Constructing Reliability Measures for Purchasing Power Parities

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ICP TAG Meeting, 17-19 May 2021

Introduction

- *What is the right measure of uncertainty around PPPs?*
- *What factors affect reliability of the estimated PPPs?*
- *Previous studies: Kravis et al. (1975), Rao (2005a, 2005b), Diewert (2005), Deaton (2012)*
- *Rao and Hajargasht (2016) provided a general framework*

Outline of the Talk

1- A brief overview of the stochastic approach to PPPs.

2- PPPs & Reliability Measures for 2011 & 2017 at above BH level.

3- Correlation Matters:

(i) Cluster Robust SEs are the One

(ii) Various arguments

(iii) Estimation using Resampling methods

(iv) Interpretation of Standard Errors

4- GEKS & other Indexes

5- 2011 vs 2017 Benchmarks

Law of One Price

Following Summers (1973), Rao (2005) and Diewert (2005), Deaton(2012) we rely on

$$p_{ij} = P_i \, PPP_j \, u_{ij}$$

price of i-th commodity in j-th country

purchasing power parity

world price of i-th commodity

random disturbance

$$E(u_{ij}) = 1$$

A Generalized Version

$$p_{ij}^{\rho} = P_i^{\rho}\, PPP_j^{\rho}\, \varepsilon_{ij}$$

or

$$\frac{p_{ij}^{\rho}}{P_i^{\rho}\, PPP_j^{\rho}} - 1 = u_{ij} \quad with \quad E(u_{ij}) = 0$$

Using the methodology developed in Rao and Hajargasht (2016) it can be shown that

$$\begin{cases} PPP_j = \left\{ \sum_{i=1}^{N} w_{ij} \left(\frac{p_{ij}}{P_i} \right)^{\rho} \right\}^{1/\rho} \\ P_i = \left\{ \sum_{j=1}^{M} w_{ij}^{*} \left(\frac{p_{ij}}{PPP_j} \right)^{\rho} \right\}^{1/\rho} \end{cases}$$

$\rho = 0$ leads to a geometric index (CPD)

$\rho = -1$ leads to a harmonic index (Ikle)

$\rho = 1$ leads to arithmetic indexes

Hajargasht and Rao (JME, 2019) provides proofs for existence of such indexes

Nonlinear Non-Additive Regression

- *Consider*

$$r(y_i, \mathbf{x}_i; \boldsymbol{\beta}) = u_i \quad i = 1, \ldots, N$$

$\boldsymbol{\beta}$ is a $K \times 1$ vector of parameters and $E(u_i) = 0$ & $Var(\mathbf{u}) = \boldsymbol{\Omega}$

- *In general Least Squares is inconsistent but if one finds instrument* $\mathbf{Z}_{N \times K}$ *then a MoM estimator can be set up as*

$$\frac{1}{N}\mathbf{Z}'\mathbf{u} = 0$$

- *It has been shown that the most efficient instrument is:*

$$\mathbf{Z}^* = E\left(\frac{\partial \mathbf{r}(\mathbf{y}, \mathbf{x}, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right)$$

Estimation with Weights & General Covariance

- *Robust estimation with weights*

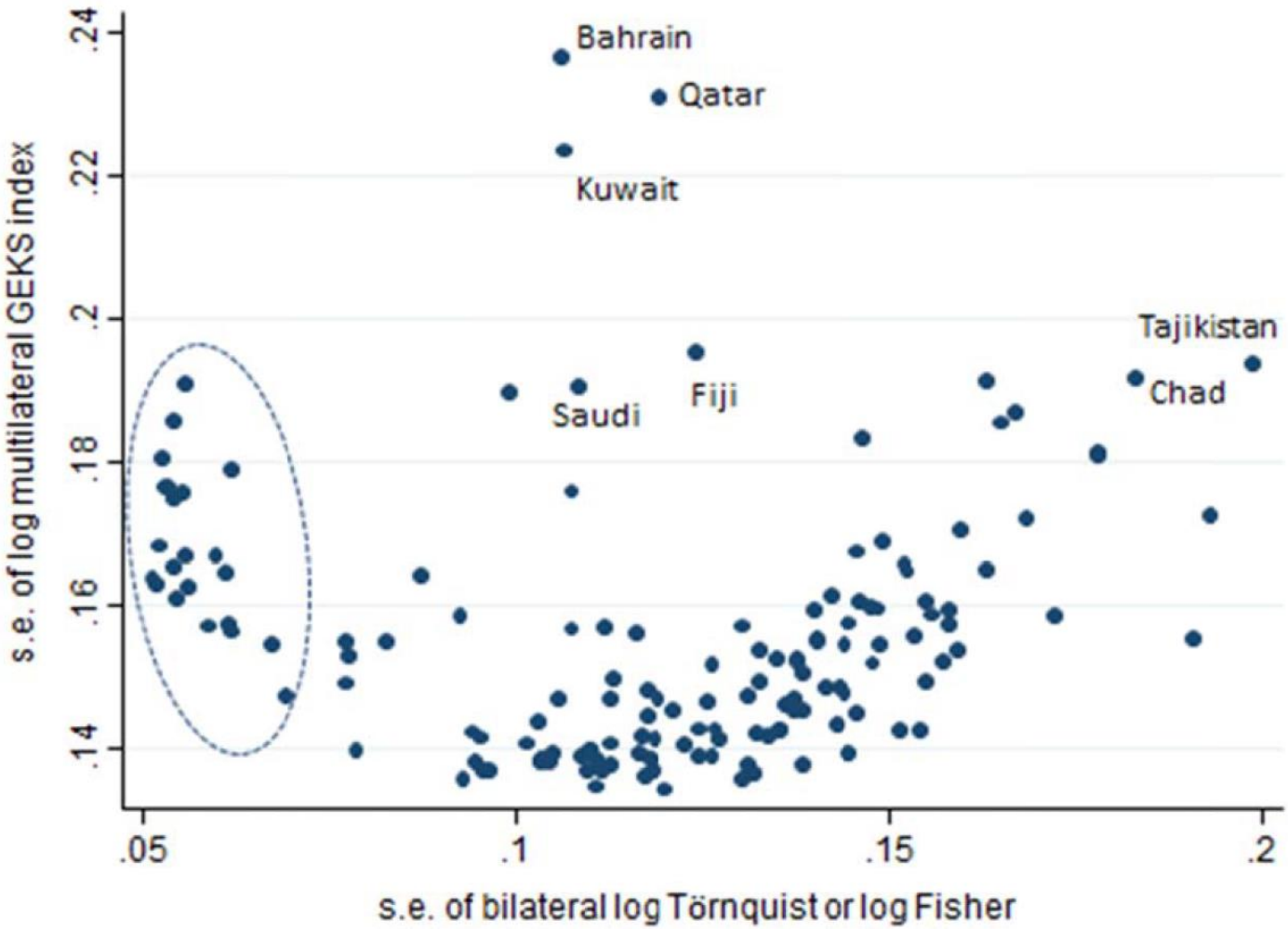
$$\frac{1}{N}\mathbf{Z}'\mathbf{W}\mathbf{r}(\mathbf{y},\mathbf{X},\hat{\boldsymbol{\beta}})=\mathbf{0}$$

variance for model with general covariance

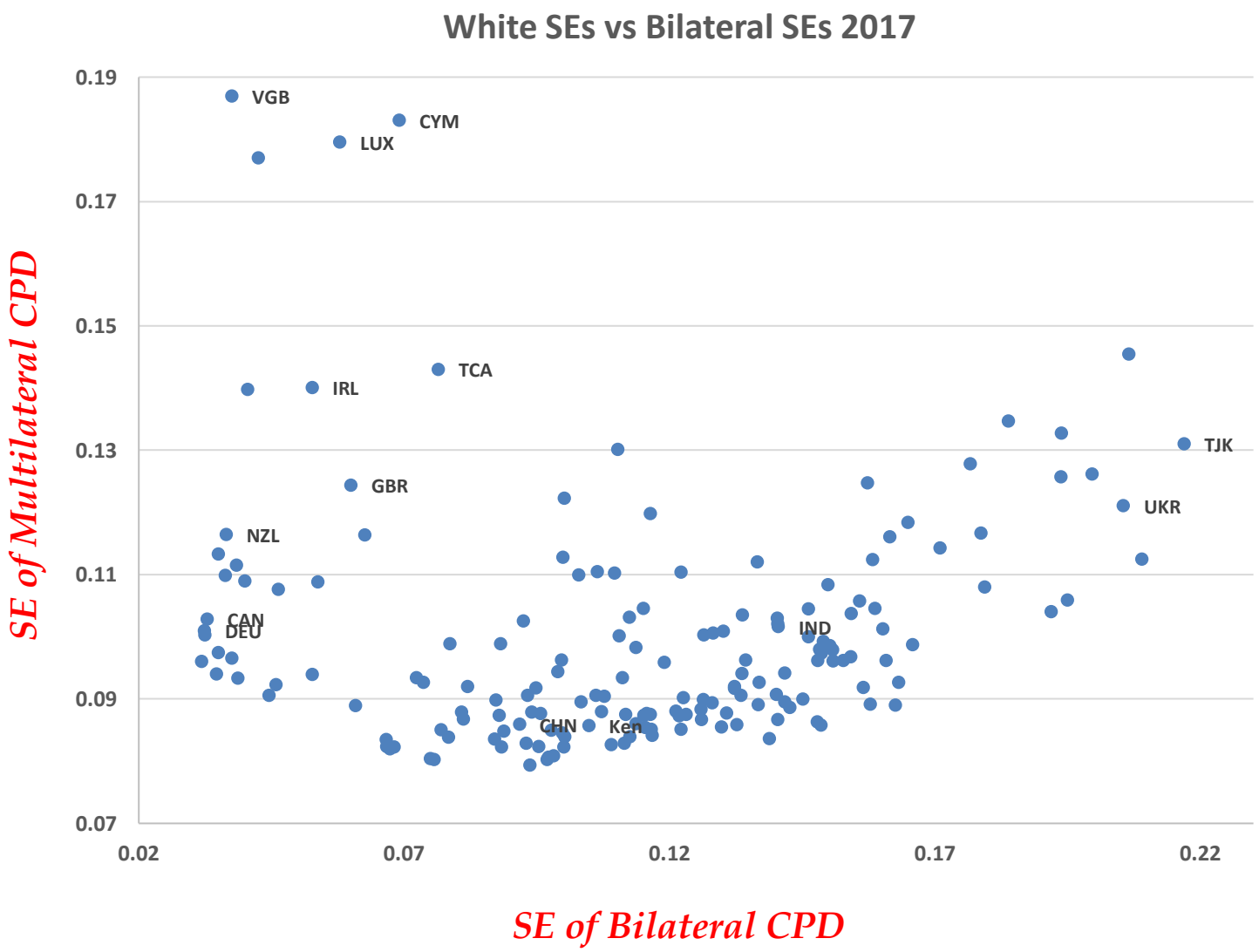
$$Var(\hat{\boldsymbol{\beta}}_{MoM})=\left[\mathbf{Z}'\mathbf{W}\mathbf{Z}\right]^{-1}\mathbf{Z}'\mathbf{W}\boldsymbol{\Omega}\mathbf{W}\mathbf{Z}\left[\mathbf{Z}'\mathbf{W}\mathbf{Z}\right]^{-1}$$

- *Different choices of r, as a result* **Z,** *and* **W** *lead to different indexes including CPD, Ikle and GK.*

SE of Log PPP (2005) by Deaton (2012)

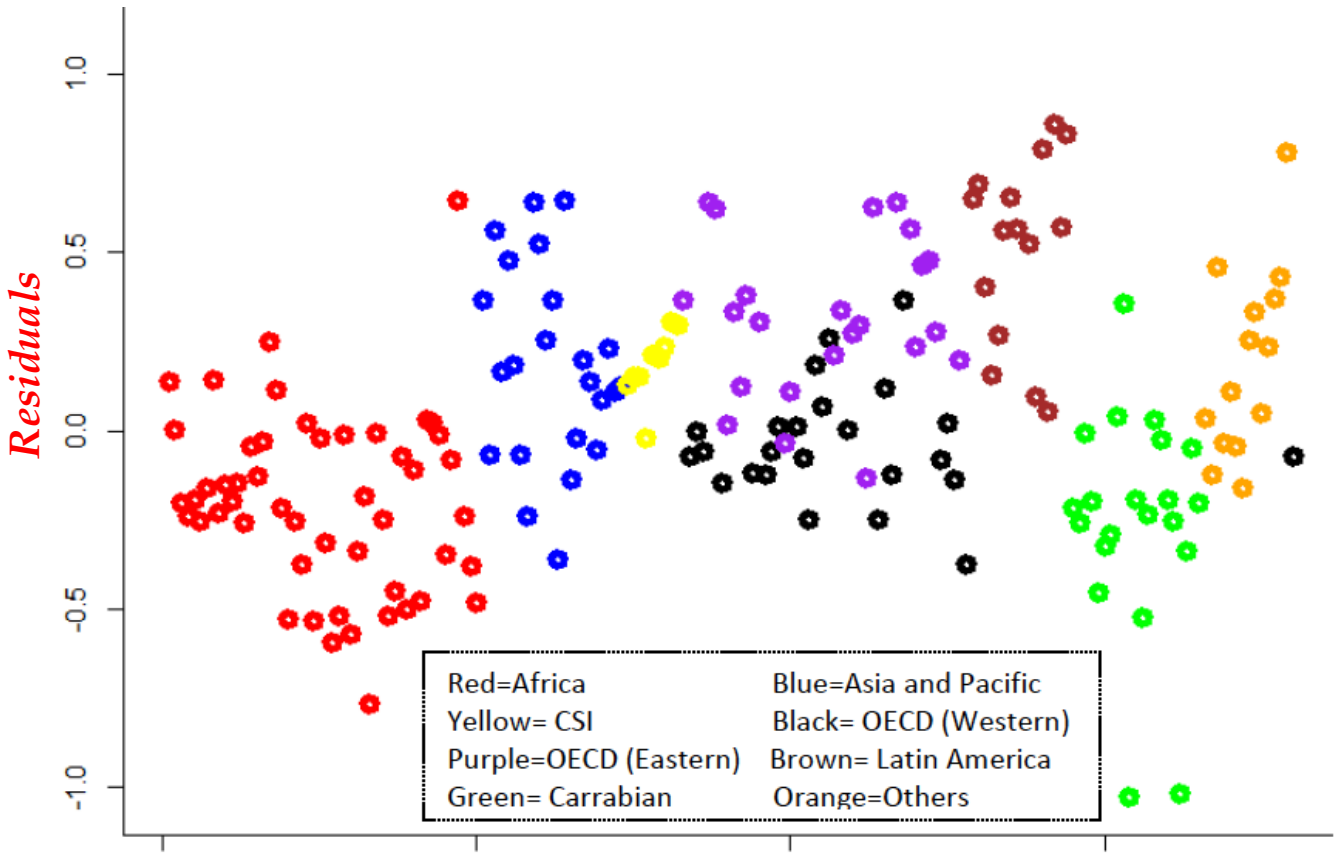


SE of Log(PPP) (2017)



The Presence of Correlation

Residuals for Clothing based on CPD model



172 Countries clustered in regions

Covariance Options

$$Cov(\boldsymbol{\varepsilon}) = \boldsymbol{\Omega}_{NM \times NM} = \begin{bmatrix} \boldsymbol{\sigma}^{1} & \boldsymbol{\sigma}^{1,2} & \cdots & \boldsymbol{\sigma}^{1,M} \\ \boldsymbol{\sigma}^{1,2} & \boldsymbol{\sigma}^{2} & \cdots & \boldsymbol{\sigma}^{2,M} \\ \vdots & & \ddots & \\ \boldsymbol{\sigma}^{1,M} & \boldsymbol{\sigma}^{2,M} & \cdots & \boldsymbol{\sigma}^{M} \end{bmatrix}$$

- *No correlation*
  Homoscedastic
  Heteroskedastic
- *With Correlation*
  Cluster with respect to Items
  Cluster with respect to Countries

Clustered Covariance

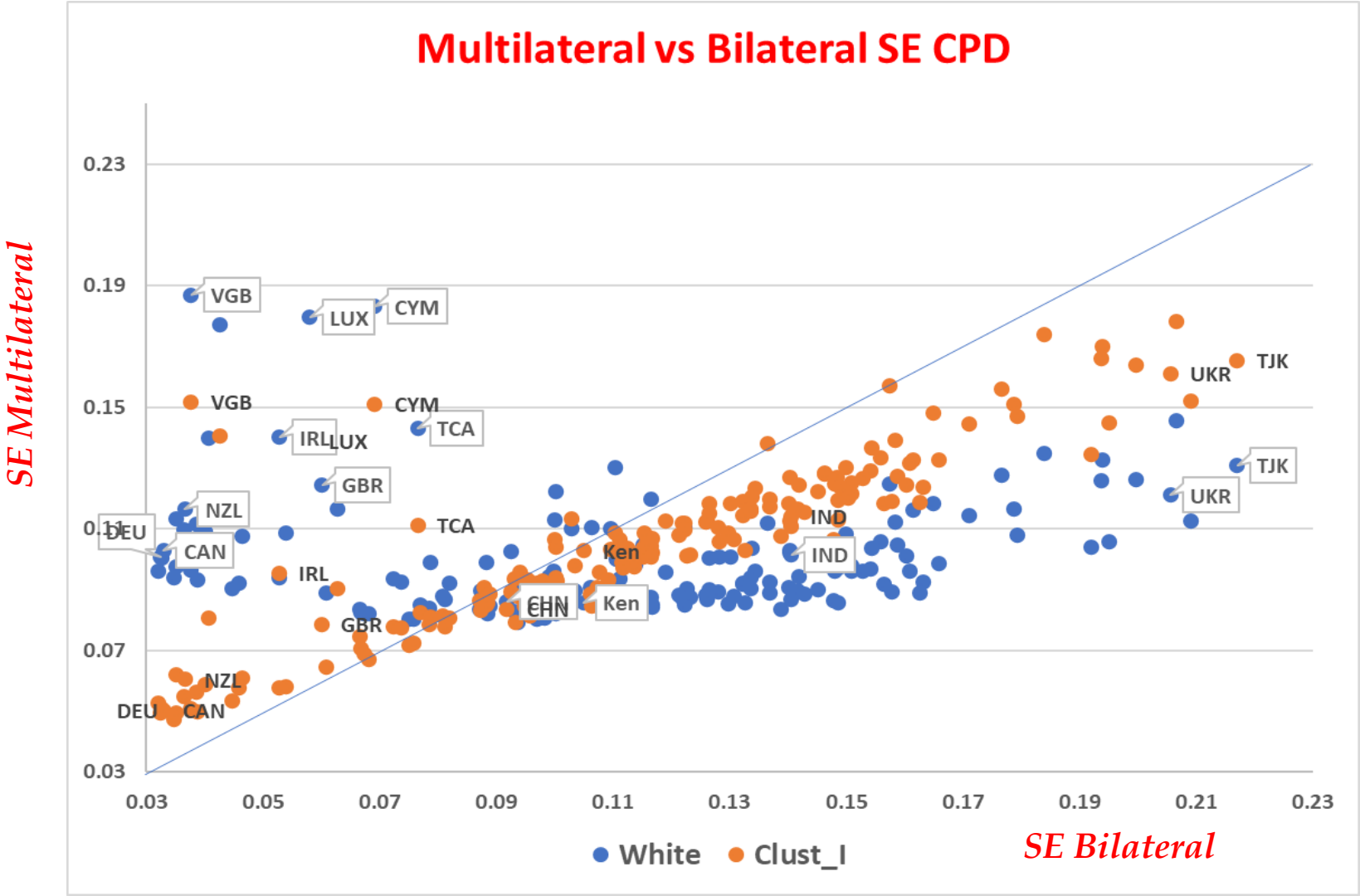
- *Cluster with respect to items*

$$\boldsymbol{\Omega}_{NM\times NM} = \begin{bmatrix} \sigma_{11}^{1} & 0 & & \sigma_{11}^{1,2} & 0 & & \cdots & \sigma_{11}^{1,M} & 0 & \\ 0 & & \sigma_{NN}^{1} & 0 & & \sigma_{NN}^{1,2} & & 0 & & \sigma_{NN}^{1,M} \\ \sigma_{11}^{2,1} & 0 & & \sigma_{11}^{2} & 0 & & \ddots & & \vdots & \\ 0 & & \sigma_{NN}^{2,1} & 0 & & \sigma_{NN}^{2} & & & & \\ & \vdots & & \ddots & & & \ddots & & & \\ \sigma_{11}^{M,1} & 0 & & & & \cdots & & \sigma_{11}^{M} & 0 & \\ 0 & & \sigma_{MM}^{M,1} & & & \cdots & & 0 & & \sigma_{NN}^{M} \end{bmatrix}$$

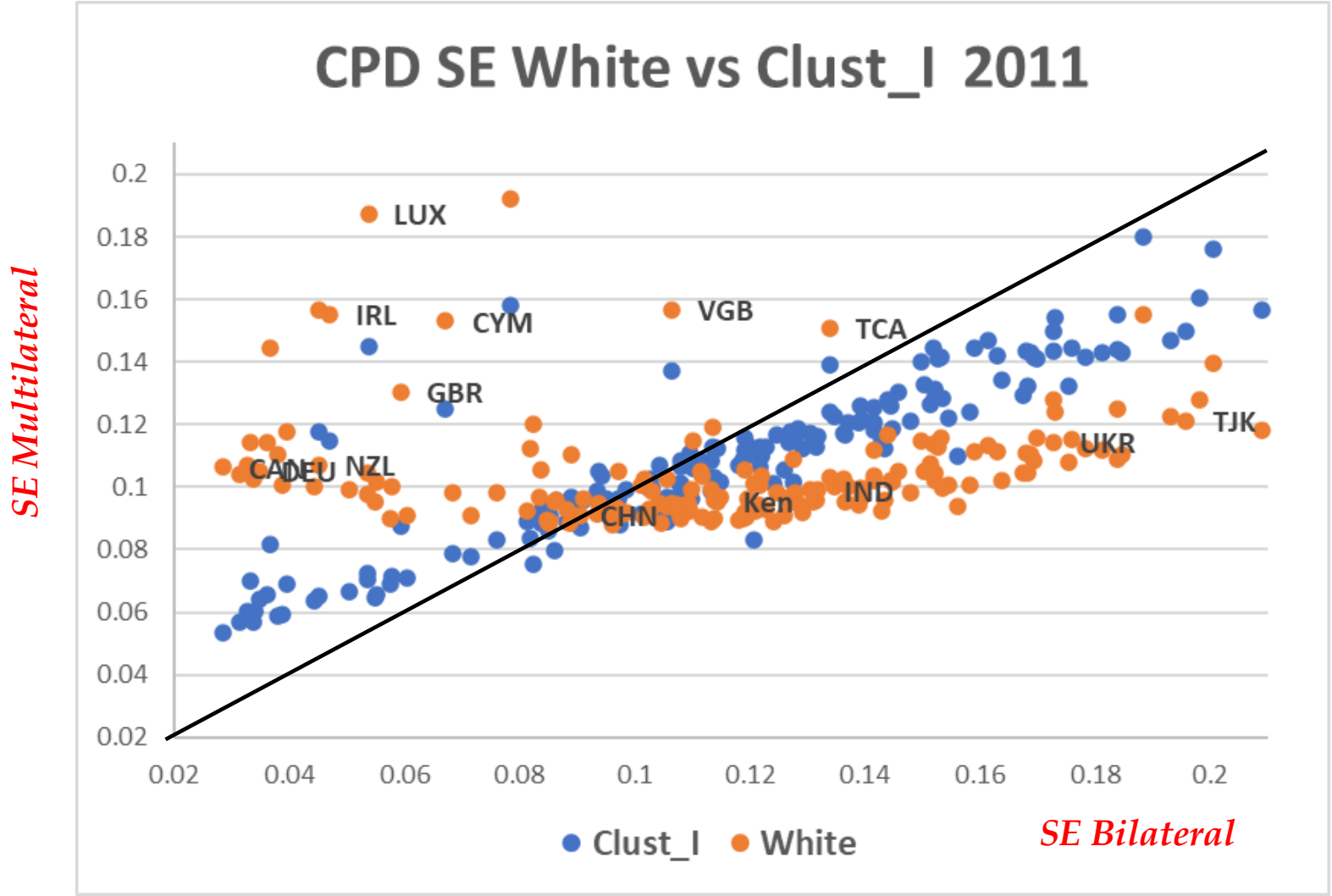
- *Cluster with respect to countries*

$$\boldsymbol{\Omega}_{NM\times NM} = \begin{bmatrix} \boldsymbol{\sigma}^{1} & 0 & \cdots & 0 \\ 0 & \boldsymbol{\sigma}^{2} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{\sigma}^{M} \end{bmatrix}$$

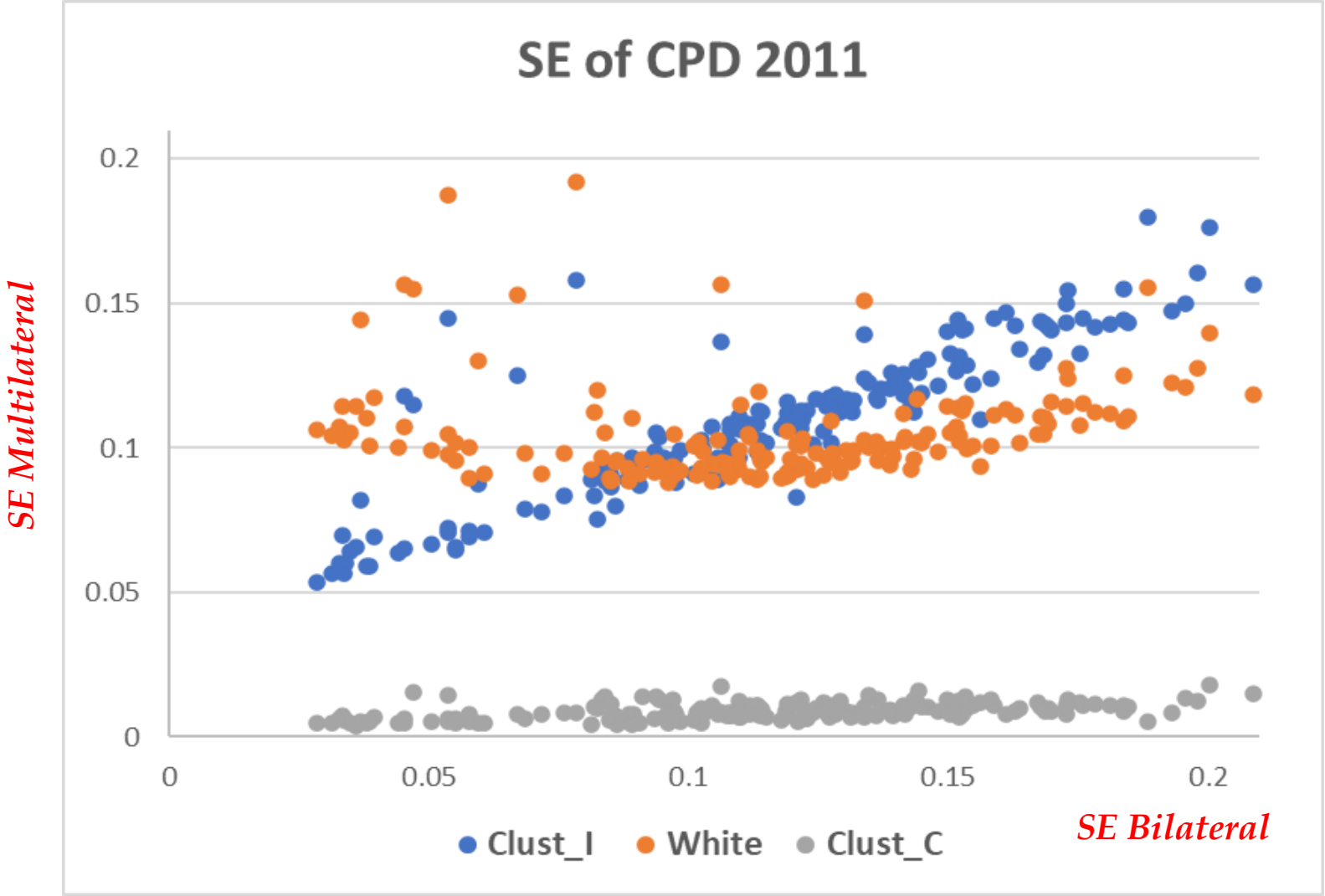
Cluster Robust Rules



Cluster Robust Standard Errors



Clust-I Rules



Can We Explain?

Assuming the special case where all weights are equal, we can derive:

Heteroskeastic:

$$Var(\ln PPP_j) = \frac{1}{N^2}\left(\sum_{i=1}^{N}\sigma_{ii}^{(j)} + \sum_{i=1}^{N}\sigma_{ii}^{(M)}\right)$$

Cluster with respect to Countries:

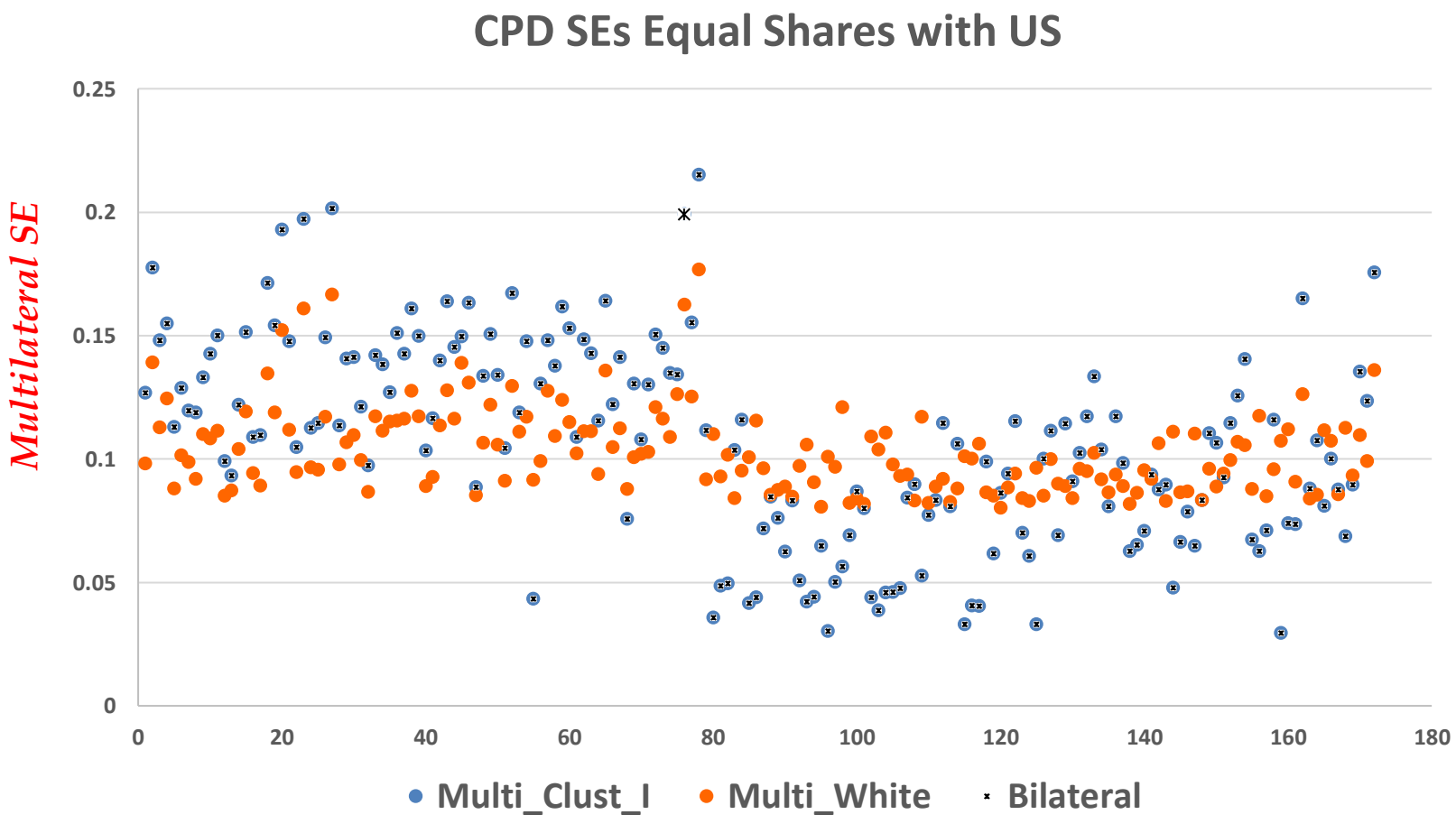
$$Var(\ln PPP_j) = \frac{\mathbf{i}'(\hat{\boldsymbol{\sigma}}^{j} + \hat{\boldsymbol{\sigma}}^{M})\mathbf{i}}{N^2} = 0$$

Cluster with respect to Items:

$$Var(\ln PPP_j) = \frac{1}{N^2}\left(\sum_{i=1}^{N}\sigma_{ii}^{(j)} + \sum_{i=1}^{N}\sigma_{ii}^{(M)} - 2\sum_{i=1}^{N}\sigma_{ii}^{(j,M)}\right)$$

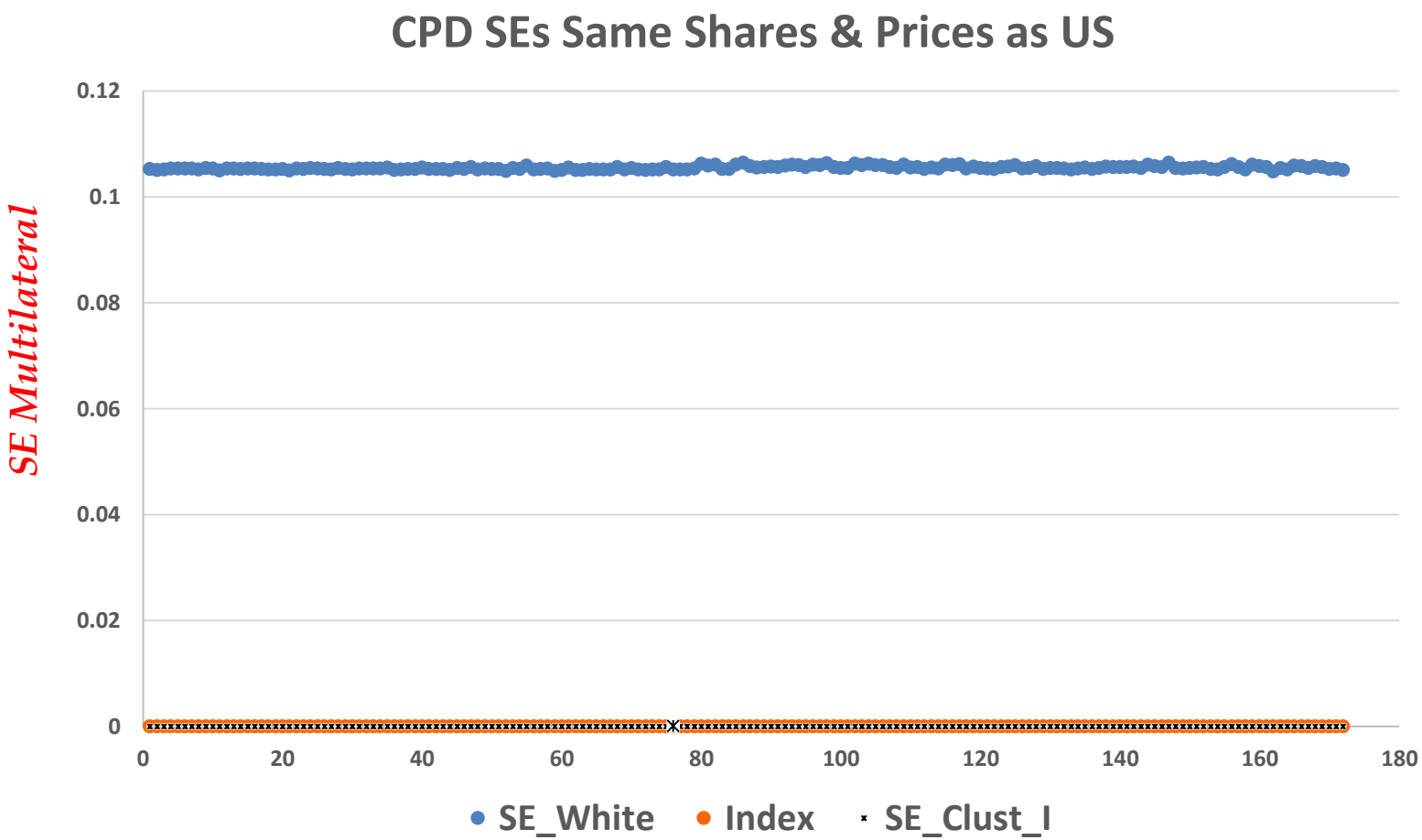
Cluster Robust is the One

Two countries with same expenditure share



Cluster Robust is the One

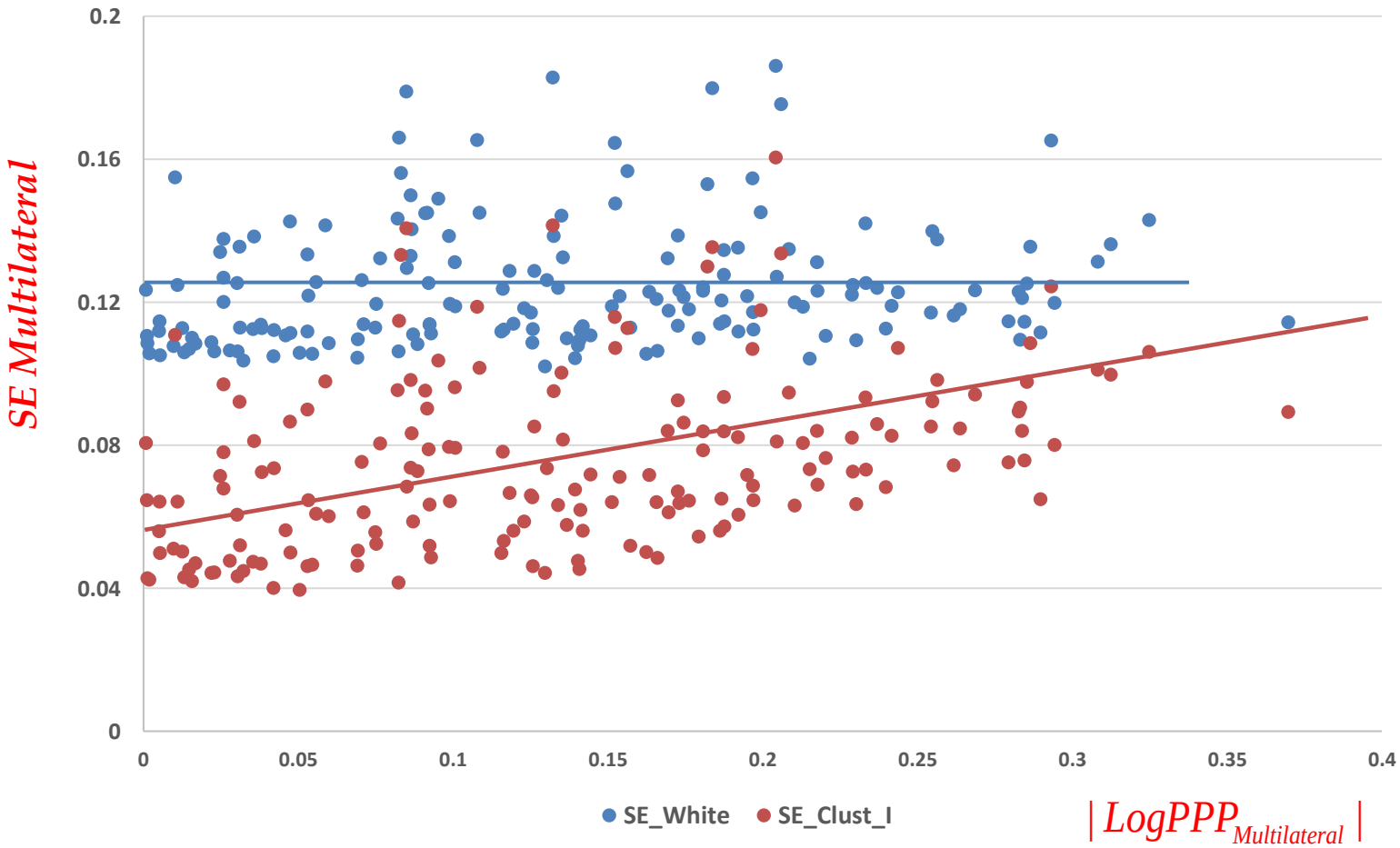
Two countries with same expenditure shares & same price



In this case both bilateral and multilateral CPD PPP for the country with respect to USA equals 1

Cluster Robust is the One

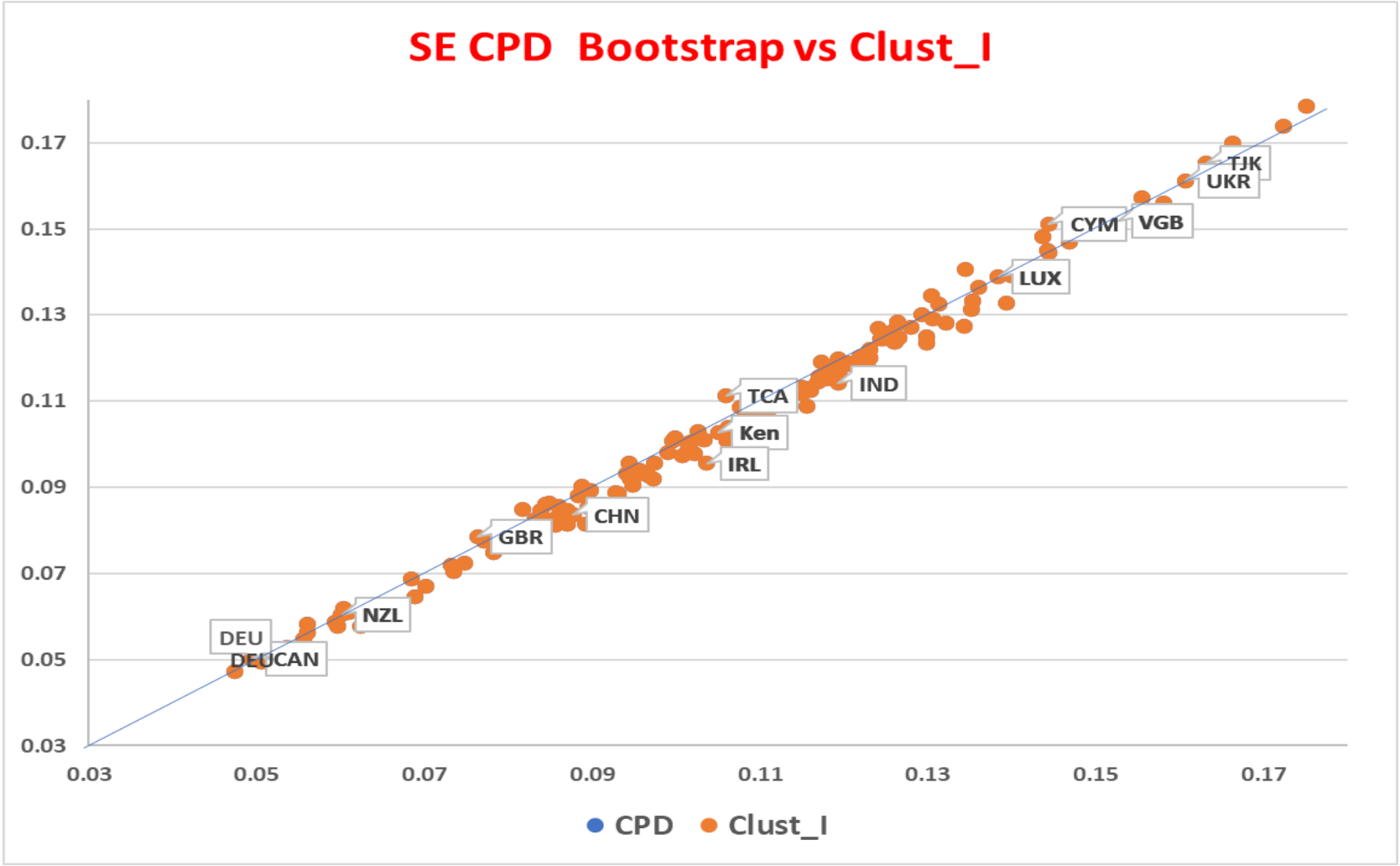
Two countries with same price



SEs based on Resampling

- *Standard errors can be estimated using resampling methods as well*
- *Resampling Methods:*
  - *Jacknife*
  - *Bootstrap*
  - *Subsampling*
- *Advantages:*
  - *Does not necessarily rely on law of one price*
  - *Give new interpretations to SEs*

Bootstrap over Items



Bootstrap SE

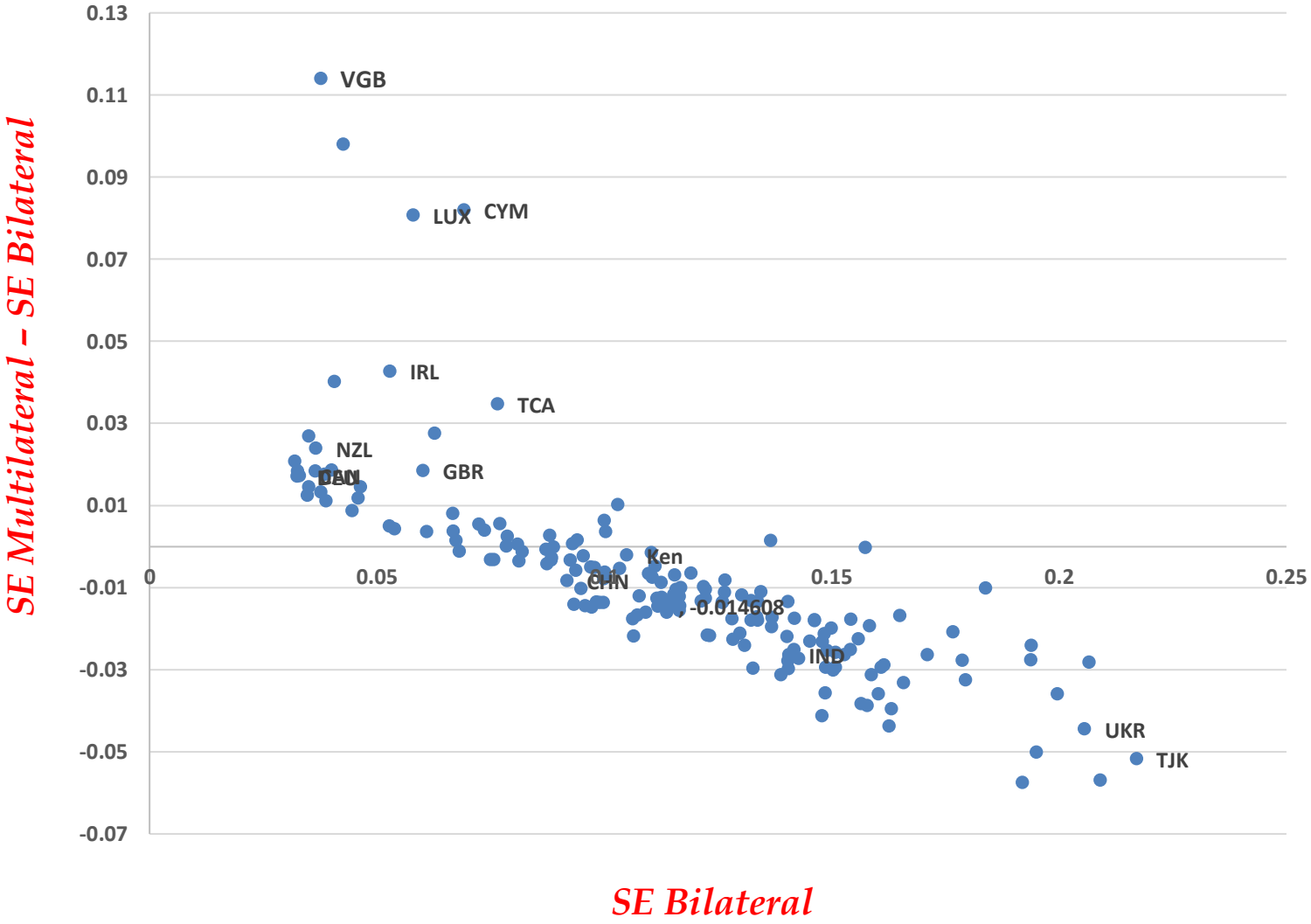
Clust_I SE

Interpretations of SEs

First Interpretation: Clust_I SEs can be interpreted as uncertainty around the estimated LogPPPs due to sampling N items out of the population of all items.

Second Interpretation: Inspired by resampling methods, Clust_I SEs can be interpreted as average sensitivity of LogPPPs to adding or dropping items.

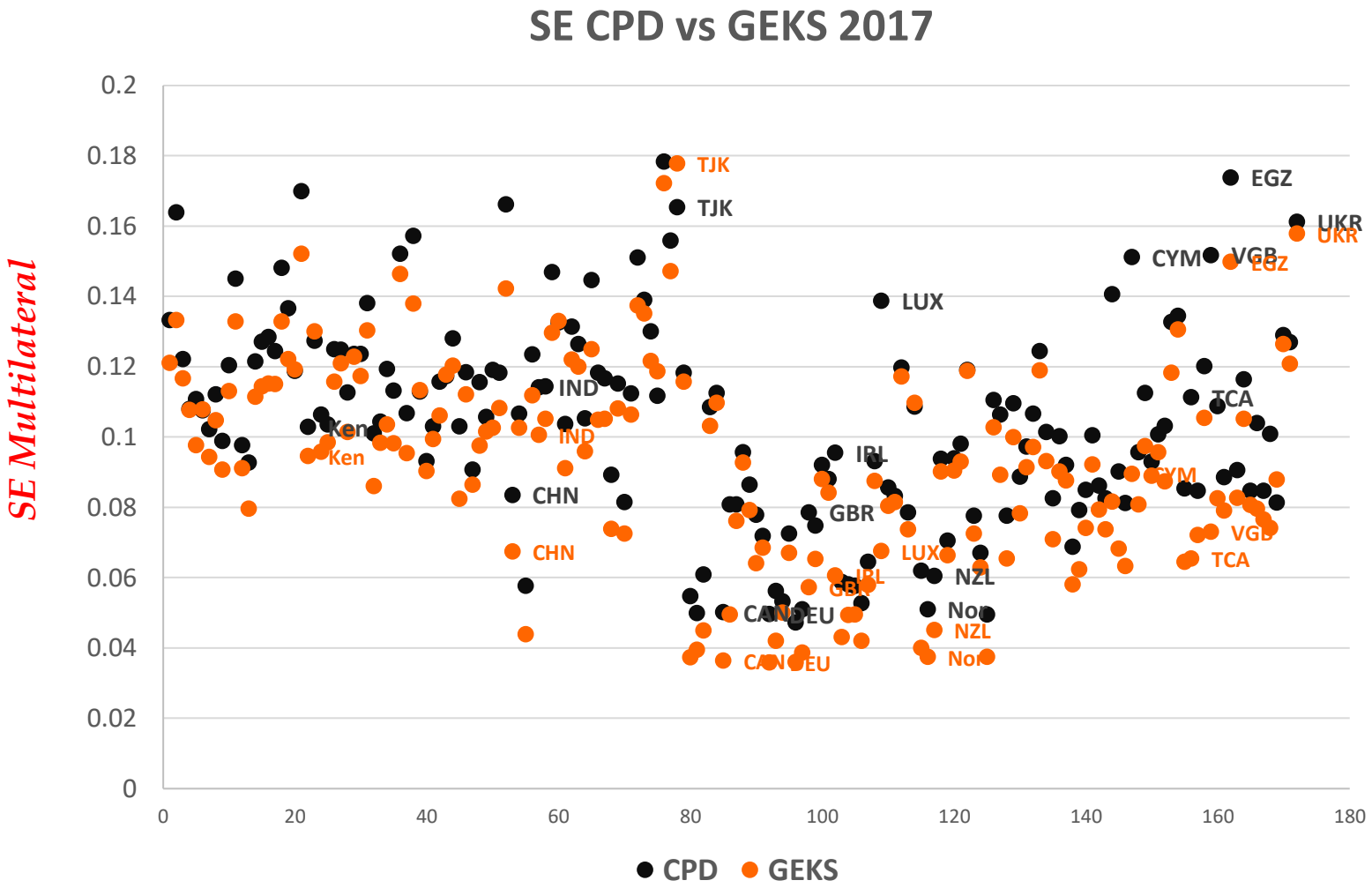
An Interesting Graph



GEKS Standard Errors

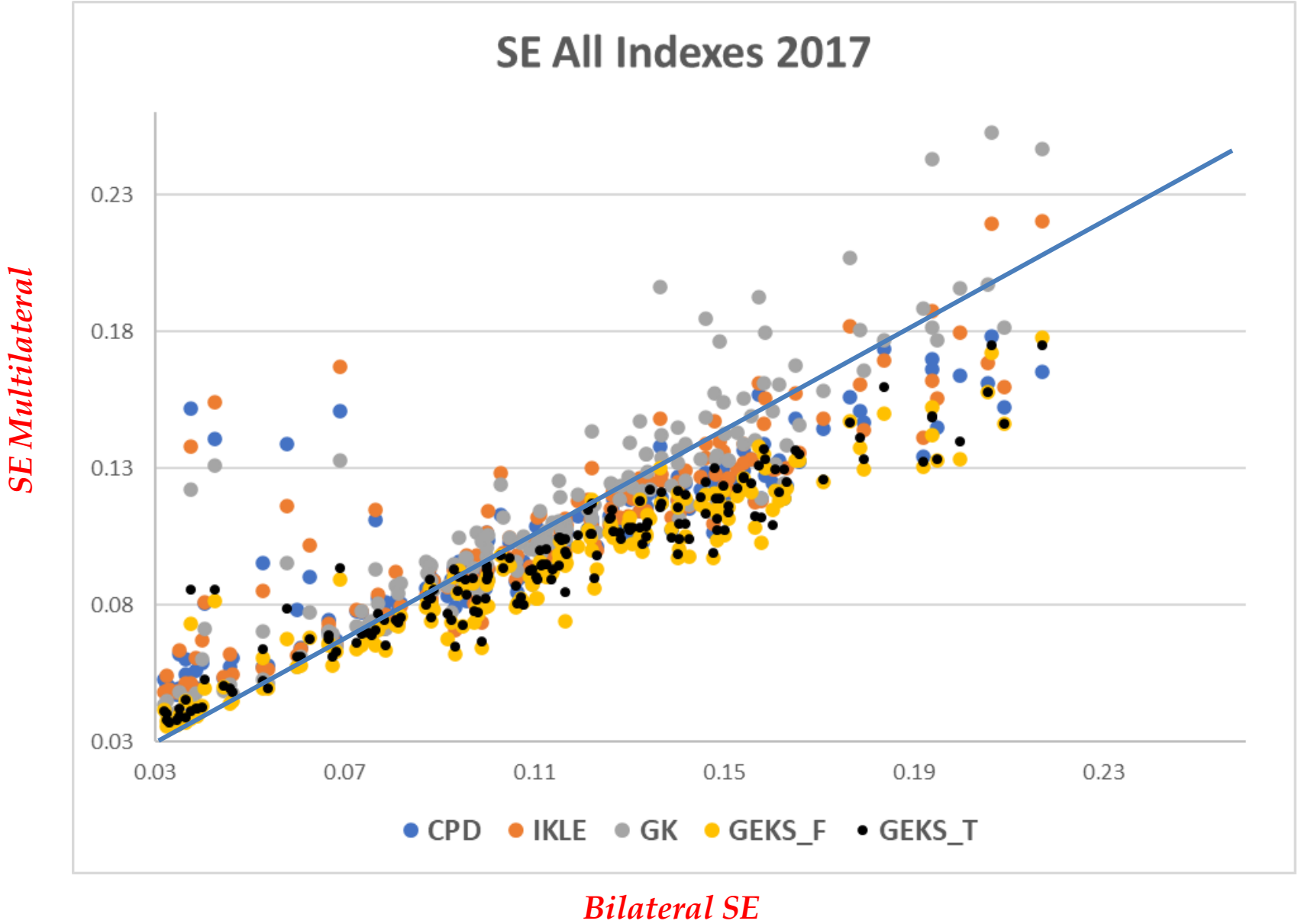
- *We have also developed a method to calculate standard errors for GEKS with Tornqvist. It is similar to Deaton's (2012)but we allow for correlation across Items.*
- *For GEKS with Fisher, SEs can be easily calculated using resampling methods.*
- *The next slide compares GEKS SEs with CPD SEs for 2017.*

CPD & GEKS (2017)

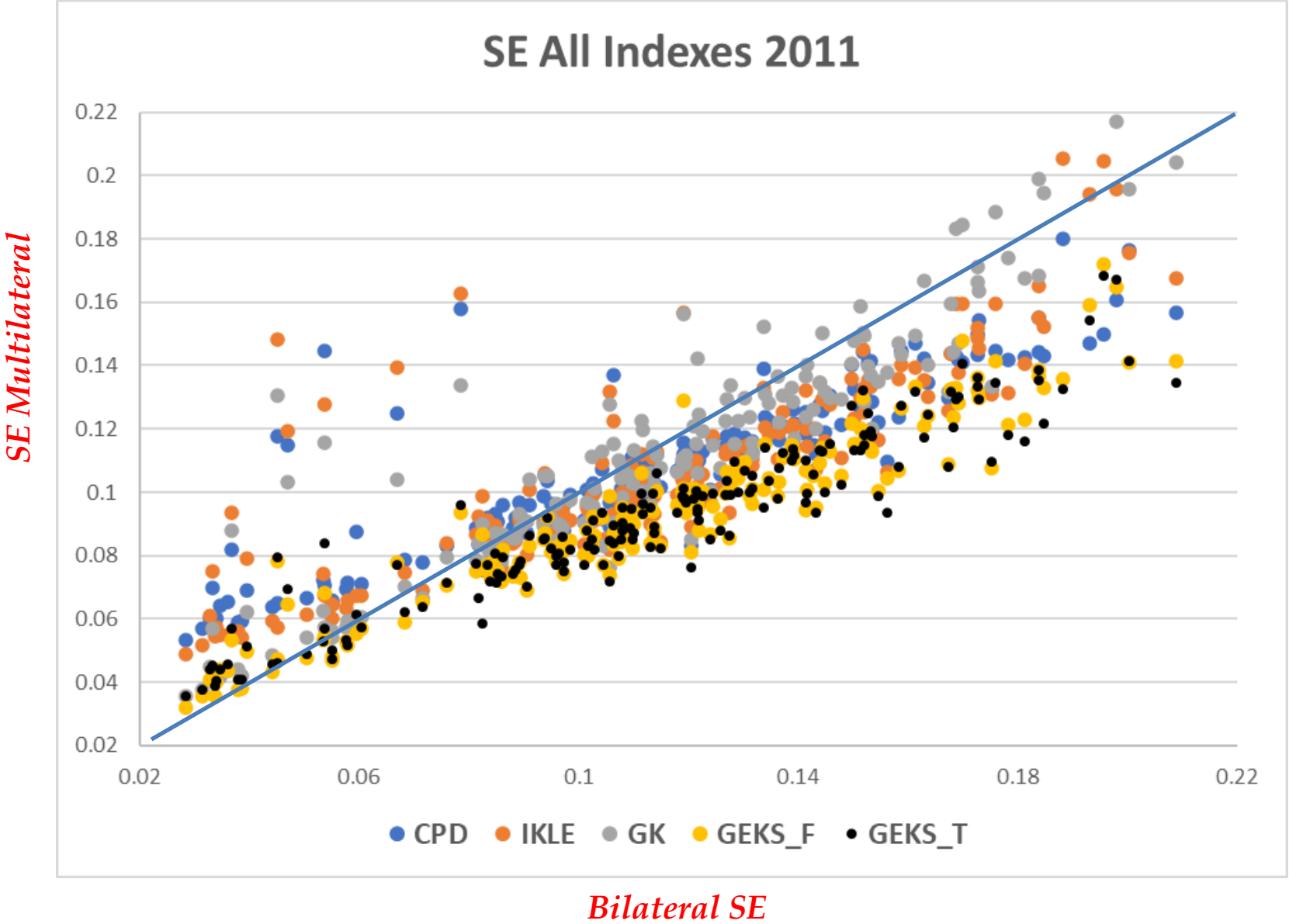


Countries from 1 to 174

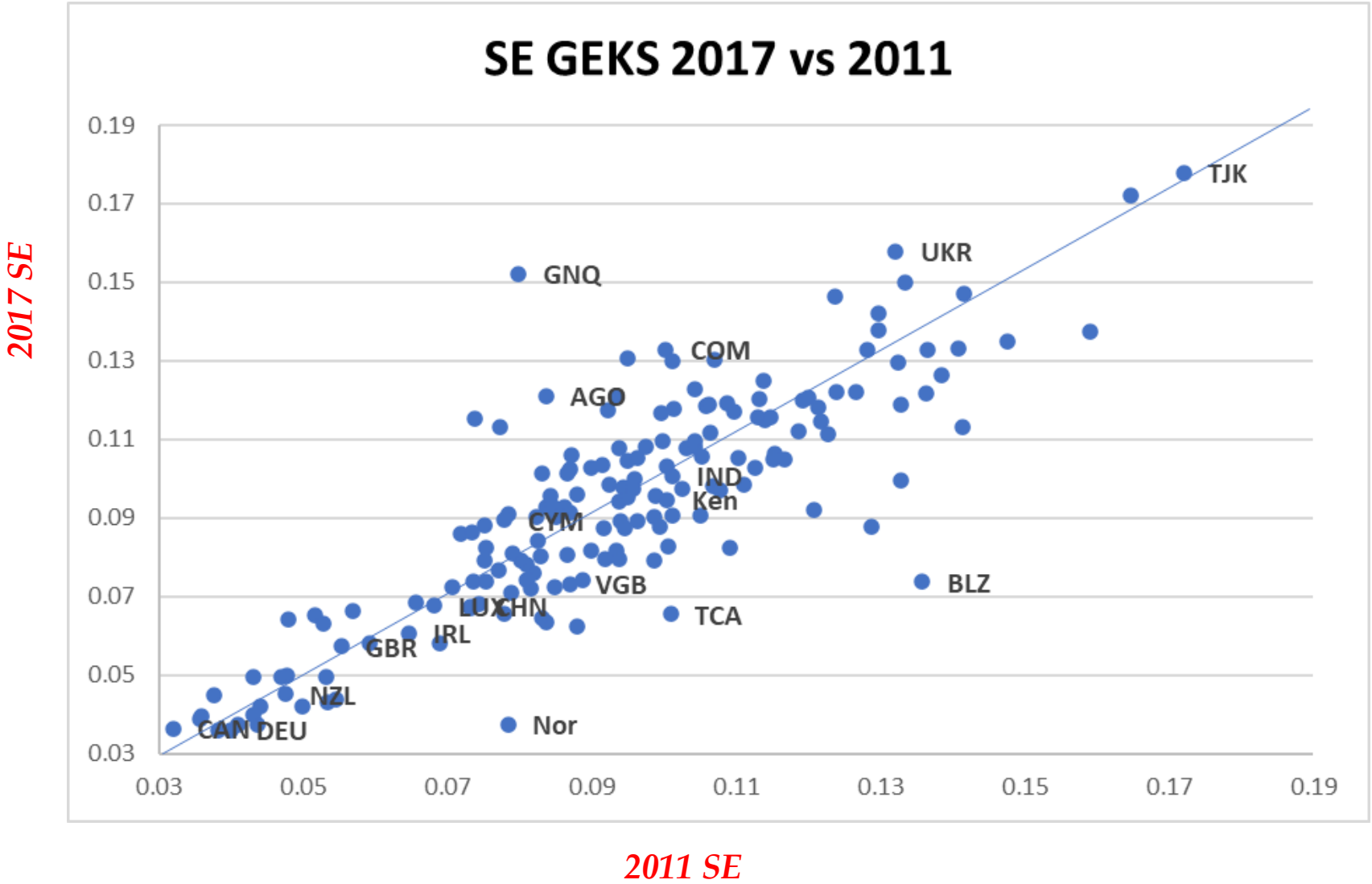
All Indexes in one Graph



All Indexes in one Graph



Comparison of Benchmarks



Conclusion

- *Demonstrated feasibility of constructing reliability measures for various PPP indexes using Statistical models and resampling methods.*
- *Main conclusion is that Clust_I Standard Errors are the most appropriate.*
- *Provided interpretations of Standard Errors.*
- *Comparing reliability of various indexes: GEKS-Fisher has lower standard errors.*
- *Comparison of reliability measures for GEKS for 2011 and 2017.*