Validation of Basic Heading and Aggregated PPPs: When Does Validation End and Estimation Begin?

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The previous chapter described the huge, complicated effort by the International Comparison Program (ICP) that goes into validation of the prices collected for over a thousand products, first at the country level and then between countries, to ensure not only that comparable products were priced but also that they were national annual averages. Although much has been written about the subsequent steps taken to aggregate basic heading purchasing power parities (PPPs) to the gross domestic product (GDP), little has been said about data validation for these steps. Therefore, this chapter adds a new dimension to data validation by examining the aggregation steps and the validation of the PPP and expenditure data used for each.[1]

The chapter begins with a review of these steps to set the stage for the data validation to be introduced at each level. It is followed by a review of different data validation tools using results from the 2005 ICP. These validation tools range from simple data plots to cluster analysis to models that account for the inherent variability in the bilateral PPPs. The chapter concludes with a discussion of when data validation should end and estimation should begin. The data analysis has two purposes: first, to point out where more validation is needed, and, second, to point out that some countries have patterns of prices and expenditures that give them the appearance of outliers in the data analysis, even using quality data. Thus arises the dilemma of when validation ends and estimation begins.

From Basic Heading PPPs to GDP: Overview of the Steps

The data validation and estimation processes described here begin with the matrix of 129 basic heading PPPs for 146 countries after all countries across the six ICP regions[2] have been linked to a common global currency. The estimation process to obtain these basic heading PPPs is described in chapter 4; the PPPs are transitive and base country–invariant. Chapter 9 describes the considerable

effort made to validate the prices underlying the basic heading PPPs. The within-country PPPs will vary across the basic headings. However, one would expect some internal consistency. Price levels in poor countries are generally lower than those in richer countries and should show a similar pattern across basic headings. The following section discusses how to examine the basic heading PPPs within countries, and then by country within each basic heading. This analysis will point out basic headings and countries in which the underlying prices should be again reviewed.

An additional matrix, 129 × 146, contains the basic heading expenditures expressed in the currency of each country. Chapter 6 describes the multistage process used to estimate global PPPs that begins with estimation of the within-region basic heading PPPs. These are then calibrated to a global currency using between-region linking factors. The final step is to average the basic heading PPPs to the GDP. The analysis presented here is based on a direct aggregation of the 129 basic heading PPPs to the GDP level, which is also described in chapter 6. The global aggregation is being used in this chapter mainly to illustrate the data validation steps to be considered.

The first step in the global aggregation process is to compute the weighted average of the basic heading PPPs using expenditure weights to obtain the PPP at the GDP for each pair of countries. Because the distribution of the expenditure shares will differ for each country, the issue is how the weights should be used in the aggregation. Chapter 5 describes how the PPPs are first averaged to the GDP using the expenditure weights for country *j,* then again for country *k.* These are the Laspeyres and Paasche indexes, respectively.

The Laspeyres index is

$$PPP_L^{j,k} = \sum_{n=1}^{N} w_n^j PPP_n^{j,k} \tag{10.1}$$

which is a weighted average of the PPPs of country *j* to country *k* across the *N* basic headings using country *j* weights. The Paasche index between the same two countries is

$$PPP_p^{j,k} = \frac{1}{\sum_{n=1}^{N} \frac{w_n^k}{PPP_n^{j,k}}} \tag{10.2}$$

using country *K* weights.

The Laspeyres and Paasche indexes result in different estimates of the PPP for the GDP of each country. As described in chapter 1, one of the fundamental principles underlying the ICP is that countries be treated symmetrically or equally. This principle is incorporated by taking the geometric average of the Laspeyres and Paasche indexes, which is the Fisher index—$PPP_F^{j,k}$—for each pair of countries. The result is a matrix of 146 × 146 Fisher indexes for every combination of two countries. Because these indexes are not transitive, the Gini-Éltetö-Köves-Szulc (GEKS) method is applied to provide transitivity. Chapters 1 and 5 describe this process. As shown in the discussion of tables 10.4 and 10.5 in this chapter, the Fisher matrix can be used to derive for each country row a set of two direct and 144 indirect PPPs. The geometric mean of the direct and indirect parities for countries *j* and *k* is the GEKS PPPs, which are then transitive and base country–invariant. Again, the respective direct and indirect PPPs are treated equally with the computation of the geometric average.

The next section reviews the basic heading PPPs and expenditure weights in order to point out the additional data validation steps that should be taken. This review is followed by a look at the Laspeyres and Paasche indexes and how they depart from the Fisher index. The penultimate

section reviews the direct and indirect PPPs using the GEKS method to achieve transitivity. The chapter concludes by considering this question: when does validation end and estimation begin?

Validating Basic Heading PPPs

The first validation is to review the variability of the basic heading PPPs within each country for the presence of outliers. The data set is the matrix of 129 basic headings times 146 countries. The analysis is based on the assumption that the within-country price levels across the basic headings are consistent—a poor country usually has lower price levels than a richer country. Recall that no expenditure or quantity weights enter into the estimation of basic heading PPPs.

For the analysis to follow, the basic heading PPPs to the U.S. dollar are standardized to the price level index (PLI)[3] for world = 100 so that the relative price levels across countries can be directly compared. The distributions of the basic heading PLIs by country are shown in figure 10.1 using box and whisker plots introduced by Tukey (1977). Box plots are nonparametric and indicate the degree of dispersion and skewness of the data and identify outliers. Construction of the box plots starts by simply sorting the basic heading PLIs from the smallest to the largest within each country. For this example, each box contains 80 percent of the basic heading PLIs for each country. Ten percent of the basic headings have PLIs larger than the top boundary of the box, and 10 percent have PLIs smaller than the bottom boundary. Each box contains a whisker, which indicates the maximum and minimum basic heading PLIs. The line shown inside each box is the PPP of the median point—half of the basic heading PLIs in each country are larger and half are smaller. Note that the median value is not always in the center of the box; the distance above or below the midpoint is an indication of skewness.

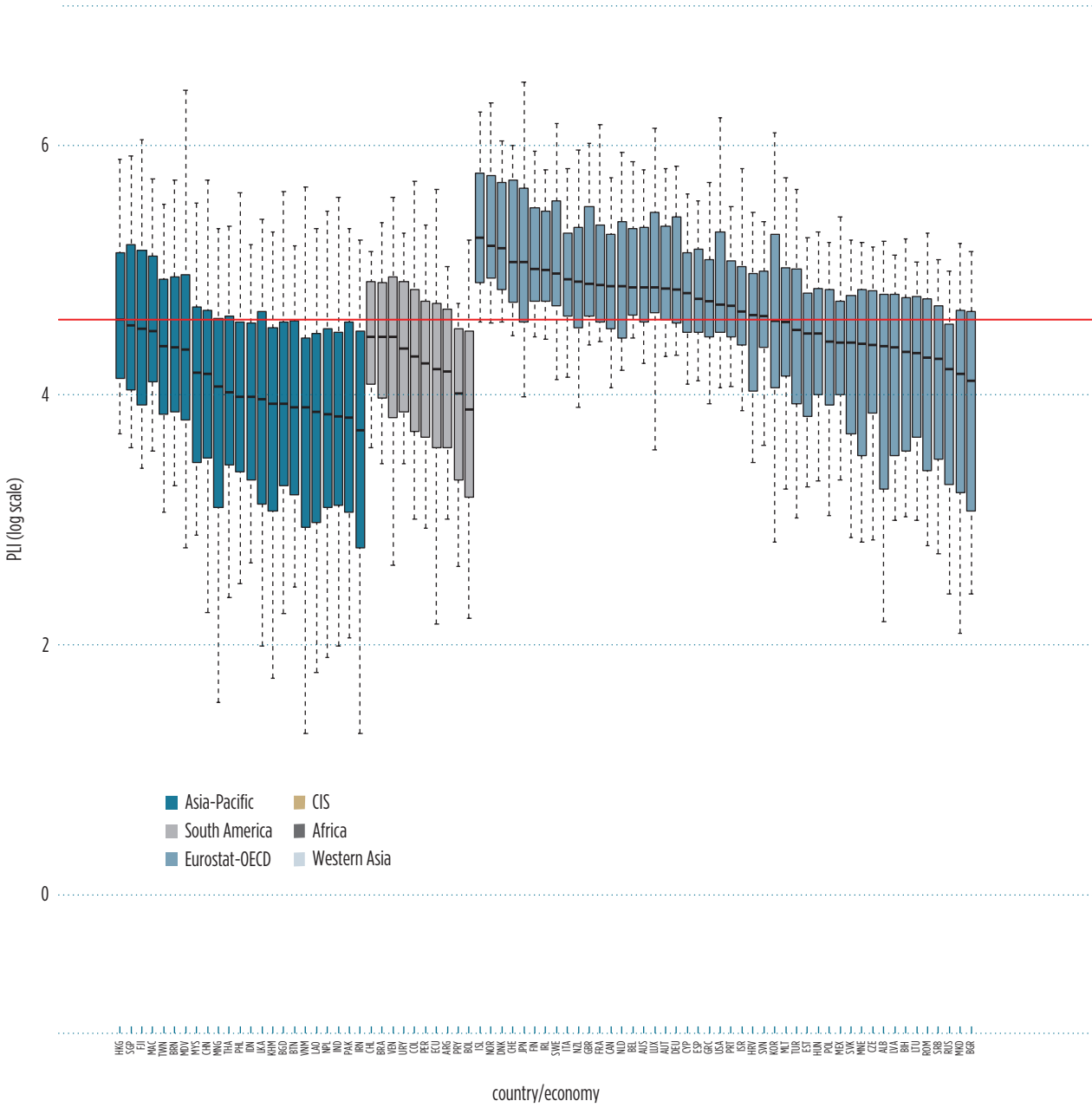
Figure 10.1 shows the countries grouped by region and then within region in order from the country with the largest median value to the smallest median value. The PLIs are shown in log scale with world = 100 (ln 100 = 4.6). Figure 10.2 shows the box plots for each of the 129 basic headings sorted by basic heading from the largest to the smallest median PLI values. Although the box plots in both figures generally show considerable consistency in the size of the boxes across basic headings and countries, there are outliers that need to be examined.

In figure 10.1, the ranking of the countries by region by median value shows, as expected, that the Eurostat–Organisation for Economic Co-operation and Development (OECD) countries have the highest price levels. However, Angola and Equatorial Guinea, which are relatively poor, have the 19th and 21st largest median values, respectively, suggesting they be examined in more detail. In both cases, the basic heading with the maximum value is "passenger transport by air." Figure 10.2 shows that this basic heading has the highest median value and also one of the largest maximum values, which is attributable to Equatorial Guinea followed closely by Angola. A closer examination of the data reveals that the PLIs for passenger transport by air provide the maximum value for more countries than for any other basic heading. This is an indication that the specifications for the pricing of this basic heading should be examined.

A similar review of the minimum values shows that they depart more from the median than do the maximum values. Several countries have minimum values that warrant additional review. Many of the minimum values are from the basic headings for compensation and medical services. These PPPs are difficult to compare across regions because not all made adjustments for productivity.

The purpose of these figures is to illustrate that even though there was an intensive data validation of the product prices, the distribution of the resulting PPPs by country and by basic heading should be examined for PLIs that do not seem plausible. For example, six countries have maximum and minimum basic heading PLIs that differ by a factor of over 100.

FIGURE 10.1 Box and Whisker Plots of Price Level Indexes by Region and Country/Economy (world = 100)



Sources: ICP 2005 and computations by Min Ji Lee, ICP Global Office.
Note: See annex to this chapter for country/economy codes.

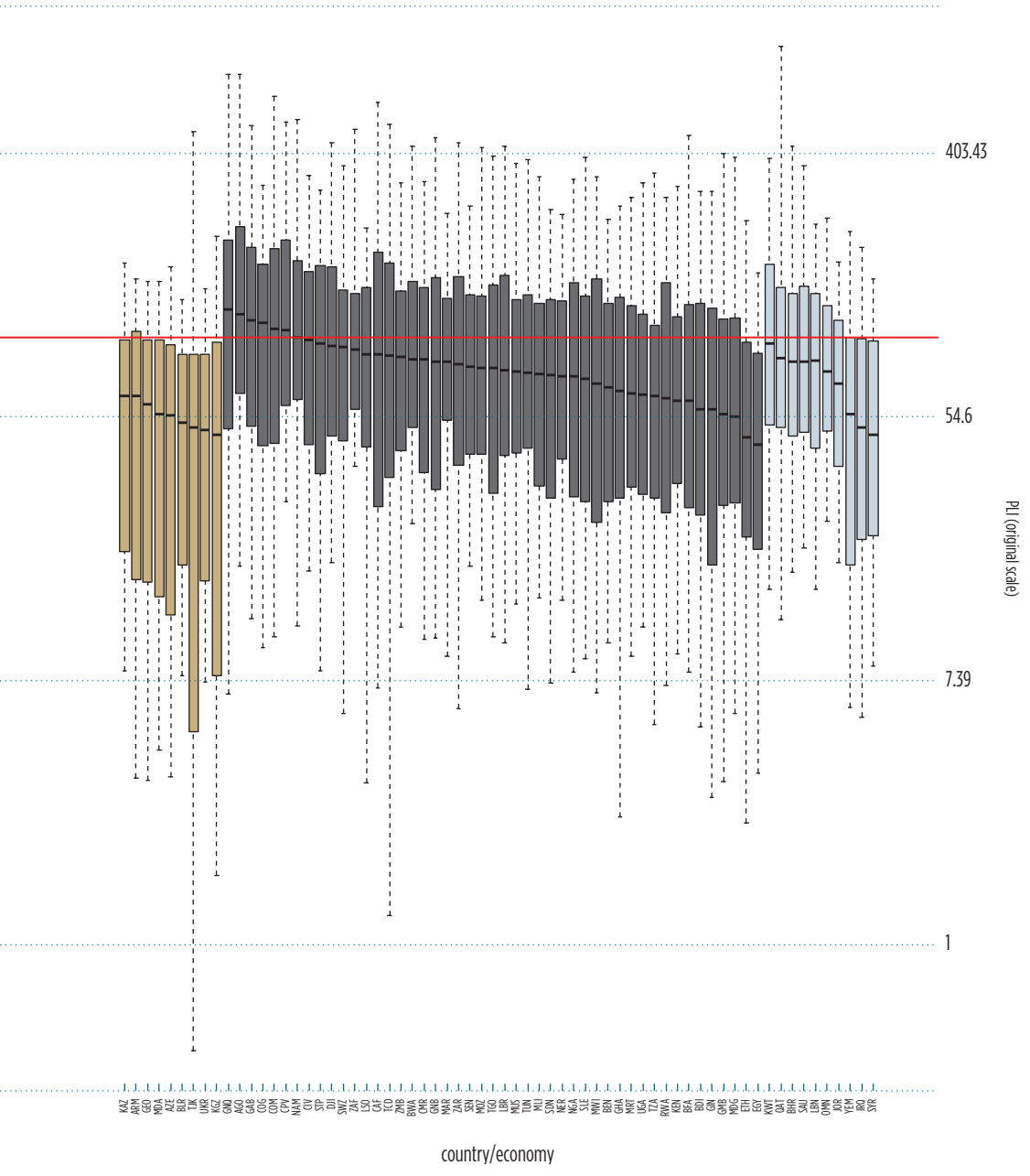
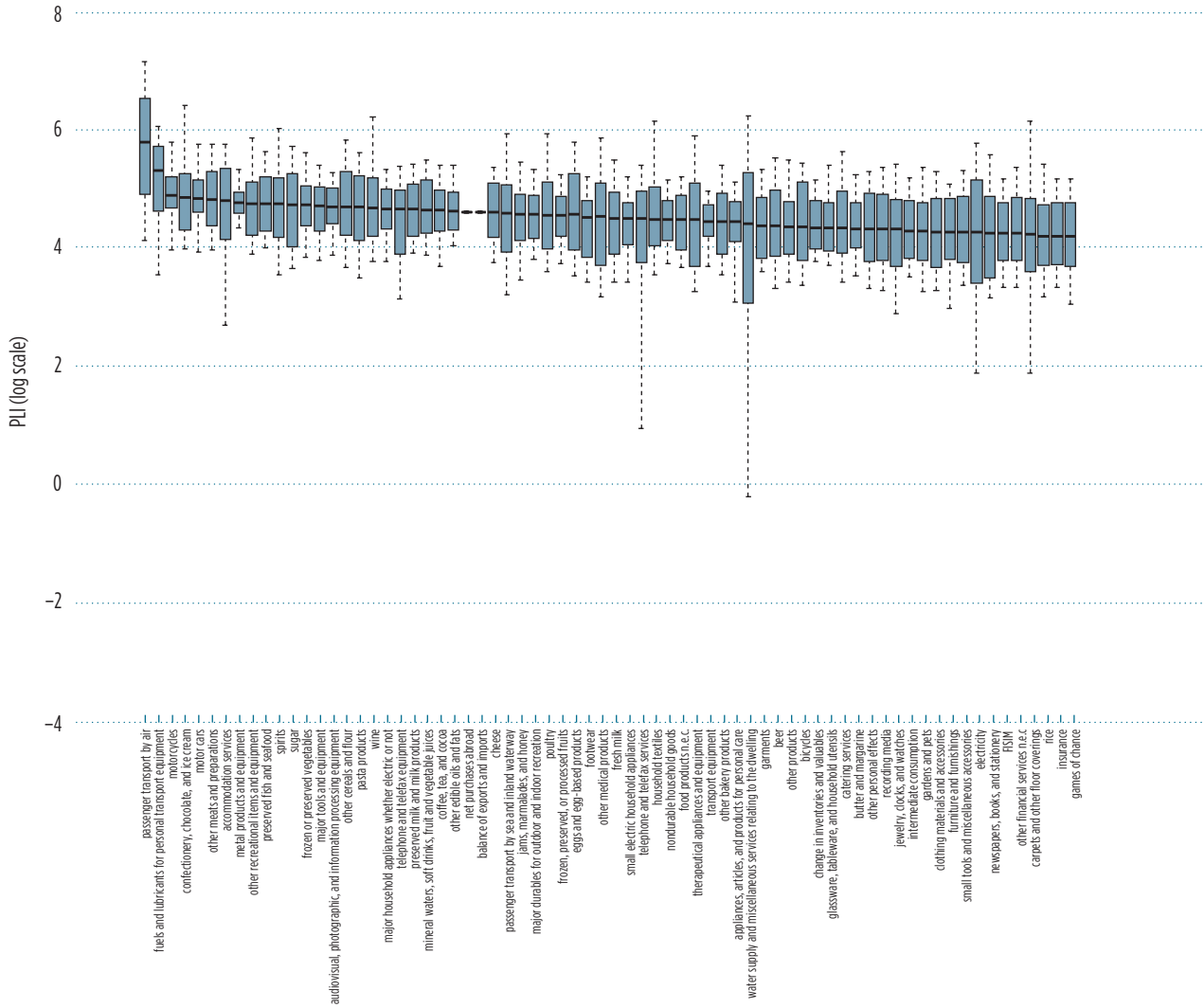

403.43
54.6
7.39
1
PLI (original scale)
KAZ ARM GEO MDA AZE BLR TJK UKR KGZ GNQ AGO GAB COG COM CPV NAM CIV STP DJI SWZ ZAF LSO CAF TCD ZMB BWA CMR GNB MAR ZAR SEN MOZ TGO LBR MDG TUN MLI SDN NER NGA SLE MWI BEN GHA MRT UGA ETZA RWA KEN BFA BDI GIN GMB MDG ETH EGY KWT QAT BHR SAU LBN OMN JOR YEM IRQ SYR
country/economy

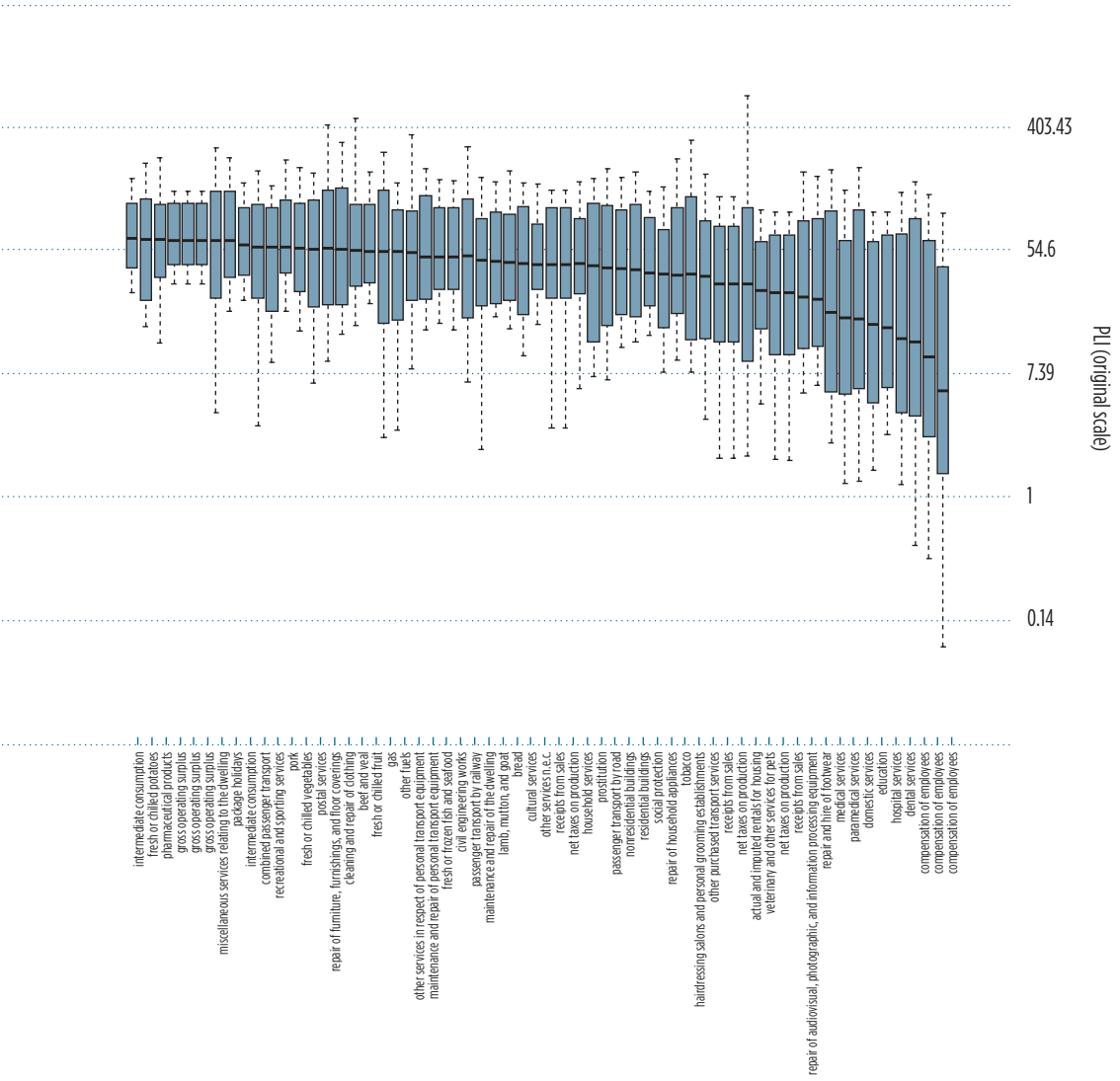
FIGURE 10.2 Box and Whisker Plots of Price Level Indexes by Basic Heading (world = 100)



Sources: ICP 2005 and computations by Min Ji Lee, ICP Global Office.

Validating Basic Heading Expenditure Weights

Neither the Quaranta nor Dikhanov tables in the previous chapter are used to validate the basic heading expenditure weights, which points to a weakness in the data validation methodology. The starting point for the proposed validation is the matrix of national expenditures by basic heading by country. The "ICP Operational Guidelines" (World Bank 2011) describe a series of validation steps, first within each country, then across countries within regions, and finally across all countries.



The within-country basic heading expenditures and shares are reviewed for

- Completeness, simply meaning that, with few exceptions, expenditures should be recorded for every basic heading
- Plausibility when comparing per capita values and expenditure shares across basic headings
- Temporal consistency with breakdowns for other years.

In each case, outliers are flagged for additional review.

The within-region and then between-region reviews compare expenditure shares, per capita nominal expenditures, and per capita indexes between countries having similar economic structures, with outliers flagged. Once the preliminary PPPs are available, per capita real expenditure values can be compared between same-cluster countries. Also, the deflated basic heading expenditures can be used to validate the respective price and quantity relationships, as discussed in the next section.

The purpose of this section is to review diagnostic procedures to identify potential basic heading expenditure values and shares that are outliers. A simple validation step begins by converting the basic heading expenditure values to expenditure shares and then reviewing the maximum and minimum shares across countries by basic heading and comparing them to the median value. The same approach can be applied to vectors of per capita real expenditure values for household consumption expenditure.

Table 10.1 provides the maximum, median, and minimum shares for basic headings for which a country reported expenditure shares greater than 10 percent of GDP. The maximum and minimum cells each represent different countries, but for the same basic heading shown in the first column. The largest expenditure share for any basic heading (21 percent) is shown by Moldova for residential buildings. The minimum share for residential buildings is 0.04 percent, shown by

TABLE 10.1 National Maximum, Median, and Minimum Expenditure Shares for Basic Headings with Maximum Values Greater than 10 Percent of GDP, and Maximum to Median and Median to Minimum Ratios

Basic heading	Maximum % share	Median % share	Minimum % share	Maximum to median ratio	Median to minimum ratio
Residential buildings	21.0	4.19	0.04	5.0	**105**
Other cereals and flour	19.6	0.63	0.01	**30.8**	**63**
Metal products and equipment	19.1	5.92	0.52	3.2	11
Rice	17.4	0.48	0.003	**36.3**	**160**
Actual and imputed rentals for housing	17.0	6.10	0.066	2.78	**92**
Nonresidential buildings	15.5	3.32	0	4.67	—
Education	15.1	1.65	0.10	9.16	16.5
Compensation of employees	14.7	4.40	1.17	3.34	3.8
Civil engineering works	14.6	3.01	0	4.86	—
Fresh milk	**11.6**	0.40	0	**29.2**	—
Transport equipment	11.4	2.27	0.08	5.0	**28**
Fresh or chilled potatoes	**11.4**	0.44	0.004	**25.8**	**110**
Catering services	10.0	1.94	0	5.14	—
Beer	**10.0**	0.34	0	**29.6**	—

Source: 2005 ICP.
Note: The numbers in boldface indicate that the basic heading expenditures for the countries with those numbers should be reviewed.

Kuwait. For other cereals and flour, the maximum (19.6 percent) and minimum (0.01 percent) shares are shown by Ethiopia and Japan, respectively. Although some basic heading expenditure shares, such as the maximum values for actual and imputed rentals for housing and civil engineering works, are plausible, questions should be raised about the values for items such as fresh milk, potatoes, and beer for countries with expenditure shares equal to or greater than 10 percent of GDP. Some of the minimum expenditure shares are also implausible, especially where countries reported zero values.

Another useful validation tool is to examine the ratios of the maximum and minimum values to the median. The median is the midpoint of the distribution, and thus its value is not affected by the maximum or minimum values. However, extreme differences of the maximum and minimum values to the median should be examined. The maximum expenditure exceeds the median by over 25 times for five basic headings in this group, indicating that expenditures for the countries reporting those values be reviewed. Table 10.1 only shows maximum to median ratios for countries with the largest maximum share values. The data validation should include a review of all basic headings with maximum to median ratios exceeding 25.

The median to minimum ratios far exceed the maximum to median ratios. The minimum values for over 78 of the basic headings are zero in at least one country. The zero values more likely indicate a failure of measurement rather than no consumption for the basic heading. From a data validation point of view, the main problem with expenditure shares is basic headings with small values. The basic heading expenditures for the countries with boldface numbers should be reviewed.

In summary, basic heading expenditures must be validated following an examination of the maximum and minimum values by basic heading and by country. The max/med and med/min ratios should also be reviewed to determine where there may be potential problems with basic heading expenditures in some countries.

The methods discussed so far to validate basic heading PPPs and expenditures treat each separately. The next section examines the results obtained when the basic heading PPPs are averaged to the GDP using expenditure values as the weights. At this and subsequent stages, the validation becomes more difficult because the PPPs and weights need to be considered together.

Evaluating the Fisher Matrix

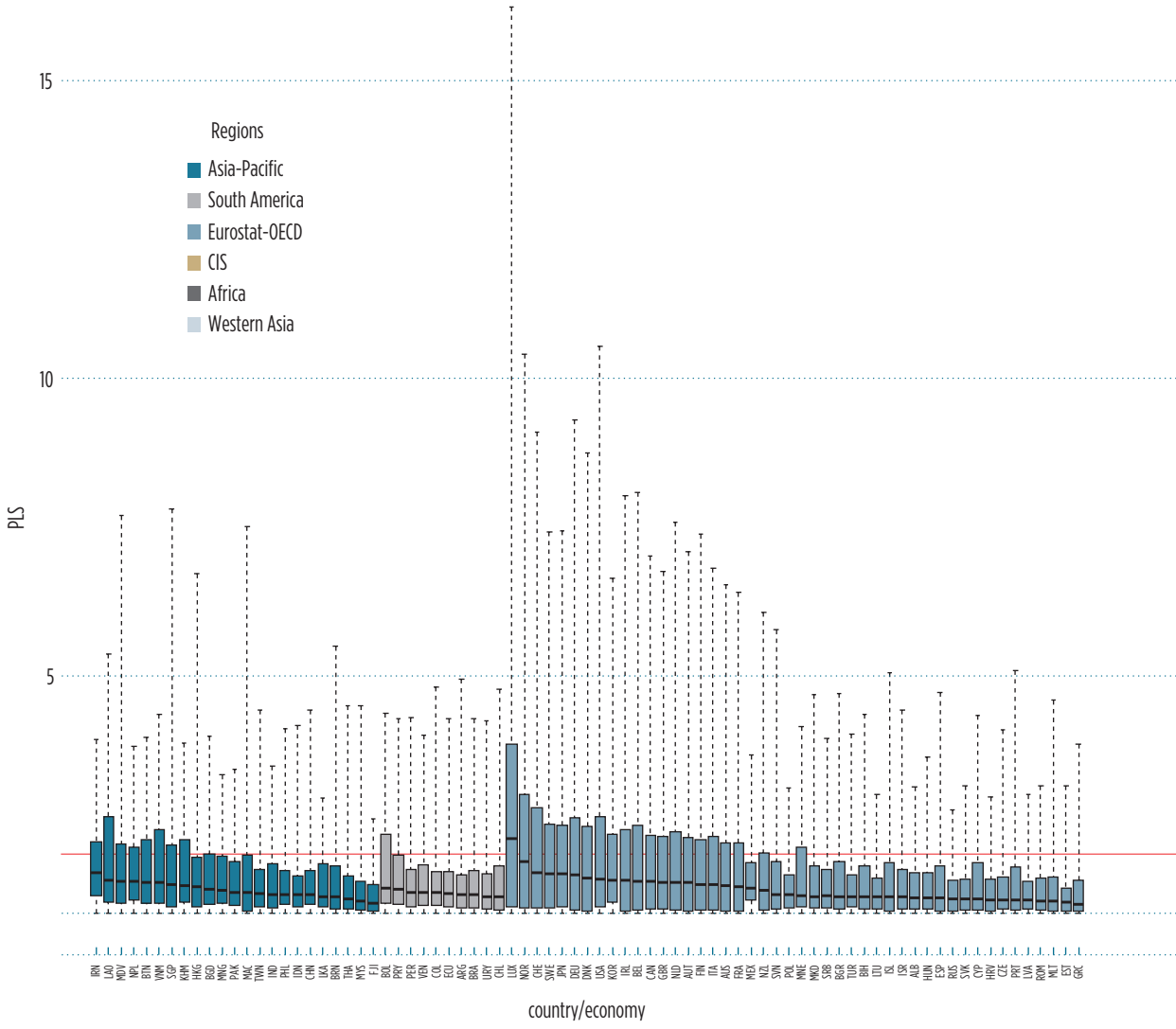
Global aggregation of the 129 basic heading PPPs to the GDP begins with estimation of the Laspeyres and Paasche indexes as shown in equations (10.1) and (10.2) for each pair of countries. The Fisher index,

$$PPP_F^{j,k} \equiv \left(PPP_L^{j,k} PPP_P^{j,k}\right)^{\frac{1}{2}}, \tag{10.3}$$

for each pair of countries results in a matrix of 146 × 146 countries with PPPs at the GDP level. The robustness of each bilateral PPP is dependent on the similarity of the price and expenditure structures between the two countries. If they are similar, the Laspeyres and Paasche results for each bilateral PPP will be similar as well. The degree of this similarity can be measured by simply using the difference between them based on the Paasche-Laspeyres spread (PLS) shown by Hill (2011) as

$$PLS_{jk}^{s} = \frac{MAX\left(P_{jk}^{p}, P_{jk}^{L}\right)}{MIN\left(P_{jk}^{p}, P_{jk}^{L}\right)}. \tag{10.4}$$

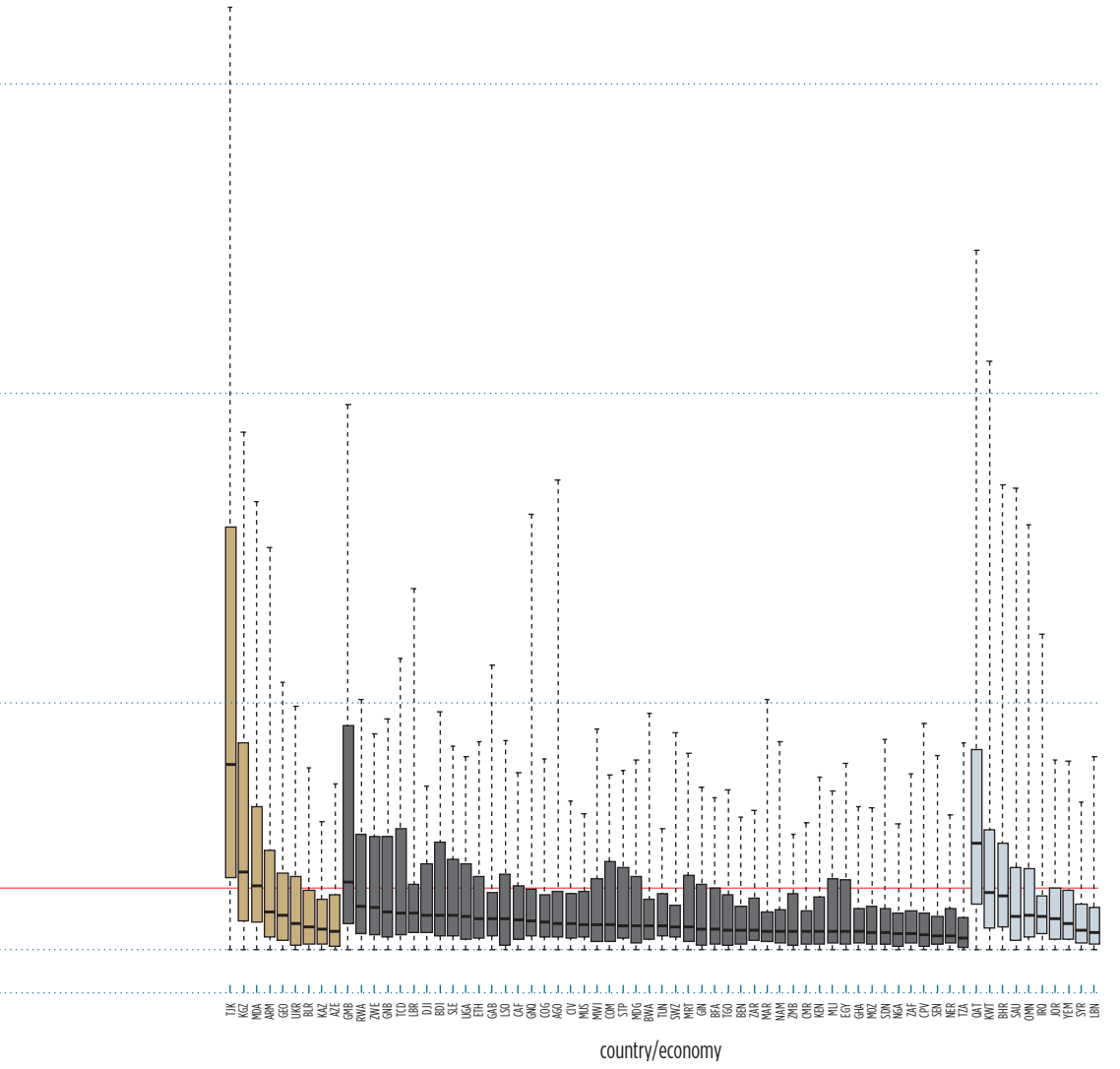
FIGURE 10.3 Box and Whisker Plots of Paasche-Laspeyres Spreads by Region and Country/Economy



Sources: ICP 2005 and computations by Min Ji Lee, ICP Global Office.
Note: See annex to this chapter for country/economy codes.

It is not unreasonable that these spreads become large for some of the bilateral comparisons because of the extreme differences in price and expenditure structures. This matter has been addressed by Diewert (2001, 2009), Aten and Heston (2009), and Hill (1999, 2011), who mainly seek ways of overcoming these differences when moving from bilateral to multilateral estimates by taking the structural differences into account in the estimation. A later section provides more about their approaches. Here, the PLS is examined first as another step in data validation.

In the box and whisker plots in figure 10.3, the countries are grouped first by region and then by the median value of the PLS. The plot for each country shows the distribution of its PLS in relation to that of the 145 other countries. The box contains 80 percent of the values and the whiskers the maximum and minimum values. The minimum value is 1.00. Therefore, the analysis focuses on the



maximum values. Luxembourg and Tajikistan have the largest maximum value (PLS = 16.23), and in this case it is the maximum bilateral PLS of the 146 countries. The maximum values across the 146 countries are represented by only three countries: Tajikistan, with the maximum value for 104 of the 146 countries, and Qatar and Luxembourg, with the maximum values of 31 and 11 countries, respectively. There are regional relationships as well. For example, a closer look at the maximum PLS for the CIS countries reveals it is always the spread with Qatar that has the largest value.

Examination of the data reveals that four countries—Tajikistan, Qatar, Kyrgyz Republic, and Luxembourg—have a bilateral PLS greater than 2.00 with 135, 112, 89, and 84 other countries, respectively. Table 10.2 shows the 10 countries with the largest number of bilateral PLSs greater than 2.00. The United States is also shown because it is the base country for the comparison.

The analysis so far points out that the price (PPPs) and quantity data for the countries in table 10.2 across the basic headings are not consistent with each other, as well as with a large number of other countries. At this stage, it is not clear whether there is a problem with the PPPs and expenditures, which would require more data validation, or whether the data are valid, which then poses an estimation issue. The following discussion provides some additional validation steps that can be used when evaluating the Fisher PPPs.

Chapter 12 in the *ICP 2005 Methodological Handbook* (World Bank 2007) defines the Laspeyres quantity index as

$$Q_L^{j,k} = \frac{\sum_{i=1}^{n} p_i^j q_i^k}{\sum_{i=1}^{n} p_i^j q_i^j} \tag{10.5}$$

which is the ratio of the real expenditures at GDP between the two countries when the quantities in both countries are valued at country *j*'s prices, and the Paasche quantity index as

$$Q_P^{jk} = \frac{\sum_{i=1}^{n} p_i^k q_i^k}{\sum_{i=1}^{n} p_i^k q_i^j} \tag{10.6}$$

which is the ratio of the real expenditures at GDP in the two countries when the quantities in both are valued at country *k*'s prices.

As with the PPPs, the Fisher quantity index is the geometric mean of the Laspeyres and Paasche quantity indexes. Hill (2011) proposes computing upper and lower price and quantity

TABLE 10.2 Paasche-Laspeyres Spreads for Countries with Largest Number of Bilateral PLSs Greater than 2.0

	No. of PLSs > 2.0	No. times max.	LUX	NOR	CHE	KGZ	MDA	TJK	BHR	KWT	QAT	GMB	USA
LUX	84	11	1.00	1.10	1.00	8.53	5.44	16.23	1.64	1.42	1.74	9.80	1.12
NOR	67		—	1.00	1.04	5.80	3.94	10.48	1.55	1.55	2.07	6.29	1.23
CHE	54		—	—	1.00	5.30	3.94	9.09	1.46	1.46	1.98	5.81	1.11
KGZ	89		—	—	—	1.00	1.08	1.18	5.71	6.39	9.36	1.87	5.54
MDA	77		—	—	—	—	1.00	1.69	4.88	5.27	8.24	1.86	3.65
TJK	135	104	—	—	—	—	—	1.00	8.51	10.50	12.30	2.75	10.53
BHR	58		—	—	—	—	—	—	1.00	1.04	1.30	5.03	1.49
KWT	66		—	—	—	—	—	—	—	1.00	1.04	5.97	1.42
QAT	112	31	—	—	—	—	—	—	—	—	1.00	7.21	1.98
GMB	79		—	—	—	—	—	—	—	—	—	1.00	7.21
USA	41												1.00

Source: ICP 2005.
Note: See annex to this chapter for country codes.

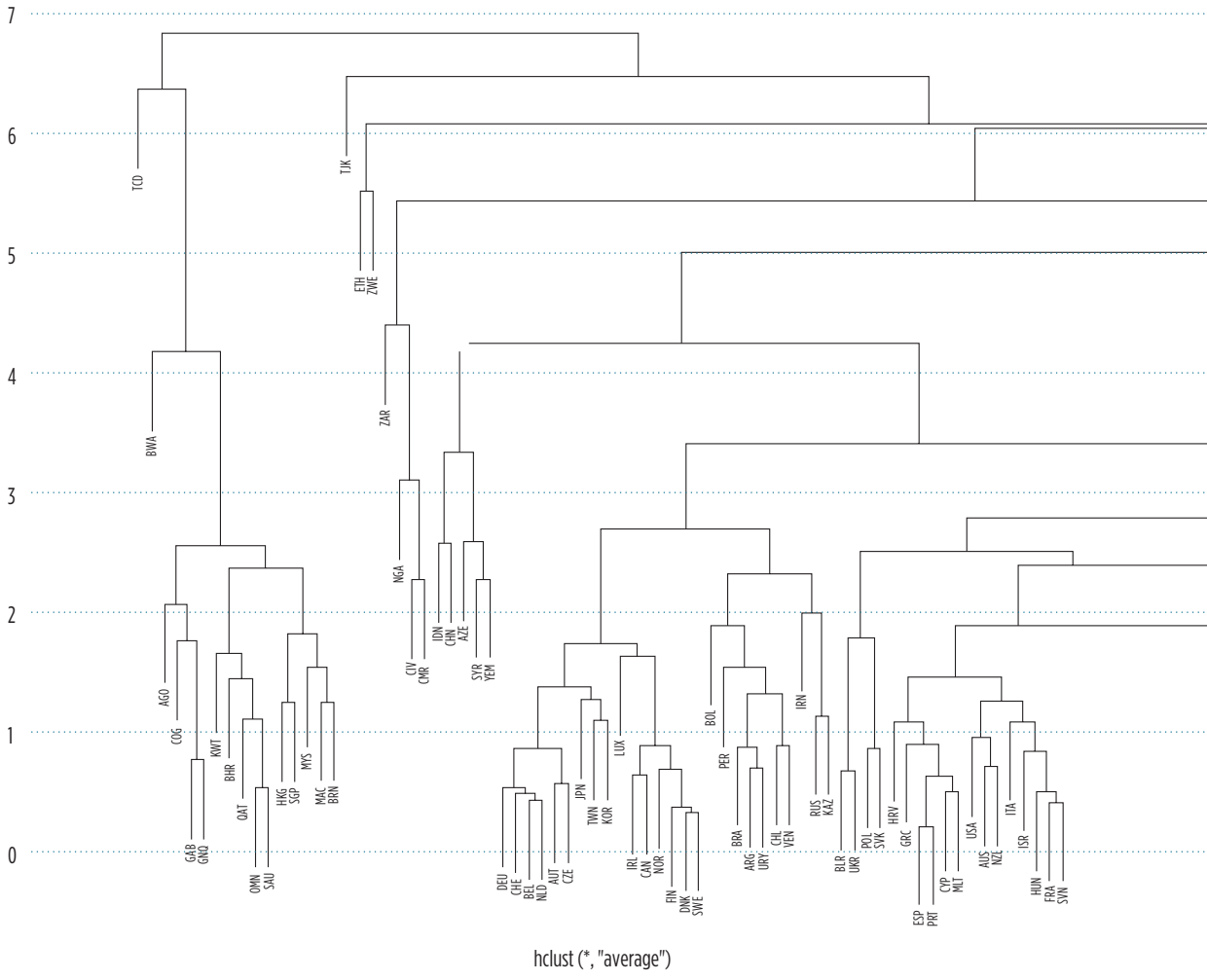
relatives to determine whether the large values of the PLS are caused by PPP or expenditure outliers. Basic headings with large upper or lower quantity or price relatives should be further examined. Hill's analysis of the 2005 ICP data for Africa shows that the extensive validation of prices led to fewer large PPP relatives than were found for quantity relatives. His other finding is that the upper quantity relatives were considerably smaller than the lower quantity relatives. Analysis of the 2005 ICP data for Asia produces similar results, which are summarized in table 10.3.

TABLE 10.3 Twenty-five Largest Upper and Lower Quantity Relatives: Asia-Pacific Region, ICP 2005

Upper quantity relatives			Lower quantity relatives		
96.7	MNG	Lamb, mutton, and goat	2,556.9	BTN	Catering services
64.8	LKA	Other purchased transport services	1,200.9	THA	Butter and margarine
52.7	FJI	Household services	1,056.3	LKA	Maintenance and repair of personal transport equipment
35.5	BTN	Cheese	731.5	KHM	Telephone and telefax services
33.3	BTN	Therapeutical appliances and equipment	681.1	LAO	Frozen, preserved, or processed fruits
32.0	PAK	Cleaning and repair of clothing	525.4	MAC	Household services
27.8	BTN	Butter and margarine	402.9	FJI	Motorcycles
25.9	IRN	Telephone and telefax services	277.1	LKA	Repair of household appliances
25.7	NPL	Fresh milk	233.5	BTN	Confectionery, chocolate, and ice cream
25.1	IRN	Gas	225.6	BRN	Other fuels
22.6	NPL	Other cereals and flour	216.5	BTN	Other recreational items and equipment
22.6	NPL	Butter and margarine	186.7	KHM	Insurance
22.5	PAK	Postal services	168.3	BTN	Lamb, mutton, and goat
21.6	LKA	Veterinary and other services for pets	142.6	THA	Lamb, mutton, and goat
20.9	LKA	Major tools and equipment	139.0	NPL	Other services in respect of personal transport equipment
20.8	PAK	Repair and hire of footwear	132.8	HKG	Other fuels
20.7	BTN	Major tools and equipment	117.2	SGP	Other fuels
20.1	PAK	Veterinary and other services for pets	90.3	HKG	Repair of furniture, furnishings, and floor coverings
19.1	PAK	Fresh milk	84.7	KHM	Telephone and telefax equipment
18.4	HKG	Telephone and telefax equipment	77.0	LAO	Therapeutical appliances and equipment
18.2	NPL	Lamb, mutton, and goat	73.2	BGD	Frozen, preserved, or processed fruits
17.8	BGD	Fresh or chilled potatoes	68.3	MNG	Fresh or frozen fish and seafood
17.7	IRN	Other fuels	67.0	TWN	Other fuels
17.3	IND	Fresh milk	65.9	MDV	Maintenance and repair of personal transport equipment
16.5	LKA	Frozen, preserved, or processed fruits	64.7	BTN	Bicycles

Sources: ICP 2005 and computations by Min Ji Lee, ICP Global Office.
Note: See annex to this chapter for country/economy codes.

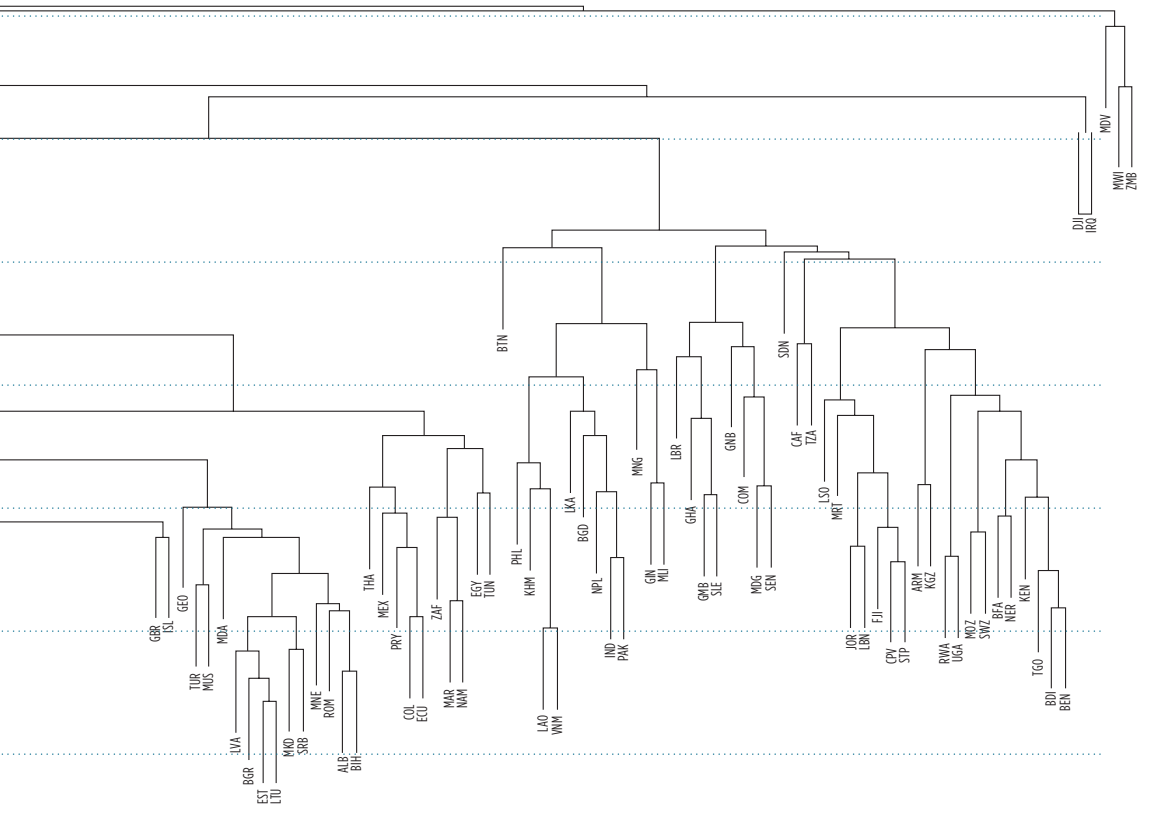
TABLE 10.4 Dendrogram Showing Clustering of 146 ICP 2005 Countries/Economies Based on Quantity Relatives



Sources: ICP 2005 and computations by Min Ji Lee, ICP Global Office.

The upper quantity relative shows the relative size of a basic heading in a country compared with the average of that of all countries when taking the size of the economy into account. The upper quantity relative of 96.7 for Mongolia means that its spending on lamb, mutton, and goat is 96 times larger than the average across countries in the Asia-Pacific region. Conversely, the lower quantity relative of 2,556.9 for Bhutan means its expenditures for catering services are $\frac{1}{2{,}556.9}$ of the average. The data for these basic headings may be correct, but they should be reviewed because they are so different. A final point is that the upper quantity relatives are considerably less than the lower quantity relatives. The conclusion reached is that basic heading expenditures that are very small should be further validated.

The PLSs and price and quantity relatives just described can be placed in a dissimilarity matrix of 146 × 146. Hill (2011) suggests using cluster analysis that seeks observation pairs with the smallest measures of dissimilarity, groups them, and then seeks the next set of



hclust (*, "average")

similar measures. This method groups country pairs that are similar in structure of prices and quantities. Those exceeding a desired value of similarity are not included, suggesting they be reviewed again.

Figure 10.4, a dendogram based on quantity relatives, shows how the countries are clustered; it is over the full set of basic headings and includes all 146 countries. Although a dozen countries are different from the rest, they are generally the same ones appearing in the diagnostics just described. The dendogram still does not answer the question of whether there is a problem with the data for some basic headings, or whether they are simply different in economic structure from the remaining countries. The basic analysis of the PPPs, expenditure weights, and PLSs as described in earlier sections of this chapter should be repeated for countries appearing as outliers.

At this stage, the issue is likely no longer a data validation one, but simply that some countries have significantly different price and expenditure structures. The issue, then, is their effect on the final estimation step, which is the GEKS procedure to achieve transitivity.

From Bilateral PPPs to Multilateral PPPs

The starting point for the GEKS method is the 146 × 146 matrix of Fisher PPPs. This matrix contains the PPPs between every pair of countries in the comparison. Table 10.4 is a partial matrix of nine countries. Each country is represented in a row and a column; the Macao SAR, China, row, for example, shows the PPP of it to each of the other countries shown in the respective columns. These PPPs are not transitive. For example, the direct PPP of Hong Kong SAR, China, to Macao SAR, China is 1.03. The indirect PPP of Hong Kong SAR, China to Macao SAR, China through India (1.141) is the PPP of Hong Kong SAR, China to India (0.38) divided by the PPP of Macao SAR, China to India (0.33) and is different from the direct PPP. Therefore, the PPPs are not transitive.

The purpose of the GEKS method as described in chapters 1, 4, and 5 is to ensure that the PPPs between any two countries can be obtained either directly or indirectly with any other country with the same results. This is achieved by first computing all of the direct and indirect PPPs for the countries in each row with US = 1 by dividing each row in table 10.4 by the USA row. There will be two direct PPPs in each row—each country to itself and with the US = 1—and $(n - 2)$ or 144 indirect PPPs. The GEKS PPP is then the geometric mean of these direct and indirect PPPs. These PPPs are transitive, which means the PPP between any two countries will equal the PPP when it is obtained through a third country. The direct and indirect PPPs are treated equally to satisfy the symmetric requirement. The consequences are discussed in the next section.

Table 10.5 shows the direct and indirect PPPs with US = 1.00 for the same countries shown in table 10.4. For example, the PPPs in the Hong Kong SAR, China row are the direct and indirect PPPs relative to the United States. HGK/HGK and HGK/USA are the direct PPPs, and the others are indirect PPPs through the country in the column heading. The final multilateral PPP for each country to US = 1.00 is obtained by taking the geometric mean of each row, which in effect gives equal weight to every country. Table 10.5 shows that the direct and indirect PPPs differ—for example, the PPP for Brunei to the United States is 1.08 when linked through Singapore and 0.76 when linked through Bhutan, a 1.44 times difference.

A final step in the data validation effort is to review the variability of the direct and indirect PPPs for each country. Table 10.6 lists countries with the largest ratios of the maximum to

TABLE 10.4 Partial Matrix of Fisher PPPs, Selected Countries/Economies

Country/ Economy	HKG	MAC	SGP	TWN	BRN	BGD	BTN	IND	USA
HKG	1.00	1.03	4.97	0.31	5.77	0.26	0.36	0.38	5.87
MAC	0.97	1.00	4.91	0.29	5.70	0.23	0.34	0.33	5.61
SGP	0.20	0.20	1.00	0.06	1.14	0.05	0.07	0.07	1.23
TWN	3.25	3.50	16.57	1.00	20.07	0.92	1.26	1.32	17.62
BRN	0.17	0.18	0.88	0.05	1.00	0.04	0.06	0.06	0.92
BGD	3.91	4.35	20.52	1.09	25.78	1.00	1.46	1.59	20.52
BTN	2.79	2.90	15.36	0.79	17.57	0.68	1.00	1.05	13.34
IND	2.62	2.99	14.32	0.76	17.03	0.63	0.95	1.00	14.01
USA	0.17	0.18	0.81	0.06	1.08	0.05	0.07	0.07	1.00

Source: ICP 2005.
Note: See annex to this chapter for country/economy codes.

TABLE 10.5 Direct and Indirect PPPs, Selected Countries/Economies (US = 1.00)

Country/ Economy	HKG	MAC	SGP	TWN	BRN	BGD	BTN	IND	USA
HKG	*5.87*	5.79	6.13	5.41	5.32	5.25	4.77	5.34	*5.87*
MAC	5.70	*5.61*	6.06	5.04	5.26	4.72	4.59	4.69	*5.61*
SGP	1.18	1.14	*1.23*	1.06	1.05	1.00	0.87	0.98	*1.23*
TWN	19.11	19.63	20.46	*17.62*	18.51	18.82	16.81	18.43	*17.62*
BRN	1.02	0.98	1.08	0.88	*0.92*	0.80	0.76	0.82	*0.92*
BGD	22.97	24.42	25.33	19.21	23.77	*20.52*	19.53	22.29	*20.52*
BTN	16.41	16.30	18.95	13.98	16.21	14.02	*13.34*	14.69	*13.34*
IND	15.41	16.79	17.67	13.40	15.70	12.90	12.72	*14.01*	*14.01*
USA	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	*1.00*

Source: ICP 2005.
Note: See annex to this chapter for country/economy codes.

TABLE 10.6 Measures of Variability of Direct and Indirect PPPs (US = 1.00) for Countries with Largest Maximum to Minimum Ratios

Country	Max/min indirect PPPs (1)	Relative standard deviation of direct and indirect PPPs (2)	Direct PPP/GEKS (3)	GEKS PLI-weighted PLS/GEKS (4)
TZA	3.59	0.18	1.19	1.01
QAT	2.78	0.15	1.18	1.05
TJK	2.76	0.15	0.83	1.09
LAO	2.63	0.14	0.93	1.00
VNM	2.56	0.14	0.99	1.04
KHM	2.56	0.13	0.94	1.00
GNQ	2.50	0.15	1.16	0.99
KGZ	2.41	0.14	0.88	1.00
GMB	2.28	0.16	0.72	0.98
MDA	2.11	0.13	0.98	1.00
BHR	1.72	0.11	1.11	0.96
LUX	1.66	0.09	1.10	1.00
KWT	1.63	0.11	1.13	1.00
NOR	1.50	0.08	1.12	1.02
CHE	1.36	0.06	1.09	0.97

Source: 2005 ICP and Aten-Heston weighted PLS/GEKS.
Note: See annex to this chapter for country codes. PLI = price level index; PLS = Paasche-Laspeyres spread; GEKS = Gini-Éltetö-Köves-Szulc.

minimum values of the indirect PPPs. Tanzania shows the largest differences, followed by Qatar and Tajikistan. Note, however, that the variability here is considerably less than that shown by the PLS spreads; the largest PLS is over 16, while the largest maximum to minimum ratio of direct and indirect PPPs is 3.59. Generally, many of the same countries appear as outliers in both cases.

Column (2) of table 10.6 lists the relative standard deviations of the direct and indirect PPPs as expressed by the median divided by the standard deviation. The values decline rapidly, indicating that there are only a small number of outlier values for these countries.

Column (3) shows the relative difference between the direct PPP for each country to the United States and the GEKS PPP, which is the geometric mean of the direct and all indirect PPPs. The real GDP for Tanzania is 1.19 times larger than if the direct PPP had been used. Ratios greater than 1.00 show the amount by which the real GDP is increased by the GEKS process; ratios less than 1.00 the amount it was reduced by the GEKS process.

Column (4), taken from Aten and Heston (2009), is discussed in the next section.

Recall that the variability measures for each country include indirect PPPs through every other country in the comparison—in this case, 146 countries. Although the data for those countries with the greatest variability should receive another review, the reality is that at this stage the differences are more likely to be caused by the extreme differences in the economic structures of the economies. The following section considers the question of whether all indirect PPPs should be given equal weight in the GEKS process.

When Does Validation End and Estimation Begin?

Countries that appear as outliers in the analysis steps described in this chapter may have quality data and are simply different in structure from the other countries. From a statistical point of view, they contribute more to measurement error than do the other countries, suggesting that they should be treated differently in the estimation process. Hill (1999, 2011), Aten and Heston (2009), Diewert (2009), and others have considered this dimension of the GEKS. Hill proposes the minimum spanning tree approach, which is a method to first compute PPPs for the countries most similar and then bring in countries less similar in a way that preserves fixity of the first set. The problem is determining the criteria for grouping the countries; the final results are very sensitive to the methods used to choose countries for each step. There is also a problem of circularity, because the final results are needed to set up the spanning tree paths.

Aten and Heston (2009) raise the question whether all Fisher indexes are equal. This question translates into whether the direct and indirect PPPs in the GEKS process should receive equal weights. Aten and Heston provide an example in which the PLS becomes a variable in estimation of the final PPPs. This is done by expressing the GEKS process as a least squares estimate and adding the PLS as a variable. Column (4) in table 10.6 is the ratio of GEKS/PLS to GEKS. Note that considerable adjustments are made for countries such as Qatar and Tajikistan that also have the largest PLS. Aten and Heston show results based on this and other methods for all countries using the variability present in the estimation process. They conclude that consideration should be given to using additional variables or weights to deal with the wide differences in economic structure across countries.

Conclusion

Several validation steps have been analyzed in this chapter, starting with those for the basic heading PPPs and expenditure weights. The outcome of this analysis is that considerable attention should be given to the validation of expenditure weights using the methods suggested by Hill (2011). The analysis also suggests that the matrix of Fisher PPPs be reviewed and validated. Analysis of the 2005 and 2011 benchmark data should continue to reveal estimation methods that better deal with the variability arising from countries of different economic structures.

A final conclusion is that there are large differences in economic structures across countries. Countries with high price levels will have different economic structures than those with low price levels. Developing countries generally have larger shares in food consumption and smaller shares in services. Over 180 countries and economies will participate in the 2011 ICP compared with the 146 that took part in the 2005 ICP. The additional countries will contribute to the variability. In response, Aten and Heston (2009) raise the question of whether there should be a departure from use of the democratic or equal weighting inherent in the GEKS process.

ANNEX

Three-Letter Country/Economy Codes, International Organization for Standardization

ABW	Aruba	CHL	Chile	GIB	Gibraltar
AFG	Afghanistan	CHN	China	GIN	Guinea
AGO	Angola	CIV	Côte d'Ivoire	GLP	Guadeloupe
ALB	Albania	CMR	Cameroon	GMB	Gambia, The
AND	Andorra	COD	Congo, Dem. Rep.	GNB	Guinea-Bissau
ARE	United Arab Emirates	COG	Congo, Rep.	GNQ	Equatorial Guinea
ARG	Argentina	COL	Colombia	GRC	Greece
ARM	Armenia	COM	Comoros	GRD	Grenada
ATA	Antarctica	CPV	Cape Verde	GRL	Greenland
ATG	Antigua and Barbuda	CRI	Costa Rica	GTM	Guatemala
AUS	Australia	CUB	Cuba	GUF	French Guiana
AUT	Austria	CYM	Cayman Islands	GUM	Guam
AZE	Azerbaijan	CYP	Cyprus	GUY	Guyana
BDI	Burundi	CZE	Czech Republic	HKG	Hong Kong SAR, China
BEL	Belgium	DEU	Germany	HND	Honduras
BEN	Benin	DJI	Djibouti	HRV	Croatia
BFA	Burkina Faso	DMA	Dominica	HTI	Haiti
BGD	Bangladesh	DNK	Denmark	HUN	Hungary
BGR	Bulgaria	DOM	Dominican Republic	IDN	Indonesia
BHR	Bahrain	DZA	Algeria	IND	India
BHS	Bahamas, The	ECU	Ecuador	IRL	Ireland
BIH	Bosnia and Herzegovina	EGY	Egypt, Arab Rep.	IRN	Iran, Islamic Rep.
BLR	Belarus	ERI	Eritrea	IRQ	Iraq
BLZ	Belize	ESP	Spain	ISL	Iceland
BMU	Bermuda	EST	Estonia	ISR	Israel
BOL	Bolivia	ETH	Ethiopia	ITA	Italy
BRA	Brazil	FIN	Finland	JAM	Jamaica
BRB	Barbados	FJI	Fiji	JOR	Jordan
BRN	Brunei Darussalam	FRA	France	JPN	Japan
BTN	Bhutan	FSM	Micronesia, Fed. Sts.	KAZ	Kazakhstan
BWA	Botswana	GAB	Gabon	KEN	Kenya
CAF	Central African Republic	GBR	United Kingdom	KGZ	Kyrgyz Republic
CAN	Canada	GEO	Georgia	KHM	Cambodia
CHE	Switzerland	GHA	Ghana	KIR	Kiribati

KNA	Saint Kitts and Nevis	NGA	Nigeria	SUR	Suriname
KOR	Korea, Rep.	NIC	Nicaragua	SVK	Slovak Republic
KWT	Kuwait	NLD	Netherlands	SVN	Slovenia
LAO	Lao PDR	NOR	Norway	SWE	Sweden
LBN	Lebanon	NPL	Nepal	SWZ	Swaziland
LBR	Liberia	NRU	Nauru	SYC	Seychelles
LBY	Libya	NZL	New Zealand	SYR	Syrian Arab Republic
LIE	Liechtenstein	OMN	Oman	TCD	Chad
LKA	Sri Lanka	PAK	Pakistan	TGO	Togo
LSO	Lesotho	PAN	Panama	THA	Thailand
LTU	Lithuania	PCN	Pitcairn	TJK	Tajikistan
LUX	Luxembourg	PER	Peru	TKM	Turkmenistan
LVA	Latvia	PHL	Philippines	TLS	Timor-Leste
MAC	Macao SAR, China	PLW	Palau	TON	Tonga
MAR	Morocco	PNG	Papua New Guinea	TTO	Trinidad and Tobago
MCO	Monaco	POL	Poland	TUN	Tunisia
MDA	Moldova	PRI	Puerto Rico	TUR	Turkey
MDG	Madagascar	PRK	Korea, Dem. People's Rep.	TUV	Tuvalu
MDV	Maldives	PRT	Portugal	TWN	Taiwan, China
MEX	Mexico	PRY	Paraguay	TZA	Tanzania
MHL	Marshall Islands	PYF	French Polynesia	UGA	Uganda
MKD	Macedonia, FYR	QAT	Qatar	UKR	Ukraine
MLI	Mali	ROU	Romania	URY	Uruguay
MLT	Malta	RUS	Russian Federation	USA	United States
MMR	Myanmar	RWA	Rwanda	UZB	Uzbekistan
MNE	Montenegro	SAU	Saudi Arabia	VEN	Venezuela, RB
MNG	Mongolia	SDN	Sudan	VGB	Virgin Islands, British
MOZ	Mozambique	SEN	Senegal	VIR	Virgin Islands, U.S.
MRT	Mauritania	SGP	Singapore	VNM	Vietnam
MSR	Montserrat	SLB	Solomon Islands	VUT	Vanuatu
MTQ	Martinique	SLE	Sierra Leone	WSM	Samoa
MUS	Mauritius	SLV	El Salvador	YEM	Yemen, Rep.
MWI	Malawi	SMR	San Marino	ZAF	South Africa
MYS	Malaysia	SOM	Somalia	ZMB	Zambia
NAM	Namibia	SRB	Serbia	ZWE	Zimbabwe
NCL	New Caledonia	SSD	Republic of South Sudan		
NER	Niger	STP	São Tomé and Príncipe		

NOTES

1. The author is grateful for the computations and data plots provided by Min Ji Lee, ICP Global Office, World Bank.
2. The five geographic ICP regions in 2005 were Africa, Asia-Pacific, Commonwealth of Independent States (CIS), South America, and Western Asia. The Eurostat-OECD members constitute a sixth region for purposes of the analysis provided in this chapter.
3. Zimbabwe was omitted from the analysis shown in figures 10.1 and 10.2 based on price level indexes because an official exchange rate was not determined due to extreme volatility during 2005.

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