the monopolist should safely ignore the difference between marginal and average returns, because the impact of one additional unit of its output on the market price falls well within the margin of the price forecasting error made by the monopolist. This illustrates the fallacy of the previous argument.

coefficient α_g for the LDC as a group. The coefficients β_j may therefore greatly differ from the coefficient β_g. Thus, in the case of cocoa, the coefficient β_g is likely to be close to unity[1/], while the coefficient β_j for a marginal exporting or importing country ($\alpha_j \simeq 0$) does not differ significantly from zero.

Let us call v_p the output/capital ratio for project p. This coefficient is equal to the annual value of the exportable supply generated by the project divided by the value of the capital lent by the agency to implement it. The difference between the rate of national return r_{jp} to country j where the project is implemented and the rate of international return r_p to the lending agency can then be expressed in relation to the difference between the coefficients β_g and β_j by:

$$r_{jp} - r_p = 100\ (\beta_g - \beta_j)\ v_p$$

Let us now illustrate this formula by returning to our example of the cocoa project implemented in a marginal exporting country. Let us assume that the capital/output ratio is equal to 2.5 for this project. With $v_p = 1/2.5 = .4$, $\beta_g = 1$ and $\beta_j = 0$, the above formula shows that $r_{jp} - r_p = 40$. Clearly, if the rate of international return were 40% lower than the rate of national return, the agency would write cocoa on its black commodity list.

1/ "The Conflict ...", pp. 20-24, op. cit.

REPRINTED FROM

European Economic Review

Volume 3, No. 2, 1972

PRIVATE, NATIONAL AND INTERNATIONAL RETURNS; AN APPLICATION TO COMMODITY LENDING

L.M. GOREUX *
Development Research Center, Washington, D.C. 20433

pp. 131 – 180

NORTH-HOLLAND PUBLISHING COMPANY – AMSTERDAM

European Economic Review 3 (1972) 131–180. © North-Holland Publishing Company

PRIVATE, NATIONAL AND INTERNATIONAL RETURNS; AN APPLICATION TO COMMODITY LENDING

L.M. GOREUX *

Development Research Center, Washington, D.C. 20433

Received December 1971

The article investigates principles of project analysis when investment decisions influence prices and the distribution of income.

The first part analyses these problems as they appear to a national policy maker; in the second part, the approach is extended to analyses of the lending policy of an International Lending Agency.

This paper investigates how some of the welfare concepts, developed for allocating public funds within a country could be extended to the allocation of international funds by an international project lending agency. By analogy with the distinction between private and social returns within a country, a distinction is drawn between national and international returns.

The case of investments affecting commodity prices is used here to illustrate conflicts between private, national and international returns. Since various parties buying and selling the commodity are affected by the price variations induced by the investment, returns of the investment are defined in relation to each affected party. These returns are measured by the impact of the investment on the value of the objective function of the party concerned. The private return to the

* This paper is based on work carried out by the author while at the International Bank for Reconstruction and Development. The author wishes to acknowledge his gratitude to Prof. A. Manne, J. Waelbroeck and B. Balassa who commented on an earlier version of the paper as well as to several economists of the Development Research Center, I. Adelman, A. Condos, P. Davis, and A. Stoutjesdijk. However, the views presented in this paper are solely the respondibility of the author and should not be interpreted as representing the official opinion of the International Bank for Reconstruction and Development.

producer (or group of producers) i is defined as the contribution of the investment to i's profit. The national return to country j is defined as the contribution of the investment to the combined producers'–consumers' surplus in country j. The international return to an international project lending agency is defined as the contribution of the agency's lending activities to the international welfare function characterizing the objectives of that agency.

By drawing a distinction among the returns of an investment to various parties, the paper shows how income distribution within a country or among countries can be influenced by national or international investments policies. The distinction between the returns to different parties is particularly relevant when the investment decision is not taken by a simple decision agent, but is influenced by several agents, each reacting to the impact of the investment on the value of his own objective function.

The paper is divided into three parts. The first deals with the conflicts among various interest groups within a country, the second with the conflicts among trading countries and, the third with the conflicts between national and international returns in the case of international commodity lending.

In the first part, international prices are taken as exogenous to the investment decision made in the country, but domestic prices are allowed to vary between a higher and a lower limit, defined by the import and export prices. The sector is divided into the project area p and the area outside the project op. The national return is subdivided into three components: the consumers' surplus c, the surplus to producers p and the surplus to producers op.

In the second part, the international commodity price is a variable endogenous to the investment decision model. Each country (or group of countries) tries to maximize its own return, taking the demand and supply curves of its trading partners as exogenous. This restriction is relaxed in the last section where interactions among trading partners are studied in the context of a dynamic game.

The last part differs from the second by the introduction of an international agency lending to the *LDC*s only. It outlines a model maximizing an international welfare function $\Sigma_j w_j Y_j$, where Y_j is country j's gain and w_j a weight negatively correlated with j's average per caput income. The indirect benefit (or loss) accruing to country j on account of the agency's lending in countries other than j are included in the country's gain Y_j for various types of investment t. In addition to contraints on availability of international funds, constraints are imposed on the minimum benefits $\bar{Y}_j$, which should accrue to each country on account of the agency's lending activities. With these model specifications, the agency should differentiate its lending rates by countries j and types of investment t. The model solution gives the matrix r_{jt} defining the cut-off rates of national returns by countries j and types of investment t. Short of taking into account interactions between j and t in an optimizing model, two separate vectors r_j and r_t could be defined. Short of defining the vector r_t, the agency could draw a black and white commodity list.

1. Private versus national return

In drawing a distinction between private and social returns, Little and Mirrlees have concentrated their attention in correcting the price distortions caused by protection and by the excess of wages over the opportunity cost of labor [1]. We follow Little and Mirrlees and measure the social return of an investment in relation to the price of international traded commodities. In the first part of the paper, we restrict our analysis to the case of commodities f for which the import and export prices are exogenous data for country j; (we assume that country j accounts for a small share of the world trade for commodities f). But we depart from Little and Mirrlees by simultaneously recognizing the difference between the import and export prices and treating country j's trading pattern as an endogenous variable. If it is not known ex ante whether country j will import commodity f, will export it or will be self-sufficient, the marginal utility of commodity f to country j's consumers is an endogenous variable bounded upwards by the import price and downwards by the export price.

If the investment induces a price decline, the national return (defined as the contribution of the investment to the combined producers'–consumers' surplus) exceeds the return to country j's producers by the gain accruing to country j's consumers. Maximizing the national return thus defined leads to the competitive equilibrium solution; when each producing agent i is a price taker, i's marginal return is identical to the prevailing market price and, consequently, to the marginal utility of commodity f to country j's consumers. But when decision agent i faces a less than infinite price elastic demand for its products, i's marginal return is lower than the marginal utility to consumers defined by the market price; this non-competitive equilibrium solution does not lead, therefore to a Pareto optimum. The difference between social and private returns is analyzed below by considering first the model of a sector producing commodity f and, second, the model of a project accounting for a substantial share of the sectoral output. Let us start with the social returns in a sectoral model.

1.1. *The sector*

Let us consider a one-commodity sector, for example, the sector producing the entire rice output of the country. This sector can sell its rice at a fixed price $\bar{P}_m$ as long as the country imports rice and at a lower price $\bar{P}_x$ if the country exports rice. Let us assume, for example, that the CIF and FOB prices are respectively equal to 100 and 80 and that the average domestic transportation cost is 10 from the producing to the consuming area, from the producing area to the port and from the port to the consuming area. In the absence of any tariffs or taxes, the fixed prices $\bar{P}_m$ and $\bar{P}_x$ would be, in this case, respectively equal to 100 and 70. If the value added by the rice sector is a small fraction of the national income, the level of the national price-quantity demand curve for rice can be considered as an exogenous datum for the rice producing sector. Conse-

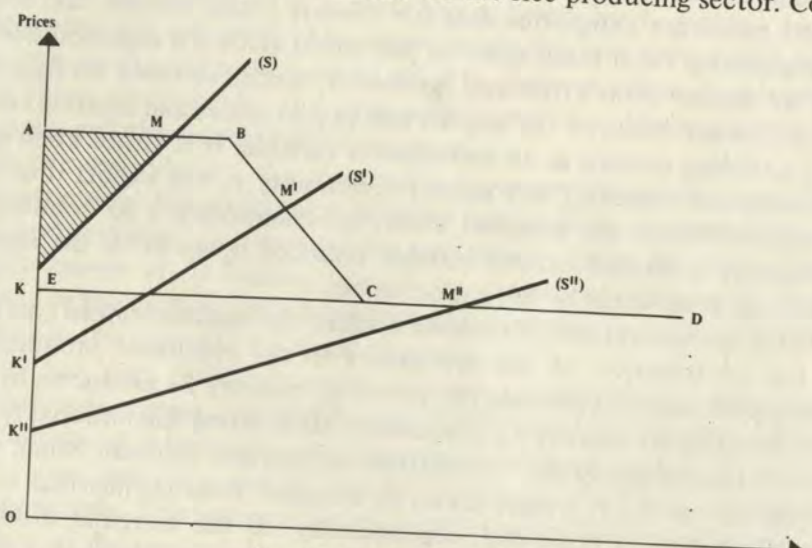

Fig. 1. Demand curve *ABCD*.

quently, the demand curve *ABCD* shown in fig. 1 can be treated as an exogenous datum for the sector.

Let us assume that a clear-cut distinction can be made between sectoral and national resources. All sectoral resources (land, water, local unskilled labor) cannot be used outside of the rice sector. All national resources (fertilizers, fungible capital, etc.) can be bought by the sector at fixed prices in unbounded quantities. The supply curves (*S*) shown in fig. 1 measure the cost of these national resources. The social utility added by the resources specific to the rice sector is then equal to the area between the demand curve *ABCD* and the sectoral supply curve. It corresponds in fig. 1 to areas *AMK*, *ABM'K'* or *ABCM''K''*, depending on whether the country imports, is self-sufficient or exports.

Had we known, ex ante, that rice would either be always imported or be always exported, we could have applied Little and Mirrlees' method. But if the structure of the trading pattern is not a datum but an unknown, we need to include in the model the demand curves *ABCD* for the product of the sector.

The sectoral model can be solved without being embedded in an economy-wide model if the sector (a) can be treated as a price-taker for the national resources it uses; (b) accounts for a modest share of the national income and therefore does not have a significant impact on the level of the demand curve for its products, and; (c) produces final goods for consumption or exports. If conditions (a) and (b) are approximately fulfilled, while condition (c) is not, the boundaries of the sector may have to be expanded, as shown in the example below.

Let us now consider the case of cotton production. Cotton can be exported as such. But it can also be sold to a domestic textile industry. We shall assume here that the textile industry faces an exogenously given demand curve for the textile goods it produces. As in the rice example above, this curve is represented in fig. 1 by *ABCD*. In this diagram, the position of the industry supply curve (*S*) depends on the price the textile industry has to pay for its cotton, because cotton is supposed to be the main input for textile production. Since the price of cotton is not the same if the country imports or exports it, the position of the supply curve (*S*) may be affected by the conditions under which cotton can be produced domestically.

In fig. 2a, the marginal cost of producing cotton domestically is represented by the curve *B'C'*. The price at which the textile industry can buy its cotton depends on the quantities it purchases. The textile

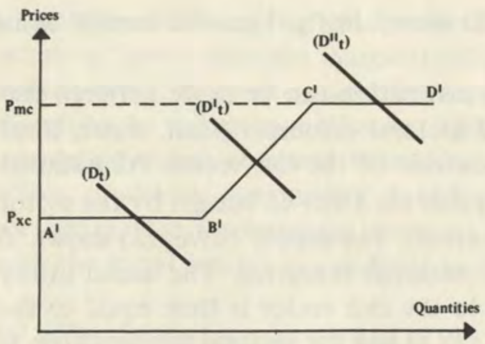

Fig. 2a. Textile industry: demand for cotton (D_t) and supply of cotton ($A'B'C'D'$).

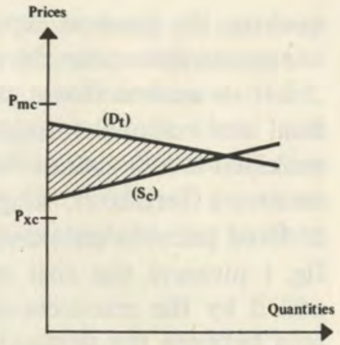

Fig. 2b. Demand for cotton by the textile industry (D_t) and cotton domestic supply (S_c).

industry therefore faces the supply curve $A'B'C'D'$ for its cotton. If, regardless of the decision made in that industry, the country would always export cotton (position D_t) or would always import it (D_t''), the cotton and the textile problems could be solved independently. However, if, depending on the decision made in the textile sector, the country can shift from an importing to an exporting position or vice versa, or can be self-sufficient in cotton (position D_t'), the cotton and the textile problems have to be solved simultaneously. In the case illustrated in fig. 2b, it is not profitable to export cotton. It is not profitable either to produce textiles for domestic consumption, if the textile industry has to buy its cotton at the import price. But it is profitable to produce some cotton and transform it into textiles for domestic consumption [7]. The social gain attached to this combined operation is represented by the hatched triangle of fig. 2b.

1.2. *The project area*

Let us now turn to the case of a project area p, which accounts for only part of the sectoral production. Let us put aside the case of sector-wide program (for example, the introduction of improved rice seeds) which is tested in p, taken as representative of the entire sector. If the model of the project area is designed to provide a representative sample of the sectoral model, the percentage increase of rice production resulting from the application of improved seeds should be the same in p and in the entire sector. The elasticity of the demand for rice in the

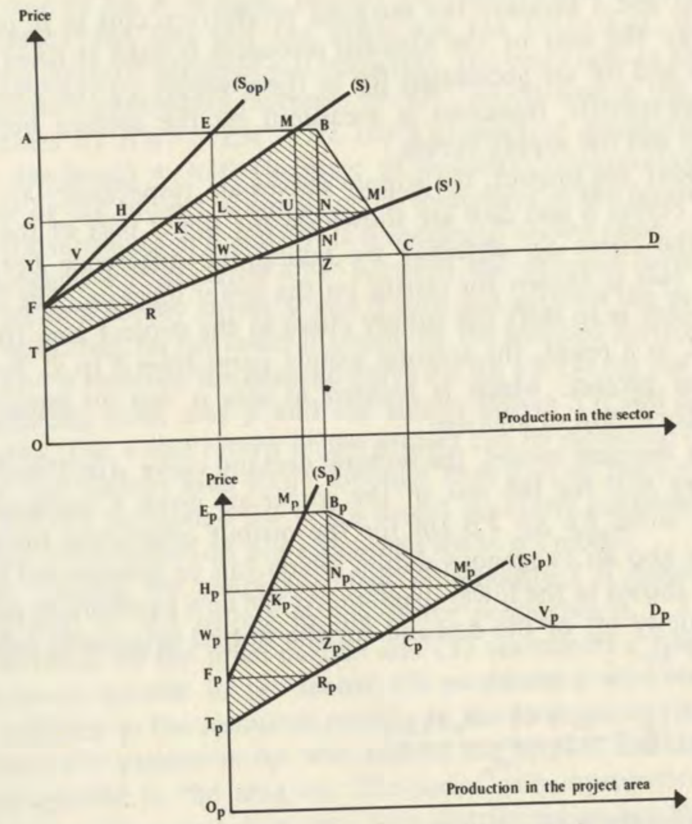

Fig. 3. Social return of project p.

model of p should therefore be the same as the elasticity of the demand for rice in the sector. We shall not elaborate on this case and we shall restrict our attention below to the case of an investment which can be implemented in p, but cannot be duplicated outside of the project area called op. For example, there is one single site for an irrigation dam and this site is located in area p and not in area op.

Areas p and op are each endowed with area-specific resources (land, water, unskilled labor, etc.), which cannot be employed outside. Areas p and op compete for the production of commodity f; the total demand for that commodity (produced in p and op) is given by the curve $ABCD$ on the upper part of fig. 3. Areas p and op are price-takers for all factors other than the area-specific resources. The supply curves

Sp, Sop and S measure the marginal production cost in p, op and $p + op$. Only the cost of the national resources bought at fixed prices by areas p and op are accounted for in these supply curves. The return to the area-specific resources is measured by the surface between the demand and the supply curves.

Without the project, the supply curves are respectively Sp, Sop and S. The curves S and Sop are shown on the upper part of the diagram, while the curve Sp obtained by taking the difference between the former two is shown for clarity on the lower diagram. The impact of the project is to shift the supply curve in the project area from Sp to $S'p$ and, as a result, the sectoral supply curve from S to S'. By assumption, the project, which is located in area p, has no impact on the supply curve Sop of area op.

In p's decision model, the sectoral demand curve $ABCD$ and the supply curve Sop for the rest of the sector are given. Consequently, the demand curve $Ep\ Bp\ Vp\ Dp$ for the output originating from project area p is also an exogenous datum which can be constructed by difference, as shown in the lower diagram. At the new equilibrium point $M'p$, the elasticity η_D^p of the demand for the output originating from area p is given [1] by:

$$\eta_D^p = \frac{\eta_D - (1-\alpha_p)\eta_{Sop}}{\alpha_p}.$$

This formula shows that the absolute value of the demand elasticity for the project area is always greater than for the sector. [2] If either the supply eleasticity in the rest of the sector η_{Sop}, or the demand elasticity for the sector, η_D is infinitely large, or the share of the project area α_p is infinitesimally small, the demand elasticity for the project area η_D^p is infinitely large and the project area can be treated as a price-taker. In fig. 3, none of these three conditions is fulfilled and the price of commodity f is an endogenous variable in p's decision model. Such a case is not unusual. Often, a large project can be treated as a price-taker for its inputs but not for its outputs. [3]

[1] See equation (15′) p. 156.

[2] Since $0 < \alpha_p < 1$ and since, under normal conditions, $\eta_D < 0$ and $\eta_{Sop} > 0$, it follows that $|\eta_D^p| > |\eta_D|$.

[3] This generally applies to large irrigation schemes which are to produce fruits and vegetables.

In the case of fig. 3, without project the country was importing the quantity MB ($=Mp\ Bp$) and the price was OA ($=Op\ Ep$). With the project, the country becomes self-sufficient, the price falls to OG ($=Op\ Hp$), domestic consumption increases by NM', production in the project area increases by $HM' - EM = HL$ (op's production displaced) + UN (imports displaced) + NM' (increase in domestic consumption). The social gain of the project is equal to the increment in the return to the area-specific resources in p and op. It is measured on the upper diagram, by the hatched surface $MBM'RTF$ between the demand curve $ABCD$ and the sectoral supply curves S' and S with and without the project. It is also [4] measured, on the lower diagram, by the hatched surface $Mp\ Bp\ M'p\ Rp\ Tp\ Fp$ between the demand curve $Ep\ Bp\ Vp\ Dp$ for the production originating from area p and the supply curves $S'p$ and Sp in the project area. The social return of the project can be measured from the model of the project area alone, because that part of the consumers' gain $AEHG$ not accounted for in p's model is exactly compensated for by the loss of producers op not accounted for in p's model.

Let us assume, for simplicity, that commodity f is consumed by nationals who do not belong to the sector producing it. [5] The three groups affected by the price decline are: (1) consumers c who do not own resources specific to the sector; (2) producers p who receive the surplus accruing to the resources specific to the project area (land, local labor, etc.); (3) producers op who receive the surplus accruing to the resources specific to the area op. The social gain represented by the hatched area of the upper diagram is then distributed among these three parties as follows:

Consumers c	$+MBM'K$		$+EMKH + AEHG$	$= ABM'G$
Producers p		$+KM'RTF - EMKH$		
Producers op			$-AEHG$	
National	$+MBM'K$	$+KM'RTF$		$= MBM'RTF$

[4] Trapezoids $MBM'K$ and $Mp\ Bp\ M'p\ Kp$ have equal areas, since they have the same height and equal basis ($AG = Ep\ Hp$, $MB = Mp\ Bp$, and $KM' = K_p\ M'_p$). For the same reason, trapezoids $KM'RF$ and $Kp\ M'p\ Rp\ Fp$ have equal areas ($GF = Hp\ Fp$, $KM' = Kp\ M'p$ and $FR = Fp\ Rp$). Finally, triangles FRT and $Fp\ Rp\ Tp$ have the same area ($FT = Fp\ Tp$ and $FR = Fp\ Rp$).

[5] This assumption can be relaxed by drawing the demand curves of consumers in p and op. This correction is essential when analysing the agricultural sector of LDCs, since a substantial part of the output is self consumed in the sector.

In the case of fig. 3, without project the country was importing the quantity *MB* (=*Mp Bp*) and the price was *OA* (=*Op Ep*). With the project, the country becomes self-sufficient, the price falls to *OG* (=*Op Hp*), domestic consumption increases by *NM'*, production in the project area increases by *HM'* – *EM* = *HL* (*op*'s production displaced) + *UN* (imports displaced) + *NM'* (increase in domestic consumption). The social gain of the project is equal to the increment in the return to the area-specific resources in *p* and *op*. It is measured on the upper diagram, by the hatched surface *MBM'RTF* between the demand curve *ABCD* and the sectoral supply curves *S'* and *S* with and without the project. It is also [4] measured, on the lower diagram, by the hatched surface *Mp Bp M'p Rp Tp Fp* between the demand curve *Ep Bp Vp Dp* for the production originating from area *p* and the supply curves *S'p* and *Sp* in the project area. The social return of the project can be measured from the model of the project area alone, because that part of the consumers' gain *AEHG* not accounted for in *p*'s model is exactly compensated for by the loss of producers *op* not accounted for in *p*'s model.

Let us assume, for simplicity, that commodity *f* is consumed by nationals who do not belong to the sector producing it. [5] The three groups affected by the price decline are: (1) consumers *c* who do not own resources specific to the sector; (2) producers *p* who receive the surplus accruing to the resources specific to the project area (land, local labor, etc.); (3) producers *op* who receive the surplus accruing to the resources specific to the area *op*. The social gain represented by the hatched area of the upper diagram is then distributed among these three parties as follows:

Consumers *c*	+ *MBM'K*		+ *EMKH* + *AEHG*	= *ABM'G*
Producers *p*		+ *KM'RTF* – *EMKH*		
Producers *op*			– *AEHG*	
National	+ *MBM'K*	+ *KM'RTF*		= *MBM'RTF*

[4] Trapezoids *MBM'K* and *Mp Bp M'p Kp* have equal areas, since they have the same height and equal basis (*AG* = *Ep Hp*, *MB* = *Mp Bp*, and *KM'* = K_p M'_p). For the same reason, trapezoids *KM'RF* and *Kp M'p Rp Fp* have equal areas (*GF* = *Hp Fp*, *KM'* = *Kp M'p* and *FR* = *Fp Rp*). Finally, triangles *FRT* and *Fp Rp Tp* have the same area (*FT* = *Fp Tp* and *FR* = *Fp Rp*).

[5] This assumption can be relaxed by drawing the demand curves of consumers in *p* and *op*. This correction is essential when analysing the agricultural sector of *LDC*s, since a substantial part of the output is self consumed in the sector.

The project brings to the country a large gain, but this gain is distributed very unequally [6] among the three groups *c*, *op* and *p*. Consumers *c* make a large gain. Producers *op* make a large loss. In this particular example, producers *p* gain, but producers $p + op$ together lose.

1.3. *Programming model*

The obvious advantage of the graphical analysis based on the demand curve for one single commodity is its simplicity. But the area between the demand and the supply curve can be interpreted as a measure of the sum of consumers' and producers' surplus only at the price of very restrictive assumptions. In particular, if the sector can produce commodities *A* and *B*, increasing the production of *A* is likely to affect the marginal cost of producing *B*. Consequently, the increment in the area between *A*'s demand and supply curves may be partly offset by a decline in the area between *B*'s demand and supply curves. The increment in the sum of producers' and consumers' surplus for commodity *A* alone, therefore, may provide a biased estimate of the net social gain. These difficulties explain why the concept of the sum of consumers' and producers' surplus which, in the days of A. Marshall, was very much in fashion, fell somewhat out of fashion. However, most of the objections made to the one-commodity analysis disappear, when the commodity demand curves described above are integrated within a large-scale multi-commodity, multi-factor programming model, which can be easily solved on modern computers [11, 12].

Remaining within linear programming techniques, two restrictive assumptions [7] still have to be made; but these restrictions on demand behavior are relatively weak. The first restriction refers to the substitutability among products. Within a product group (say, cereals) perfect substitutability is permitted within bounds among products (say, between rice and wheat) by allowing any convex combination of a predominantly rice basket and a predominantly wheat basket. But among product groups (say, between cereals and fruits) substitutability is not

[6] This was the result obtained in a two-district agricultural programming model of the Ivory Coast, pp. 57–59 [12]. It could also apply to the impact of large irrigation schemes in California (*p*) on cotton growers in the South of the United States (*op*); the result was to accelerate migrations of black workers into the cities.

[7] The income effect on the level of the demand curve can be introduced by iterations between the sectoral and the central models.

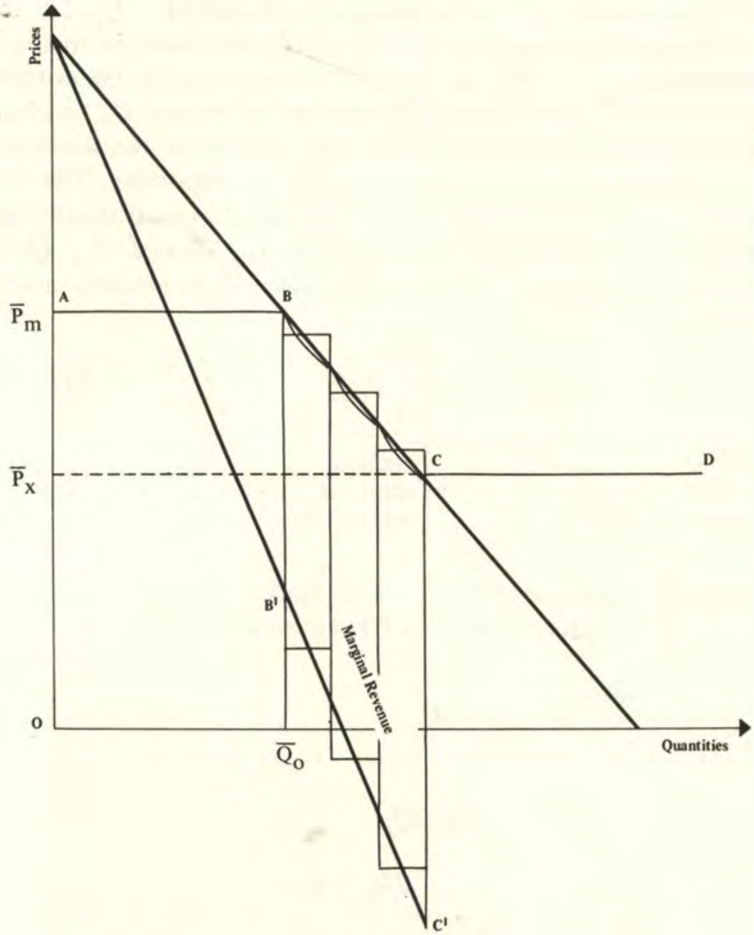

Fig. 4. Linear approximation of the price and marginal revenue curves.

permitted. The second restriction is that the amount of utility derived from product (or product group) A has no effect on the utility derived from product (or product group) B [11].

By allowing the model to import at the price $\overline{P}_m$ and to export at the price $\overline{P}_x$, the relevant part of the domestic demand curve is limited to the segment BC. This segment BC, together with the segment $B'C'$ along the marginal revenue curve, are approximated by staircases [8], as shown

[8] It may be noted that approximating the section $B'C'$ of the marginal revenue curve by horizontal steps is equivalent to approximating the price-quantity demand curve by segments of equilateral hyperbolae.

on fig. 4. Each step s (1, ..., n) is bounded in length $Q_s \leqslant \bar{Q}_s$. The utility of the additional consumption is $\Sigma_{s=1}^{s=n} \bar{P}_s Q_s$, the increase in the producers' surplus is $\Sigma_{s=1}^{s=n} - \bar{M}\bar{R}_s, Q_s$ – costs. The increase in the consumers surplus is $\Sigma_{s=1}^{s=n} (\bar{P}_s - \bar{M}\bar{R}_s) Q_s$. The volume of imports Q_m is obtained by constraining the model to fulfill the minimum requirement $\bar{Q}_0$ (length of segment AB) by either producing or importing. The volume of exports Q_x is given by the length of the last step used along segment CD. The utility provided by the quantity $\bar{Q}_0$ *exceeds* $\bar{P}_m$ $\bar{Q}_0$ by a constant. Neglecting this constant, the sum of producers' and consumers' surplus can be written:

$$U = \bar{P}_m \bar{Q}_0 - \bar{P}_m (\bar{Q}_0 - Q_0) + \sum_{s=1}^{s=n} \bar{P}_s Q_s + \bar{P}_x Q_x - C(Q)$$

$$\begin{bmatrix}\text{utility of}\\ \text{minimum}\\ \text{requirement}\end{bmatrix} - \begin{bmatrix}\text{cost of}\\ \text{imports}\\ \end{bmatrix} + \begin{bmatrix}\text{utility of}\\ \text{additional}\\ \text{consumption}\end{bmatrix} + \begin{bmatrix}\text{receipt}\\ \text{from}\\ \text{exports}\end{bmatrix} - \begin{bmatrix}\text{cost of}\\ \text{production}\\ \end{bmatrix}$$

Considering the segments AB and CD as steps [9] 0 and n+1, the utility added U and the producers' surplus PS can be written:

$$U = \sum_{s=0}^{s=n+1} \bar{P}_s Q_s - C(Q)$$

$$PS = U - \sum_{s=1}^{s=n} (\bar{P}_s - \bar{M}\bar{R}_s) Q_s$$

with

$$Q_s \leqslant \bar{Q}_s \qquad s(0, ..., n)$$
$$Q_m + Q_0 - \bar{Q}_0 = 0$$
$$Q_x - Q_{n+1} = 0$$
$$Q - \sum_{s=0}^{s=n+1} Q_s = 0.$$

To measure the social return of an investment, a distribution has to be made among three types of commodities:

(a) Commodities f_m, which will be always imported at the price $\bar{P}_{f_m}$

[9] Without loss of accuracy, the number of rows can be reduced by using convex combinations of activities selling the cumulated amounts $\bar{C}\bar{Q}_s = \Sigma_{r=0}^{r=s} \bar{Q}_r$ for a utility $\bar{U}_s = \Sigma_{r=0}^{r=s} \bar{P}_r \bar{Q}_r$. The two rows required are then the convexity constraint $\Sigma_s \lambda_s = 1$ and the commodity balance $Q - \Sigma_s \lambda_s \bar{C}\bar{Q}_s = 0$. The utility U is then given by $U = \Sigma_s \lambda_s U_s$ [11].

(b) Commodities f_x, which will be always exported at the price $\bar{P}_{f_x}$
(c) Commodities f which may be imported or exported or for which the country may be self-sufficient.

The contribution of the investment to the sum of producers' and consumers' surplus ΔPS and to the consumers' surplus ΔCS can be written:

$$\Delta U = \sum_{f_m} \bar{P}_{f_m} \Delta Q_{f_m} + \sum_{f_x} \bar{P}_{f_x} \Delta Q_{f_x} + \sum_{f} \sum_{s} \bar{P}_{fs} \Delta Q_{fs} - \text{costs},$$

$$\Delta PS = \Delta U - \sum_{f} \sum_{s} (\bar{P}_{fs} - \overline{MR}_{fs}) \Delta Q_{fs} ,$$

$$\Delta CS = \Delta U - \Delta PS .$$

If the investment affects only the output of commodities f_m and f_x, the social gain ΔU is equal to the gain of the producers ΔPS. If it affects the output of commodities f, conflicts of interest may arise among the various agents involved in the decision-making process

Let us start with the case of a cartel which faces the demand curve *ABCD* and has the marginal cost curve S' (see fig. 3). For a volume of production *AB*, the cartel's marginal cost is lower than the import price but higher than the export price. It is also higher than the cartel's marginal return from domestic sales. The cartel's interest is to produce only *AB*. In relation to the free competition solution, the social loss is represented on fig. 3 by the area of triangle *BM'N'*. If the demand is price-inelastic and the supply is price-elastic, this loss remains modest, but the change in income distribution can be very substantial. In relation to the free competition solution, the consumer's loss is represented by area *ABM'G* and the producer's gain by rectangle *ABNG* minus triangle *NM'N'*. If, at production level *AB*, the marginal production cost were lower than the export price, the cartels' interest would be to act as a discriminating monopolist by selling to domestic consumers at price *OA* and selling abroad at price *OY*. The programming model optimizing the cartels' return will therefore fulfill *AB* first, then skip *BC* and go directly to *CD*.

Let us now turn to the case of a government who has to make the decision of building or not building an irrigation dam. Clearly, the government (unlike the monopolist) has to include the consumer surplus in measuring the social return (ΔU) to the investment. But the ways in which the government should recover the initial capital cost depends on the distribution of the producers'–consumers' surplus among the various parties. If most of the social gain goes to consumers in the form of lower food prices, there is an argument for financing most of the dam from the general budget. However, reducing the price of water would increase the gains of the producers in area *p* whose income level might be satisfactory even without subsidies. Furthermore, it would not help producers in area *op* who lose because of the scheme and who, before the scheme, might have been poorer [10] than producers in area *p*. In selecting public investments, the government should therefore give attention to the income distribution effects of these investments.

We have contrasted a cartel maximizing income accruing to its members with a government agency maximizing national consumers' and producers' surplus. In practice, there are few watertight cartels and few governments whose sole objective is to maximize the sum of consumers' and producers' surplus. There are often pressure groups, lobbys and considerations of income distribution which affect the nature of the government objective function or introduce additional constraints in the government decision model. It is therefore probably more important to analyze the trade-offs among objectives than to choose the optimal investment pattern on the basis of a single objective.

2. Trading partners maximizing their national returns

In the previous part, we assumed that an increase in the production of a commodity did not affect the world price level. The analysis was therefore limited to the impact of domestic price variations on various groups within the country. In this part, we assume that an increase in the production of commodity *k* in country *j* has an impact on the world price of that commodity and, consequently, on the various countries trading that commodity with *j*.

Within a country, we had previously drawn a distinction among three interest groups: the producers *p* in the project area, the producers *op* outside this area, and the consumers *c*. Now, we shall draw a distinction

[10] This could be illustrated by comparing in Mexico the rich farmers in the North-West irrigated perimeters with the poor farmers of the high plateaus.

among the exporting country *j* which has to make an investment decision, the other exporting countries *oj* and the importing countries. Exporting country *j* replaces, in this part of the paper, the group of producers *p* considered in the previous part. The problem remains basically the same; the solution is different because the roles of the decision agents *p* and *j* are quite different. In the previous part, the main decision agents were the individual farmers and the government. The group of farmers in the project area *p* was generally a loose unit which could influence government decisions only through lobbying. In this part, country *j* is a major decision agent who can raise export taxes and import duties. Previously, agent *p* was able to influence government decisions, as the weak baron was able to influence the will of an absolute monarch. Now, agent *j* is the medieval baron who does not obey any king.

The difference in the decision making process is reflected in the formulation of the model. Before, social welfare was generally used as the objective function, while the return to group *p* was introduced only as an accounting row or a constraint. Now the return to country *j* is the objective function.

Due to the existence of several optimizing agents, the analysis is conducted in two stages. In the first section, we optimize country *j*'s decision, taking the demand and supply curves of countries other than *j* as exogenous to country *j*'s decision. In the second section, we analyze the interactions among the decisions of the various trading partners within the context of game theory.

2.1. *Comparative statics*

Before considering the pricing of commodities in a national programming model, despite all the restrictive assumptions required, let us start with the one-commodity graphical analysis.

2.1.1. *Graphical analysis.* In the absence of any tax or tariff, the equilibrium point M (Q, P) is at the intersection of the world demand and supply curves (D) and (S) shown in the upper diagram of fig. 5. The quantity A_jB_j (upper diagram) exported by country *j* is equal to the quantity A_oB_o (middle diagram) imported by countries *o*. Let us now depart from this equilibrium by launching a project in island *p* which had never consumed nor produced the commodity before. The appearance of this new exporter on the world market has the effect of shifting

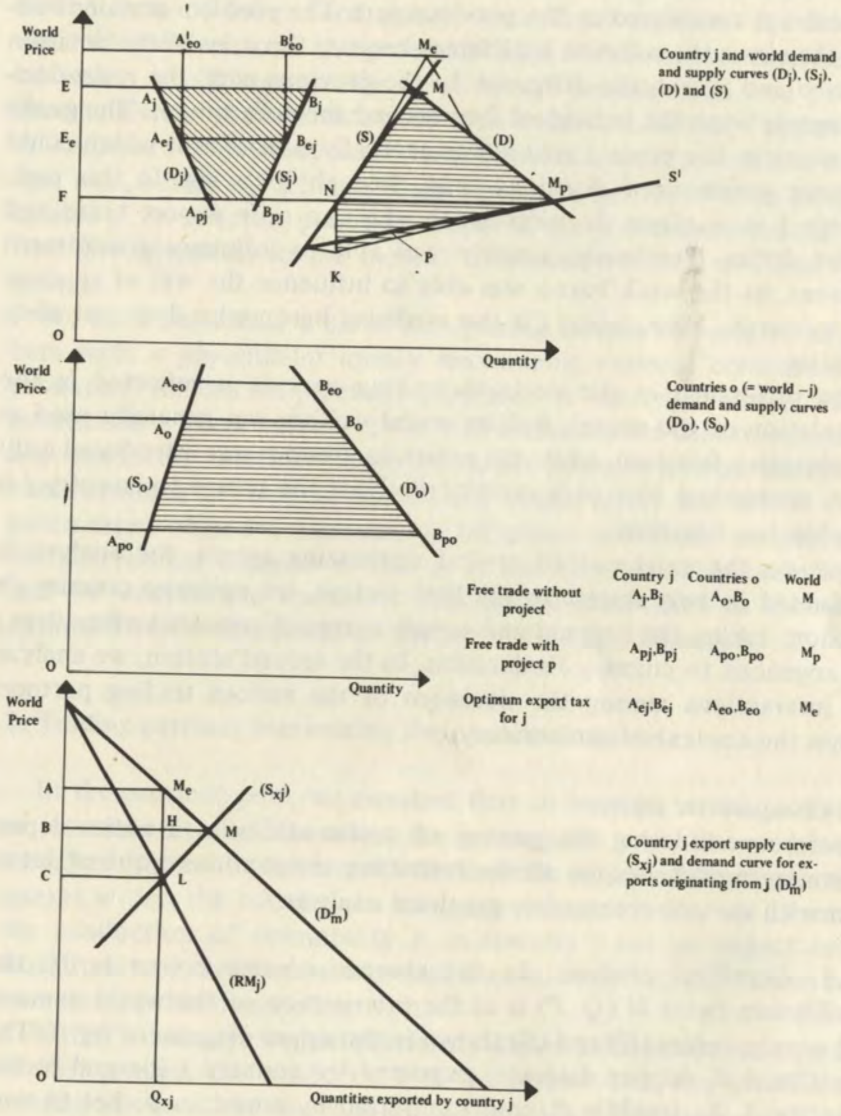

Fig. 5. Impact of project p under free trade and of the optimum export tax for country j.

the world supply curve from S to S' and of reducing the world price from OE to OF. The impact of this shift on the combined producers'–consumers' surplus of island p and of countries j and o is as follows:

Island p	$+NKM_p$		$=NM_pK$
Country j	$-A_jB_jB_{pj}A_{pj}$		$=-A_jB_jB_{pj}A_{pj}$
Countries o	$+A_jB_jB_{pj}A_{pj}$	$+NMM_p$	$=A_oB_oB_{po}A_{po}$
World	$+NKM_p$	$+NMM_p$	$=NMM_pK$

If "world welfare" is defined as the sum of the national consumer–producer surpluses, project p has increased world welfare. But the gain in 'world welfare' has not been distributed equally between the three parties j, o and p. Country j has suffered a loss. Countries o have captured j's loss and have, in addition, shared the net world gain together with island p. Let us now imagine that island p belongs to country j. By equalizing the marginal production cost in the project area p with the world price which would prevail with the project (net of transportation costs), country j would lose if, as in this case, area $A_j B_j B_{pj} A_{pj}$ exceeds the area KNM_p. Clearly, country j's interest is not to implement project p.

To compare the free trade solution and the optimum solution for country j, it is convenient to shift to the lower diagram of fig. 5. S_{xj} is the marginal cost curve for country j's exports, D^j_m is the demand curve for exports originating from j and RM_j is the marginal return curve from j's exports. The free trade equilibrium point M is at the intersection of S_{xj} and D^j_m. The optimum equilibrium for country j is at point L, where S_{xj} and RM_j intersect. By establishing the export tax LM_e, country j exports a smaller quantity AM_e at a higher world price OA; as appears on the upper diagram, it consumes a larger quantity $E_e\ A_{ej}$ at a lower internal price OE_e. In country j, both the consumers and the government gain from the export tax, while the producers lose. Country j as a whole gains because the government could redistribute the profit of the tax so as to make both consumers and producers of country j better off with than without the export tax. The impact of the tax on the various trading countries can be summarized as follows:

Country j	$+HBAM_e - HML$	
Countries o (rest of the world)	$-HBAM_e$	$-HMM_e = -MBAM_e$
World		$-HML-HMM_e = -MLM_e$

Country j's export tax results in a second best solution from the world point of view. With free trade, the world's combined producer–consumer surplus would have been higher; consequently the winning countries could have "bribed" the losing ones so that all countries, including country j, could have been better off with free trade. But, country j is better off with an export tax than without an export tax and without a bribe.

2.1.2. *National programming model.* In the programming model, we have considered earlier the case of commodities f for which country j was a price-taker on the world market. Let us now turn to the case of commodity k, the international price of which is endogenous as illustrated in fig. 5. This commodity is produced for domestic consumption and for export. In country j's social objective function, domestic consumption has to be valued according to its utility by the area under the demand curve D_j on the upper diagram. But, the return from exports has to be valued not according to its utility for the importers (area [11] under the demand curve D^j_m on the lower diagram) but by its return to country j which is equal to the area under curve RM_j in the lower diagram.

The domestic demand curve D_j and the marginal revenue curve from exports RM_j have to be approximated by two staircases, the height $P_{k_d s}$ and $RM_{k_x s}$ of each step being the same on both staircases. If Q_k refers to domestic production, Q_{k_d} to domestic consumption, Q_{k_x} to export, s being a subscript characterizing the steps along the two staircases, the commodity balance can be written:

$$Q_k - \sum_s Q_{k_d s} - \sum_s Q_{k_x s} = 0$$

with

$$Q_{k_d s} \leq \bar{Q}_{k_d s}$$
$$Q_{k_x s} \leq \bar{Q}_{k_x s}$$

and the entries in the social objective function:

$$\sum_{k_d} \sum_s \bar{P}_{k_d s}\, Q_{k_d s} + \sum_{k_x} \sum_s \overline{MR}_{k_x s}\, Q_{k_x s} - \text{costs.}$$

[11] This would apply to the discriminatory monopolist.

The optimal solution will then be to expand production up to the point where the marginal production cost is equal to both the marginal return from exports and the utility from domestic consumption. The level of the optimum export tax will be the vertical distance LM_e between the last step used on RM_j and the point on curve D^j_m with the same abscissa.

By analogy, if country j were a major importer affecting the import price P_{k_m} of commodity k_m, the marginal [12] import cost curve MC_j and the domestic demand curve D should be approximated by staircases. The commodity balances and the entries in the social objective function could then be written:

$$Q_k + \sum_s Q_{k_m s} - \sum_s Q_{k_d s} = 0$$

$$\sum_{k_d} \sum_s \bar{P}_{k_d s}\, Q_{k_d s} - \sum_{k_m} \sum_s \overline{MC}_{k_m s}\, Q_{k_m s} - \text{costs.}$$

The model would equalize the marginal cost of production to both the marginal cost of imports and the utility of domestic consumption. The level of the optimum import tax would then correspond to the vertical distance between the last step used in the marginal import cost curve and the import supply curve.

2.1.3. *Multi-country model.* Let us now consider an indivisible project p in a one-commodity n-country model. The problem is to measure the impact of project p on each country and to define under which conditions country j (or group of countries g) is better off with than without the project.

Assumptions

(1) The project producing commodity k has no impact on the price of commodities other than k. The international prices of commodities other than k are used, therefore, as numeraire for measuring the benefits of countries j (or groups of countries g) on account of the project p.

(2) Country j pays for its imports or receives for its exports of commodity k price $P_j = P_x + T_j$, where P_x is the world export reference price and T_j is a country-specific transportation cost differential. The

[12] Assuming the country is not a discriminatory monopolist.

project p induces a variation in the world reference price from P_x to $P_x + \Delta P_x$ and in the country-specific price from P_j to $P_j + \Delta P_x$; the country-specific margin T_j therefore remains unaffected by the price change ΔP_x.

(3) Import demand and export supply curves M_j and X_j are defined for each country j in relation to the world reference price P_x and the levels of these curves is not [13] affected by ΔP_x.

(4) γ_{xj} P_{xj} measures the opportunity cost of the resources released by reducing country j's exports by one marginal unit. Similarly, γ_{mj} P_{mj} measures the opportunity cost of country j's marginal unit of imports. The coefficients γ_{xj} and γ_{mj} would be equal to unity with neutral effective protection, full employment and perfect mobility of country j's resources. In practice, the coefficient γ_{xj} is substantially lower than unity for tropical export crops.

(5) The marginal opportunity costs $\gamma_{xj} P_{xj}, \gamma_{mj} P_{mj}$ and the average opportunity cost $C_p^{(j)}$ per unit of projects output are measured in relation to international prices using the Little and Mirrlees' method or the programming approach described earlier.

(6) Within the margin of price variation from P_x to $P_x + \Delta P_x$ induced by the project, the export supply and import demand curves can be approximated linearly. Similarly, the marginal opportunity cost can be approximated linearly from $\gamma_j P_j$ to $\gamma_j (P_j + \Delta P_x)$.

(7) The net gain (+) or loss (–) of group g is the algebraic unweighted [14] sum of the individual gains or losses of every country j belonging to g.

After having reviewed the assumptions, the reader may skip the algebra and go directly to the implications on page 158.

The first subscript x or m characterizes exports or imports. The absence of the first subscript indicates either that the formula applies regardless of whether the country imports or exports, or summation over gross imports (–) and gross exports (+). The second subscript p, j or g characterizes the project p, the country j or the group of countries g. The absence of the second subscript refers to the world as a whole.

The main symbols are summarized below:

Q_{xj} = volume of j's gross exports (+)
Q_{mj} = volume of j's gross imports (–)

[13] This assumption will be relaxed in section 2.2.
[14] This assumption will be relaxed in sections 3.1.3 and 3.2.

Q_{xg} = $\Sigma_{j\in g} Q_{xj}$ = volume of g's gross exports (+)
Q_x = $\Sigma_j Q_{xj}$ = volume of world gross exports (+)
Q_m = Q_x = volume of world gross imports (–)
α_{xj} = Q_{xj}/Q_x = share of j's gross exports in relation to world exports (+)
α_{mj} = Q_{mj}/Q_x = share of j's gross imports in relation to world exports (–)
α_{xg} = Q_{xg}/Q_x = share of g's gross exports in relation to world exports (+)
α_{mg} = Q_{mg}/Q_x = share of g's gross imports in relation to world exports (–)
α_g = share of g's net exports (+) or net imports (–) in relation to world exports (+)
P_{xj} = $P_x + T_j$ = price j receives for its exports
P_{mj} = $P_x + T_j$ = price j pays for its imports
P_x = world reference export price
$\gamma_{xj} P_{xj}$ = opportunity cost of the resources released by reducing j's exports by one marginal unit
$\gamma_{mj} P_{mj}$ = opportunity cost of the marginal unit of imports
ΔP_x = $\Delta P_j = \epsilon P_x$ = price variation induced by the project
η_j = elasticity of quantities imported or exported by country j in relation to variations of the world reference price P_x
η_m = elasticity of the world import demand in relation to P_x (–)
η_x = elasticity of the world export supply excluding the project in relation to P_x (+)
η = $-\eta_m + \eta_x$ = (+)
$\alpha_p Q_x$ = volume of exports generated by the project
V_p = $\alpha_p Q_x (P_x + \Delta P_x)$ = exports generated by the project valued at the prevailing world reference price
$C_p^{(j)}$ = average opportunity cost per unit of the project output in country j
$\pi_p^{(j)}$ = direct gain (+) or loss (–) from the project accruing to country j
π_j = indirect gain (+) or loss (–) incurred by country j on account of the price variation induced by the project

All quantities Q and shares α are counted positively when they refer to gross or net exports, and negatively when they refer to gross or net imports. ΔP_x is counted negatively for price declines and positively for

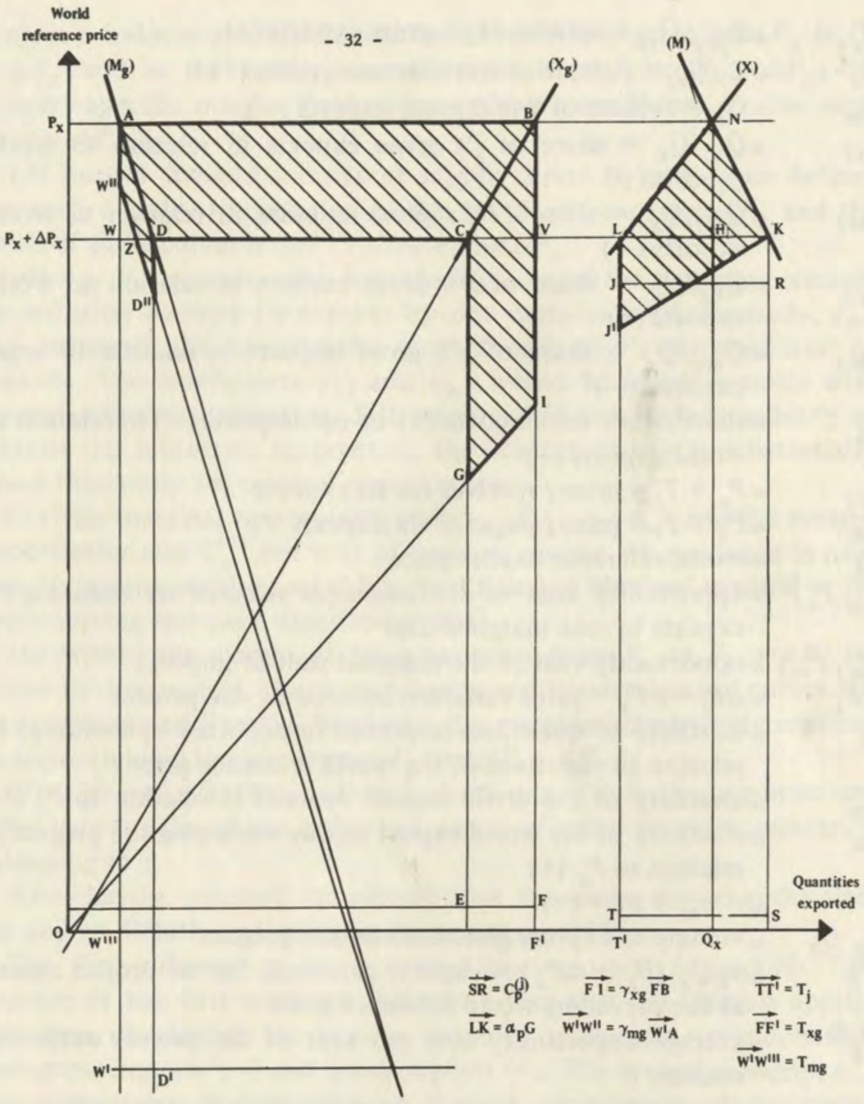

Fig. 6. Impact of project p on group g ($\alpha_g > 0$).

price increase (eradication schemes). π always stands for gain. If the computed value of π is positive, it has to be interpreted as a net gain. If it is negative, it has to be interpreted as a net loss. With these sign conventions, the formulae are valid regardless of whether the project induces a price decline ($\alpha_p > 0$) or a price increase ($\alpha_p < 0$), whether

the group g has a net export surplus ($\alpha_g > 0$) or a net import surplus ($\alpha_g <$ 0), whether the opportunity cost coefficients γ are smaller or larger than unity. Fig. 6 is used only for illustrative purpose.

Project p

Without the project, the equilibrium point corresponds in fig. 6 to the point N where the world import demand and export supply curves M and X intersect. The quantity of world exports is Q_x and the world export reference price is P_x. The project p generates the quantum of exportable supplies **LK** $= \alpha_p\, Q_x$ and induces the fall **NH** in the world reference price from P_x to $P_x + \Delta P_x$. The price decline ($\Delta P_x < 0$) results in a reduction $|\Delta Q_x|$ in the world exports originating from outside the project and an increase $|\Delta Q_m|$ in world imports. With the sign convention used, both ΔQ_x and ΔQ_m are negative. ΔQ_x is a reduction in the volume of exports counted as +. ΔQ_m is an increase in the value of imports counted as −.

Without the project, the world trade balance was:

$$Q_x + Q_m = 0\,. \tag{1}$$

The impact of the project on this balance is:

$$\alpha_p Q_x + \Delta Q_x + \Delta Q_m = 0 \tag{2}$$
$$(\ \mathrm{LK} + \mathrm{HL} + \mathrm{KH} = 0),$$

where ΔQ_x and ΔQ_m are defined by:

$$\frac{\Delta Q_x}{Q_x} = \eta_x \frac{\Delta P_x}{P_x} \tag{3}$$

$$\frac{\Delta Q_m}{Q_m} = \eta_m \frac{\Delta P_x}{P_x}\,. \tag{4}$$

By combining (1), (2), (3) and (4), the relative price decline ϵ can be written:

$$\epsilon = \frac{\Delta P_x}{P_x} = \frac{-\alpha_p}{\eta_x - \eta_m} = \frac{-\alpha_p}{\eta}\,. \tag{4'}$$

The size of the project can therefore be characterized either by its share α_p of world exports before the project or by the relative price decline ϵ induced by the project. If the project output $\alpha_p\, Q_x$ is sold at the world reference price $P_x + \Delta P_x$, by using equation (4′), the export value generated V_p can be written:

$$V_p = \alpha_p Q_x (P_x + \Delta P_x) = -\epsilon(1+\epsilon)\eta P_x Q_x \,. \tag{5}$$

If the project output is exported at the price $P_x + \Delta P_x + T_j$, where T_j accounts for transportation cost differential, the export value $V_p^{(j)}$ of the project output is:

$$V_p^{(j)} = \alpha_p Q_x (P_j + \Delta P_j) = \alpha_p Q_x (P_x + \Delta P_x + T_j) = V_p + \alpha_p Q_x T_j \,. \tag{6}$$

If $C_p^{(j)}$ is the average opportunity cost per unit of the project output (measured in terms of internationally tradable commodities), the direct profit $\pi_p{}^{(j)}$ of the project (area LKJ' = area $LKRJ$ in fig. 6) is:

$$\pi_p^{(j)} = V_p^{(j)} - \alpha_p Q_x C_p^{(j)} = V_p \frac{P_j + \Delta P_j - C_p^{(j)}}{P_x + \Delta P_x} \,. \tag{7}$$

Country j where the project p is not located

j's gain resulting from the impact of the price decline $\Delta P_x = \Delta P_j$ on j's export earnings (or import bill) from (or for) commodity k is measured by:

$$(P_j + \Delta P_j)(Q_j + \Delta Q_j) - P_j Q_j = Q_j \Delta P_x + (P_j + \Delta P_x)\Delta Q_j \,. \tag{8}$$

To this first effect has to be added the saving resulting from the opportunity cost of the resources released by reduction in j's exports or the gain from j's additional imports. If this marginal gain is measured (in terms of internationally tradeable commodities) by $\gamma_j P_j$, when the price declines from P_x to $P_x + \Delta P_x$, country j's gain is:

$$-\gamma_j \int_{Q_j}^{Q_j+\Delta Q_j} P_j(Q_j)\,\mathrm{d}Q_j = -\gamma_j \frac{P_j + \Delta P_j + P_j}{2}\Delta Q_j \,. \tag{9}$$

The minus sign is due to the fact that a price decline [15] ($\Delta P_x < 0$) induces a positive gain and implies $\Delta Q_j < 0$. In the case of fig. 6, the gain from the resources released corresponds to the trapezoid $EGIF$

[15] On the opposite $\Delta P_x > 0$ implies $\Delta Q_j > 0$ and induces a loss.

with

$$\mathrm{FI} = \gamma_{xj} P_j \,, \qquad \mathrm{EG} = \gamma_{xj}(P_j + \Delta P_j) \,, \qquad \mathrm{FE} = \Delta Q_{xj} \,.$$

By adding (8) and (9), j's gain π_j can be written:

$$\pi_j = Q_j \Delta P_x + [(1-\gamma_j)P_j + \Delta P_x - \gamma_j \frac{\Delta P_x}{2}]\,\Delta Q_j \,. \tag{10}$$

Noting that:

$$\frac{\Delta Q_j}{Q_j} = \eta_j \frac{\Delta P_x}{P_x} \,,$$

replacing ΔP_x by ϵP_x and introducing the share coefficients α and V_p from (5)

$$P_x \Delta Q_j = \alpha_j \eta_j \epsilon P_x Q_x = -\frac{V_p}{1+\epsilon}\frac{\eta_j}{\eta}\alpha_j$$

$$Q_j \Delta P_x = \alpha_j \epsilon P_x Q_x = -\frac{V_p}{1+\epsilon}\frac{\alpha_j}{\eta} \,,$$

π_j can be rewritten in relation to V_p by defining a coefficient β_j.

$$\pi_j = -\beta_j V_p = -V_p \frac{\alpha_j}{\eta(1+\epsilon)}\left[1 + \eta_j[(1-\gamma_j)\frac{P_j}{P_x} + \epsilon(1-\tfrac{1}{2}\gamma_j)]\right] . \tag{11}$$

Country j would be better off with than without the project provided the sum of the direct project gain $\pi_p^{(j)}$ and of the indirect impact π_j on country j is positive. Combining (7) with (11) the condition $\pi_p^j + \pi_j > 0$ can be written:

$$C_p^{(j)} < P_j + \Delta P_j - \beta_j (P_x + \Delta P_x) \,, \tag{12}$$

where P_j refers to the import or export price, depending on whether j imports or exports and β_j is a coefficient defined by equation (11). When the relative price decline tends towards zero, the value of this coefficient tends towards the limit β_j^*:

$$\beta_j^* = \frac{\alpha_j}{\eta}[1 + \eta_j(1-\gamma_j)\frac{P_j}{P_x}] \,. \tag{13}$$

Equations (14) and (15) show the values of β_j^* for two values of γ_j. For $\gamma_j = 1$, the opportunity cost of the marginal unit traded is identical to the price (or received) for it. For $\gamma_j\ (P_j + \Delta P_j) = C_p^{(j)}$, the saving made by reducing the output of the established producers is identical to the production cost of the new producers; production of commodity k is therefore optimally allocated within country j between the project area and the rest of the country.

$$\beta_j^* = \frac{\alpha_j}{\eta} \qquad \text{for} \qquad \gamma_j = 1\,, \tag{14}$$

$$\beta_j^* = \frac{\alpha_j}{\eta - \alpha_j \eta_j} \qquad \text{for} \qquad \gamma_j (P_j + \Delta P_j) = C_p^{(j)}\,. \tag{15}$$

In the case of an exporting country ($\alpha_j > 0$), calling η_{xoj} the elasticity of exports originating from countries other than j and noting that $\eta = -\eta_m + \eta_x = -\eta_x + \alpha_j\ \eta_{xj} + (1-\alpha_j)\ \eta_{xoj}$, (15) can be rewritten in relation to the elasticities in countries other than j:

$$\beta_{xj}^* = \frac{\alpha_j}{-\eta_m + (1-\alpha_j)\eta_{xoj}} \qquad \text{for} \qquad \gamma_j (P_j + \Delta P_j) = C_p\,. \tag{15'}$$

Group of countries g

The indirect return π_j to country j is measured in (11) in terms of the international prices of commodities other than k. Assuming that one dollar gain or loss accruing to any country j, belonging to g, has the same value, the indirect return π_g to group g is defined by:

$$\pi_g = \sum_{j \in g} \pi_j\,. \tag{16}$$

Countries j belonging to g are stratified between exporters xj and importers mj. Average coefficients α_g, η_g, γ_g and P_g are defined for each stratum. For the export stratum, the definitions are given in (17):

$$\alpha_{xg} = \sum_{xj \in g} \alpha_{xj} \qquad \eta_{xg} = \frac{\sum_{xj \in g} \alpha_{xj}\ \eta_{xj}}{\alpha_{xg}}$$
$$\gamma_{xg} = \frac{\sum_{xj \in g} \alpha_{xj}\ \eta_{xj}\ \gamma_{xj}}{\alpha_{xg}\ \eta_{xg}} \qquad P_{xg} = \frac{\sum_{xj \in g} \alpha_{xj}\ \eta_{xj}\ \gamma_{xj}\ P_{xj}}{\alpha_{xg}\ \eta_{xg}\ \gamma_{xg}}\,. \tag{17}$$

For the import stratum, the coefficients can be obtained simply by replacing subscript x by subscript m. The share of net trade α_g is defined by $\alpha_g = \alpha_{xg} + \alpha_{mg}$.

Replacing π_j in (16) by its value given in (11) and using the definitions given in (17), π_g (which corresponds to the hatched area $W''ABIGCDD''$ in fig. 6) can be written:

$$\pi_g = -\beta_g V_p = -\frac{V_p}{1+\epsilon}\left[\frac{\alpha_g}{\eta} + \alpha_{xg}\frac{\eta_{xg}}{\eta}\,[(1-\gamma_{xg})\frac{P_{xg}}{P_x} + \epsilon(1-\tfrac{1}{2}\gamma_{xg})]\right.$$
$$\left. + \alpha_{mg}\frac{\eta_{mg}}{\eta}\,[(1-\gamma_{mg})\frac{P_{mg}}{P_x} + \epsilon(1-\tfrac{1}{2}\gamma_{mg})]\right]. \tag{18}$$

$$\begin{aligned} -|\text{area } W''ABIGCDD''| &= -|\text{area } ABVW| - |\text{area } CVFE| + |\text{area } GIFE| \\ &\quad -|\text{area } DWW'D'| + |\text{area } D''W''W'D'| \\ &= -|\text{area } ABVW| - |\text{area } CVIG| - |\text{area } DZD''| \\ &\quad +|\text{area } ZWW''|\,. \end{aligned}$$

By combining eq. (18) and (7), it follows that group g is better off with than without the project ($\pi_g + \pi_p^j > 0$) if:

$$C_p^{(j)} < P_j + \Delta P_j - \beta_g (P_x + \Delta P_x)\,. \tag{19}$$

P_j is the specific export or import price for country j where the project is located and β_g is the coefficient defined in (18).

For particular values of γ, the values of π_g and β_g can be simplified. Thus, for $\gamma_{xg} = \gamma_{mg} = 1$, π_g (which corresponds to area $ABCD$ in fig. 6) takes the form (20):

$$\pi_g = -\frac{V_p}{1+\epsilon}\left[\frac{\alpha_g}{\eta} + \frac{\epsilon}{2}(\alpha_{xg}\frac{\eta_{xg}}{\eta} + \alpha_{mg}\frac{\eta_{mg}}{\eta})\right] \tag{20}$$

$$\begin{aligned} -|\text{area } ABCD| &= -|\text{area } ABVW| + |\text{area } CVB| \\ &\quad + |\text{area } DWA|\,. \end{aligned}$$

With $\gamma_{xg} = \gamma_{mg} = 0$, π_g measures the loss in net export earnings.

$$\pi_g = -\frac{V_p}{1+\epsilon}\left[\frac{\alpha_g}{\eta} + \alpha_{xg}\frac{\eta_{xg}}{\eta}\;\frac{P_{xg}+\Delta P_{xg}}{P_x} + \alpha_{mg}\frac{\eta_{mg}}{\eta}\;\frac{P_{mg}+\Delta P_{mg}}{P_x}\right] \quad (21)$$

$$-|\text{area } W'ABFECDD'| = -|\text{area } ABVW| - |\text{area } CVFE| - |\text{area } DWW'D'|\,.$$

When ϵ tends towards zero, the coefficient β tends towards β^*

$$\beta_g^* = \frac{\alpha_g}{\eta} + \alpha_{xg}\frac{\eta_{xg}}{\eta}(1-\gamma_{xg})\frac{P_{xg}}{P_x} + \alpha_{mg}\frac{\eta_{mg}}{\eta}(1-\gamma_{mg})\frac{P_{mg}}{P_x}\,, \quad (22)$$

which can be written:

$$\beta_g^* = \frac{\alpha_g}{\eta} \quad \text{for} \quad \gamma_{xg} = \gamma_{mg} = 1\,. \quad (23)$$

If group g is extended to the entire world $\alpha_g = 0$, $\alpha_{xg} = 1$ and $\alpha_{mg} = -1$. It follows that in equation (23) β_g^* becomes zero and that equation (19) becomes $C_p^{(j)} < P_j + \Delta P_j$. We end up with the well-known result of the free trade model: the production cost in the marginal project should equate the prevailing world price after adjustment for transportation costs ($P_j - P_x = T_j$).

Implications

The difference between the gain π_j accruing to country j implementing project p and the gain π_g of the group g to which country j belongs is $\pi_j - \pi_g = (\beta_g - \beta_j)\,V_p$. If country j is a marginal exporter, this difference becomes $\beta_g V_p$, where V_p is the value of the gross output of the project. For a cartel or an international project lending agency representing the interests of group g, it is important to take into account the difference $\pi_j - \pi_g$ and therefore to compute β_j and β_g.

The computation of β_j and, in particular, that of β_g from (18) may appear difficult. However, in most practial cases, the output of the project is small in relation to the volume of world exports. Consequently, the relative price decline (ϵ) induced by the project is also small and β_g^* (or β_j^*) provides a satisfactory [16] approximation for β_g (or β_j). Thus, for groups g, the criterion for project selection given in equation (19)

[16] The reader may wish to calculate from formula (18) the size of the projects α_p above which $|\beta_g - \beta_g^*|$ exceeds the permissible margin of error.

can be replaced by:

$$C_p^{(j)} < P_j - \beta_g^* P_x \,, \tag{24}$$

where the right-hand side measures the marginal return MR_g of a unit of production to group g. The coefficient β_g^* is equal, therefore, to the difference between the price P_j at which the commodity can be exported from (or imported into) country j and the marginal return to group g (MR_g) divided by the world reference price (P_x):

$$\beta_g^* = \frac{P_j - MR_g}{P_x} \,. \tag{25}$$

To apply the criterion (24), the relative price decline (ϵ) induced by the project does not even need to be computed, but the coefficient β_g^* has to be computed. The reason is that, when ϵ tends towards zero, $P_j - MR_g$ tends towards the non-zero limit $\beta_g^* P_x$.

Under the perfect market assumption ($\gamma_{xg} = \gamma_{mg} = 1$), β_g^* can be easily estimated from (23) $\beta_g^* = \alpha_g/(-\eta_m + \eta_x)$. The numerator α_g measures group g's net exports (+) or net imports (−) over world gross exports (+). The denominator measures the sum of the absolute values of the price elasticities of world import demand and world export supply. If the perfect market assumption is removed, β_g^* has to be estimated from equation (22) instead of (23). This will often result in increasing $|\beta_g^*|$ as illustrated by the two following examples, where g stands for the *LDC*s.

For tropical export crops ($\beta_g^* > 0$), the difference between equations (22) and (23) is mainly due to the term in $1 - \gamma_{xg}$, since α_{xg} is large and $|\alpha_{mg}|$ is small. The opportunity cost of the resources released by reducing g's exports by a marginal unit is generally substantially lower than the export price ($\gamma_{xg} < 1$). The reasons may be: imperfect resource mobility; market wages exceeding opportunity cost of unskilled labor; over-evaluation of currency; export taxes.

For most industrial products ($\beta_g^* < 0$), the difference between (22) and (23) is due to the term in $1 - \gamma_{mg}$ since, this time, $|\alpha_{mg}|$ is large and α_{xg} is small. In the case of those industries for which the *LDC*s are overprotected by tariff barriers, the opportunity cost of reducing imports by a marginal unit would exceed the import price ($\gamma_{mg} > 1$). As for tropical export crops, equation (22) would give a higher $|\beta_g^*|$ than (23).

When the trade shares (α_j or α_g) of each decision agent are infinitely small and when the price-elasticity coefficients have non-zero values, all coefficients β are equal to zero. With these assumptions characterizing the "free competition case", each decision agent maximizes his profit by equalizing his marginal cost to the prevailing world price. When the trade shares (α_j or α_g) of some of the decision agents differ significantly from zero and when the price elasticities have finite values, the coefficients β differ from zero for some decision agents. We then depart from the "free competition case" and conflicts of interests between trading partners arise.

2.1.4. *Optimum export tax.* To compute the optimum export tax for country j, we shall assume that the resources devoted to the production of the commodity concerned are optimally allocated within that country and therefore use formula (15′) for β_j^*. Since the level of the optimum export tax, expressed as the percentage e_j of the export price P_j, must be such that $\beta_j^* P_j = 0.01\ e_j P_j$, it follows that $e_j = 100\ \beta_j^* = 100\alpha_j/[-\eta_m + (1-\alpha_j)\eta_{xoj}]$. The level of the tax depends therefore on the share of country j in world exports (α_j) the price elasticity of world import demand ($-\eta_m$) and the price elasticity of export supplies in countries other than j (η_{xoj}).

On the basis of this equation, the value of the optimum export tax e_j has been computed in table 1 for selected values of the three parameters η_m, η_{xoj} and α_j. Since the marginal production cost can never become negative, country j can never reach a share of world exports exceeding the absolute value of the price elasticity of the demand for its exports. In particular, if the absolute value of the price elasticity of the world import demand is lower than unity, a country can never reach a perfect monopolistic position whatever its comparative advantage is.

The northeast corner of table 1 remains blank because the existence of a profit maximizing country with a very high share of world exports is inconsistent with very low price elasticity coefficients. Thus, if the profit maximizing country j accounts for 90% of world exports and if the price elasticity of supply in the other exporting countries (η_{xoj}) is equal to 1.5, the absolute value of the price elasticity of the world import demand (η_m) should exceed 0.75. This can be illustrated by the historical experience of Brazil. With a 60% share of the world coffee

Table 1
Optimum export tax for country j acting as a profit maximizer monopolist.
$100\ \beta_j^* = e_j = 100\alpha_j/[-\eta_m + [1-\alpha_i]\eta_{xoj}]$.

	Price elasticity of		α_j Country j's exports as fraction of world exports						
	World import demand η_m	Export supply in countries other than j η_{xoj}	0	0.03	0.10	0.30	0.60	0.90	1.0
(1)	−0.3	+0.1	0	8	16	81			
(2)	−0.3	+0.2	0	6	21	68			
(3)	−0.4	+0.2	0	5	17	56			
(4)	−0.4	+0.6	0	3	11	37	94		
(5)	−0.4	+1.0	0	2	8	27	75		
(6)	−0.4	+1.5	0	2	6	21	60		
(7)	−1.0	+0.2	0	3	8	26	56	88	
(8)	−1.0	+0.5	0	2	7	22	50	86	
(9)	−1.5	+1.0	0	2	5	18	43	82	
(10)	−1.5	+1.5	0	1	4	12	29	55	67
(11)	−2.0	+1.5	0	1	3	10	23	42	50
(12)	Infinity		0	0	0	0	0	0	0

exports, Brazil should have raised an export tax equal to 75% of the world price, if the long-term price elasticities were those shown in line (5). But exporting countries with a 3 or 10% share should have established taxes of only 2 or 8% of the world price. Although Brazil was the most efficient coffee producer, its comparative advantage vis-a-vis its competitors was not all that large. Brazil had therefore no choice but to reduce the volume of its exports when prices fell and its share shrank from 60 to 40 percent.

Brazil has established a high export tax on coffee through the mechanism of multiple exchange rates. For cocoa, the major exporters, (Nigeria, Ivory Coast and Ghana in particular) have also established large export taxes. If the long-term elasticities for cocoa were those shown in line (5), the interest of Ghana (with a 30% share) would be to establish an export tax equal to 27% of the world price according to the formula. Although the actual level of the export tax may differ from the one shown in table 1, the fact remains that, for primary commodities with price inelastic demand and supply, major exporting countries have established substantial export taxes which are consistent with the formula.

It is important to draw a distinction between short and long-term price elasticities, especially in the case of tree crops. Thus for cocoa, while the long-term price elasticities are likely to be close to those shown in line (5), the short-term elasticities are more likely to correspond to the values shown in line (2). In the short term, the tree population being given, production can only respond to changes in the application of current inputs, such as pesticides. A number of technicians, who claim that pesticide applications are highly profitable, have criticized Ghana for having cut pesticide imports when cocoa prices fell sharply around 1965. We shall see how this behavior may be rationalized within a short-term profit maximization horizon.

Let us assume that, when cocoa sells at 30 cents a pound, one additional dollar's worth of pesticide gives an additional cocoa production worth $ 4.00. By applying pesticides, the opportunity cost of producing an additional pound of cocoa is only 25% of the price at which this pound of cocoa sells on the world market. With the values shown in line (2), Ghana is better off by exporting more since the opportunity cost of production is less than 32% of the world price. Consequently, when cocoa sells at 30 cents a pound, Ghana is better off when applying pesticides, even with a short-term profit maximization horizon. But when cocoa sells at only 20 cents, the opportunity cost of an extra pound of cocoa saved by pesticides reaches 37.5% of the world price, which exceeds the threshold of 32%. If the government takes a short-term profit maximization horizon, it is better off by cutting pesticides imports.

2.2. *Dynamic game*

We have discussed in the previous section the level of the optimum export tax for country *j*, assuming no retaliation from country *j*'s trading partners. But if a decision taken by country *A* can hurt country *B* and vice versa, country *B* may try to retaliate against country *A*, which may, in turn, retaliate. The threat of retalation may sometimes be a sufficient form of dissuasion to avoid an escalation which could harm both parties. Countries *A* and *B* may even decide to join in an alliance. Eventually, the main trading partners may end up in a cooperative game, called a commodity arrangement. It is generally during the course of the price downswing that the factors conductive to such an arrangement reach the required critical mass.

During the price upswing, producing countries play a non-cooperative game; each one tries to increase supply as fast as it can. When the price starts to fall, the major producing country is the first to reduce the volume of its exports, since it is the only one which can cut its losses by unilateral reduction in the volume of its exports. The bargaining power of the major producers vis-à-vis other producers may be strengthened by the possession of large stocks, as was the case of Brazil for coffee. If the price fall was temporary, the major producing country is then at the top of its strength. If the price downswing continues, rather than to carry alone the burden of supporting world prices, the main producer tries to convince other producers to join in an alliance.

If the price elasticities are low, producers as a group always gain from a cartel restricting export supplies. The practical problem is the redistribution of the cartel's gain among its members. The most acute problem is probably that of the newcomer (*N*) which has a low share today but can increase its production very substantially at low cost. Let us assume that during the three years preceding the agreement, *N* has exported on the average a quantity of 1, but that he could export an average quantity of 3 during the three years to be covered by the agreement. Let us further assume that the price would be 100 without agreement and 150 with agreement, while *N*'s production cost is only 50. What is the level of the quota for which *N* is better off by joining the agreement?

Obviously, *N* is not interested in the agreement if he receives a quota equal to 1. Without an agreement *N* would gain 3(100–50) = 150; with agreement he would gain only 1 (150–50). If *N* is offered a quota of 2, he has to make a choice between two strategies. With the first, joining the agreement, he will gain 2(150–50) = 200. With the second, not joining, *N* would gain 3(100–50) = 150 if no agreement is reached, but 3(150–50) = 300 if an agreement is established without him and if he can still sell 3 while taking advantage of the high price resulting from the agreement. *N* has therefore to compare the gain of 200 with a probability p_1 of gaining 150 and a probability $1 - p_1$ of gaining 300. Depending on his assessment of the probability p_1, on his attitude towards risk and on his desire for cooperation, he may choose to join or not to join. But, if *N* was offered a quota of 3, he would not hesitate in joining the agreement.

The situation of the major producer is different. Without agreement, he would have to reduce exports if his marginal return is lower than his marginal cost. With an agreement, he will have to reduce his exports further, but he will then benefit from the reductions made by a number of other producers and, consequently, will receive more. Since the strategy "joining" is usually strictly dominant for the major exporter, the newcomer N will have an excellent bargaining power and, if he is a hard bargainer, he may succeed in pushing his quota close to 3.

If there are only a few countries like N, the major producer can convince them to join by offering them a large enough quota. But even then the cartel cannot raise prices too much, since this would attract newcomers. In some manufacturing industries, the existence of large economies of scale provides the cartel with a protection against newcomers; but, this is not generally the case for primary commodities. For those commodities, producers' cartels are therefore not very stable without the cooperation of the importing countries in enforcing the agreement. Bringing the importers in, increases the stability of the agreement, but it, obviously, reduces the scope for raising prices.

3. International lending agency

The case for international commodity agreements on tropical export crops is often argued as a way of raising world prices (within reasonable limits) and consequently of improving income distribution among countries. Similarly, the case for concessional lending to developing countries is generally argued on the basis of international welfare considerations. We shall consider here an international agency making project loans under concessional terms to developing countries only. In the case of projects affecting world commodity prices, we shall analyze project selection criteria in relation to international welfare. In the first section, we shall consider three simple alternative lending criteria. In the second section, we shall optimize the lending activities of the agency by maximizing an international welfare function subject to minimum income constraints for the agency's member countries.

3.2. *Three simple criteria*

3.1.1. *Maximizing the return to the borrowing country only.* For the agency the simplest is to consider the impact of the project financed in country A on country A only. However, the application of this criterion may have very unfavorable international welfare implications, as shown by the example below.

World exports originate exclusively from the LDCs and are distributed equally among the four countries A,B,C,D. One-tenth of world exports is imported by developing country E and the remaining nine-tenths by developed countries. In relation to world prices, the elasticity η_x of the export supply in countries $A,B,C,$ and D is equal to +0.8, while the elasticity η_m of the import demand in country E and in the developed countries is equal to −0.4. This set of assumptions is summarized in the first two columns of table 2, using the notations explained in section 2.1.3.

Table 2

Impact of small projects (p): $\alpha_p = 0.001, Q = 1000, P = 100, V_p = 100$.

Countries	Assumptions: Price elasticities η_{xj} for exporters, η_{mj} for importers, $\eta = -\eta_m + \eta_x$ for world	Assumptions: Trade shares α_j	Net gain (+) or loss (−) π_j on projects financed by the agency: Project located in A	Net gain (+) or loss (−) π_j on projects financed by the agency: Rounds of projects located successively in A, B, C and D
Developing countries		(+0.90)	(−51.7)	(−206.8)
A exporting	+0.8	+0.25	+15	− 60
B exporting	+0.8	+0.25	−25	− 60
C exporting	+0.8	+0.25	−25	− 60
D exporting	+0.8	+0.25	−25	− 60
E importing	−0.4	−0.10	+ 8.3	+ 33.2
Developed countries	−0.4	−0.90	+75	+300
World total	+1.2	0.00	+23.2	+ 93.2

With optimal resources allocation within each exporting country, equation (15′) can be used to compute the coefficients:

$$\beta_A^* = \beta_B^* = \beta_C^* = \beta_D^* = \frac{\alpha_j}{-\eta_m + (1-\alpha_j)\eta_{xoj}} = \frac{+0.25}{0.4 + (1-0.25)0.8} = +0.25\,.$$

Each exporting country taken in isolation therefore gains by implementing a project with unitary cost of production lower than 75% of the world price.

We shall now make the following assumptions: (a) The agency finances a "good project" for which the cost of production is only 60% of the world price $C_p^{(j)} = 0.6P$; (b) The country cannot implement this project without the help of the agency, because the latter brings in, not only financing, but also technical expertise; (c) The exportable supply generated by the project is only equal to 0.01% of world exports (α_p = 0.001), and; (d) the transportation cost differentials T_j are negligible and $\gamma_{mj} = 1$.

Taking the volume of world exports without the project as $Q = 1000$ and the world price without the project as $P = 100$, the value $V_p = \alpha_p PQ$ of the project output is equal to 100. The project induces a decline of world prices equal to one-twelfth of one percent, as appears from the application of equation (4):

$$\frac{-\Delta P}{P} = \frac{\alpha_p}{-\eta_m + \eta_x} = \frac{0.001}{+0.4++0.8} = \frac{0.01}{12}\,.$$

The direct gain from the project calculated from equation (7) is 40:

$$\pi_p^{(j)} = (1 - \frac{C_p^{(j)}}{P})\, V_p = 0.4 \times 100 = 40\,.$$

The indirect impact $\pi_j = -\beta_j V_p$ resulting from the price decline can be calculated as follows:

$$\beta_E^* = \frac{\alpha_E}{\eta} = \frac{-0.1}{1.2} = -\frac{1}{12}\,, \qquad \pi_E = -\beta_E^* V_p = \frac{100}{12} = +8.3$$

$$\beta_{Dd}^* = \frac{\alpha_{Dd}}{\eta} = \frac{-0.9}{1.2} = -\frac{3}{4}\,, \qquad \pi_{Dd} = -\beta_{Dd}^* V_p = +75\,.$$

If the project is implemented in country A, the other exporting countries B, C and D with $\beta^* = +0.25$ lose:

$$\pi_B = \pi_C = \pi_D = -0.25 \times 100 = -25\,,$$

while country A gains:

$$-\beta_A^* V_p + (1 - \frac{C_p^{(j)}}{P})\, V_p = -25 + 0.4 \times 100 = +15\,.$$

If the agency takes into account only the impact of the project in country A, it will finance the project since country A gains 15. The agency can then turn to country B and, using the same criteria, finance in B a project identical to the one previously financed in A. The agency can next move to country C and finally to country D. After this first round has been completed, the world price has declined by only one-third of one percent. On the one hand, each of the countries A, B, C and D has gained 15 once and lost 25 three times; the net loss of each country is therefore equal to 60. On the other hand, country E has gained 33.2 and the developed countries have gained 300. The agency can then proceed to a second round.

In the classical international trade model, which assumes an equilibrium or a fixed gap [4] in the trade balance, there is a selfcorrecting mechanism. The fall in the terms of trade for tropical export crops reduces the developing countries' import demand for exports originating from developed countries, while it stimulates the developed countries' import demand for *LDC*s exports. These two factors tend to reverse the initial fall in the *LDC*s terms of trade. However, if the initial reduction of the *LDC*s import capacity is compensated for by higher lending from the agency, which itself borrows its funds from the developed countries, the self-correcting mechanism is replaced by a self-generating lending process inducing an ever growing debt from the *LDC*s to the developed countries.

3.1.2. *Maximizing the return to the LDCs as a group.* The agency maximizes this time the unweighted sum of the gains (+) and losses (−) accruing to the borrowing countries on account of the agency's activities. The agency does not take into account the distribution of the total gain among the borrowing countries nor the impact on the developed countries. Assuming the agency has no constraint on the volume of its total lending, it will finance a project if the cost of production per unit of the project output $C_p^{(j)}$ satisfies equation (24):

$$C_p^{(j)} < (1 - \beta_g^*) P_x$$

Table 3
Numerical values of $100\beta_g^*$ ($\gamma_{mg} = 1, P_{xg} = P_x$).

	Elasticity in relation to world price		Large export surplus $\alpha_g = 0.95, \alpha_{xg} = 1.0, \alpha_{mg} = -0.05$		Import deficit $\alpha_g = -0.4, \alpha_{xg} = 0.1, \alpha_{mg} = -0.5$	
	Imports	Exports	Opportunity cost of resources released			
	$\eta_m = \eta_{mg}$	$\eta_x = \eta_{xg}$	$\gamma_{xg} = 0.5$	$\gamma_{xg} = 1.0$	$\gamma_{xg} = 0.5$	$\gamma_{xg} = 1.0$
(1)	−0.3	+0.1			−99	−100
(2)	−0.3	+0.2			−78	− 80
(3)	−0.4	+0.2			−65	− 67
(4)	−0.4	+0.6		95	−37	− 40
(5)	−0.4	+1.0		68	−25	− 29
(6)	−0.4	+1.5	90	50	−17	− 21
(7)	−1.0	+0.2	87	79	−32	− 33
(8)	−1.0	+0.5	79	63	−25	− 27
(9)	−1.0	+1.0	73	48	−18	− 20
(10)	−1.5	+1.5	57	32	−11	− 13
(11)	−2.0	+1.5	49	27	− 9	− 11
(12)	Infinity		0	0	0	0

with P_x = world price, $\gamma_{mg} = 1.0$, $P_{xg} = P_x$ equation (22) can be written

$$\beta_g^* = \frac{\alpha_g}{\eta}\,[1 + \frac{\alpha_{xg}}{\alpha_g}(1-\gamma_{xg})\,\eta_{xg}]\,.$$

Table 3 gives numerical values of the coefficients β_g^* under alternative assumptions. Twelve different combinations of the price elasticities are shown along the lines. Two different combinations of the trade shares associated with two different values of γ_{xg} (γ_{xg} = 0.5 and γ_{xg} = 1.0) are shown along the columns. The first two columns correspond to the *LDC* trade in cocoa. The last two columns correspond to that in wheat.

Let us consider the case of cocoa and assume that the price elasticities are those shown in line (5). If the opportunity cost of the resources released from cocoa production was equal to the prevailing world price (γ_{xg} = 1.0), the agency would finance a cocoa project only if the unitary cost of production was lower than (100−68 =) 32% of the world price. If the opportunity cost of the resources released per unit of production displaced was equal to only half of the prevailing world price (γ_{xg} = + 0.5), which seems more likely, the agency would never finance a cocoa project with the price elasticities shown in line (5). With the price elasticities shown in line (6) the agency would finance a cocoa project only if the cost of production per unit of the project output was less than (100−90 =) 10% of the world price. In practice, the agency would not finance cocoa projects.

Table 4
LDCs share in world market for selected agricultural commodities, 1963–1965.

	LDCs net exports (+) or net imports (−) over world exports $\alpha_g + \alpha_{xg} + \alpha_{mg}$	*LDCs* exports over world exports α_{xg}	*LDCs* imports (−) over world exports α_{mg}
Cocoa	0.95	0.99	−0.04
Coffee	0.93	0.98	−0.05
Sisal	0.92	0.95	−0.03
Abaca	0.91	0.97	−0.06
Bananas	0.90	0.94	−0.04
Copra	0.84	1.00	−0.16
Groundnuts	0.92	0.90	−0.08
Natural rubber	0.79	0.97	−0.18
Palmoil	0.79	0.96	−0.17
Jute	0.77	0.95	−0.18
Tea	0.74	0.96	−0.22
Coconut oil	0.74	0.89	−0.15
Linseed oil	0.74	0.84	-0.10
Groundnut oil	0.63	0.82	−0.19
Fishmeal	0.57	0.62	−0.05
Sugar	0.55	0.78	−0.23
Cotton	0.44	0.64	−0.20
Maize	0.15	0.24	−0.09
Timber	0.10	0.18	−0.08
Linseed	0.10	0.12	−0.02
Lamb	0.02	0.06	−0.04
Rice	−0.08	0.70	−0.78
Wheat	−0.44	0.10	−0.54

Let us now turn to the last two columns of table 3. This time, the *LDCs* account for half of the world gross imports and one-tenth of world gross exports with the price elasticities shown in line (4) (η_m = −0.4 and η_x = + 0.6) and γ_{xg} = +0.5, the agency would finance a commodity project as long as the unitary production cost does not exceed the world price by more than 37%. With γ_{xg} = +1.0, the threshold price would increase only from 37 to 40%.

The crucial importance of the *LDCs* trade shares appears most clearly if we assume $\gamma_{xg} = \gamma_{mg} = 1.0$, since in this case

$$\beta_g^* = \frac{\alpha_g}{\eta}\,.$$

By maximizing the return to the *LDC*s as group, the agency could draw up a black and white list of commodities. Table 4 shows that for 16 of the 23 agricultural commodities listed, the coefficient α_g is greater than half. Most[17] of these (which account for the bulk of *LDC*s agricultural export earnings) would be on the black list. Few agricultural commodities for export, but most of the industrial products, would be on the white list. The lending implications would be clear-cut because the grey area does not contain many products having a large weight in *LDC* trade.

3.1.3. *Maximizing the weighted sum of national gains.* Taking the same weight for all developing countries and a zero weight for all developed countries is equivalent to the criterion just described. It leads to ranking commodities k according to a scale vector β_k. However, if the agency were to use weights negatively correlated with average income per capita, this would lead to establishing a matrix β_{kj} by commodity and by country. Assume now that the agency is asked to finance two projects. One is in country A, which is very poor. The other is country B, which is not so poor. With this criterion, the agency may have to lend to A nut not to B. If the agency could not finance any other project in B, that country would incur a net loss on account of the agency's lending activities.

This last criterion might provide an acceptable rule of thumb to deal with most commodity producing projects. It is, however, worth analyzing the problem of interaction among countries within an optimizing model, since it raises more general problems associated with the allocation of scarce international funds.

3.2. *Programming model*

The objective function reflects the mission assigned to the agency by its members collectively. Since the agency would not be able to per-

[17] The coefficient β_g depends not only on the share α_g but also on the price elasticity coefficients η and the opportunity cost parameter γ_g. If natural and synthetic rubber were perfect substitutes, the coefficient β_g should be computed by treating natural and synthetic rubber as a single product.

Table 5
Lending model.

	Rows \ Columns	Subsidy S_{jt}	Lending activities X_{jts}	Production of commodity k V_k	Gain (+) or loss (–) of developing country j Y_j	Gain (+) or loss (–) of developed country i Y_i	R.H.S.
(1) Step constraints reflecting decreasing returns to capital	X_{jts}	+	$+X_{jts}$				$\leq \bar{X}_{jts}$
(2) Production of commodities k affecting world prices	V_k		$+\sum_j v_{jk} \sum_s \delta_{kjs} X_{kjs}$	$-V_k$			$= 0$
(3) Gain of developing country j on account of the agency's activities	Y_j	$\sum_t S_{jt}$	$+\sum_t \sum_s \delta_{jts}(r_{jts} - r_h) X_{jts}$	$-\sum_k \beta_{jk}^* V_k$	$-Y_j$		$= 0$
(4) j's minimum gain					$+Y_j$		$> \bar{Y}_j$
(5) Minimum gain for project t in j	Y_{jt}	S_{jt}	$+\sum_s \delta_{jts}(r_{jts} - r_h - \beta_{jt}^* v_{jt}) X_{jts}$				$\geq \bar{Y}_{jt}$
(6) Gain or loss to developed country i on acccount of the agency's activity	Y_i			$-\sum_k \beta_{ik}^* V_k$		$-Y_i$	$= 0$
(7) i's maximum loss						$+Y_i$	$\geq \bar{Y}_i$
(8) Subsidy to j	S_j	$\sum_t S_{jt}$	$-(g_\sigma - g_h) \sum_t \sum_s X_{jts}$				≤ 0
(9) Maximum subsidies	S	$\sum_j \sum_t S_{jt}$					$\leq \bar{S}$
(10) Maximum lending	X		$+\sum_j \sum_t \sum_s X_{jts}$				$\leq \bar{X}$
(11) objective function	Y				$+\sum_j w_j Y_j$	$+\sum_i w_i Y_i$	max

r_{jts} rate of return on investment
r_h rate of interest on hard loans
g_σ grant content of soft loans
g_h grant content of hard loans
v value of exportable supply per dollar lent
δ conversion from streams into present values
w welfare weights
β^* coefficient defined in section 2.1.3
k subscript for commodities affecting world prices

form its mission satisfactorily without the individual cooperation of its members, lower bounds on the gains of each member on account of the agency's activities are introduced in the model. The first section outlines the structure of the model. The second considers the implications regarding lending criteria for projects located in country *A* and having an impact on countries other than *A*.

3.2.1. *Structure of the model.* Table 5 outlines a static model optimizing the agency's lending activities over a single period. A distinction is made between the developed member-countries *i* to which the agency cannot lend and the developing member-countries *j* to which the agency can lend; no account is taken of the non-member countries. Developed countries *i* can be affected by projects financed by the agency only indirectly through price effects. Developing countries *j* can be affected both directly and indirectly. The impact of the agency's activities on a given country (*i* or *j*) is measured by the present value (Y_i or Y_j) of the discounted stream of gains and losses incurred by that country on account of all the loans extended by the agency to all countries *j*.

The objective function shown in row (11) is the weighted sum of the gains and losses *Y* incurred by each member country. The weights (w_i and w_j) are a decreasing function of the national per capita income which each country would have reached in the absence of the agency. The minimum gain ($\bar{Y}_j$>0) to be insured to a given developing country *j* appears as a constraint in rows (4). The maximum permissible loss ($\bar{Y}_i$<0) to a given developed country *i* appears as a constraint in rows (7).

The agency can lend in the form of hard (*h*) and soft (σ) concessional loans. The grant contents g_h and g_σ of these two types of loans are defined in relation to the terms of non-concessional loans available to the developing countries in unbounded quantities. The coefficient g_h measures the difference between the present value of the stream of repayments per dollar of non-concessional loan and the present value of the stream of repayments per dollar of concessional loan made by the agency on hard terms. By definition, the grant content is greater for soft than for hard concessional loans ($g_\sigma - g_h > 0$).

The agency extends to country *j* the mix ($X_{\sigma j}$, X_{hj}) between soft and hard loans. This mix is translated in the model's language by the mix between the total volume of lending X_j (= $X_{\sigma j}$+ X_{hj}) on hard terms and the volume of straight subsidies S_j (=($g_\sigma - g_h$) $X_{\sigma j}$). The agency can

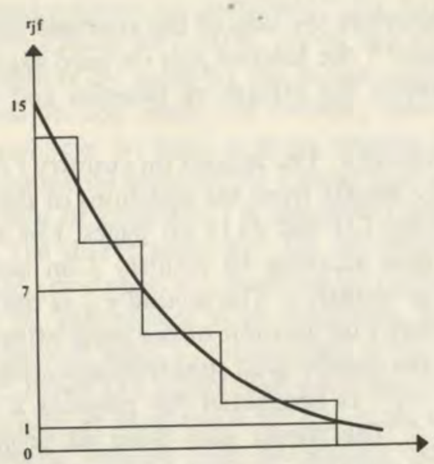

Fig. 7. Decreasing return to foreign capital inflow.

modify the allocation of subsidies among countries only by modifying the country mix between hard and soft loans ($X_{hj}/X_{\sigma j}$). The level of the subsidy activity S_j allocated to country *j* is therefore constrained by $0 \leq S_j \leq (g_\sigma - g_h)X_j$. The upper bound of this constraint is introduced in rows (8). The overall financial constraints imposed on the agency in terms of total lending $\bar{X}$ (= $\bar{X}_h + \bar{X}_\sigma$) and total subsidies $\bar{S}$ [=($g_\sigma - g_h$) $\bar{X}_\sigma$] are shown in rows (10) and (9).

The agency is assumed to be the only one extending concessional loans to developing countries. Its lending activities are fully defined by the matrices S_{jt} and X_{jt} characterizing respectively the allocation of subsidies and the volume of lending by countries *j* and by types of project *t*. Among projects *t*, a distinction is drawn between projects *k* which affect world commodity prices (for example, a cocoa project) and projects *f* which do not (for example, an education project).

Let us consider the projects *f* first and call *r* (X_{jf}) the rate of return of the foreign capital inflow X_{jf} in relation to the value of the objective function of country *j*, defined as the combined consumers'–producers' surplus of that country. If we approximate the curve *r* (X_{jf}) by the step function shown in fig. 7, the present value of the increase in the country's objective function resulting from the investment X_{jf} can be written:

$$\pi_f^{(j)} = \sum_s \delta_{jfs}(r_{jfs} - r_h)\, X_{jfs} \,.$$

The subscript s characterizes the step of the staircase shown in fig. 7. The parameter r_h measures[18] the interest rate on hard agency loans and the coefficient δ_{jfs} converts the stream of benefits and costs into its present value.

Let us now turn to projects k. The impact on country j of all projects k financed by the agency results from the additions of the two components defined by equations (7) and (11) on pages 154 and 155. The first $\pi_k^{(j)}$ is the direct gain accruing to country j on account of the projects k implemented in country j. The second π_{jk} is the indirect gain (or loss) accruing to country j on account of the price effect induced by the project k financed by the agency in all countries including j.

The first component $\pi_k^{(j)}$ is computed for projects k in the same manner as for projects f. The direct gain from all projects f and k financed by the agency in country j is obtained, therefore, by summing equation (7′) over all projects t (f and k), as shown in line (3) column (X) of table 5. The step[19] constraints $X_{jts} < \bar{X}_{jts}$ characterizing the decreasing return to foreign capital inflow in country j appear in row (1).

The second component π_{jk} is equal to $-\beta^*_{jk} V_k$ where β^*_{jk} is the approximation of β for small projects given in equation (13) and V_k is the discounted export value generated by all projects k financed by the agency in all countries (j included). The values of V_k are computed in rows (2) by multiplying the volume of the agency's lending X_{jk} by the coefficient ν_{kj} characterizing the value of exportable supplies generated in country j per dollar lent and by the coefficient δ_{jks}, which transforms the stream into present value. The program remains linear by measuring ν_{kj} in relation to the price which would have prevailed without the agency's intervention and by using the approximated formula for β^*.

[18] Let us call ρ the rate at which the country discounts gains or losses and let us characterize the repayment flow for an agency's hard loan by the value x_τ to be repaid in year τ. This hard loan has the same present value that a loan of X with a constant yearly charge of $r_h X$ for interest and indefinite repayment period for the principal. The rate r_h is given by:

$$r_h = \rho \sum_{\tau=1}^{\tau=\infty} \frac{x_\tau}{(1+\rho)^\tau}.$$

Since $\Sigma_{\tau=1}^{\infty} (1/(1+\rho)^\tau) = 1/\rho$, it can be easily verified that the equality applies if $x_\tau \equiv r_h$. For an investment X generating a steady stream of value added V, the rate of return on the investment is $r = V/X$ and the coefficient δ is $1/\rho$. If $r = r_h$, the profit Π is $((r - r_h)/\rho)X$.

[19] The number of rows (1) could be reduced to one per type of project in country j by following a device similar to the one described for the demand curves in footnote 9, p. 142.

These approximations are justified[20] for two reasons. First, for commodities on the black list (coffee, cocoa, tea, etc.), the agency will finance very few projects by applying the model outlined here. Second, for the commodities on the white list (wheat, industrial products, etc.), the agency is unlikely to have a large impact on world prices, because developing countries account for a small part of world output.

The net gain of countries j is computed in rows (3) by adding up the subsidy S_j, the direct gains $\pi_k^{(j)}$ for all projects financed in j and the indirect gains (or losses) π_{jk} incurred by j on account of all the projects k financed by the agency in all countries. Rows (4) insure that countries j will gain at least $\bar{Y}_j$ on account of all the lending operations of the agency. Rows (5) insure that each project t implemented in j will bring to j a gain at least equal to Y_{jt}. If all the $\bar{Y}_{jt}$ were equal to zero, rows (5) would express that no project can be unprofitable for the country which has to implement it.

For developed countries i, rows (6) adds up only the indirect gains (or losses) on account of the projects k financed in all countries j. Rows (7) insure that i's net loss cannot exceed $-\bar{Y}_i$.

3.2.2. *Implications.* To comprehend the lending implications of the model, it is convenient to start from the simplest case discussed below under Al and to go progressively to the most complex case B2, the structure of which was outlined in the previous section.

A. *No externalities* ($\beta_g \equiv \beta_j$)

In the absence of commodities k, rows (2), (5), (6) and (7) as well as columns V_k and Y_i disappear from table 5. The distinction among types of projects t becomes irrelevant. The agency needs only to know the curve of fig. 7, which defines for all types of investments the marginal efficiency of foreign capital $r\ (X_j)$ in each potential borrowing country j.

A.1. *Efficiency only* ($w_j \equiv 1, \delta_j = \delta$, no $\bar{Y}_j$ constraint)

Since the distribution of the subsidy S among countries j does not affect the value of the objective function $Y = \Sigma_j Y_j$, the agency can use for every country the same blend between soft and hard loans and therefore the same lending rate $\bar{r}_d$. If r_j is the marginal return to country j, the contribution of a variation $d\,X_j$ in the amount lent to j is $d\,Y = \delta\ (r_j - \bar{r}_d)\ dX_j$. In the optimal solution, the ratio dY/dX_j, and

[20] If it were not β would be approximated by a staircase.

consequently the marginal rate of return r_j, must be identical for every country j.

The common cut-off rate of return r is the ordinate of the point where the curve $r(X)$ (aggregated over all projects t and all countries j) intersects the vertical of abscissa $\bar{X}$ (maximum amount which the agency can borrow). If the agency could borrow unlimited amounts at a fixed rate of interest rate $\bar{r}$, the model would become redundant. Financing any project with a rate of return $r_j \geqslant \bar{r}$ would always be optimal.[21]

A.2. *Efficiency versus equity* ($w_j \neq 1$, no $\bar{Y}_j$ constraint)

With a single lending rate $\bar{r}_d$, the marginal utility of lending to country j is $dY/dX_j = \delta_j w_j (r_j - \bar{r}_d)$. In the optimal solution, the cut-off rate of national return for country j is $r_j = \bar{r}_d + (1/\delta_j w_j)(dY/dX)$. The cut-off rates should therefore be higher in the rich than in the poor countries.

For a given grant content, the agency can raise Y by differentiating its terms of lending. Let us consider a poor country A and a rich country B with $w_A - w_B > 0$ and $\delta_A w_A - \delta_B w_B > 0$. Subsidies must be allocated by priority to A, since, in Y, S_A is more heavily weighted than S_B. Assuming all subsidies go to A, the optimal lending allocation between A and B requires $r_B - r_A = (1/\delta_B w_B - 1/\delta_A w_A)(dY/dX)$. As was the case before with the single lending rate r_d, with differential lending rates the cut-off rates should still be higher in the rich than in the poor countries.

A.3. *Efficiency* $(r_j - \bar{r}_h)$ *versus equity* (w_j) *with national income constraints* $\bar{Y}_j$.

The shadow prices of the binding income constraints $\bar{Y}_j$ measure the international welfare cost of satisfying individual countries. The prices obtained in the first iteration would help in adjusting the $\bar{Y}_j$ in relation to the institutional weights of those countries.

B. *Externalities* $(\beta^*_{gk} \neq \beta^*_{jk})$

Let us turn to the case of projects producing commodities k for which the world import demand and the world export supply are price inelastic. The indirect price effects of project k financed by the agency in country j on countries other than j enters now in Y.

[21] This may not be true in dynamic models due to future repayment constraints, which reflect the limited ability of the country to earn (or save) more foreign exchange in latter years.

B.1. *Developing countries as a group* ($w_j \equiv 1, w_i \equiv 0, \delta_j \equiv \delta$)

The difference between the contributions of the loan dX_{jk} to the value of the objective function Y_j of country j where the project is located and to the value of the international objective function Y is given by:

$$dY_j - dY = \delta v_{jk}(\beta^*_{gk} - \beta^*_{jk})(dX_{jk}) = \delta(r_{jk} - r_k)(dX_{jk}) ,$$

where β^* is the coefficient defined in section 2.1.3, δ a coefficient transforming streams into present values and v_{jk} the output/capital ratio (annual value of the exportable supply generated by the project divided by the value of the capital lent by the agency).

The difference between r_{jk} (rate of national return to country j) and r_k (rate of international return to the agency) is:

$$r_{jk} - r_k = v_{jk}(\beta^*_{gk} - \beta^*_{jk}) .$$

This difference can be illustrated numerically by considering a cocoa project implemented in a marginal exporting country j, for which therefore $\beta^*_{jk} = 0$. With a capital/output ratio equal to 2, $v_{jk} = 0.5$; for $\beta^*_{gk} = 1$ (see p. 167), $r_{jk} - r_k = 0.5$. The rate of international return of this cocoa project would then be 50% lower than the rate of national return to the country where the project is located.

B.1.1. *The commodity black list* ($\overline{S} = 0$, no $\overline{Y}_j$ income constraints)

Let us assume a 10% cut-off rate for international returns. Among projects f which do not induce externalities, the model would select those which have a rate of national return r_{jf} larger than 10%. But, the cocoa project described above would be selected only if its rate of national return r_{jk} were higher than 60%.

It would not be very sensible for the agency to finance projects with such high rates of national return. If such projects were not financed by the agency, they would be implemented by country j from other financial sources. Due to the fungibility of capital, these agency loans would ultimately be used to implement other projects, which would remain unknown to the agency.

Without using a model, the agency could draw a commodity black list including all commodities k for which the difference between the national and international rates of return exceeds the acceptable threshold (for example, 15%).

B.1.2. *Selected trade strategy* ($\bar{S} > 0$, $\bar{Y}_j$ and $\bar{Y}_i$ income constraints)

In the previous case, the agency avoided causing a deterioration in the *LDC*s terms of trade by not investing in particular fields. Now, the agency can contribute to an improvement in the *LDC*s terms of trade by subsidizing investments for which the international rate of return exceeds the national rate of return.

Let us consider the case where the agency has a choice in country j between a cocoa project C (with $\beta^*_{gc} > 0$) and a pulp and paper project P for which the *LDC*s have a large and growing import deficit (with $\beta^*_{gp} < 0$). Due to price externalities, the rate of international return is higher for P than for C, ($r_p > r_C$). If the rate of return to country j is higher for C than for P, ($r_{jp} < r_{jC}$), there is a conflict. Let us consider the following case:

$r_{jP} < r_{jC} < \bar{r}_j$, $\quad \bar{r}_j$ = interest rate at which country j can borrow commercially

$\bar{r}_\sigma < r_{jP} < \bar{r}_h$, $\quad \bar{r}_\sigma$ and $\bar{r}_h$ = interest rates on soft and hard agency loans

$r_C < r < r_P$, $\quad r$ = cut-off rate of international return.

The agency prefers P to C. But country j would select C, if it were to receive the same terms of lending for P and C. Lending for P on soft terms is a way of solving the conflict. Since $r_{jP} - \bar{r}_\sigma > 0$, country j is better off with than without project P. The model optimally allocates the subsidies S_{jk} by project and country in maximizing Y. Due to the constraints $Y_{jk} \geqslant \bar{Y}_{jk} > 0$, it provides the country enough incentive for implementing projects k.

Let us now assume that the agency has no choice among projects. For example, the agency cannot find anything to finance in Burundi except a coffee project. To fulfill Burundi's minimum income constraint, the agency has to finance the coffee project. Minimizing the size of the project by financing it with soft loans will then generally be optimal.

B.2. *Efficiency* (r_{jt}) *versus equity* (w_j), *policy considerations* (Y_j) and *externalities* ($\beta^*_{gk} - \beta^*_{jk}$)

This case is the most general one outlined in table 5. To maximize the value of its objective function, the agency has to allocate subsidies according to a matrix S_{jt} and to select projects according to the matrix r_{jt} defining the cut-off rate of national return by countries j and types of projects t.

This two-way classification by income levels and by types of projects is consistent with the allocation of public funds within a Welfare State.

On the one hand, subsidies are given to low income groups while high income groups are taxed. On the other, subsidies are given to sectors such as education and health where the social return exceeds the private return, while taxation is imposed on polluting industries where the social cost exceeds the private cost.

Short of taking into account the interaction between these two types of criteria and short of building the full matrix r_{jt}, the Welfare State can use the two criteria independently. It can extend soft loans to individuals or regions falling below a given income level and to sectors for which the social return substantially exceeds the private return. Similar simplifications could be made by the international lending agency. Thus, short of computing the matrices S_{jt} and r_{jt}, the agency could draw a black list of the types of projects not to be financed even with hard loans and a white list of the types of projects to be financed with soft loans. Such a list would not have to be used as a Bible, but as "a presumption".

A differentiation by types of projects is not trivial for an agency lending to all developing countries and only to developing countries. Most countries at an early stage of development have fairly similar trade patterns and, for many of the export products common to those countries are low. This clearly applies to tropical non-competing commodities, such as coffee, cocoa and tea. It may also apply to a number of competing manufactured products with a high labor content, such as cotton textiles and clothing, because developed countries may unfortunately impose quota restrictions to protect their "depressed domestic sectors", once the level of imports exceeds a critical mass. By measuring the coefficients β in relation to a forward projection of international trade, the agency could integrate a trade strategy for the *LDC*s in the criteria for project selection.

The case of commodity projects was used in this paper to illustrate the impact of a given type of external economies. Research applied to the particular conditions prevailing in a number of developing countries could provide another illustration.

The research on improved wheat and rice varieties, conducted respectively in Mexico and the Philippines is a classical example of external economies. Research on capital–labor substitution could be another promising area. For a single labor-surplus country, it may be more profitable to borrow industrial technologies directly from the shelves of the rich countries rather than to adjust them to local conditions. But, for the international community, it may be worthwhile using international subsidies to develop technologies better fitted to the resource endowments prevailing in the *LDC*s.

References

[1] Ian M.D. Little and James A. Mirrlees. Manual of industrial project analysis of developing countries, Vol. II. OECD Development Center, Paris, 1969.

[2] J.R. Hicks, Value and capital, London, 1965, Note to Chapter II Consumer's Surplus, pp. 38–41.

[3] E.J. Mishan, What is producer's surplus? American Economic Review. December 1968, pp. 1269–1282.

[4] M.C. Kemp, The gain from international trade, The Economic Journal, Dec. 1962, pp. 803–819.

[5] P.A. Samuelson, The gains from international trade once again, The Economic Journal, December 1962, pp. 820–829.

[6] R. Duncan Luce and Howard Raiffa, Games and decisions: introduction and critical survey, New York, Wiley, 1957.

[7] H.B. Chenery, The interdependence of investment decisions in: Readings in welfare economics, 1961, pp. 336–371.

[8] M. Bruno, The optimal selection of export-promoting and import substituting projects, Planning the External Sector: Techniques Problems and Policies, New York, U.N., 1967.

[9] L.M. Goreux, Price stabilization policies in world markets for primary commodities: an application to cocoa, IBRD, 1970.

[10] A.S. Manne, Lending criteria for an ideal international development agency, IBRD, Jan. 1971.

[11] J.H. Duloy and R.D. Norton, CHAC, a programming model of Mexican agriculture, ch. IV.1 in Agricultural sector model: Multilevel planning: case studies in Mexico, edited by L.M. Goreux and A.S. Manne, North-Holland Publishing Company, Amsterdam, to be published in 1972.

[12] R. Vaurs, A. Condos and L.M. Goreux, An agricultural model from the Ivory Coast programming study, March 1972, Development Research Center, IBRD. Working Paper 125.