Capital Embodied Structural Change *

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Abstract

Structural change is a significant empirical pattern in economic development. Capital-embodied technical change (CETC), or the decline in the price of capital relative to consumption, plays a crucial role in driving economic growth. So what is the role of CETC for structural change? We build new measures of the relative price of capital used for production in different sectors in the US between 1948 and 2020, and document faster decline in services, than in industry than in agriculture. The passthrough between structural change and this decline is mediated by the dynamics of the labor share and output per worker in each sector, through equilibrium output prices. We show that CETC, when paired with the observed dynamics of the labor share, can trace the dynamic of the price of output in agriculture relative to industry, and to a less extent in services relative to industry. To quantify the role of CETC for structural change we build a parsimonious model that accommodates sector-specific CETC through the usage of distinct bundles of equipment across sectors and where each sectorial labor share is endogenous to the path of CETC. Through counterfactuals, we find that CETC is the primary driver of the reallocation of output away from agriculture relative to industry, and accounts for a third of the reallocation towards services. These findings emphasize sector-specific factor-biased technical change and underscore the importance of investment and embodied technology as drivers of structural change.

JEL codes: O13, O47, Q10.
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1 Introduction

The reallocation of economic activity across broadly defined sectors, i.e. structural change, is a robust feature of the process of economic development. There is a growing consensus in the literature that the main drivers of such reallocation are disparities in productivity growth across sectors and shifts in consumers' demand due to income effects (Ngai and Pissarides, 2007; Comin *et al.*, 2021; Boppart, 2014). At the same time, capital embodied technological change (CETC) has been identified as an important driver of economic growth (Greenwood *et al.*, 1997). The treatment of CETC within the framework of structural change has predominantly concentrated on scenarios where capital services can be easily transferred between sectors, assuming the existence of a single capital good. This approach appears somewhat peculiar considering the evident disparities in the types of capital utilized across different sectors, as well as the varying rates of technological advancements experienced by such capital. It is only logical to ponder the extent to which the reallocation of economic activity is influenced by the presence of embodied technology. This paper answer the question.

Solow defines embodied technology as a shift in the marginal cost of producing investment relative to consumption, and factor neutral technological change, as a shift in output for a given level and composition of the capital stock, (Hercowitz, 1998). Greenwood *et al.* (1997) maps Solow's definition to investment-specific technical change, which is a consistent way of defining embodiment in a two-sector economy. However, in the context of multi-sector economies, we revert to the broader concept of a shift in the marginal cost of investment relative to consumption. This is because the nature of the investment composite can alter the relative price when it differs from the consumption composite.

Using this definition we document systematic disparities in CETC across sectors that are driven by systematic disparities in the type of equipment used in production across sectors. Our empirical analysis focuses on the US time series and the goal is to describe the relationship between CETC and labor productivity, which is in turn proportional to relative output prices. The elasticity of sectorial output to productivity embodied in capital depends on the intensity of use of capital in production through the capital expenditure share. Hence, whether fast CETC in a sector translate in faster labor productivity depends on how intensively the sectors uses capital relative to others. Under the assumption of constant returns technologies and competitive markets, we show how to link the path of relative prices to sector-specific user cost of capital and, through it, to CETC. We document that CETC tracks the path of the relative price of agriculture to industry and the relative price of services to industry quite well, although the magnitudes in the latter are small than in the data.

Then, we construct a model economy that accommodates these disparities, while being consistent with long-run balanced growth, and use it to run an accounting exercise on the role of CETC for structural change. We find that CETC is the chief driver of the reallocation of employment out of agriculture (88%) and accounts for about a third of the flow of employment into services (37%) in the US, between 1948 and 2016.

Bringing CETC to the forefront of the drivers of sectoral productivity growth yields a new emphasis to the role of capital accumulation for structural change. Unpacking the drivers of sectorial productivity growth is important for two reasons. First, disparities in productivity growth across sectors may not be stable. Post-second world war evidence suggest that in developed economies productivity growth is the fastest in agriculture, and it is faster in industry than in services (Herrendorf *et al.*, 2014). However, we document that CETC is the slowest in agriculture, and slower in industry than in services. CETC is a contributor to labor productivity growth. As sectors become more capital intensive, CETC becomes increasingly important as a driver of productivity growth and therefore current trends in productivity growth may reverse.[1] Second, most of the equipment in developing countries is imported and hence, the ability of these countries to improve sectorial productivity may depend critically on their ability to import given types of equipment (Eaton and Kortum, 2001, Mutreja *et al.*, 2018).

To measure CETC we require an assumption on the technology that transforms consumption goods into investment, which we assume linear, following the long tradition of (Hulten, 1992; Greenwood *et al.*, 1997). Then, quality-adjusted prices of investment relative to consumption yield an empirical measure of the inverse of the rate of productivity growth in this linear technology, a measure of CETC. We use quality-adjusted prices across 24 equipment categories when available, and complement them with NIPA equipment deflators when that is not the case. We then construct sector specific deflators for new capital by weighting equipment prices across these 24 equipment categories using nominal investment shares per equipment type and sector from the BEA. We document that the cost of new capital relative to consumption has declined at an annualized rate of 1.5% in agriculture, 2.4% in industry and 4.8% in services between 1948 and 2016. These are sizable differences in the rate of CETC across sectors which are driven by systematic differences in the composition of equipment across types in each sector.

The passthrough between capital services and labor productivity is mediated by the expenditure share of capital in value added. We measure this share residually after computing

[1] Some early evidence of this phenomenon is documented by Alvarez-Cuadrado and Poschke (2011) in agriculture and, more recently, BEA estimates suggests that productivity growth in services may be surpassing that of industry.

the expenditure share of labor following the methodology in Herrendorf *et al.* (2015). Their method allocates proprietors' income equally between capital and labor within a sector. This is an important source of returns to labor in agriculture and some services where workers are mostly self-employed. The path of the capital share within each sector is consistent with the extensive literature that documents a decline in the labor share in industry since 1980s, which has accelerated in the 2000s (Sahin *et al.*, 2013). The capital share in services is flat and the capital share in agriculture increases in the first half of the sample and starts declining after that. We show that these movements in the capital share at the sectoral level are quantitatively important for the correlation that we uncover between sectoral labor productivity and CETC.

The relative price of agriculture to industry falls in the data, suggesting that labor productivity in agriculture raises faster than in industry. So how does CETC track this movement given that CETC in agriculture is slower than in industry? Agriculture is more capital intensive than industry at the beginning of the sample, when disparities in CETC across these two sectors are not that large. The relative price of services to industry raises in the data, suggesting that labor productivity growth in services is slower than in industry. So how that CETC tracks this movement, in particular the acceleration in the pace of increase since 1980s if CETC in services is the fastest? The capital share in industry raises dramatically since the 1980s, compensating for faster CETC in services.

Armed with these empirical patterns, we move to quantifying the role of CETC for structural change in a general equilibrium model. For this purpose, we first take on the task of constructing a model of structural change where sector-specific CETC has a role in shaping the path of sectorial labor productivity. In the model, technical change is both factor neutral (as in Ngai and Pissarides, 2007) and embedded in capital (CETC, as in Greenwood *et al.*, 1997). The latter emerges as a result of sectors using capital of different types in different combinations to produce output. Sectorial output is a composite of output from different activities, which can be performed with capital-intensive or labor-intensive technologies. The elasticity of substitution across these activities determines the elasticity of substitution between capital and labor in equilibrium in the aggregate sectorial production function. Decisions related to the usage of capital-intensive or labor-intensive technologies, or the share of activities performed by each technology, affect the bias between labor and capital within the constant elasticity production function. In the limit, when the elasticity of substitution across activities approaches 1, the technology is Cobb-Douglas and the factors shares are proportional to the measure of activities performed with capital-intensive and labor-intensive activities.

Importantly, in our model, capital deepening in production occurs through two mechanisms: on the one hand, CETC generates higher capital services per unit invested, lowering the user cost of capital and increasing capital services; on the other hand, sectoral producers may switch to more capital intensive technologies altogether. Non-monotonic movements in capital expenditure shares paired with a steady decline in the relative price of capital informs this modelling choice. In other words, the ability to switch to more capital intensive technologies may be linked to innovation in processes that do not directly map into the cost of capital. This feature allows us to match the non-monotonicity of the expenditure shares despite the steady decline in the user cost of capital. For a given set of technologies, the usage of more capital services is directly linked to its user cost.

We then quantify the model economy to measure the role of CETC in driving structural change by asking how much of the reallocation of labor across sectors observed in the US between 1948 and 2016 would have occurred absent CETC. To do so, we exploit the limiting case of our model to Cobb-Douglas production functions for sectoral output, with sectoral specific capital shares.[2] We parameterize such model to match sectoral capital, output, and employment in 1984 as well as the path of sectoral prices between 1984 and 2016. Importantly, under such parameterization, the model is consistent with the path of structural change observed in the data: it generates an employment outflow from agriculture of the same magnitude as that observed in the data as well as 61% of the reallocation of employment out of industry. Then, we run counterfactual exercises where we shut down, one at a time, the two exogenous forces driving the path of relative prices in the model: sectoral factor-neutral productivity growth and sectoral CETC. Via these exercises, we quantify that CETC is an important factor driving structural change: it accounts for 88% of the movement out of agriculture, for 27% of the movement out of industry, and for 37% of the movement into services. In particular, factor-neutral productivity growth alone cannot generate the decline in the relative price of agriculture to industry and, therefore, this force only generate 1/4 of the reallocation of employment out of agriculture.

There is a growing literature that documents structural change within investment, i.e. shifts in the composition of goods used to produce investment goods in the economy (García-Santana *et al.*, 2021 and Herrendorf *et al.*, 2020). Differently, we focus on sectorial heterogeneity in the type of capital goods used in production and show that such a heterogeneity is first order in explaining the process of structural change. This heterogeneity is particularly important in poor and middle-income countries, which import the bulk of the equipment used for production. Our proposed framework can be readily extended to allow for a rich vintage structure for capital, diffusion waves as well as to think about policies geared towards

[2]We are working on a calibrated version of the economy with arbitrary (and possibly heterogeneous) elasticities of substitution in each sector.

boosting investment.

Our facts relate to an extensive literature that duels with the nature of technical change and the extent to which its nature is embodied or disembodied from capital or other factors of production (Intriligator, 1992). To the extent that embodied technology is reflected in the decline in the relative price of equipment to consumption, our findings indicate that in relative terms, CETC is of first order importance for agricultural productivity growth (supporting previous cross-country findings in Caunedo and Keller, 2020, where either technology embodied in labor or disembodied is relatively more important in the service sector).

From a modeling stand-point, we build a framework that is consistent with factor-biased technical change and balanced growth. Acemoglu (2003) focuses on the incentives to innovate on technologies that augment labor or capital. Conceptually, the main difference to this work is that growth stems from the introduction of goods that are more capital or labor intensive. In our framework, the set of goods is kept constant but the productivity of factors of production changes making it cheaper to produce a good and so inducing growth. We take the innovation process as given but our framework can be readily extended to allow for endogenous innovation. Jones and Liu (2022) show a strategy to reconcile balanced growth with production technologies that display non-unitary elasticities of substitution between capital and labor. In their framework, technical change is factor biased but not necessarily factor embodied, and whether technical change is embodied is important for measurement and accounting. Indeed, as a consequence of this disparity, we are able to distinguish between labor productivity growth, which can be capital-biased and occurs whether investment is undertaken or not; and CETC, which can only be taken advantage of via investment. Finally, we show that our model of structural change displays a limiting BGP despite having allocations where capital-augmenting technical change is constant in production. The reason is that capital-expenditure shares do not map one-to-one to the capital-augmenting term (as they do in Jones and Liu, 2022). Instead, they depend on the relative price of sectorial output to consumption, which is only unitary when a sector overtakes the economy.

2 Sectoral CETC

This section measures sectorial growth rates in relative prices, which are proportional to sectorial labor productivity. Under the assumptions of competitive markets and constant returns to scale technologies in each sector, these relative prices are a function of sectorial CETC. We show that CETC varies systematically across sectors due to disparities in the composition of the capital stock used in the sector and that the incidence of CETC on labor productivity is also heterogeneous across sectors, through disparities in the expenditure share of capital.

We measure capital-embodied technical change in each capital type from the (quality-adjusted) relative price of investment to consumption. This is a valid measurement for the efficiency units of new capital when the technology that transforms goods into new capital is linear. We construct a time-series of the quality-adjusted prices exploiting the estimates in Gordon (1990) (which are available from 1948 to 1983) and then extrapolate his measures using the prices reported by the BEA.[3] Because we abstract from structures in the analysis that follows, we remove the real state and construction sectors for the sectorial aggregation. We also abstract from the government sector. Sectors are aggregated following Herrendorf *et al.* (2014) into three categories: agriculture, industry (including mining and industry) and services.

Let I_s be a sector-specific aggregator of investment in J capital types, $I_s(\{X_{js}(t)\}_j)$. The aggregator displays constant returns to scale so the change in the price of sectorial investment is a weighted average of the price of each investment type, with weights equal to the sectorial investment shares for each type $\kappa_{js}(t) \equiv \frac{P_j^x(t)X_{js}(t)}{\sum_j P_j^x(t)X_{js}(t)}$,

$$\frac{\dot{P}_s^x}{P_s^x} = \sum_j \kappa_{js}(t)\frac{\dot{P}_j^x}{P_j^x}.^{4} \tag{1}$$

If we add the assumption of a linear technology in the production of investment goods, the (quality-adjusted) price of each investment good relative to consumption is the inverse of CETC.[5] Hence, equation 1 defines CETC at the sectorial level, A_s^x, as an investment weighted average of CETC for each equipment type.

We start by reporting the change in the relative price of investment to consumption in each of our three sectors. The decline in the price of capital is slower in agriculture than in industry and both of them are slower than in services. Disparities in these declines are purely a consequence of the compositional differences in the investment bundle across sectors and

[3] We could have alternatively followed the methodology in Cummins and Violante (2002) which projects quality-adjusted series on BEA deflators. However, using a linear projection method over more than 30 years of data stretches the validity of the assumptions of the econometric model. Quantitatively, the difference between the quality-adjustment and the BEA series is not too large, in part because for the equipment with arguably fastest shifts in quality, i.e computers and communication equipment, BEA introduces quality adjustments and we use the raw series rather than an extrapolation.

[4] This is the correct aggregator when the composite of investment in each sector displays constant returns to scale, Oulton and Srinivasan (2003).

[5] Appendix C displays the path of prices without quality adjustment (NIPA). Most of the differences between the series occur at the beginning of the series and are concentrated in agriculture, consistent with the findings in Manuelli and Seshadri (2014).

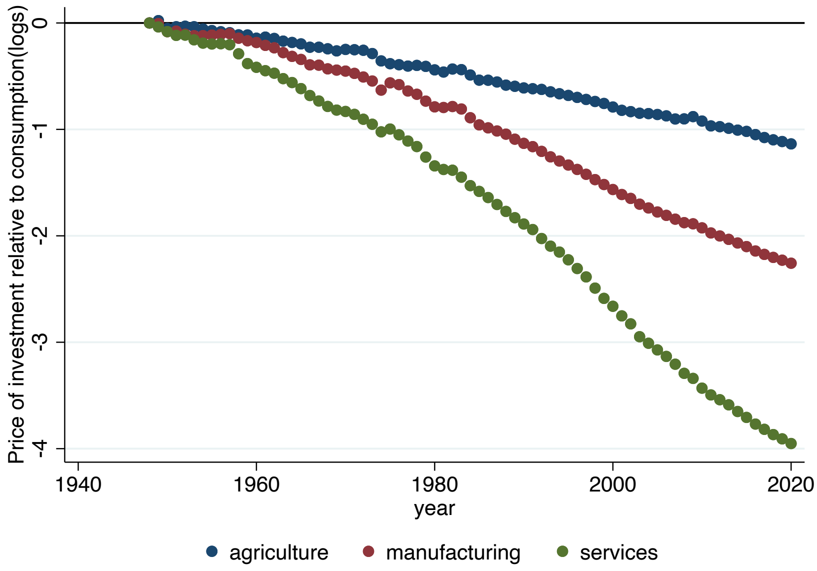


Figure 1: Log-price of investment relative to consumption.

The price of investment relative to consumption is normalized to 1 in 1948 and the picture displays log of prices. Source: BEA and own computations.

time, κ_{js}. Hence, an implication of these differences is that the service sectors are relatively more intensive in capital types with strong declines in the relative price of investment to consumption. These average differences are certainly present in the data, but in addition, the service sector has changed the bundle composition of capital towards equipment types with faster CETC.[6]

These differences in CETC amount to an annualized growth rate of embodiment of 1.5% in agriculture, 3.2% in industry and 4.3% in services between 1948 and 2016.

agriculture	industry	services
1.5%	3.2%	4.3%

Table 1: CETC

Decline in the relative price of investment to consumption between 1948-2016, annualized growth rates. Source: BEA and own computations.

Let production technologies be $Y_s(t) \equiv A_s(t)F_s(K_s(t), N_s(t))$ where A_s is total factor

[6]Figure 7 presents the series of prices when we fix investment weights to their values at the beginning of the sample and at the end of the sample. The decline in prices is slower than in the benchmark if fixing weights at the beginning of the sample.

productivity, and F is a production technology that displays constant returns to scale. In a standard model of structural change —i.e. F_s has common factor shares across sectors, and unitary elasticities of substitution between capital and labor, e.g. Ngai and Pissarides (2007)— differences in total factor productivity map into differences in output per worker and inversely to relative output prices across sectors. Hence, faster productivity growth in agriculture relative to industry translates into a decline in the relative price of agriculture to industry. When factor shares are different across sectors but the elasticity of substitution in capital and labor is unitary, relative prices are also inversely proportional to TFP but now capital-output ratios need not be equalized across sectors. Capital-deepening (either through investment or aggregate CETC) shifts labor productivity and differentially so across sectors that are more or less capital intensive, as in Acemoglu and Guerrieri (2008).

We showed however, systematic disparities in the bundles of capital used by different sectors, which in turn leads to systematic differences in the cost of capital across sectors. In other words, there is no such a thing as an aggregate capital stock from which sectors rent services. Disparities in the the cost of capital through the type of equipment used for production have a direct impact on capital-output ratios, beyond disparities in factor shares.

Often the path of productivity across sectors is inferred from relative output prices, assuming factor price equalization in the market for labor across sectors. In particular,

$$\frac{P_s(t)}{P_{s'}(t)} = \frac{1-\alpha_{s'}(t)}{1-\alpha_s(t)} \overbrace{\frac{Y_{s'}(t)/N_{s'}(t)}{Y_s(t)/N_s(t)}}^{\text{labor prod.}} = \frac{1-\alpha_{s'}(t)}{1-\alpha_s(t)} \frac{A_{s'}(t)}{A_s(t)} \frac{F_{s'}(K_{s'}(t)/N_{s'}(t),1)}{F_s(K_s(t)/N_s(t),1)} \tag{2}$$

where $1-\alpha_s \equiv \frac{F_n N}{F}$. Hence, the mapping between total factor productivity and relative output prices depends on disparities in labor expenditure shares, $1-\alpha_s$, as well as disparities in capital-labor ratios, both of which are potentially affect by sectorial CETC.

We labor expenditure shares from the ratio of the compensation of employees in each sector and value added, as reported in the components of GDP by Industry reported by the BEA and following the methodology of Herrendorf *et al.* (2015), see Figure 2. We find systematic differences in the level of expenditure shares across sectors, consistent with Valentinyi and Herrendorf (2008); and importantly, differences in their paths across sectors. As it is well known in the literature, the labor share in industry declines post 1980s and this decline accelerates post 2000s. Perhaps less known is that the labor share in agriculture also falls sustainedly in the first half of our sample period, pre 1980s, and remains stable afterwards. These movements affect the pass-through between capital-labor ratios and relative output prices, which we quantify next.

Relative output prices and CETC. To make the link between relative prices and

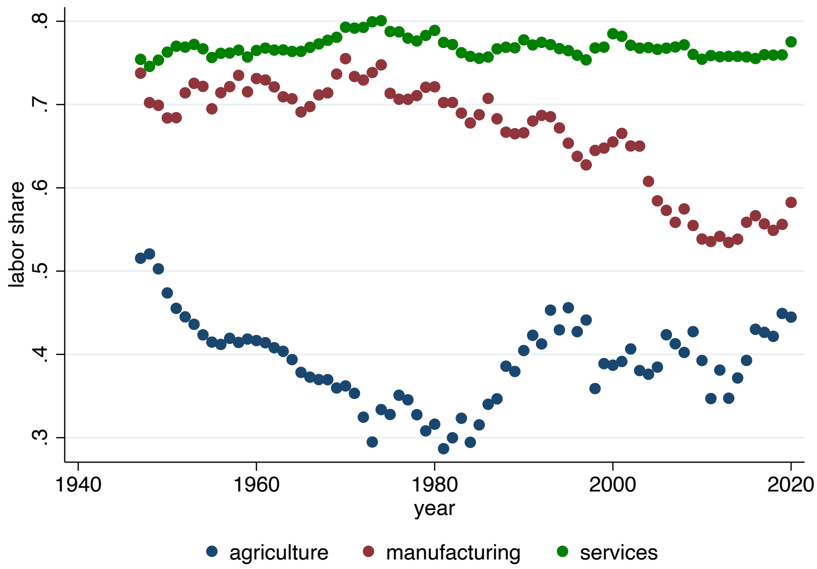


Figure 2: Labor expenditure shares

Labor compensation as a function of total value added in the sector. Estimates include proprietors income, which has been distributed equally between labor and capital. Construction and real state are abstracted away. Source: GDP by industry reported BEA and own computations.

CETC explicit we add the assumption of competitive input markets. Capital-labor ratios are:

$$\frac{K_s(t)}{N_s(t)} = \frac{W(t)}{R_s(t)}\frac{F_{k,s}(t)}{F_{n,s}(t)} = \frac{W(t)}{R_s(t)}\frac{\alpha_s(t)}{1-\alpha_s(t)},$$

where the last equality exploits the homogeneity assumption on technology F, so that its marginals are homogeneous of degree zero in inputs.

Proposition 1 *The growth rate of relative prices across sectors satisfies*

$$\frac{\dot{P}_s}{P_s} - \frac{\dot{P}_{s'}}{P_{s'}} = \frac{1}{1-\alpha_{s'}(t)}\frac{\dot{\mathcal{A}}_{s'}}{\mathcal{A}_{s'}} - \frac{1}{1-\alpha_s(t)}\frac{\dot{\mathcal{A}}_s}{\mathcal{A}_s} + \frac{\alpha_s(t)}{1-\alpha_s(t)}(\frac{\dot{R}_s}{R_s} - \frac{\dot{P}_s}{P_s}) - \frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)}(\frac{\dot{R}_{s'}}{R_{s'}} - \frac{\dot{P}_{s'}}{P_{s'}})$$

where $\frac{\dot{\mathcal{A}}_s}{\mathcal{A}_s}$ *corresponds to the change in sectorial TFP as well the weighted changes in expenditure shares, i.e.* $(1-\alpha_s(t))\frac{\dot{\overline{1-\alpha_s}}}{1-\alpha_s} + \alpha_s(t)\frac{\dot{\alpha}_s}{\alpha_s}$.

Proofs can be found in Appendix B.

Proposition 1 shows that the difference in the change in relative output prices depends on the difference in the change in factor neutral technical change and in the marginal product of capital, i.e. the ratio between the user cost of capital and the sectorial output price.

This condition is satisfied in any equilibrium where firms minimize costs and it is useful for measurement purposes.

The link between the marginal product of capital and CETC requires further structure on the model, namely an equilibrium path where the interest rate in the economy is constant. This implies that the marginal product of capital is constant and therefore, that one can link the path of the usercost of capital to the price of investment.

Corollary 1.1 *The growth rate of relative prices across sectors in a balanced growth path where one sector overtakes the economy satisfies*

$$\frac{\dot{P}_s}{P_s} - \frac{\dot{P}_{s'}}{P_{s'}} = \frac{1}{1-\alpha_{s'}(t)}\frac{\dot{\mathcal{A}}_{s'}}{\mathcal{A}_{s'}} - \frac{1}{1-\alpha_s(t)}\frac{\dot{\mathcal{A}}_s}{\mathcal{A}_s} + \frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)}\frac{\dot{A}^x_{s'}}{A^x_{s'}} - \frac{\alpha_s(t)}{1-\alpha_s(t)}\frac{\dot{A}^x_s}{A^x_s}.$$

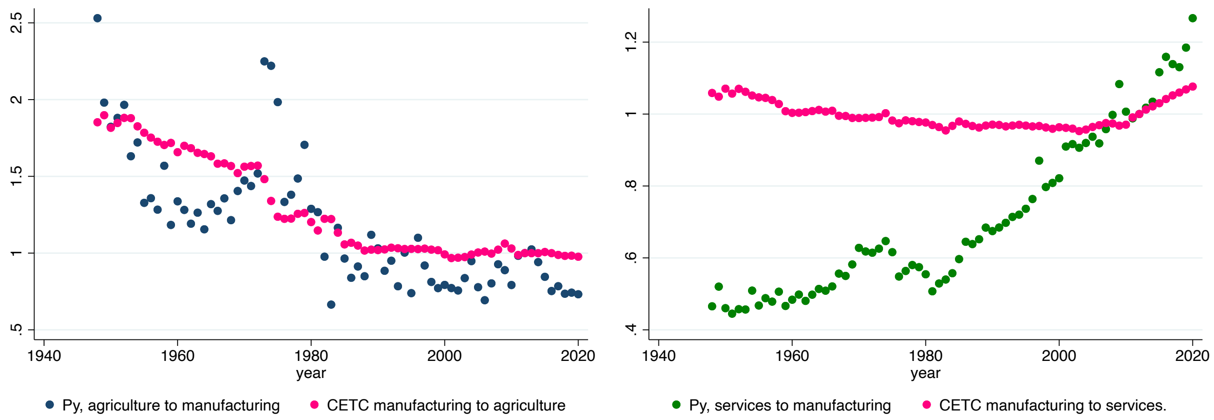
CETC affects relative output prices both directly, through the rate of sectorial embodiment, $\frac{\dot{A}^x_s}{A^x_s}$, and indirectly, through movements in the capital expenditure shares induced by shifts in the usercost of capital. The result in Corollary 1.1 requires that one sector overtakes the economy. The reason is that the marginal product of capital is a measure of the usercost relative to the price of sectorial output while CETC is a measure of the usercost of capital relative to the price of consumption. Hence, if the economy displays structural change, the price of sectorial output and the price of consumption will in general differ. While the measurement of CETC is unaffected, the components of the residual variation in relative output prices that are not explained by CETC now includes the price of sectorial output relative to consumption.

For measurement purposes, it is useful to disaggregate the measure of sectorial CETC as an investment weighted measure of CETC in each equipment type that comprises the sectorial aggregator, so

$$\begin{aligned}\frac{\dot{P}_s}{P_s} - \frac{\dot{P}_{s'}}{P_{s'}} &= \frac{1}{1-\alpha_{s'}(t)}\frac{\dot{\mathcal{A}}_{s'}}{\mathcal{A}_{s'}} - \frac{1}{1-\alpha_s(t)}\frac{\dot{\mathcal{A}}_s}{\mathcal{A}_s} \\ &\quad + \frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)}\sum_j \kappa_{js}(t)\frac{\dot{A}^x_j}{A^x_j} - \frac{\alpha_s(t)}{1-\alpha_s(t)}\sum_j \kappa_{js}(t)\frac{\dot{A}^x_j}{A^x_j}. \qquad (3)\end{aligned}$$

Equation 3 is our main accounting measure. We observe relative prices, as well as construct measures of the last two terms which correspond to the contribution of CETC for the shift in relative prices.[7] In addition, relative prices are proportional to labor productivity given a constant returns technology for production, so the above condition also measures the

[7] Away from the balanced growth path, the residual term in equation 3 will also include shifts in sectorial prices adjusted by the expenditure shares. See Proposition 1.



(a) Agriculture

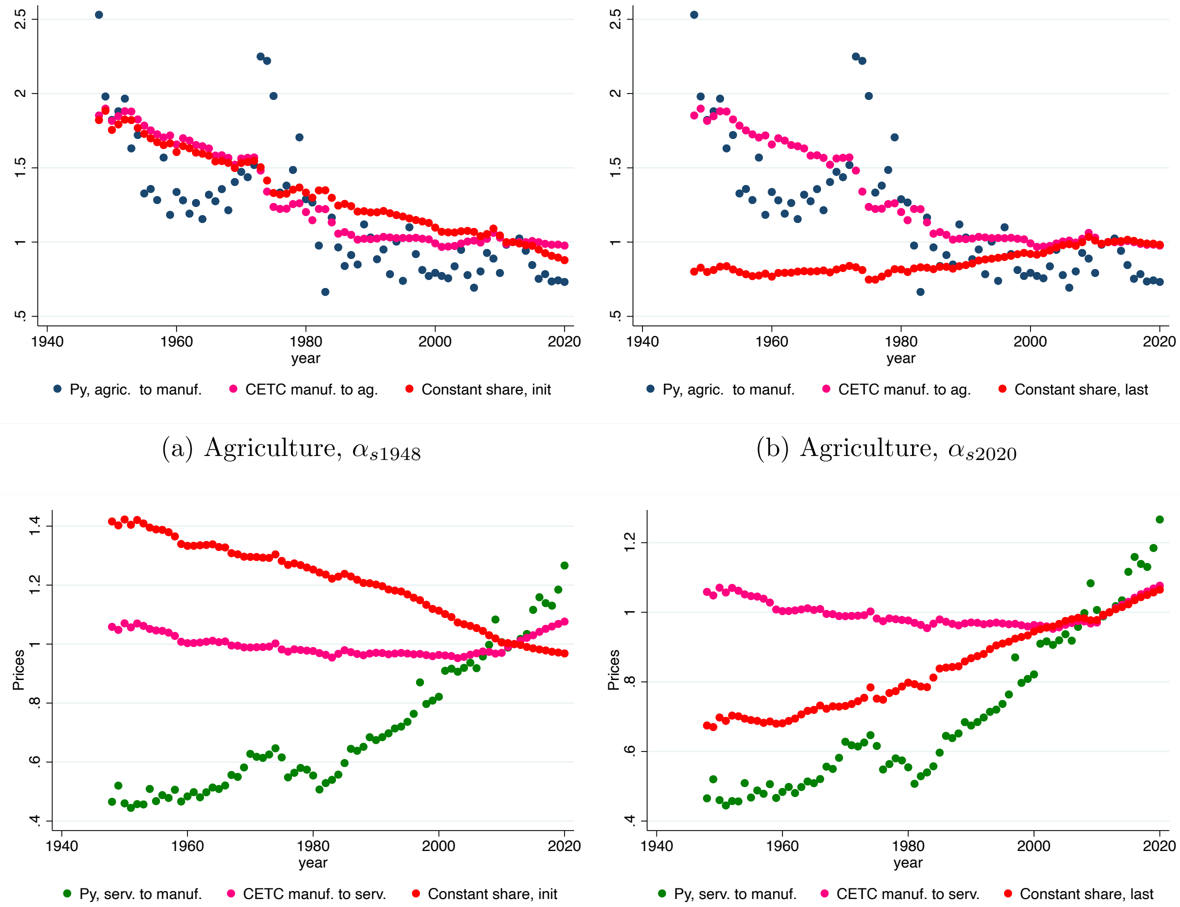
(b) Services

Figure 3: Relative prices and CETC.

The price of value added in a sector relative to industry and CETC in industry relative to a given sector. Source: BEA and own computations.

contribution of CETC to labor productivity. Because we are interested in changes through time, both measures are normalized to 1 in the base year 2012.

Figure 3 Panel (a) displays the path for the relative of agriculture to industry as well as the contribution of CETC. We find that movements in CETC track closely the path of relative surprises, suggesting that in an accounting sense, CETC was likely the main driver of differences in labor productivity across sectors. In other words, the residual variation in relative output prices that is **not** accounted by CETC (adjusted by the capital expenditure share) is relatively small. Figure 3 Panel (b) displays the paths for services relative to industry. Qualitatively, the measure of relative CETC tracks the dynamics of relative prices in the second half of the sample, particularly post-2000. This is in part driven by the increased importance of high-skill services within the sector and the fact that CETC in those sectors better tracks their relative prices, Figure **??**. In low-skill services, CETC is comparable to that of high-skill services but the measured capital expenditure share is closer to that in industry, pushing relative prices to decline rather than to raise as observed in the data. Quantitatively, the changes in measured CETC (corrected by the expenditure shares) are substantially lower in magnitude than the change in relative output prices, which implies higher residual variation accounted for disparities in total factor productivity.



(a) Agriculture, α_{s1948} (b) Agriculture, α_{s2020}

(c) Services, α_{s1948} (d) Services, α_{s2020}

Figure 4: Contribution of CETC to labor productivity, constant α_s.

Panel (a) displays the relative price of output in agriculture and Panel (b) the relative price of output in services relative to industry. In pink, we show the path of CETC in industry relative to each of these sectors. In red, we report the contribution of CETC when fixing the capital shares to their value in their initial and final sample years. Source: BEA and own computations.

2.1 Discussion

Capital expenditure shares. These findings are somehow surprising given that the ranking of CETC across sectors is opposite to the path of total factor productivity across sectors that has been widely documented in the literature, where productivity growth in agriculture is fastest in agriculture than in industry and in services (Herrendorf *et al.*, 2014). Hence, the direction of the component of relative prices accounted for by CETC is driven by the expenditure share adjustment.

To assess the role of time-varying factor shares for the contribution of CETC to relative labor productivity, we compute a counterfactual path of CETC fixing the labor share to their initial and final levels in the sample, see top panel of Figure 4 for agriculture and bottom for services. On average, agriculture is the most capital-intensive sector followed by industry and services, with capital expenditure shares of 61%, 33% and 23% respectively. These qualitative disparities remain through the sample, but its magnitude do not. industry becomes more capital intensive (and closer to agriculture), which explain the upward pressure on the relative price of agriculture to industry when computed with end-of-sample expenditure shares. The same capital intensification, makes services and industry more dissimilar in terms of expenditure shares, again generating upward pressure on relative output prices as observed in the data.

Investment aggregator. Our main accounting allows for an arbitrary constant returns to scale aggregator of investment in each sector. In addition, our benchmark accounting assumes that, aside from an equipment-specific linear rate of transformation between consumption and investment, there are no differences in the input aggregator across sectors i.e. the final output aggregator is the same for consumption and investment goods. There is a growing literature that argues that the nature of the production technology for investment goods is systematically different than that of consumption goods and that this technology shifts along the process of structural change, (García-Santana *et al.*, 2021 and Herrendorf *et al.*, 2020). A natural question is to what extent the protracted decline in the price of capital relative to consumption is driven by the process of structural change itself. Given our definition of embodied, i.e. a decline in the relative price of investment to consumption, shifts in factor neutral-technical change that has first order implications for this relative price, are also embodied.

While the question of the core drivers of such relative prices is an interesting one, this paper focuses on accounting for the direct effects of the systematic differences in this decline for employment and output reallocation across sectors. Indeed, our main accounting, equation 3, does not require assumptions on the nature of the final output aggregators. The introduction of a distinct investment and consumption aggregator affects the mapping between the relative price of investment in each equipment type and the residual productivity of the investment sector, see Corollary 1.1.

To adjust this mapping, we can augment our framework for an aggregator of final output that differs between consumption and investment as in Buera *et al.* (2020). The output of the investment aggregator is then linearly transformed into services of different capital types. Cost minimization implies that the relative price of investment to consumption in each equipment type is a combination between CETC and the sectorial output prices weighted by their role in the investment aggregator, i.e. a common trend. Since there are systematic differences in the expenditure shares of capital across sectors, the common component in the price investment would affect relative prices, exactly as in Acemoglu and Guerrieri (2008). For example, suppose industry is relatively more capital intensive than services, and that the price of the good produced by the investment aggregator falls. Then the price of of the more capital-intensive sector (industry) will fall relative to the less capital-intensive sector (services), everything else equal. Because the disparity in aggregator technologies between consumption and investment have first order effect on the relative price of investment to consumption, this effect is embodied in capital for the purposes of our accounting.

3 A model of capital-embodied structural change

To study the role of CETC for structural change, we build a parsimonious multi-sector model where each sector uses heterogeneous bundles of capital, and capital expenditure shares are endogenous to the price of each of these bundles relative to the cost of labor. Labor is homogeneous and frictionlessly allocated across sectors. Final goods are a composite of goods produced by these sectors and can be allocated to consumption or investment. A unit of final output invested towards capital of type j generates $A_j^x(t)$ capital services.

3.1 Demand structure

Consider a standard continuous time problem of a representative household with constant intertemporal elasticity preferences over a consumption aggregate $C(t)$. The household inelastically supplies $N(t)$ units of labor which earns a wage $W(t)$, and invest in capital of different types $j \in \{1, , .., J\}$, $X_j(t)$. This investment is distributed into three different sectors for the accumulation of sectorial capital, which can be rented to firms in the economy at a rental rate, $R_s(t)$. Capital in each sector depreciates at a rate $\delta_s \in (0, 1)$. A bond $B(t)$, which is priced and pays off in units of consumption, is in zero net supply. This bond prices the (consumption-based) interest rate, $R(t)$, given the rate of discount υ.

The household's problem is therefore:

$$\max_{C(t),X_j(t),X_{js}(t),K_s(t),B(t)} \int_{t=\tau}^{\infty} e^{\upsilon(t-\tau)} \frac{C(t)^{1-\theta}}{1-\theta},$$

subject to

$$P_c(t)C(t) + \sum_j P_j(t)X_j(t) + P_c(t)\dot{B}(t) = W(t)N(t) + \sum_s R_s(t)K_s(t) + r(t)P_c(t)B(t),$$

$$\dot{K}_s(t) = I_s(\mathbf{X_s}(t)) - \delta_s K_s(t), \tag{4}$$

$$X_j(t) = \sum_s X_{js}(t),$$

where $\mathbf{X_s}$ is a vector of investment of each equipment type in sector s, $\mathbf{X_s} \equiv [X_{1s}, X_{2s}, ..., X_{Js}]$ and I_s is a homogeneous of degree one aggregator.

The amount of household labor allocated to different sectors should be consistent with the labor supply,

$$\sum_s N_s(t) = N(t). \tag{5}$$

To this consumption-investment problem, we add structural transformation. Define an output aggregator

$$Y(t) = (\sum_{s=a,m,s} \omega_{ys}^{\frac{1}{\sigma_y}} Y_s(t)^{\frac{\sigma_y - 1}{\sigma_y}})^{\frac{\sigma_y}{\sigma_y - 1}}$$

for an elasticity of substitution, $\sigma_y \in [0, +\infty)$. Final output can be used to produce final consumption goods or inputs for investment goods, $\chi_j(t)$,

$$C(t) + \sum_j \chi_j(t) = Y(t).^{8} \tag{6}$$

3.2 Production structure

Producers in the investment sector maximize profits $P_j(t)X_j(t) - P_c(t)\chi_j(t)$ subject to a linear technology for production

$$X_j(t) = A_j(t)\chi_j(t), \tag{7}$$

with productivity trend, $\dot{A}_j(t) = \gamma_j(t)A_j(t)$.[9]

Producers of sectorial goods need to perform a measure 1 of activities to generate output. They choose which activities to perform with each factor of production and how much of each factor to allocate to a given activity:

$$\max_{k_i(t), n_i(t), m(t)} P_s(t)F(k_i(t), n_i(t)) - R_s(t) \int_0^1 k_i(t) - W(t) \int_0^1 n_i(t) di.$$

[8] The model can be readily extended to a non-homothetic final output aggregator, as well as distinct output aggregators for consumption and investment goods.

[9] In the extensive tradition that studies investment-specific technical change, e.g. Greenwood *et al.* (1997) and followers, the rate of technical change is assumed constant, $\gamma_j(t) = \bar{\gamma}_j$. At this level of generality, we allow for a time-varying growth rate, and later narrow its path as we characterize equilibrium allocations.

subject to their production technology

$$Y_s(t) \equiv F(k_i(t), n_i(t)) = \left[\int_o^1 y_i(t)^\rho di\right]^{\frac{1}{\rho}},$$

$$y_i(t) = \zeta_i(t)^{\frac{\rho-1}{\rho}} Z_s(t)^{\frac{\rho-1}{\rho}} n_i(t) \qquad i \in [0,1],$$

$$y_i(t) = k_i(t) \qquad i \in [0, m(t)],$$

$$\dot{Z}_s(t) = \gamma_{Z_s} Z_s(t).$$

Capital is equally productive across different activities, but labor is not: there exist a profile ζ_i, increasing in the activity index i, which characterizes the productivity of labor in each activity. In addition, there is a common component of labor productivity across activities in the sector, Z_s. Differences across economies in the supply of skills that are relevant to a given sector would map into this common trend, while the relative productivity of workers across production processes would better map to the profile ζ_i.

3.3 Optimal allocations

Key features of the allocation in this economy include the amount of labor allocated to each sector, the capital-labor ratios, the capital-expenditure shares, and the shares of output in total value added.

Expenditure shares across activities. We start characterizing the input allocation in each sector. Optimality requires

$$W(t) = p_i(t)(Z_s(t)\zeta_i(t))^{\frac{\rho-1}{\rho}},$$

$$R_s(t) = p_i(t),$$

for a price of each activity that follows $p_i(t) = P_s(t)\left(\frac{Y_s(t)}{y_i(t)}\right)^{1-\rho}$. An implication of this optimal allocation is that expenditure shares in all activities within a sector are the same.

Measure of mechanized activities. When the cost of performing an activity is lower with capital than with labor, the activity is mechanized.

$$W(t) Z_s(t)^{\frac{1-\rho}{\rho}} \zeta_i(t)^{\frac{1-\rho}{\rho}} \geq R_s(t). \tag{8}$$

Any activity below $m(t)$ that satisfies such a condition will be allocated no labor and the optimal amount of capital, i.e. the activity is "mechanized".

This condition presents the key tradeoff between the rate of decline in the usercost of capital, which is linked to CETC; and the improvements in labor productivity.

Capital and labor allocation per activity. A consequence of the equalization of the expenditure shares across activities is that all mechanized activities get the same amount of capital

$$k_i(t) = \frac{K_s(t)}{m_s(t)}. \tag{9}$$

Output across non-mechanized activities depends on labor productivity and so does the labor allocated to them,

$$n_i(t) = \frac{\frac{1}{\zeta_i(t)}}{\mathcal{Z}_s(t)} \frac{N_s(t)}{1 - m_s(t)}, \tag{10}$$

where $\mathcal{Z}_s \equiv \frac{1}{1-m_s(t)} \int_{m_s(t)}^1 \frac{1}{\zeta_i(t)} d_i$, a geometric average of the labor productivity across activities. The amount of labor allocated to activities with higher labor productivity is relatively lower.

It is indeed low enough that output in non-mechanized activities with higher labor productivity is lower than in activities with lower labor productivity. This is again a consequence of the equalization of expenditure across activities.

$$y_i(t) = \frac{P_s(t)^{\frac{1}{1-\rho}} Y_s(t)}{W(t)^{\frac{1}{1-\rho}} (Z_s(t)\zeta_i(t))^{\frac{1}{\rho}}}, \tag{11}$$

$$y_i(t) = \frac{P_s(t)^{\frac{1}{1-\rho}} Y_s(t)}{R_s(t)^{\frac{1}{1-\rho}}}. \tag{12}$$

Capital expenditure share. The expenditure share of capital, which is the sum of the expenses across mechanized activities, can be described using equation 9

$$\alpha_s(t) \equiv \int_o^{m_s(t)} \frac{p_i k_i}{P_s(t) Y_s(t)} = \frac{R_s(t) K(t)}{P_s(t) Y_s(t)}.$$

We can further describe the capital expenditure share in sector s as proportional to the mechanization rate, $m(t)$ and a function of the marginal product of capital using equation 12:

$$\alpha_s(t) = m_s(t) \left(\frac{P_s(t)}{R_s(t)} \right)^{\frac{\rho}{1-\rho}}. \tag{13}$$

When there are complementarity in production across activities $\rho < 0$, the capital expenditure share increases in the marginal product of capital, $\frac{R_s}{P_s}$, as well as the mechanization rate. In a one sector economy, the marginal product of capital is simply a function of CETC because the sectorial price equals the price of consumption. In an economy with a non-trivial and heterogeneous path for sectorial prices, the marginal product is not only a function of CETC but also of the relative price of output to consumption.[10]

Sectorial output and factor augmenting technology. Given the optimal capital and labor demand, output in each sector can be written as

$$Y_s(t) = \left[(\underbrace{(1 - m_s(t))^{\frac{1-\rho}{\rho}} A_s^n(t)}_{b^n(t)=\text{labor augmenting}} N_s(t))^\rho + (\underbrace{m_s(t)^{\frac{1-\rho}{\rho}} A_s^k(t)}_{b^k(t)=\text{capital augmenting}} \tilde{K}_s(t))^\rho \right]^{\frac{1}{\rho}}, \tag{14}$$

where the elasticity of substitution between capital and labor services is $\frac{1}{1-\rho} < 1$, and where capital has been described as product between the stock in units of the top technology available, and the efficiency of the top technology $A_s^k(t)$, i.e. $K_s = A_s^k(t)\tilde{K}_s$. Then, the capital-augmenting term is a function of the share of activities that are mechanized, and the efficiency of the top technology. The labor-augmenting term is in turn a function of the average labor productivity in activities that are not mechanized, as well as its common component within the sector, $A_s^n(t) \equiv \frac{Z_s(t)}{\mathcal{Z}_s(t)}^{\frac{\rho-1}{\rho}}$.

Proposition 2 *When $\rho \to 0$, the technology converges to a Cobb-Douglas form with output elasticity to capital equal to m_s and output elasticity to labor equal to $1 - m_s$, for arbitrary $m_s \in (0, 1)$.*

Hence, in the limit when $\rho \to 0$ in all sectors, the economy converges to Acemoglu and Guerrieri (2008).

Perhaps most important to the properties of allocations along the development path, one can use the expression for output in each sector and the optimality condition for labor (in terms of the labor expenditure share) to describe:

$$Y_s(t) = \frac{b_s^n(t)}{(1 - \alpha_s(t))^{\frac{1}{\rho}}} N_s(t). \tag{15}$$

This representation of output highlights the key departures from an economy where the output elasticity of capital is constant. Output per worker is a function of the labor augmenting term as in the plain vanilla model of structural change. But output per worker

[10] Note that Jones and Liu (2022) sets up an economy with structural change with no differences in the prices of sectorial output. The authors assume this when imposing a linear aggregation of real output into GDP.

is also a function of the endogenous cost-share of capital, which responds to the cost of capital relative to labor. When capital and labor are complementary, output per worker is lower in sectors with higher labor share.

Households' savings decisions. The dynamic decisions of the household are characterized by the Euler equation associated to the accumulation of capital of type s,

$$\theta \frac{\dot{C(t)}}{C(t)} = R(t) - \upsilon = \frac{R_s(t)\iota_{js}(t)}{P_j(t)} - \delta_s - \upsilon + \left(\frac{\dot{P}_j(t)}{P_j(t)} - \frac{\dot{P}_c(t)}{P_c(t)} - \frac{\dot{\iota_{js}(t)}}{\iota_{js}(t)}\right)$$

where $\iota_{js}(t) \equiv \frac{\partial I_s(\mathbf{X_s})}{\partial X_{js}}$, i.e. the partial of the investment aggregator in sector s to capital of type j in sectorial capital s. The return to a unit invested in equipment j at cost P_j which yields ι_{js} additional units of sectorial capital is $\frac{R_s(t)\iota_{js}(t)}{P_j(t)}$ net of the effective discount rate, $\delta_s + \upsilon$, and the change in the value of the sectorial capital.

The relative price of investment to consumption can be computed from the zero profit condition in the production of investment goods, so that $\frac{\dot{P}_j(t)}{P_j(t)} - \frac{\dot{P}_c(t)}{P_c(t)} = -\frac{\dot{A_j(t)}}{A_j(t)}$.

$$\theta \frac{\dot{C(t)}}{C(t)} = \frac{R_s(t)\iota_{js}(t)}{P_j(t)} - \delta_s - \upsilon - \frac{\dot{A_j(t)}}{A_j(t)} + \frac{\dot{\iota_{js}(t)}}{\iota_{js}(t)}$$

Cost minimization in the investment aggregator implies that $\frac{P_s^x(t)}{P_j(t)}\iota_{js}(t) = 1$. The change in the value of capital is then $\left(\frac{\dot{P}_j(t)}{P_j(t)} - \frac{\dot{P}_c(t)}{P_c(t)} - \frac{\dot{\iota_{js}(t)}}{\iota_{js}(t)}\right) = \frac{\dot{P}_s^x(t)}{P_s^x(t)} - \frac{\dot{P}_c(t)}{P_c(t)}$.

$$\theta \frac{\dot{C(t)}}{C(t)} = \frac{R_s(t)}{P_s^x(t)} - \delta_s - \upsilon - \frac{\dot{A_s^x(t)}}{A_s^x(t)} \tag{16}$$

In other words, we can describe the dynamics for the accumulation of capital in the economy as a function of the sectorial capital composite, and its rate of embodiment. Likewise, these Euler equations impose a series of no-arbitrage conditions on the value of the marginal product of capital across sectors.

We can describe consumption in units of investment of an arbitrary sector as $\tilde{C}_s(t) \equiv \frac{P_c(t)}{P_s^x(t)}C(t)$. Then, the Euler equation(s) read

$$\theta \frac{\dot{\tilde{C}}_s(t)}{\tilde{C}_s(t)} = \frac{R_s(t)}{P_s^x(t)} - \delta_s - \upsilon - (1-\theta)\frac{\dot{A_s^x(t)}}{A_s^x(t)}. \tag{17}$$

Then, with logarithmic preferences, $\theta = 1$ the Euler equation is independent of the relative price of investment to consumption, i.e. CETC, similarly to Herrendorf *et al.* (2020).

3.4 Equilibrium

Definition: *The* ***competitive equilibrium*** *is fully characterized by the Euler Equation, 16, the law of motion for capital in each sector, 4, the optimal capital and labor allocation 9 and 10, the optimal measure of mechanized activities in each sector, 8 and the feasibility constraints of the economy, 5, 7 and*

$$C(t) + \sum_j \chi_j = Y(t);$$

as well as the transversality condition for each capital type, $\lim_{t\to\infty} e^{vt} C(t)^{-\theta} K_j(t) = 0$.

With the definition of equilibrium at hand, we can explore what features of the economy would admit a generalized balanced growth path as described below. In what follows we assume logarithmic preferences for final consumption.

Definition: *A* ***generalized balanced growth path (GBGP)*** *is an allocation where the interest rate in the economy is constant,* $\bar{R} \equiv R(t) = \frac{R_s(t)}{P_s^x(t)} - \delta_s$.

The existence (or lack of) a GBGP depends critically on the properties of the rental rate of capital in units of investment. Due to complementarities in the output aggregator, the sector with slowest output growth overtakes the economy, call this sector s. The optimality condition for capital implies that the rental cost of capital (in units of investment) is

$$\frac{R(t))}{P^x(t)} = \frac{P_s(t)}{P_c(t)} \left(\frac{Y_s(t)}{\tilde{K}_s(t)} \right)^{1-\rho} (b_s^k)^\rho \tag{18}$$

where $\frac{P_s(t)}{P_c(t)} = \left(\frac{Y(t)\omega_s}{Y_s(t)} \right)^{\frac{1}{\sigma_y}}$. Hence, the sector with slowest output growth displays the fastest growth of capital in efficiency units $\tilde{K}_s(t)$ whenever the elasticity of substitution between capital and labor is higher than the elasticity of output across sectors, $\sigma_y < \sigma_{kn} \equiv \frac{1}{1-\rho}$, which is the empirically relevant case. Conversely, the sector with the fastest growth in output should display the slowest growth in capital in efficiency units. But a permanently shrinking capital stock in efficiency units (sectorial output) would eventually hit the non-negativity constraint for stocks (output), so it is not feasible along a GBGP. Hence, the BGP is only achieved in the limit.

Before describing the long-run allocations, it is important to highlight the main differences between our economy and others studied in the literature. Consider a version with no disparities in CETC across sectors, $\gamma_s^x = \gamma^x$, but where sectorial output follows the CES structure posed before. Balanced growth for this class of technologies requires pure labor-augmenting technical change Uzawa (1961), or $b_s^k(t) = b_s^k$.

Proposition 3 *Along an allocation with constant interest rate and capital-augmenting technical change,* ***relative output*** *follows*

$$\frac{Y_s(t)}{Y_{s'}(t)} = \frac{\omega_{s'}}{\omega_s}\left(\frac{\phi_s}{\phi_{s'}}\frac{\alpha_s(t)}{\alpha_{s'}(t)}\right)^{-\sigma_y\frac{1-\rho}{\rho}},$$

where $\phi_s \equiv \left(\frac{R}{P^x}\frac{1}{b_s^{k\frac{1}{\rho}}}\right)^{\frac{\rho}{1-\rho}}$ *is a constant.*

Proposition 3 implies that the limiting sector has also the slowest increase in the capital-expenditure share, which occurs whenever capital and labor are complementary, $\rho < 0$). The representation of output in this Proposition also highlights the main difference to Jones and Liu (2022), where constant capital-augmenting technical change maps one-to-one to a constant capital share. This is a consequence of the 1-sector nature of their economy and the fact that they study a static allocation, i.e. capital fully depreciates.[11] Indeed, the representation of the capital expenditure share in equation 13, makes it clear that the expenditure share is a function of the price of sectorial output, which differs from the price of consumption. In a one-sector economy, these two prices are identical.

3.5 Long run properties

We now characterize what happens along the limiting BGP.

Sectorial Output. In the steady-state the growth rate of output in the sector follows from equation 15,

$$g_{Y_s} = \frac{1-\rho}{\rho}(g_{1-m_s} + g_{A_s}) = \frac{\rho-1}{\rho}(g_{Z_s} - g_{\int_{m_s}^1 \frac{1}{\zeta_i} d_i})). \tag{19}$$

Given that $\rho \leq 0$ in the limiting sector, growth occurs as A_s declines. It does so for three reasons: (a) there are less activities performed by labor, (b) the productivity of labor in newly mechanized activities is higher (lower $\frac{1}{\zeta_{m_s}(t)}$ on the margin), (c) the productivity of labor in all activities grows exogenously at rate g_{Z_s}. the first two reasons imply that average productivity across activities is falling, $g_{\int_{m_s}^1 \frac{1}{\zeta_i} d_i} < 0$.

Aggregate Output and Wages. To compute the equilibrium wage, aggregate over the optimal demand for labor in sector. Assuming a common elasticity of substitution between

[11] In this case, the rental rate of capital equals the price of investment and 13 can be rewritten as $\alpha_s(t) = m_t(\frac{P_s(t)}{P_c(t)}\frac{P_c(t)}{P_s^x}\frac{P_s^x}{R_s(t)})^{\frac{\rho}{1-\rho}} = m_t(\frac{P_s(t)}{P_c(t)}A_s^k(t)\frac{P_s^x}{R_s(t)})^{\frac{\rho}{1-\rho}}$. In the setup of Jones and Liu (2022) the latter equation equals, $\alpha_s = b_s^{k\frac{1}{1-\rho}}$.

capital and labor across sectors, wages in the long-run converge to

$$W(t) \to \frac{1}{N(t)^{1-\rho}} (\omega_{ys})^{\frac{1}{\sigma_y}} \left(\frac{Y_s(t)}{Z_s(t)} \int_{m_s}^{1} \frac{1}{\zeta_i(t)} d_i \right)^{1-\rho}$$

for the sector s that overtakes the economy. Using the growth rate for sectorial output 19 we can compute the equilibrium growth rate of wages, which as expected, follows the growth rate of output:

$$g_w = \frac{\rho - 1}{\rho} (\gamma_{Z_s} - g_{\int_{m_s}^{1} \frac{1}{\zeta_i} d_i}) \tag{20}$$

which is an increasing function of the labor augmenting term for $\rho < 0$. In other words,

$$g_w = g_Y = g_{Y/N}.$$

Usercost of Capital and Mechanization. Using the ratio of the expenditure shares in capital and labor, we can compute the growth rate in the usercost of capital as a function of the mechanization rate and the growth rate in labor augmenting technology

$$\frac{\alpha_s}{1 - \alpha_s} = \frac{R_s(t) K_s(t)}{W(t) N_s(t)} = \frac{m_s(t)}{\int_{m_s}^{1} \frac{1}{\zeta_i} d_i} \left(\frac{W(t)}{R_s(t)} \right)^{\frac{\rho}{1-\rho}}.$$

so that

$$g_{R_s} = \frac{1 - \rho}{\rho} (g_m - (g_{\int_{m_s}^{1} \frac{1}{\zeta_i} d_i} - \gamma_{Z_s})) + g_w$$

$$g_{R_s} = \frac{1 - \rho}{\rho} g_m. \tag{21}$$

Hence, the reason for which in this model the capital share is constant is that when the cost of capital falls, there is a proportional increase in the share of newly automated tasks, compensating for the upward pressure on the capital expenditure share from more mechanized activities. Notice that the impact of mechanization on the efficiency of labor generates even more upward pressure on the expenditure ratio, but this effect is exactly compensated with an increase in wages.

The growth rate of the usercost of capital is linked to CETC through the Euler equation, equation 16, which in turn pins down g_m as a function of factor neutral productivity and

CETC, i.e. $g_{R_s} = -\gamma_{A_s^k}$; so that

$$g_{m_s} = \frac{\rho}{\rho - 1}\gamma_{A_s^k} > 0.^{12} \tag{22}$$

By definition, as activities become mechanized, the measure of unmechanized activities shrinks. Incidentally, the rate of shrinkage is also the rate at which $m_s(t)$ reaches its limit

$$g_{1-m_s} = \frac{\dot{m}_s}{1 - m_s(t)} = -g_{m_s}\frac{m_s(t)}{1 - m_s(t)}$$

or what is the same

$$g_{m_s} = -g_{1-m_s}\frac{1 - m_s(t)}{m_s(t)}. \tag{23}$$

There are two takeaways from this expression. The second term in the LHS is monotonically decreasing so two scenarios can realize: (a) if the growth rate of mechanized activities is constant, then the growth rate of unmechanized activities has to be monotonically increasing; or (b) the growth rate of unmechanized activities is constant, and the growth rate of mechanized activities is monotonically decreasing. The first case cannot arise because m_t is bounded between 0 and 1. Hence, if $g_{m_s} > 0$ is declining then so is the rate of decline in the usercost of capital, see equation 21, $g_{m_s}, g_{R_s} \to 0$. A slow-down in the rate of decline in the usercost of capital, also imposes restrictions on the rate of CETC through the Euler equation.

The optimality condition for the set of automated tasks implies,

$$g_w + \frac{1-\rho}{\rho}(\gamma_{Z_s} + g_{\zeta_{m_s}}) = g_{R_s}.$$

Replacing the expressions for the growth rates of wages and the usercost, equations 20 and 21,

$$g_{\zeta_{m_s}} = g_{m_s} - g_{\int_{m_s}^1 \frac{1}{\zeta_i} d_i} \tag{24}$$

Let the trend in labor productivity be $-q = (g_{\int_{m_s}^1 \frac{1}{\zeta_i} d_i} - \gamma_{Z_s})$. Along the BGP, q should be constant (Uzawa, 1961). Then, the rate of shrinking of the productivity across activities is proportional to the growth in productivity within the sector.[13]

[12] The usercost of capital is only exactly proportional to CETC when the rate of embodiment is constant, the interest rate is constant and the expenditure share in each equipment type in the investment aggregator is constant. The latter occurs along the transition path if the investment aggregator displays unitary elasticity across capital types. Still, the economy's interest rate might not be constant away from the limiting BGP.

[13] Before convergence in m_s, the threshold is also a function of CETC, $g_{\zeta_{m_s}} = \frac{\rho}{\rho-1} g_{A_s^x} - g_{\int_{m_s}^1 \frac{1}{\zeta_i} d_i}$.

Totally differentiating the expression for labor productivity and using the Leibniz rule,

$$-\frac{\dot{m_s}(t)}{\zeta_{m_s}(t)} = -q + \gamma_{Z_s}$$

Then,

$$\zeta_{m_s}(t) = \frac{g_{1-m_s}(1 - m_s(t))}{q + \gamma_{Z_s}},$$

or what is the same, $\gamma_{\zeta_{m_s}} = g_{1-m_s}$. Using equation 24, we conclude that $q = g_{1-m_s} + \gamma_{Z_s}$ and constant.

Finally, using the expression of the usercost of capital is easy to see that in the limit, capital and output grow at the same rate.

3.6 Detrending and steady state

Sectorial output in each sector should be detrended by labor productivity, $A_s^n(t)$. Define the trend in aggregate output as

$$a(t) \equiv \left(\sum_s \omega_s^{\frac{1}{\sigma_y}} \left(b_s^n(t)\right)^{\frac{\sigma_y - 1}{\sigma_y}} \right)^{\frac{\sigma_y}{1-\sigma_y}}$$

, for $b_s^n(t) \equiv (1 - m_s(t))^{\frac{1-\rho}{\rho}} A_s^n(t)$.

The trend in sectorial capital is

$$a_s^k(t) \equiv a(t) A_s^k(t).$$ [14]

With these trends, we can define detrended consumption, output, sectorial output and sectorial capital as follows

$$c(t) \equiv \frac{C(t)}{a(t)}, \quad y(t) \equiv \frac{Y(t)}{a(t)}, \quad y_s(t) \equiv \frac{Y_s(t)}{a(t)}, \quad k_s(t) \equiv \frac{K_s(t)}{a_s^k(t)}.$$

[14] Here we take a shortcut and describe the trend in sectorial CETC, but this trend is an investment weighted sum across CETC of different equipment types.

3.7 Transition dynamics

With the detrended economy, the transition dynamic of the problem is summarized by the following conditions

$$\theta \frac{\dot{\tilde{c}}_s(t)}{\tilde{c}_s(t)} = r_s(t) - \delta_s - \upsilon$$

where $\tilde{c}$ corresponds to the detrended consumption expenditure in units of investment and $r_s(t)$ is the detrended value of the marginal product of capital in each sector, which satisfies

$$p_s(t) \frac{y_s(t)}{k_s(t)}^{1-\rho} A_s^x(t)^\rho m_s(t)^{1-\rho} (\frac{a(t)}{a_s(t)})^\rho = r_s(t).$$

The detrended sectorial output satisfies

$$y_s(t) = \left[N_s^\rho + m_s(t)^{1-\rho} \left(\frac{a(t)}{a_s(t)} \right)^\rho A_s^k(t)^\rho k_s(t)^\rho \right]^{\frac{1}{\rho}}.$$

The dynamics of capital in the detrended economy are

$$\frac{\dot{k}_s(t)}{k_s(t)} = \frac{y(t)}{k_s(t)} - \frac{c(t)}{k_s(t)} - \sum_{s' \neq s} \frac{x_{s'}(t)}{k_s(t)} - \delta_s - \gamma_{A_s^k} - \gamma_a$$

for $x_{s'} \to 0$ for any sector that is not the one overtaking the economy.

Prices of sectorial output satisfy:

$$p_s(t) = \omega_s^{\frac{1}{\sigma y}} (\frac{a_s(t)}{a(t)})^{\frac{\sigma y - 1}{\sigma y}} \left(\frac{y(t)}{y_s(t)} \right)^{\frac{1}{\sigma y}}$$

Combining the optimality condition for capital with that of labor, we obtain a characterization of the static allocation, and equilibrium capital-labor ratios.

Finally, the system is closed with a transversality condition for each sectorial capital, $\lim_{t\to\infty} \exp(-(\upsilon + (1-\theta)\gamma_a - \gamma_{A^x})t) c(t)^{-\theta} k_s(t) = 0$. For log utility, the transversality condition requires $\upsilon > \gamma_{A^x}$, which is trivially satisfied absent CETC in the limit, or if agents have a relatively high discount for time.

4 The role CETC for structural change

To quantify the role of CETC for structural change, we run an accounting exercise on a calibrated version of the model described in the previous section. We start our exercise

Table 2: Calibration.

Parameter		Value			Target
output elast.	σ_y		0.4		.
discount factor	β		0.98		Buera *et al.* (2020)
		Agr.	Manuf.	Serv.	
elast. of substitution, k-l	$\frac{1}{1-\rho}$	1.6	0.8	0.75	Herrendorf *et al.* (2015)
depreciation rate	δ	0.14	0.21	0.17	data
output shares	ω_{ys}	0.10	0.40	0.50	employment shares, 1948
mechanization rate	$m_s(t)$				capital share
labor aug. tech: relative	$b_s^n(t)/b_m^n(t)$				relative prices
labor aug. tech: level	$b_m^n(t)$				output per worker

Note: The table shows the value of the calibrated parameters and their targets. Sectoral values are reported in the following order: agriculture, industry, services.

by feeding the path of observed sectoral capitals to the model and calibrate the remaining parameters to the path of relative prices and average output growth in the data. We then run counterfactual exercises where we measure the contribution of CETC to the reallocation of labor across sectors – that is, our measure of structural change.

4.1 Calibration

We calibrate our model to the US economy between 1948 and 2020. In our accounting exercise, we take the path of sectoral capital as given. We use the Euler equation for each sectoral capital good to measure the path of CETC in the sector and allow for a wedge in the sectoral capital demand, $\tau_s(t)$, (i.e. equation 18):

$$\tau_s(t)R_s(t) = P_s(t)m_s(t)^{1-\rho}\left(\frac{Y_s(t)}{K_s(t)}\right)^{1-\rho}, \tag{25}$$

where the price of consumption is normalized to 1. This approach allows us to get a perfect fit on the path of sectoral capital along the process of structural change.

Our calibration strategy is summarized in Table 2 and described below.

Pre-set parameters. We measure the sectoral depreciation rates δ_s directly from the data as described in Section 2. We set the discount factor, β, in line with Buera *et al.* (2020) and the elasticity of substitution across occupational outputs, σ_y, to 0.4. We allow for the elasticity of substitution between capital and labor to differ across sectors and parameterize it according to Herrendorf *et al.* (2015): capital and labor are measured substitutes in agriculture and complements in the remaining two sectors. Lastly, as mentioned above we measure the user cost of capital $R_s(t)$ from the Euler equation as described in Section 2.

Targeted moments. We calibrate the remaining parameters to match relevant features of the structural transformation process in the US between 1948 and 2020. In the first year of our sample, 1948, we target the sectoral employment allocation using the sectoral shares in final good production, ω_{ys}, and sectoral labor productivity, using labor augmenting technology $b_s^n(t)$, using:

$$\left(\frac{N_s(t)}{N_u(t)}\right)^{\frac{1}{\sigma_y}} = \left(\frac{\omega_{ys}}{\omega_{yu}}\right)^{\frac{1}{\sigma_y}} \left(\frac{b_s^n(t)}{b_u^n(t)}\right)^{\rho} \left(\frac{Y_s(t)/N_s(t)}{Y_u(t)/N_u(t)}\right)^{1-\rho-\frac{1}{\sigma_y}},$$

and the sectoral production function.

Then, in all other years in our sample, we target (a) the capital share, to parameterize the mechanization rate $m_s(t)$ for observed capital output ratios in each sector:

$$\alpha_s(t) = m_s(t)^{1-\rho} \left(\frac{Y_s(t)}{K_s(t)}\right)^{-\rho};$$

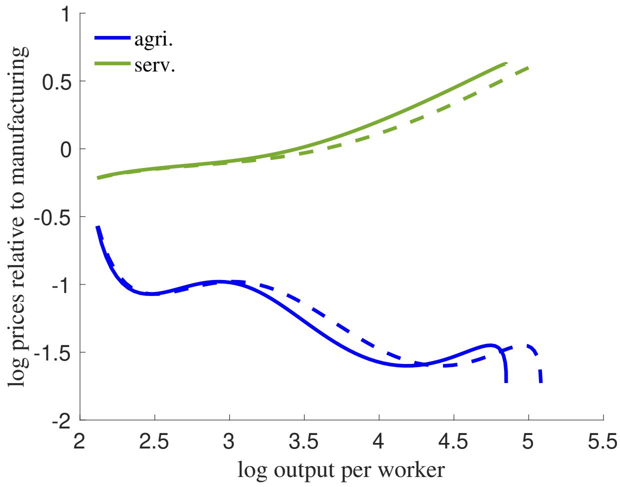
(b) the path of relative output prices to parameterize the path of sectoral labor-augmenting technology relative to the industry sector, using the optimality conditions for sectorial output demand and the industry sector as baseline

$$g_{\frac{P_s}{P_m}} = -\frac{1}{\sigma_y} g_{\frac{Y_s}{Y_m}};$$

(c) the path of labor productivity in the industry sector to parameterize the path of labor-augmenting technology in the sector, using the sectoral production function. Finally, the model-implied wedge on sectoral capital demand can be recovered from equation 25.

Outcomes. Figure 5 shows the fit of the model on the paths of relative sectoral prices. The model generates the same trend in relative prices as in the data. Table 3, columns *Model* and Data shows the model fit on the change in the allocation of employment between 1948 and 2020. This path is not a target of our calibration exercise and it is a measure of structural change. We find that the model generates a decline of 3.31p.p. in the share of agricultural employment, compared to the 4.24p.p. decline observed in the data. At the same time, the model generates an increase in the share of employment in services of 17.55p.p. compared to the observed increase of 35.28p.p.. Hence, the model generates 78% of the reallocation of employment out of agriculture and 50% of the reallocation toward services.[15]

[15] A missing source of structural change in our set-up are income effects (see Herrendorf *et al.*, 2014). We



(a) Target

Figure 5: Model fit on relative sectoral prices.

The figure shows the model fit on the targeted trend in relative prices. Data moments are in solid lines and model moments are in striped lines.

Our next step is to isolate the drivers of these relative prices and, therefore, of structural change, via counterfactual exercises. Before that, we describe the paths of the two additional sources of technological changes that, along with CETC, shape allocations in the model: labor-augmenting technology and mechanization rates. Figure 6 displays the path of mechanized activities in agriculture, industry and services. Two patterns arise: (a) the rate of mechanization has been faster in agriculture, with a pick in the 2000s; (b) the rate of mechanization in services and industry proceeds at similar speed throughout the period of analysis. The right panel of Figure 6 displays the paths of labor-augmenting technology in each sector. The growth rate of labor-augmenting technology has been stronger in agriculture, followed by industry and then services, consistently with the gold-standard calibration outcomes of models of structural change.

4.2 Accounting exercise

Three exogenous forces drive the paths of relative prices in the model, and so, of structural change: sectoral labor-augmenting technical change, $b_s^n(t)$, sectoral CETC, $A_s^k(t)$, and sectoral changing mechanization rates $m_s(t)$. We run counterfactual exercises to isolate the role of the latter two forces, where we held constant each force, one at a time. In our first counterfactual, "*no* A_s^k", we shut down sectoral differences in CETC – that is, we set $A_s^k(t) = 1$

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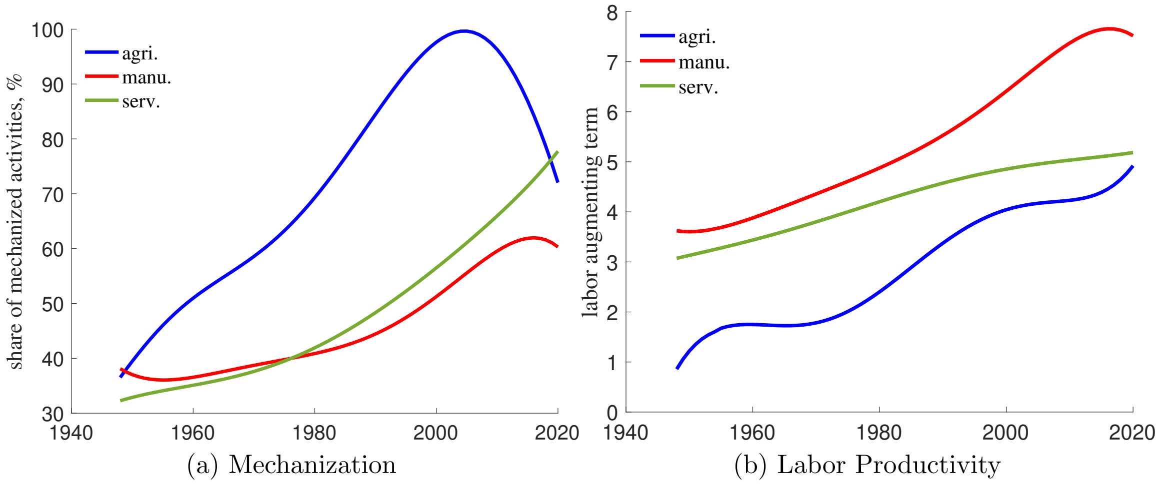


Figure 6: Paths of mechanization and labor-augmenting technology.

Panel (a) displays the mechanization rates $m_s(t)$ while Panel (b) displays labor-augmenting technology, $b_s^n(t)$. Source: Own computations.

Table 3: Counterfactuals.

Sector	Data	Model	no A_s^k	no m_s	both	no A_m^k
Agriculture	-4.24	-3.31	-2.26	-2.2	-0.1	-3.1
industry	-31.05	-14.23	-13.3	-17.4	-16.3	-5.5
Services	35.28	17.55	15.6	19.6	16.5	8.7

Note: The table shows the change between 1948 and 2016 in sectoral relative prices and employment shares in the data, as predicted by teh model, and as predicted in the two counterfactual exercises described in the text. Values are in percent.

$\forall s, t$. In our second counterfactual, "*no* m_s", we shut down sectoral differences in the path of mechanization. Lastly, we run a third counterfactual where we shut down both forces at the same time, "both". Table 3 shows the change in sectoral employment shares between 1948 and 2020, as predicted by each of the three counterfactual exercises. We measure that CETC accounts for 32% of the movement of employment out of agriculture and for 11% of the movement toward services. Mechanization accounts for the same fraction of the outflow of employment from the agricultural sector but, differently from CETC, it generates and outflow of employment from the service sector. Last, we run an additional set to counterfactual exercises where we isolate the effect of sectoral CETC – that is, we separately shut down CETC in each sector. In column "no A_m^k "of Table 3, we report the results for when $A_m^k(t) = 1$. CETC in the industry sector is a strong driver of the outflow of employment from industry into services: it accounts for 61% of the former and for 50% of the latter.

5 Final Remarks

We document systematic disparities in the bundle of capital used for production across sectors. These disparities induce sector-specific rates of capital-embodied technical change which we can directly measure in the data using data on the cost of capital relative to consumption.

CETC differentials map into labor productivity differences through capital-expenditure shares, which we also show have been changing within sectors through time. We build a structural model that can accommodate those movements endogenously, and therefore propose a theory for the endogenous link between CETC and labor productivity.

Our preliminary results show that labor productivity differentials across sectors (fastest in agriculture, slower in industry and slowest in services) may reverse as CETCbecomes ever more important for output production. Hence, our findings may have implications for Baumol's cost disease.

Overall, our findings draw attention to the nature of investment for the path of economic development.

References

Acemoglu, D. (2003). Labor- and capital-augmenting technical change. *Journal of the European Economic Association*, **1** (1), 1–37.

— and Guerrieri, V. (2008). Capital deepening and nonbalanced economic growth. *Journal of Political Economy*, **116** (3), 467–498.

Alvarez-Cuadrado, F. and Poschke, M. (2011). Structural change out of agriculture: Labor push versus labor pull. *American Economic Journal: Macroeconomics*, **3** (3), 127–58.

Boppart, T. (2014). Structural change and the kaldor facts in a growth model with relative price effects and non-gorman preferences. *Econometrica*, **82** (6), 2167–2196.

Buera, F. J., Kaboski, J. P., Mestieri, M. and O'Connor, D. G. (2020). *The Stable Transformation Path*. NBER Working Papers 27731, National Bureau of Economic Research, Inc.

Caunedo, J., Jaume, D. and Keller, E. (2021). *Occupational Exposure to Capital-Embodied Technical Change*. CEPR Working Papers 15759, CEPR.

— and Keller, E. (2020). Capital Obsolescence and Agricultural Productivity*. *The Quarterly Journal of Economics*, **136** (1), 505–561.

Comin, D., Lashkari, D. and Mestieri, M. (2021). Structural Change With Long-Run Income and Price Effects. *Econometrica*, **89** (1), 311–374.

Cummins, J. G. and Violante, G. L. (2002). Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences. *Review of Economic Dynamics*, **5** (2), 243–284.

Eaton, J. and Kortum, S. (2001). Trade in capital goods. *European Economic Review*, **45** (7), 1195–1235.

García-Santana, M., Pijoan-Mas, J. and Villacorta, L. (2021). Investment demand and structural change. *Econometrica*, **89** (6), 2751–2785.

Gordon, R. (1990). *The Measurement of Durable Goods Prices*. National Bureau of Economic Research, Inc.

Greenwood, J., Hercowitz, Z. and Krusell, P. (1997). Long-Run Implications of Investment-Specific Technological Change. *American Economic Review*, **87** (3), 342–62.

Hercowitz, Z. (1998). The 'embodiment' controversy: A review essay. *Journal of Monetary Economics*, **41** (1), 217–224.

Herrendorf, B., Herrington, C. and Ákos Valentinyi (2015). Sectoral Technology and Structural Transformation. *American Economic Journal: Macroeconomics*, **7** (4), 104–133.

—, Rogerson, R. and Valentinyi, A. (2014). Growth and structural transformation. In *Handbook of Economic Growth*, vol. 2, *06*, 1st edn., Elsevier, pp. 855–941.

—, — and Valentinyi, (2020). Structural Change in Investment and Consumption—A Unified Analysis. *The Review of Economic Studies*, **88** (3), 1311–1346.

Hulten, C. R. (1992). Growth Accounting When Technical Change Is Embodied in Capital. *American Economic Review*, **82** (4), 964–80.

Intriligator, M. D. (1992). Productivity and the embodiment of technical progress. *The Scandinavian Journal of Economics*, **94**, S75–S87.

Jones, B. F. and Liu, X. (2022). *A Framework for Economic Growth with Capital-Embodied Technical Change*. Working Paper 30459, National Bureau of Economic Research.

Manuelli, R. and Seshadri, A. (2014). Frictionless Technology Diffusion: The Case of Tractors. *American Economic Review*, **104** (4), 1368–91.

Mutreja, P., Ravikumar, B. and Sposi, M. (2018). Capital goods trade, relative prices, and economic development. *Review of Economic Dynamics*, **27**, 101–122.

Ngai, L. R. and Pissarides, C. A. (2007). Structural change in a multisector model of growth. *American Economic Review*, **97** (1), 429–443.

Oulton, N. and Srinivasan, S. (2003). Capital stocks, capital services, and depreciation: an integrated framework.

Sahin, A., Elsby, M. and Hobijn, B. (2013). *The Decline of the U.S. Labor Share*. Working Paper Series 2013-27, Federal Reserve Bank of San Francisco.

Uzawa, H. (1961). Neutral inventions and the stability of growth equilibrium. *The Review of Economic Studies*, **28** (2), 117–124.

Valentinyi, and Herrendorf, B. (2008). Measuring factor income shares at the sectoral level. *Review of Economic Dynamics*, **11** (4), 820–835.

A Data construction.

VA price deflators by sector are Torqvinst price indexes constructed weighting the price deflator of output in each sector (variable VA_P) by nominal value-added weights, (variable VA).

Real measures of value added (in national currency) are the ratio between total value added by sector (aggregated linearly) and the price deflator for output in the sector.

Measures of **employment** correspond to counts of people and total hours of those employed (variables $EMPE$ and H_EMPE). I have also included measures of labor compensation LAB.

Measures of capital include capital compensation rk, (variable CAP) and KLEMS also produces a measure of stock which is a quantity index with base year 2010.

B Proofs

Proof. Proposition 1.

Let $k_s(t) \equiv \frac{K_s(t)}{N_s(t)}$, and define output in terms of capital per worker as $f(k_s(t)) \equiv F(\frac{K_s(t)}{N_s(t)}, 1)$. The, wages can be written as,

$$P_s(t)(1-\alpha_s(t))f(\frac{W_t}{R_s(t)}\frac{\alpha_s(t)}{1-\alpha_s(t)}) = W_t.$$

The above expression can be linearized around the steady state of the economy as

$$\ln(P_s^\star) + \frac{P_s - P_s^\star}{P_s^\star} + \ln(A_s^\star) + \frac{A_s - A_s^\star}{A_s^\star} + \ln(1-\alpha_s^\star) + \frac{(1-\alpha_s)-(1-\alpha_s^\star)}{(1-\alpha_s^\star)} + \ln(f(k^\star)) + \frac{f_k k^\star}{f}\frac{(k-k^\star)}{k^\star}$$
$$= \ln(W^\star) + \frac{W - W^\star}{W^\star}$$

where $x^\star$ corresponds to the s.s. level of variable x.

$$d\ln(P_s) + d\ln(1-\alpha_s) + d\ln(A_s) + \frac{f_k k^\star}{f} d\ln(k) = d\ln(W)$$

using the optimal capital labor ratios

$$d\ln(P_s) + d\ln(1-\alpha_s) + d\ln(A_s) + \frac{f_k k^\star}{f}(d\ln(W) - d\ln(R_s) + d\ln(\alpha_s) - d\ln(1-\alpha_s)) = d\ln(W).$$

The term $\frac{f_k k^\star}{f} = \alpha_s(t)$ by definition, and therefore

$$(1-\alpha_s(t))(d\ln(P_s)+d\ln(A_s))+d\ln(1-\alpha_s)(1-\alpha_s(t))+\alpha_s(t)d\ln(\alpha_s)+\alpha_s(t)(d\ln(P_s)-d\ln(R_s)) = d\ln(W)(1-$$

$$d\ln(P_s)+d\ln(A_s)+d\ln(1-\alpha_s(t))+\frac{\alpha_s(t)}{1-\alpha_s(t)}d\ln(\alpha_s)+\frac{\alpha_s(t)}{1-\alpha_s(t)}(d\ln(P_s)-d\ln(R_s)) = d\ln(W)$$

Computing the difference between sectors yields the result. ■

Proof. Corollary 1.1 We rewrite this condition as a function of CETC using the relationship between the user cost of capital and CETC through the Euler equation in the long-run:

$$\frac{\lambda_t}{\beta\lambda_{t+1}} - \frac{P^x_{st+1}}{P_s(t)^x}(1-\delta_s) = \frac{r_{st+1}}{P_s(t)^x}$$

for P^x_s the price of investment in sector s

$$\frac{\lambda_t}{\beta\lambda_{t+1}} - \frac{P^x_{st+1}}{P_s(t)^x}(1-\delta) = \frac{r_{st+1}}{P_s(t)}\frac{P_{ct}}{P_s(t)^x}\frac{P_s(t)}{P_{ct}}.$$

The first term is the interest rate in the economy R_t whereas the second term is a function of CETC $\frac{P^x_{st+1}}{P_s(t)^x}$. Both of them are constant along a (limiting) BGP. The term P_{ct} corresponds to the deflator of final output. Along the (limiting) BGP, $\frac{P_s(t)}{P_{ct}}$ is a constant, and therefore the user cost of capital relative to the price of output grows at the same rate as the price of investment in the sector relative to consumption:

$$d\ln(R_s(t)) - d\ln(P_s(t)) = d\ln(P^x_s(t)) - d\ln(P_{ct}) = -d\ln(A^x_s(t)).$$

We replace this long run condition into equation **??** and use the condition for wage equalization across sectors to obtain equation **??**.[16]

■

Proof. Proposition 2.

Apply logs to equation 14 and take the limit when $\rho \to 0$.

$$\lim_{\rho\to 0}\ln(Y_s(t)) = \lim_{\rho\to 0}\frac{1}{\rho}\ln\left((1-m_s(t))\left(\frac{A^n_s(t)N_s(t)}{1-m_s(t)}\right)^\rho + m_s(t)\left(\frac{A^k_s(t)\tilde{K}_s(t)}{m_s(t)}\right)^\rho\right)$$

[16]Along the transition to the limiting BGP, the mapping between the usercost of capital and CETC becomes non-linear, but equation **??** still provides a valid link between the growth rate in sectorial prices (and labor productivity), and the usercost of capital.

Because $m_s(t) + 1 - m_s(t) = 1$ we can apply L'hopital rule and compute the limit.

$$\lim_{\rho \to 0} \ln(Y_s(t)) = (1 - m_s(t)) \ln\left(\frac{A_s^n(t) N_s(t)}{1 - m_s(t)}\right) + m_s(t) \ln\left(\frac{A_s^k(t)\tilde{K}_s(t)}{m_s(t)}\right)$$

which is just a Cobb-Douglas form.

$$Y_s(t) = \frac{A_s^n(t)^{1-m_s(t)}}{(1 - m_s(t))^{1-m_s(t)} m_s(t)^{m_s(t)}} N_s(t)^{1-m_s(t)} (A_s^k(t)\tilde{K}_s(t))^{m_s(t)} \tag{26}$$

■

Proof. Proposition 3.

We can compute **relative output** as the ratio of output in different sectors, equation 15.

$$\frac{Y_s(t)}{Y_{s'}(t)} = \left(\frac{b_{s'}^n(t)}{b_s^n(t)}\right)^{-\frac{1}{\rho}} \left(\frac{1 - \alpha_s(t)}{1 - \alpha_{s'}(t)}\right)^{-\frac{1}{\rho}} \frac{N_s(t)}{N_{s'}(t)} \tag{27}$$

The labor allocation follows

$$\frac{N_{s'}(t)}{N_s(t)} = \frac{Y_{s'}(t)}{Y_s(t)}^{1-\frac{1}{\sigma_y}\frac{1}{1-\rho}} \left(\frac{\omega_s}{\omega_{s'}}\right)^{\frac{1}{\sigma_y}\frac{1}{1-\rho}} \left(\frac{b_{s'}^n(t)}{b_s^n(t)}\right)^{\frac{1}{1-\rho}}$$

Replacing back in the equation for relative output, we obtain

$$\left(\frac{Y_s(t)}{Y_{s'}(t)}\right)^{\frac{1}{\sigma_y}\frac{1}{1-\rho}} = \left(\frac{b_s^n}{b_{s'}^n}\right)^{\frac{1}{\rho}} \left(\frac{1 - \alpha_s(t)}{1 - \alpha_{s'}(t)}\right)^{-\frac{1}{\rho}} \left(\frac{\omega_{s'}}{\omega_s}\right)^{\frac{1}{\sigma_y}\frac{1}{1-\rho}} \left(\frac{b_s^n(t)}{b_{s'}^n(t)}\right)^{\frac{1}{1-\rho}}$$

$$\left(\frac{Y_s(t)}{Y_{s'}(t)}\right) = \left(\frac{b_{s'}^n}{b_s^n}\right)^{-\frac{\sigma_y}{\rho}} \left(\frac{1 - \alpha_s(t)}{1 - \alpha_{s'}(t)}\right)^{-\sigma_y\frac{1-\rho}{\rho}} \left(\frac{\omega_{s'}}{\omega_s}\right) \tag{28}$$

Output in the limiting sector will grow slower than elsewhere if labor augmenting productivity growth is slower in this sector (slower decline in b_s^n), or if the labor share falls faster in that sector. In the data however, we have seen a faster decline in the labor share in industry relative to services, which would push in the opposite direction.

To sign the dynamics of the **labor share** in both sectors, we need the dynamics of the capital-labor ratios since

$$\frac{1 - \alpha_s(t)}{\alpha_s(t)} = \frac{b_s^n(t)}{b_s^k(t)} \left(\frac{N_s(t)}{\tilde{K}_s(t)}\right)^{\rho}. \tag{29}$$

Using the optimality conditions for labor describe the ratio of **capital-labor ratios** as

$$\frac{P_s}{P_{s'}}\left[\frac{Y_s(t)/\tilde{K}_s(t)}{Y_{s'}(t)/\tilde{K}_{s'}(t)}\right]^{1-\rho}=\left[\frac{\tilde{K}_{s'}(t)/N_{s'}(t)}{\tilde{K}_s(t)/N_s(t)}\right]^{1-\rho}\frac{b^n_{s'}(t)}{b^n_s(t)}$$

The optimality condition for capital 18 implies that the left hand side is a constant along a GBGP if b^k is constant, equal to

$$\frac{R_s(t)/P^x_s(t)}{R_{s'}(t)/P^x_{s'}(t)}\frac{b^k_{s'}}{b^k_s}=\left[\frac{\tilde{K}_{s'}(t)/N_{s'}(t)}{\tilde{K}_s(t)/N_s(t)}\right]^{1-\rho}\frac{b^n_{s'}(t)}{b^n_s(t)} \tag{30}$$

and therefore the capital labor ratios move inversely to the labor augmenting terms b^n. Faster labor productivity growth in a sector implies a shrinking labor augmenting term and therefore higher capital-labor ratios.

Hence, combining equation 30 and 29

$$\left(\frac{R_s(t)/P^x_s(t)}{R_{s'}(t)/P^x_{s'}(t)}\frac{b^k_{s'}}{b^k_s}\frac{b^n_s}{b^n_{s'}}\right)^{\frac{1}{1-\rho}}=\left(\frac{1-\alpha_s(t)}{\alpha_s(t)}\frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)}\frac{b^k_s(t)}{b^n_s(t)}\frac{b^n_{s'}(t)}{b^k_{s'}(t)}\right)^{\frac{1}{\rho}}$$

Hence,

$$\frac{1-\alpha_s(t)}{\alpha_s(t)}\frac{\alpha_{s'}(t)}{1-\alpha_{s'}(t)}=\left(\frac{R_s(t)/P^x_s(t)}{R_{s'}(t)/P^x_{s'}(t)}\right)^{\frac{\rho}{1-\rho}}\left(\frac{b^k_{s'}}{b^k_s}\frac{b^n_s}{b^n_{s'}}\right)^{\frac{1}{1-\rho}}$$

which is only a function of the labor productivity trends along a BGP where the value of the marginal product of capital is constant. If labor augmenting productivity is slower in the limiting sector, then the capital share grows slower in the limiting sector than elsewhere in the economy (assuming capital and labor are complementary, $\rho<0$).

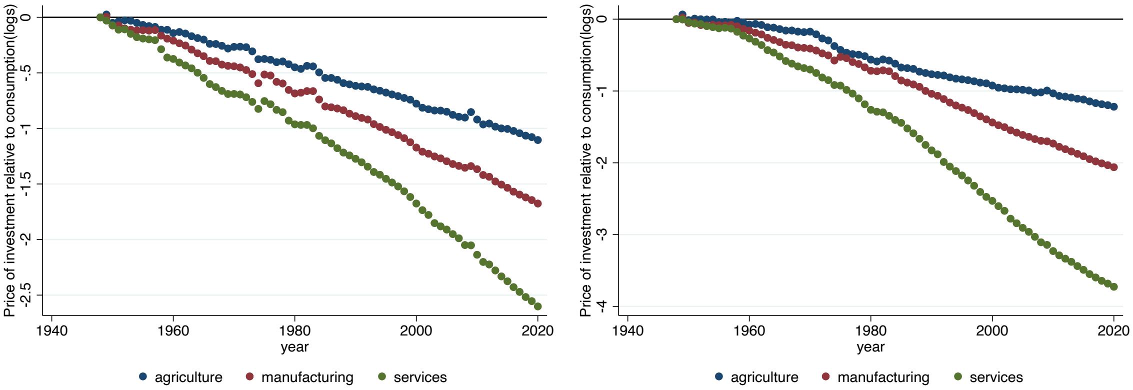
Define the constant $\phi_s\equiv(R_s(t)/P^x_s(t))^{\frac{\rho}{1-\rho}}\frac{1}{{b^k_s}^{\frac{1}{1-\rho}}}$ so that we can write the ratio of the labor shares as

$$\frac{1-\alpha_s}{1-\alpha_{s'}}=\frac{\phi_s}{\phi_{s'}}\left(\frac{b^n_s}{b^n_{s'}}\right)^{\frac{1}{1-\rho}}\frac{\alpha_s}{\alpha_{s'}}$$

which can be replaced into 28 as

$$\left(\frac{Y_s(t)}{Y_{s'}(t)}\right)=\left(\frac{b^n_{s'}}{b^n_s}\right)^{-\frac{\sigma_y}{\rho}}\left(\frac{\phi_s}{\phi_{s'}}\left(\frac{b^n_s}{b^n_{s'}}\right)^{\frac{1}{1-\rho}}\frac{\alpha_s}{\alpha_{s'}}\right)^{-\sigma_y\frac{1-\rho}{\rho}}\left(\frac{\omega_{s'}}{\omega_s}\right)$$

$$\left(\frac{Y_s(t)}{Y_{s'}(t)}\right)=\left(\frac{\phi_s}{\phi_{s'}}\frac{\alpha_s(t)}{\alpha_{s'}(t)}\right)^{-\sigma_y\frac{1-\rho}{\rho}}\left(\frac{\omega_{s'}}{\omega_s}\right).$$



(a) Initial weights, κ_{js1948} (b) Last weights, κ_{js2020}

Figure 7: Sectorial CETC for different investment weights.

The price of investment relative to consumption is normalized to 1 in 1948 and the picture displays log of prices. Source: BEA and own computations.

C Additional Tables and Figures.

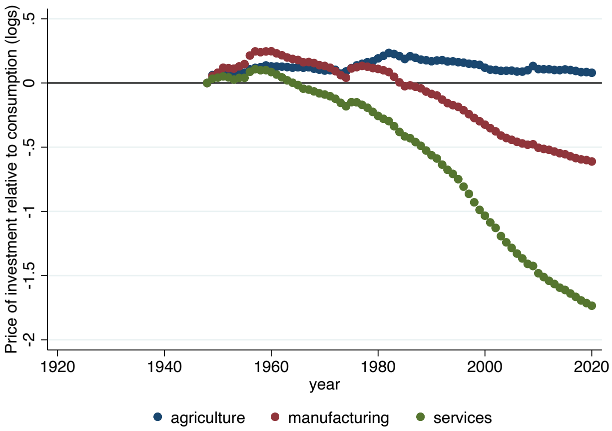


Figure 8: Relative price of investment to consumption (logs). Raw series (BEA), without quality adjustment.