Constructing Reliability Measures for Purchasing Power Parities

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Introduction

- **What is the right measure of uncertainty around PPPs?**

- **What factors affect reliability of the estimated PPPs?**

- **Previous studies:** Kravis et al. (1975), Rao (2005a, 2005b), Diewert (2005), Deaton (2012)

- **Rao and Hajargasht (2016) provided a general framework**
Outline of the Talk

1- A brief overview of the stochastic approach to PPPs.
2- PPPs & Reliability Measures for 2011 & 2017 at above BH level.
3- Correlation Matters:
   (i) Cluster Robust SEs are the One
   (ii) Various arguments
   (iii) Estimation using Resampling methods
   (iv) Interpretation of Standard Errors
4- GEKS & other Indexes
5- 2011 vs 2017 Benchmarks
Following Summers (1973), Rao (2005) and Diewert (2005), Deaton (2012) we rely on

\[ p_{ij} = P_i \cdot PPP_j \cdot u_{ij} \]

- price of i-th commodity in j-th country
- purchasing power parity
- world price of i-th commodity
- random disturbance

\[ E(u_{ij}) = 1 \]
A Generalized Version

\[ p_{ij}^\rho = P_i^\rho \ PPP_j^\rho \, \varepsilon_{ij} \]

or

\[ \frac{p_{ij}^\rho}{P_i^\rho \ PPP_j^\rho} = -1 = u_{ij} \quad \text{with} \quad E(u_{ij}) = 0 \]

Using the methodology developed in \textit{Rao and Hajargasht (2016)} it can be shown that

\[
\begin{align*}
\text{PPP}_j &= \left\{ \sum_{i=1}^{N} w_{ij} \left( \frac{p_{ij}}{P_i} \right)^\rho \right\}^{1/\rho} \\
\text{P}_i &= \left\{ \sum_{j=1}^{M} w_{ij}^* \left( \frac{p_{ij}}{\text{PPP}_j} \right) \right\}^{1/\rho}
\end{align*}
\]

\( \rho = 0 \) leads to a geometric index (CPD)

\( \rho = -1 \) leads to a harmonic index (Ikle)

\( \rho = 1 \) leads to arithmetic indexes

\textit{Hajargasht and Rao (JME, 2019)} provides proofs for existence of such indexes
Nonlinear Non-Additive Regression

• Consider

\[ r(y_i, x_i; \beta) = u_i \quad i = 1, \ldots, N \]

\( \beta \) is a \( K \times 1 \) vector of parameters and \( E(u_i) = 0 \) \& \( Var(u) = \Omega \)

• In general Least Squares is inconsistent but if one finds instrument \( Z_{N \times K} \) then a MoM estimator can be set up as

\[
\frac{1}{N} Z' u = 0
\]

• It has been shown that the most efficient instrument is:

\[
Z^* = E \left( \frac{\partial r(y, x, \beta)}{\partial \beta} \right)
\]
Robust estimation with weights
\[ \frac{1}{N} \mathbf{Z}' \mathbf{W} \mathbf{r}(y, \mathbf{X}, \hat{\beta}) = 0 \]

Variance for model with general covariance
\[ \text{Var}(\hat{\beta}_{\text{MoM}}) = \left[ \mathbf{Z}' \mathbf{W} \mathbf{Z} \right]^{-1} \mathbf{Z}' \mathbf{W} \Omega \mathbf{W} \mathbf{Z} \left[ \mathbf{Z}' \mathbf{W} \mathbf{Z} \right]^{-1} \]

Different choices of \( r \), as a result \( \mathbf{Z} \), and \( \mathbf{W} \) lead to different indexes including CPD, Ikle and GK.
SE of Log PPP (2005) by Deaton (2012)
White SEs vs Bilateral SEs 2017

SE of Multilateral CPD vs SE of Bilateral CPD
The Presence of Correlation

Residuals for Clothing based on CPD model

172 Countries clustered in regions
Covariance Options

\[ \text{Cov}(\epsilon) = \Omega_{NM \times NM} = \begin{bmatrix} \sigma^1 & \sigma^{1,2} & \ldots & \sigma^{1,M} \\ \sigma^{1,2} & \sigma^2 & \ldots & \sigma^{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^{1,M} & \sigma^{2,M} & \ldots & \sigma^M \end{bmatrix} \]

- **No correlation**
- Homoscedastic
- Heteroskedastic

- **With Correlation**
  - Cluster with respect to Items
  - Cluster with respect to Countries
Clustered Covariance

- Cluster with respect to items

\[
\Omega_{NM \times NM} = \\
\begin{bmatrix}
\sigma_{11}^1 & 0 & \cdots & 0 \\
0 & \sigma_{NN}^1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_{MM}^M
\end{bmatrix}
\]

- Cluster with respect to countries

\[
\Omega_{NM \times NM} = \\
\begin{bmatrix}
\sigma^1 & 0 & \cdots & 0 \\
0 & \sigma^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma^M
\end{bmatrix}
\]
Cluster Robust Rules

Multilateral vs Bilateral SE CPD
Clust-I Rules

SE of CPD 2011

- Clust_I
- White
- Clust_C
Assuming the special case where all weights are equal, we can derive:

**Heteroskeastic:**

\[
\text{Var}(\ln PPP_j) = \frac{1}{N^2} \left( \sum_{i=1}^{N} \sigma_{ii}^{(j)} + \sum_{i=1}^{N} \sigma_{ii}^{(M)} \right)
\]

**Cluster with respect to Countries:**

\[
\text{Var}(\ln PPP_j) = \frac{i'(\hat{\sigma}^j + \hat{\sigma}^M)i}{N^2} = 0
\]

**Cluster with respect to Items:**

\[
\text{Var}(\ln PPP_j) = \frac{1}{N^2} \left( \sum_{i=1}^{N} \sigma_{ii}^{(j)} + \sum_{i=1}^{N} \sigma_{ii}^{(M)} - 2 \sum_{i=1}^{N} \sigma_{ii}^{(j,M)} \right)
\]
Cluster Robust is the One

Two countries with same expenditure share

CPD SEs Equal Shares with US

- Multi_Clust_I
- Multi_White
- Bilateral
Cluster Robust is the One

Two countries with same expenditure shares & same price

CPD SEs Same Shares & Prices as US

In this case both bilateral and multilateral CPD PPP for the country with respect to USA equals 1
Cluster Robust is the One

Two countries with same price
SEs based on Resampling

• Standard errors can be estimated using resampling methods as well

• Resampling Methods:
  – Jacknife
  – Bootstrap
  – Subsampling

• Advantages:
  – Does not necessarily rely on law of one price
  – Give new interpretations to SEs
Interpretations of SEs

First Interpretation: Clust_I SEs can be interpreted as uncertainty around the estimated LogPPPs due to sampling N items out of the population of all items.

Second Interpretation: Inspired by resampling methods, Clust_I SEs can be interpreted as average sensitivity of LogPPPs to adding or dropping items.
GEKS Standard Errors

• We have also developed a method to calculate standard errors for GEKS with Tornqvist. It is similar to Deaton’s (2012) but we allow for correlation across Items.

• For GEKS with Fisher, SEs can be easily calculated using resampling methods.

• The next slide compares GEKS SEs with CPD SEs for 2017.
CPD & GEKS (2017)

SE CPD vs GEKS 2017

Countries from 1 to 174
All Indexes in one Graph

SE All Indexes 2017

Bilateral SE
All Indexes in one Graph

Bilateral SE

SE Multilateral

SE All Indexes 2011
Comparison of Benchmarks

SE GEKS 2017 vs 2011

2017 SE

2011 SE
Conclusion

• Demonstrated feasibility of constructing reliability measures for various PPP indexes using Statistical models and resampling methods.

• Main conclusion is that Clust_I Standard Errors are the most appropriate.

• Provided interpretations of Standard Errors.

• Comparing reliability of various indexes: GEKS-Fisher has lower standard errors.

• Comparison of reliability measures for GEKS for 2011 and 2017.