International Coordination of Macro-Prudential and Monetary Policies.*

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Abstract

We investigate the question of international coordination of monetary and macro-prudential policies in the context of a general equilibrium model with two regions where banks choose the optimal mix between ex ante and ex post liquidity. In each region, a monetary policy maker determines the interest rate, i.e. the cost of ex post liquidity and a macro-prudential policy maker controls bank leverage, i.e. the demand for ex ante liquidity. In this framework, we derive three results. First, for given macro-prudential policies, welfare gains to monetary policy cooperation are necessarily asymmetric one region always experiencing lower welfare under cooperation. Second with Nash monetary policies, all regions enjoy strictly positive welfare gains from macro-prudential policy coordination relative to the Nash equilibrium. Last, under cooperative monetary policies, the way macro-prudential policy is conducted is just irrelevant. In other words, coordinating or not macro-prudential policy makes no difference under cooperative monetary policies.

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1 Introduction

The purpose of this paper is twofold. First, it aims at providing provide a framework that lends itself the study of policy cross-border cooperation. Second, it aims at understanding how cooperation or the absence of such cooperation influences welfare when policy makers face different policy instruments on which they may or may not coordinate. Specifically, we investigate the question of international coordination of monetary and macro-prudential policies in the context of a general equilibrium model with two regions where agents choose the optimal mix between ex ante and ex post liquidity. Ex ante liquidity comes in the form of cross-border capital flows, by which agents -henceforth banks- in one region can buy shares in projects run by banks of the other region. Given that returns on banks' projects display negative correlation across regions, banks can sell claims on future output as a risk sharing device. The size of cross-border capital flows therefore determines the extent to which banks can insure against domestic risks. The market for risk sharing however suffers two frictions. First, banks are unable to pledge their total investment to outside investors. They hence face a limit on the amount of insurance they can provide through cross-border risk sharing. Second risk sharing comes at a cost which introduces a wedge between the cost of issuing claims for risk sharing purposes and the return on such assets. Banks can however get around the frictions affecting the market for risk sharing as they can rely on ex post liquidity through the market for funding which opens once aggregate uncertainty has unraveled. On this market, banks in need for liquid funds may issue claims and sell them to banks in excess of liquid funds. As is the case on the market for risk sharing, banks issuing claims on the market for ex post funding are still unable to fully pledge reinvestment. However contrary to the market for risk sharing, there is no wedge between the cost to issue claims on the market for ex post funding and the return on such claims. This captures the fact that frictions on debt markets -e.g. the market for ex post funding- tend to be weaker than those on equity markets -e.g. the market for risk sharing.

We introduce monetary and macro-prudential policy by assuming that authorities have two policy levers. Monetary policy consists in setting the return on an ex post deposit risk free facility that any bank can access and deposit its funds into. Macro-prudential policy consists in setting the maximum amount of claims banks can sell ex ante on the market for risk sharing. Macro-prudential policy can therefore limit or expand how much risk domestic banks can share with the other region, and thereby how much capital flows in and out of each region.

At the heart of the model is a set of trade-offs for policy makers that can be described as follows. For monetary policy, high interest rates allow on the one hand domestic banks to earn a larger return when they are on the lending side of the market for ex post funding. But on the other hand, high interest rates have two drawbacks: banks need to pay a higher cost for ex post liquidity when they need to carry out reinvestment, i.e. when they are on the borrowing side of the market for ex post funding. And in addition, ex ante insurance through cross-border risk sharing also becomes more expensive when central banks set higher interest rates. Optimal interest rates therefore tend to be lower when banks have issued more claims for risk sharing purposes, i.e. when banks are more leveraged. Turning to macro-prudential policy, the potential trade-off lies between capital inflows and financial conditions. One the one hand, issuing risk sharing claims is profitable because such claims can be sold at a high price, given their risk sharing properties. On the other hand, when macro-prudential policy makers allow domestic banks to issue more risk sharing claims, there are both more capital inflows into the region and but also less capital outflows, i.e. less capital inflows into the other region. This last development leads monetary policy makers in the other region to optimally set higher interest rates, which may hurt banks' profits in the first region.[1]

1.1 Main takeaways

In this framework, we provide three main results. First looking at monetary policy, the model shows that under exogenous macro-prudential policies, gains from cooperation, i.e. the difference in welfare under cooperative vs. Nash monetary policies, is always asymmetric. In other words, moving from Nash to cooperative monetary policies is never a Pareto improvement as one region always enjoys strictly lower welfare under cooperation. The intuition for this result is simple. When monetary policies are determined in a Nash equilibrium, the equilibrium cost of ex post liquidity that comes out of the game between the two

[1] Hence, notwithstanding the usual role assigned to macro-prudential policy in keeping systemic risk under control or ensuring banking and financial stability, this model stresses the trade-off facing macro-prudential policy in controlling financial conditions at the cost of possibly limiting beneficial trades with the rest of world.

central banks, is optimal for one region but too high for the other one. As a result, moving to cooperation -which shifts the cost of ex post funding down- is bound to hurt the region for which it was optimal under Nash.[2]

Second, when monetary policies are conducted in a non-cooperative way, then there are gains to coordinate macro-prudential policies. Moreover, unlike the case of monetary policy, moving to cooperative macro-prudential policies is a Pareto improvement as all regions experience strictly positive welfare gains. Specifically, cooperative macro-prudential policies display a larger degree of risk sharing, i.e. larger cross-border capital flows, and a lower global interest rate. Why so? Under cooperation, macro-prudential policy makers care, not only about *domestic* capital inflows, but about *global* capital flows, i.e. capital inflows and outflows. However at the decentralized equilibrium, these two are positively correlated: when macro-prudential policy makers in one region allow domestic banks to issue more claims for risk sharing, this permits macro-prudential policy makers in the other region to allow for larger capital inflows. This *general equilibrium externality*, which policy makers fail to internalize in the Nash equilibrium leads to inefficiently low levels of risk sharing when there is no cooperation on macro-prudential policy. Conversely under cooperative macro-prudential policy, each macro-prudential policy maker is willing to increase capital inflows, i.e. its supply for risk sharing, because even if this were to reduce domestic profits, the benefits for the other region are so large that they compensate for any possible loss on domestic profits and expected profits at the global level are always larger.

Last we look at the benefits of coordinating macro-prudential policy under cooperative monetary policies and show that those benefits simply do not exist. The intuition is very simple: When monetary policy is conducted cooperatively, the global interest rate is too high from a domestic perspective for one region and too low for the other. Yet, under cooperative monetary policies, the global interest rate depends on global capital flows, which each macro-prudential policy maker has only some limited influence on as was mentioned above. Pushing the argument to the limit, if the world economy was made of a large number of economies,

[2] Note that irrespective of how cooperation shifts the equilibrium interest rate relative to the Nash case, the region for which the Nash equilibrium maximizes domestic profits is bound to lose. In this model cooperation shifts the equilibrium interest rate down, but a similar result would apply if cooperation led to a higher interest rate.

each domestic macro-prudential policy maker would have virtually no influence on the global interest rate, which would end up being "almost" like an exogenous variable. In this setting, it is straightforward to see that optimal macro-prudential policy always consists in maximizing capital flows, whether in the Nash or in the cooperative equilibrium. Figure 1 below wraps up the three main results of the model.

MACRO-PRUDENTIAL POLICY \ MONETARY POLICY	NASH	COOPERATION
EXOGENOUS	$W(N,.)$	$W(C,.) \geq W(N,.)$ $W_i(C,.) < W_i(N,.)$
NASH	$W(N,N)$	$W(C,N)$
COOPERATION	$W(N,C) \geq W(N,N)$	$W(C,C) = W(C,N)$

Figure 1: Welfare matrix

For given macro-prudential policies, global welfare W is by definition larger under cooperative monetary policy, $W(C,.) > W(N,.)$. Yet, one region, here region i, always enjoys lower welfare under cooperative monetary policy, $W_i(C,.) < W_i(N,.)$. This result holds for given or exogenous macro-prudential policies. Then when monetary policy is Nash, coordinating macro-prudential policy provides a welfare both at the global and at the regional level, $W(N,C) \geq W(N,N)$ and $W_i(N,C) \geq W_i(N,N)$. Conversely, when monetary policy is cooperative, coordinating macro-prudential policy yields no welfare gain whatsoever, $W(C,C) = W(C,N)$ and $W_i(C,C) = W_i(C,N)$. From this matrix, it is also straightforward to note that under Nash macro-prudential policies, there are gains to monetary policy cooperation since by construction we have $W(N,N) \leq W(N,C) \leq W(C,C) = W(C,N)$. However these gains go away under cooperative macro-prudential policies since we have $W(N,C) = W(C,C)$.

1.2 Literature review

This paper relates to three different strands of literature. First, this paper is tightly related to the literature on gains from cross-border policy coordination. While it has been established using new open economy macroeconomic models that benefits from monetary policy cooperation are rather limited (Obstfeld and Rogoff 2002), it is an open question as to whether such result still holds given recent significant changes in the conduct of monetary policy, e.g. the heavy use of unconventional monetary policy tools, or changing constraints and evolving limits on (the effectiveness of) monetary policy, as illustrated by the debate on dilemma vs. trilemma (Rey 2015). Moreover it remains equally open to figure out if the same type of result extends to macro-prudential policy. Last and not least, understanding how the benefits to cooperate on one policy lever depend on the conduct of other policies has remained largely an unexplored territory. Second, a growing body of empirical evidence has highlighted the sizeable cross-border impact of monetary and macro-prudential policies, particularly in the recent years where central banks in advanced economies have used extensively unconventional monetary policy tools. Bowman et al (2015) provide evidence that emerging market economies (EME) sovereign bond yields react strongly to the use of unconventional monetary policy in the US. For the ECB, Fratzscher et al. (2014) document that spillover from non-standard monetary policy measures include a positive impact on global equity markets in both advanced economies (AE) and EME as well as lower credit risk among banks and sovereigns. On the modelling side, Bagliano and Morana (2012) using a large macroeconometric model to show that asset prices as well as international trade are the key channels through which financial disturbances in the US transmit to the rest of the world. More specifically on macro-prudential policy, Cerutti et al. (2017) shows that usage of macro-prudential policy tools tend to be associated with lower credit growth, meanwhile cross-border borrowing tends to go up. Focusing on economies from the Asia-Pacific region, Bruno et al. (2017) find that capital flow management tools are effective in curbing banking and bond inflows, a finding that is very consistent with our modelling of macro-prudential policy.

Third our analytical framework builds on the seminal Hölmstrom and Tirole (1998) model of liquidity provision where firms may require the provision of outside liquidity -supplied by the government- to face aggregate shocks. In our model, such outside liquidity is provided by the rest of the world, hence giving rise to mutually advantageous cross-border capital flows. Our framework is also close in spirit to Jeanne and Korinek (2013) where macro-prudential policy comes ex ante as a constraint on agents' choices while monetary policy intervenes ex post, like in our framework, to set the price of liquidity on the market for ex post funding. Theoretically, macro-prudential policy intervention is justified either as a response to aggregate demand externalities (Farhi and Werning (2013) and Korinek and Simsek (2016) or in the presence of pecuniary externalities (Gromb and Vayanos (2002), Caballero and Krishnamurthy (2003), Lorenzoni (2008), Jeanne and Korinek (2010), Stein (2012)). In our framework - which belongs to the latter category-, the use of macro-prudential policy derives from two features of the model. First monetary policy makers choose to set interest rates ex post in relation to the composition of banks balance sheets. Affecting such balance sheets through macro-prudential policy can therefore help adjust funding costs in line with the needs of the economy. Second, because of the presence of a global capital market with frictionless arbitrage, countries may lose control of domestic financial conditions. Macro-prudential policy can therefore be useful as policy makers can recover some influence on domestic financial conditions by affecting the composition of domestic banks assets and liabilities and thereby affecting the equilibrium of the global market where banks exchange claims.

The road map for the paper is as follows. The next section presents the analytical framework and its main assumptions. Section 3 derives the decentralized equilibrium for given monetary and macro-prudential policies. Then section 4 determines optimal monetary policies under the Nash and the cooperative equilibria while section 5 investigates optimal macro-prudential policies with and without cooperation. Welfare gains from policy cooperation are quantified in section 6 and finally conclusions are drawn in section 7.

2 Timing and Technology

2.1 Framework

We consider a world economy consisting of two regions -region i and region $-i$-, lasting for three periods, 0; 1 and 2, and populated by risk neutral banks maximizing date-2 expected profits. At date 0, each bank starts with one unit of funds and can invest in a risky asset.

Investing I in the risky asset at date 0 delivers I one period later with a probability $\frac{1}{2}$. But with a probability $\frac{1}{2}$, the date-0 investment in the risky asset yields no output at date 1. Still, it can then deliver output at date 2 if reinvestment takes place at date 1. Specifically, reinvesting J at date 1 yields $\min\{J; j_i\}$ at date 2, j_i being the maximum reinvestment capacity for banks of region i ($j_i > 0$). The shock affecting the timing of the return on risky assets is observable, verifiable and perfectly correlated *within* regions, but imperfectly correlated *across* regions. For simplicity we assume a perfect *negative* correlation *across* regions: When region i bank risky assets pay-off at date 1 then region $-i$ bank risky assets do not pay anything at date 1 and vice-versa. In what follows, banks whose risky assets do not pay-off at date 1 will be called *distressed* banks while banks whose risky assets pay-off at date 1 will be called *intact* banks. Both investment and reinvestment face financial frictions: Region i (region $-i$) banks, i.e. banks located in region i (region $-i$), can finance externally only a fraction ϕ_i (ϕ_{-i}) of date-0 investment and a fraction ϕ_1 of date-1 reinvestment.

Intact banks -those whose risky assets pay-off at date 1- can lend their funds to distressed banks located in the other region seeking reinvestment and in need for liquidity. Alternatively, they can always store their funds in one of the region's risk free technologies which return θ_i in region i and θ_{-i} in region $-i$. Last, when risky assets do not pay-off at date 1, a distressed bank faces with a probability q, a variable-scale reinvestment ($j_i = +\infty$ but $\phi_1 = \phi < 1$). Alternatively, with a probability $1 - q$, a distressed bank faces a fixed-scale reinvestment ($j_i = \overline{j_i}$ but $\phi_1 = 1$).

2.2 Contracts

First, banks can enter into cross-border risk sharing agreements at date 0. Region i banks can take shares in the risky projects held by region $-i$ banks and vice-versa. L_i then denotes the amount of funds a bank from region i raises from banks from region $-i$ at date 0 and $R_{i,1}$ denotes the return banks from region i pay at date 1 to banks from region $-i$ if they are intact. Importantly when banks from region i pay a return $R_{i,1}$, banks from region $-i$ only earn $\beta R_{i,1}$, with $\beta < 1$, the difference $(1 - \beta) R_{i,1}$ being a cost that scales the friction that the market for risk sharing suffers from.[3] Note that when banks of region i are distressed,

[3] For instance, this cost may be paid for ex ante certification or ex post verification purposes.

then region $-i$ banks' assets L_i on banks from region i become worthless.

Second, distressed banks can borrow ex post from intact banks to take opportunity of their reinvestment opportunity. D_i then denotes the amount of funds distressed banks from region i raise at date 1 and $r_{i,2}$ denotes the return distressed banks from region i pay at date 2 on liabilities D_i raised at date 1.

2.3 Monetary and macro-prudential policies

We introduce monetary and macro-prudential policies as follows. Monetary policy consists in setting the return to the domestic risk free technology in each region between date 1 and date 2, which is akin to setting the return on a domestic deposit facility.[4] In what follows we will call these returns on the risk free technologies, interest rates and denote θ_i (resp. θ_{-i}) the return set by the central bank from region i (resp. region $-i$) with $0 < \theta_i; \theta_{-i} < 1$. Turning to macro-prudential policy, authorities in each region determine the maximum amount of liabilities domestic banks can issue for risk sharing purposes. They can do so by setting the maximum fraction ϕ_i of date-0 investment banks can pledge to outside investors, as a larger fraction ϕ_i allows banks from region i to issue more liabilities L_i for risk sharing purposes. Importantly we assume that macro-prudential authorities set their decision -on the amount of liabilities banks can issue at date 0- first, i.e. before monetary policy makers choose the returns on their respective domestic deposit facilities, which is consistent with the timing of the model as macro-prudential policy decisions should be made ex ante, at date 0 while monetary policy decisions can be made either at date 0, i.e. before uncertainty on risky assets is resolved or at date 1, once uncertainty on risky assets has unraveled.[5] Indeed, to simplify the analysis we will consider that monetary policy decisions are made ex ante at date 0 under perfect commitment.[6]

[4] Limiting the availability for banks of the deposit facility between date 1 and date 2 ensures that monetary policy can influence the cost of capital without having full control on it. This amounts to assuming that the central bank can affect the return on risk free assets only in the short-run.

[5] Another reason justifying this specific sequencing relates to the different frequencies of monetary and macro-prudential policies. Given that the frequency of the former is higher than the frequency of the latter, the case where monetary policy decisions are taken while macro-prudential policies are given tends to happen more often than the opposite.

[6] Although it simplifies to a great extent the analytics of the model, it is fair to say this assumption is not realistic, because central banks set interest rates in response to shocks affecting their economy. Section 7.3 in the appendix therefore derives the model when interest rate are decided at date 1, after uncertainty is resolved. It shows that the logic of the model is unchanged if the model is enriched with an adverse selection problem for borrowers on the market for ex post liquidity which implies that the interest rate charged to borrowers faces an upper bound that depends negatively on the internal funds of the borrowers, hence replicating the trade-off for macro-prudential policy between allowing larger capital inflows and facing less favorable financing conditions.

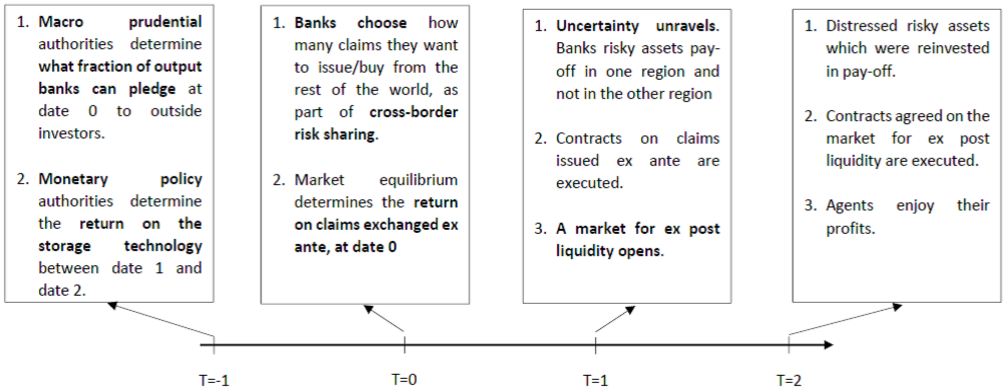

Figure 2: Timing of the model

Once these two sets of decisions are made in both regions, then banks determine their investment plans, splitting their endowment between investment in domestic risky assets and buying shares into the other region's risky assets. They also can issue risk sharing claims to the other region. Uncertainty then unravels, risk sharing contracts are executed and a market for ex post liquidity opens. Distressed banks can borrow from intact banks and reinvest in their risky assets. Finally distressed risky assets deliver output, distressed banks pay back intact banks and all banks enjoy their profits.

As is usual in this kind of model, we use backward induction for the resolution. We start by determining the decentralized equilibrium and its properties. Once this is established, we work out the optimal monetary policies under two different scenarios: the Nash and the cooperative equilibrium. Finally assuming some game for optimal monetary policies, we look at optimal macro-prudential policies. Here again, we consider the Nash and the cooperative solutions.

3 The decentralized equilibrium

3.1 Optimal portfolio allocation decisions.

Let us consider a bank from region i. At date 0, this bank invests L_{-i} in risky assets of banks from region $-i$ and receives L_i from banks of region $-i$ seeking to invest in risky assets of banks from region i. The bank therefore invests $I_i = 1 + L_i - L_{-i}$ in its risky assets at date 0. With a probability $\frac{1}{2}$, the risky asset pays early and it reaps I_i at date 1 and pays $r_{i,1}L_i$ to banks of region $-i$. The bank from region i hence earns at date 1 a profit

$$\Pi_{i,1} = I_i - R_{i,1}L_i \tag{1}$$

It can then use these profits to make loans at date 1 to distressed banks of region $-i$ on the market for ex post funding, with a return $R_{-i,2}$. Alternatively, with a probability $\frac{1}{2}$, risky assets of region i do not deliver at date 1. The bank then neither reaps any output nor pays anything to shareholders from region $-i$. However, it can enjoy the proceeds $r_{-i,1}L_{-i}$ of the shares held on risky assets of banks of region $-i$ and use these funds as inside equity to reinvest. Assuming the bank under consideration borrows D_i (at a cost $r_{i,2}$), it can reinvest $J_i = \beta R_{-i,1}L_{-i} + D_i$ and provided $J_i \leq j_i$, profits from reinvestment write as

$$\Pi_{i,2} = J_i - R_{i,2}D_i \tag{2}$$

The date-0 problem for a bank from region i therefore consists in choosing an amount of risky assets L_{-i} to buy (from banks of region $-i$) and an amount of risky assets L_i to sell (to banks of region $-i$) which maximize expected profits Π_i, taking as given the returns $(R_{i,1}; R_{-i,1})$ and $(R_{i,2}; R_{-i,2})$:

$$\max_{L_i;L_{-i}} \Pi_i = (I_i - R_{i,1}L_i)\, R_{-i,2} + E\left[J_i - R_{i,2}D_i\right]$$
$$\text{s.t.} \begin{cases} I_i = 1 + L_i - L_{-i} \text{ and } L_i \leq \phi_i I_i \\ J_i = \beta R_{-i,1}L_{-i} + D_i \text{ and } D_i \leq \phi J_i \text{ with prob. } q \\ \text{or} \\ J_i \leq \overline{j_i} \text{ and } D_i \leq \overline{j_i} - \beta R_{-i,1}L_{-i} \text{ with prob. } 1-q \end{cases} \tag{3}$$

The constraint $L_i \leq \phi_i I_i$ stipulates that at date 0, a bank from region i cannot sell to banks of region $-i$ an amount of shares L_i which would exceed a fraction ϕ_i of the initial investment I_i. Similarly, the constraint $D_i \leq \phi J_i$ states that a distressed bank from region i faces a borrowing limit such that the amount borrowed D_i cannot exceed a fraction ϕ of date-1 reinvestment J_i. Conversely, when it faces a technological limit on a reinvestment, a distressed bank does not need to borrow more than the difference between maximum reinvestment j_i and its own funds $R_{-i,1}L_{-i}$.

To solve this problem we start with optimal borrowing at date 1. Assuming it is profitable for distressed banks of region i to borrow, i.e. $R_{i,2} \leq 1$, optimal borrowing and reinvestment satisfy

$$
\begin{gathered}
D_i = \tfrac{\phi}{1-\phi}\beta R_{-i,1}L_{-i} \text{ and } J_i = \tfrac{1}{1-\phi}\beta R_{-i,1}L_{-i} \text{ if } j_i = +\infty \\
\text{or} \\
D_i = \overline{j_i} - \beta R_{-i,1}L_{-i} \text{ and } J_i = \overline{j_i} \text{ if } j_i = \overline{j_i}
\end{gathered} \tag{4}
$$

Then denoting $1-\nu_i = (1-q)\overline{j_i}$, and $\varphi_i = \frac{\phi_i}{1-\phi_i}$, the problem for banks of region i ends up writing as:

$$
\begin{gathered}
\max_{L_i;L_{-i}} \left(1 - L_{-i} + \left(1 - R_{i,1}\right)L_i\right)R_{-i,2} + \left(1 - R_{i,2}\right)\left(1-\nu_i\right) + \left[\tfrac{q}{1-\phi} + \left[1 - \tfrac{q}{1-\phi}\right]R_{i,2}\right]\beta R_{-i,1}L_{-i} \\
\text{s.t. } L_i \leq \varphi_i\left(1 - L_{-i}\right)
\end{gathered} \tag{5}
$$

Here it is important to note that a bank from region i is restricted in the shares L_i it can sell -because of the issuance limit imposed by macro–prudential policy makers, but not in the shares L_{-i} it can buy, except for physical limits. Then, assuming $q+\phi=1$, the first order conditions imply that banks of region i optimal supply for claims L_i^s and optimal demand for claims L_{-i}^d (issued from banks of region $-i$) write as:

$$
L_i^s = \mathbf{1}\left[R_{i,1} \leq 1\right]\varphi_i\left(1 - L_{-i}\right) \text{ and } L_{-i}^d = \begin{cases} 0 \text{ if } \beta R_{-i,1} \leq R_{-i,2} \\ 1 \text{ if } \beta R_{-i,1} > R_{-i,2} \end{cases} \tag{6}
$$

Banks of region i issue liabilities at date 0, i.e. $L_i^s > 0$, if and only if this is profitable, i.e. $R_{i,1} < 1$. Conversely, they choose to buy shares on risky assets of banks of region $-i$, i.e. $L_{-i}^d > 0$, if and only if the return earned on these claims $\beta R_{-i,1}$ is sufficiently large. Indeed, the condition $\beta R_{-i,1} > R_{-i,2}$ simply states that the marginal benefit $\beta R_{-i,1}$ of holding shares on risky assets of banks from region $-i$ must be larger than the opportunity cost $R_{-i,2}$ of doing so.

A bank from region i buying one share of risky assets of banks of region $-i$ incurs a loss $R_{-i,2}$ when it is intact, since this unit could have yielded a return $R_{-i,2}$, had it not been used to buy shares. But, similarly when a bank of region i buys a unit of shares in risky assets of banks in region $-i$, and it is distressed, then the region i bank earns a return $\beta R_{-i,1}$, hence the condition $\beta R_{-i,1} > R_{-i,2}$ stating that the marginal benefit should exceed the opportunity cost for banks from region i to hold shares in risky assets of banks from region $-i$.

3.2 Equilibrium cross-border capital flows.

Using the optimal demand and supply for cross-border claims on risky assets (6), we can derive the equilibrium amount of shares exchanged at date 0 and the equilibrium return on these shares. According to the optimal supply and demand for claims defined in (6), shares are exchanged at date 0 from region $-i$ to region i if and only if the returns $R_{i,1}$ satisfy $R_{i,1} \leq 1$ and $\beta R_{i,1} > R_{i,2}$. The equilibrium amount of shares L_k sold by banks of region k to banks of region $-k$ therefore satisfies

$$L_k = \varphi_k \left(1 - L_{-k}\right) \mathbf{1} \left[R_{k,2} \leq \beta\right] \text{ for } k = \{i; -i\} \tag{7}$$

and equilibrium returns on capital flows exchanged at date 0 satisfy

$$\beta R_{k,1} = R_{k,2} \text{ for } k = \{i; -i\} \tag{8}$$

3.3 Equilibrium of the market for funding.

Once date-0 cross-border flows have been cleared, banks invest in risky assets. Then at date 1, uncertainty unravels and distressed banks can raise funds -to carry out reinvestment- from intact banks located in the other economy or from distressed banks located in the same economy which hold funds in excess of their reinvestment capacity. Considering the case where banks from region i are distressed and need to carry out reinvestment, the equilibrium of the market for funding at date 1 writes as

$$I_{-i} = \frac{q}{1-\phi} \beta R_{-i,1} L_{-i} + (1-q)\, j_i \tag{9}$$

On the left hand side of (9), we have the supply for funds from banks from region $-i$. For these banks, risky assets pay early. They hence earn at date 1 an output I_{-i}. Moreover banks from region $-i$ have also access to the deposit facilities in region i and region $-i$. They will hence supply their funds on the market only if the return $R_{i,2}$ they can earn on the market for funding satisfies $R_{i,2} \geq \max(\theta_i; \theta_{-i})$.

On the right hand side of (9), we have the demand for funds from banks of region i: a fraction q of these banks is credit constrained and can reinvest $\frac{1}{1-\phi} R_{-i,1} L_{-i}$ while a fraction $1-q$ is reinvestment constrained and reinvests j_i. Moreover banks from region i are willing to raise funds to finance reinvestment only if it is profitable to do so, i.e. $R_{i,2} \leq 1$. Hence assuming the funding cost $R_{i,2}$ satisfies $\max(\theta_i; \theta_{-i}) \leq R_{i,2} \leq \beta$ and using the property that $I_{-i} = 1 - L_i + L_{-i}$, and the pricing equation (8), the equilibrium of the market for funding when region i banks' risky assets pay late implies that the equilibrium cost of funds $R_{i,2}$ satisfies

$$R_{i,2} = \begin{cases} 1 \text{ if } L_i > \nu_i + (1 - R_{-i,2}) L_{-i} \\ \max(\theta_i; \theta_{-i}) \text{ if } L_i \leq \nu_i + (1 - R_{-i,2}) L_{-i} \end{cases} \tag{10}$$

The equilibrium condition (10) shows that the return $R_{i,2}$ on the market for ex post liquidity depends positively to the capital inflows L_i and negatively on capital outflows L_{-i}. Every thing else equal, higher capital flows L_i reduce the amount of funds available for ex post refinancing when risky assets of banks of region i turn out to pay late. Conversely, higher capital flows L_{-i} increase the amount of funds available for ex post refinancing when risky assets of banks of region i turn out to pay late. However in addition higher capital flows L_{-i} also increase the amount of funds available to banks of region i -through ex ante risk sharing contracts- which means these banks have more funds to reinvest and borrow against. Yet given the assumption $q + \phi = 1$, the first effect always dominates and the funding cost $R_{i,2}$ is always decreasing in L_{-i}.

3.4 Characterizing the decentralized equilibrium

Using expressions (8) and (10) for the return on cross-border capital flows and the cost of ex post funding, we can now determine the equilibrium funding costs $(R_{i,2}; R_{-i,2})$ and the equilibrium cross-border ex ante capital flows $(L_i; L_{-i})$ as a function of the primitives of the models, $(\theta_i; \theta_{-i})$ and $(\phi_i; \phi_{-i})$.

Proposition 1 *Denoting* $\theta = \max(\theta_i; \theta_{-i})$, *there is a pure strategy decentralized equilibrium in which claims* $(L_i; L_{-i})$ *exchanged at date* 0 *and funding costs* $(R_{i,2}; R_{-i,2})$ *satisfy*

$$L_i = \varphi_i \frac{1-\varphi_{-i}}{1-\varphi_i\varphi_{-i}} \text{ and } L_{-i} = \varphi_{-i} \frac{1-\varphi_i}{1-\varphi_i\varphi_{-i}} \text{ and } (R_{i,2}; R_{-i,2}) = \theta \tag{11}$$

if and only if the conditions

$$L_i \leq \nu_i + (1-\theta) L_{-i} \text{ and } L_{-i} \leq \nu_{-i} + (1-\theta) L_i \text{ and } \theta \leq \beta \tag{12}$$

hold together.

Proof. First, it can easily be shown that the case where the funding costs satisfy $R_{i,2} = R_{-i,2} = 1$ cannot be an equilibrium. This is because banks then perfer not to issue any risk sharing claims: $L_i = L_{-i} = 0$. As a result, the conditions under which the equilibrium with $R_{i,2} = R_{-i,2} = 1$ hold -$L_i > \nu_i + (1 - R_{-i,2}) L_{-i}$ and $L_{-i} > \nu_{-i} + (1 - R_{-i,2}) L_i$- cannot be satisfied. There can hence be no decentralized equilibrium with $R_{i,2} = R_{-i,2} = 1$.

Second, when the funding costs satisfy $R_{i,2} = R_{-i,2} = \theta$, then banks are better-off exchanging risk sharing claims if and only if $\theta \leq \beta$. When $\theta > \beta$ and banks do not exchange claims for risk sharing purposes, the equilibrium conditions -$L_i \leq \nu_i + (1 - R_{-i,2}) L_{-i}$ and $L_{-i} \leq \nu_{-i} + (1 - R_{-i,2}) L_i$- naturally hold. Conversely, when $\theta \leq \beta$, banks exchange claims L_i and L_{-i} such that $L_i = \varphi_i (1 - L_{-i})$ and $L_{-i} = \varphi_{-i} (1 - L_{-i})$ which yields (11). And this equilibrium holds if and only if interest rates $(\theta_i; \theta_{-i})$ and capital flows $(L_i; L_{-i})$ satisfy

$$\theta \leq \beta \text{ and } L_{-i} \leq \nu_i + (1-\theta) L_i \text{ and } L_i \leq \nu_{-i} + (1-\theta) L_{-i} \tag{13}$$

■

The decentralized equilibrium has two important properties. First, the decentralized equilibrium always features an excess funding supply, i.e. the funding costs $(R_{i,2}; R_{-i,2})$ are always at their lower bound θ. The reason is as follows: According to (8), the return $\beta R_{i,1}$ to selling insurance should equate the return $R_{i,2}$ on ex post funding. As a result, when the latter goes up -because for instance of a scarcity of ex post funding-, then the cost of selling insurance $R_{i,1}$ needs to increase disproportionately (by $1/\beta > 1$), which makes insurance so costly to sell that banks prefer to stop doing so. As a result, the demand for ex post funding goes down and the scarcity of funding assumed at the beginning disappears. To put it in a nutshell, the friction on the market for ex ante risk sharing prevents any scarcity of ex post funding at the equilibrium.[7]

Second, capital flows, L_i and L_{-i} are *negatively* related to each other through macro-prudential policy parameters φ_i and φ_{-i}, but positively related to each other through the equilibrium conditions $L_i \leq \nu_i + (1 - \max(\theta_i; \theta_{-i})) L_{-i}$ and $L_{-i} \leq \nu_{-i} + (1 - \max(\theta_i; \theta_{-i})) L_i$. When macro-prudential policy in region i is loosened, i.e. φ_i goes up, then capital inflows into region i, L_i go up but capital inflows into region $-i$, L_{-i} go down because more capital inflows into region i imply lower investment in region $-i$ and hence a tighter constraint on the ability to issue claims from banks of region $-i$. Conversely, when capital inflows into region i, L_i, go up and banks from region $-i$ end up distressed, then both the funding supply $1 - L_{-i} + L_i$ and the funding demand $\beta R_{i,1} L_i + 1 - \nu_{-i}$ go up as intact banks from region i can provide more liquidity given that they invested more while distressed banks from region $-i$ can also borrow more. However the supply for funding increases more than the demand for funding because the return that banks from region i earn on capital inflows is larger than the return they pay to banks from region $-i$. As a result, the increase in capital inflows L_i exacerbates the situation of excess supply. Banks from region i can then afford to invest more in risky assets of banks from region $-i$, hence the positive relationship between capital flows L_i and L_{-i}, without compromising the excess supply on the market for ex post funding. To put it in a nutshell, when capital inflows into one region increase, then the cost of ex post funding need not change, provided

[7] The reason for this paradox is relatively clear: introducing a friction on the market for risk sharing reduces the demand for ex post liquidity while it does not affect the supply, hence the lower likelihood of a scarcity of ex post funding. This can hold, for example, when the costs associated with cross-border risk sharing are only borne at the latest stage, so that the proceeds used to cover these costs are actually reinvested on the market for ex post liquidity. In this case, the supply for ex post liquidity would be unaffected by the presence of a friction on cross-border risk sharing while the demand for ex post would be reduced given that distressed banks which are credit constrained can borrow less.

capital inflows into the other region follow the same increasing pattern.

Last, the equilibrium conditions $L_i \leq \nu_i + (1 - \max(\theta_i; \theta_{-i})) L_{-i}$ and $L_{-i} \leq \nu_{-i} + (1 - \max(\theta_i; \theta_{-i})) L_i$ imply that larger capital flows in a given region L_i can be sustained when interest rates $(\theta_i; \theta_{-i})$ are lower. Indeed, lower interest rates reduce the return to cross-border capital flows and hence the demand for funding. As a result, the excess supply situation is compatible with larger capital inflows $(L_i; L_{-i})$.[8]

4 Optimal interest rate policy.

We now move to the question of interest rate determination. To answer this question we consider two polar cases. First, the case of a Nash equilibrium. In this equilibrium, a monetary authority sets in each region the interest rate, i.e. the return to the domestic risk free technology, with the aim to maximize domestic welfare, taking as given the interest rate in the other region. Second, we consider the case of cooperation in which interest rates are set to maximize global welfare, i.e. the sum of region i and region $-i$ banks profits.

4.1 The non-cooperative equilibrium.

In the non-cooperative equilibrium, each monetary authority determines the optimal domestic interest rate -that maximizes domestic banks' expected profits- as a best response to the other monetary authority's interest rate decision. Then, the problem for monetary authorities in region k $(k = \{i; -i\})$ consists in choosing the interest rate θ_k to solve

$$\begin{gathered} \max_{\theta_k} \quad \pi_k(\theta_k) = \left[1 + \left(1 - \tfrac{\theta}{\beta}\right) L_k\right] \theta + (1 - \theta)(1 - \nu_k) \\ \text{s.t. } \theta = \max(\theta_i; \theta_{-i}) \text{ and } L_i \leq \nu_i + (1 - \theta) L_{-i} \text{ and } L_{-i} \leq \nu_{-i} + (1 - \theta) L_i \end{gathered} \tag{14}$$

The following proposition details the non-cooperative equilibrium in interest rates:

[8] Note also that the property that lower interest rates allow for larger capital flows at the equilibrium is due to the presence of both fixed- and variable-scale reinvestment. If there were no banks with variable-scale reinvestment, then these conditions would write as $L_i \leq \nu_i + L_{-i}$ and $L_{-i} \leq \nu_{-i} + L_i$ and changes in interest rates $(\theta_i; \theta_{-i})$ would have no implication. Conversely, if there were no banks with fixed-scale reinvestment, then these conditions would write as $L_i \leq 1 + (1 - \max(\theta_i; \theta_{-i})) L_{-i}$ and $L_{-i} \leq 1 + (1 - \max(\theta_i; \theta_{-i})) L_i$ but given that capital flows cannot exceed the unit endowment, i.e. $L_i; L_{-i} \leq 1$, the equilibrium constraints would just be irrelevant.

Proposition 2 *Denoting i the region such that $L_i \leq \frac{\nu_i}{\nu_{-i}} L_{-i}$, optimal interest rates in the symmetric non cooperative equilibrium satisfy*

$$\theta_i = \theta_{-i} = \min\{\beta; \theta_n\} \text{ with } \theta_n = \frac{\beta}{2}\left[1 + \frac{\nu_i}{L_i}\right] \tag{15}$$

if and only if capital flows L_i and L_{-i} satisfy

$$L_i \leq \nu_i + (1 - \min\{\beta; \theta_n\}) L_i \text{ and } L_{-i} \leq \nu_{-i} + (1 - \min\{\beta; \theta_n\}) L_{-i} \tag{16}$$

Proof. The interest rates θ_i and θ_{-i} which respectively maximize expected profits π_i and π_{-i} for banks of region i and banks of region $-i$ satisfy

$$\theta_i = \max\left\{\theta_{-i}; \frac{\beta}{2}\left[1 + \frac{\nu_i}{L_i}\right]\right\} \text{ and } \theta_{-i} = \max\left\{\theta_i; \frac{\beta}{2}\left[1 + \frac{\nu_{-i}}{L_{-i}}\right]\right\}$$

Moreover, interest rates should satisfy $\theta_i; \theta_{-i} \leq \beta$, so that the equilibrium conditions $R_{k,1} = \beta R_{k,2} \leq 1$ hold for $k = \{i; -i\}$. Hence when $\frac{\nu_{-i}}{L_{-i}} \leq \frac{\nu_i}{L_i}$, the equilibrium interest rate when monetary policy makers play a Nash game writes as

$$\theta_i = \theta_{-i} = \min\{\beta; \theta_n\} \text{ with } \theta_n = \frac{\beta}{2}\left[1 + \frac{\nu_i}{L_i}\right]$$

Last, applying proposition 1, this situation is an equilibrium if and only if capital flows L_i and L_{-i} are such that (16) holds. ■

Monetary authorities face a relatively simple trade-off: On the one hand, setting a high interest rate raises domestic banks' profits when domestic risky assets pay-off early, because domestic banks reap a higher return when lending to distressed banks from the other region. On the other hand, setting a high interest rate raises the cost to sell insurance which reduces the amount of liquidity banks can provide on the market for funding when their risky assets pay early and hence their profits. In addition, setting a higher interest rate reduces the profits of domestic banks when they are distressed.

The equilibrium interest rate actually depends on the region where banks issue less liabilities ex ante L_i for risk sharing purposes. The intuition behind this result is relatively simple: when the global interest rate is set by, say for example monetary authorities of region i, then expected profits for banks of region $-i$ are weakly increasing in the domestic interest rate θ_{-i}. This is because, the global interest rate is exogenous for monetary authorities in Foreign. As a result, there is no cost to setting a higher domestic interest rate. Monetary authorities in region $-i$ therefore set θ_{-i} at its maximum value which is here by definition, θ_i. By contrast, there is a cost for monetary authorities in region i to setting a high domestic interest rate θ_i because this raises the return paid on claims L_i issued ex ante for risk sharing purposes. In addition, this also raises the cost of funding raised ex post in case of distress, which cuts profits for distressed banks which end up facing a reinvestment constraint. And the cost of setting a higher interest rate θ_i is a positive function of the amount of claims L_i issued ex ante and a negative function of the fraction of banks q which end up facing a credit constraint when distressed. The optimal interest rate set by the monetary authority in region i, is hence a decreasing in L_i and increasing in ν_i. In the special case where $\nu_i = \nu_{-i}$ and capital flows are exogenous, the global interest rate is set by monetary authorities of the region in which banks buy more shares into the other region's risky assets than they sell of their own risky assets, i.e. the region which has a positive net foreign asset position.[9] Turning to expected profits those of banks of region i write as

$$\pi_{k,n} = 1 - \nu_k + \min\{\beta; \theta_n\} \left[\nu_k + \left(1 - \min\left\{ 1; \frac{\theta_n}{\beta} \right\} \right) L_k \right] \tag{17}$$

As was noted above, the interest rate that would maximize expected profits for banks in region k writes as $\min\left\{\beta; \frac{\beta}{2}\left[1 + \frac{\nu_k}{L_k}\right]\right\}$. The global interest rate under the Nash equilibrium θ_n^s is therefore optimal for banks of region i but too high for banks of region $-i$.

[9] We will see that when we allow regions to be asymmetric, i..e $\nu_i > \nu_{-i}$ and capital flows and thereby macro-prudential policy are endogenous, this result will be reversed: the interest rate setting region under Nash monetary policies will actually be the one running a negative, not a positive, net foreign asset position, the reason being that optimal capital inflows L_k $(k = \{i; -i\})$ into region k are an increasing with the parameter ν_k.

4.2 The cooperative equilibrium.

In the cooperative equilibrium, each monetary authority still determines the optimal domestic interest rate as a best response to the other monetary authority's interest rate decision. The only difference with the non-cooperative equilibrium is that the two monetary authorities now maximize a common criterion, global welfare, i.e. the sum region i and region $-i$ banks' expected profits. Given that the funding costs ($R_{i,2}$ and $R_{-i,2}$) can take two values, either $\max\{\theta_i;\theta_{-i}\}$ or 1, the problem in the cooperative equilibrium consists in choosing the interest rates θ_i and θ_{-i} which solve

$$\begin{gathered}\max_{\theta_i;\theta_{-i}} \quad \pi(\theta_i;\theta_{-i}) = \pi_i(\theta_i) + \pi_{-i}(\theta_{-i}) \\ \text{s.t. } L_i \leq \nu_i + (1-\theta) L_{-i} \text{ and } L_{-i} \leq \nu_{-i} + (1-\theta) L_i\end{gathered} \tag{18}$$

The following proposition then details the cooperative equilibrium based on the solution to this problem.

Proposition 3 *Denoting $\nu = \nu_i + \nu_{-i}$ and $L = L_i + L_{-i}$, optimal interest rates in the symmetric cooperative equilibrium write as $\theta_i = \theta_{-i} = \theta_c$ with*

$$\theta_c = \min\{\beta;\theta_c\} \;\; \textit{with } \theta_c = \frac{\beta}{2}\left[1+\frac{\nu}{L}\right] \tag{19}$$

if and only if cross-border capital flows L_i and L_{-i} satisfy

$$L_{-i} \leq \nu_{-i} + (1-\min\{\beta;\theta_c\}) L_i \;\; \textit{and } L_i \leq \nu_{-i} + (1-\min\{\beta;\theta_c\}) L_{-i} \tag{20}$$

Proof. The interest rates θ_i and θ_{-i} which maximize the sum of expected profits $\pi_i + \pi_{-i}$ satisfy

$$\theta_i = \max\left\{\theta_{-i};\frac{\beta}{2}\left[1+\frac{\nu}{L}\right]\right\} \text{ and } \theta_{-i} = \max\left\{\theta_i;\frac{\beta}{2}\left[1+\frac{\nu}{L}\right]\right\}$$

Moreover, as in the case of the Nash equilibrium, interest rates should satisfy $\theta_i; \theta_{-i} \leq \beta$. As a result, the equilibrium interest rates when monetary policy makers play a cooperative game write as

$$\theta_i = \theta_{-i} = \min\{\beta; \theta_c\} \text{ with } \theta_c = \frac{\beta}{2}\left[1 + \frac{\nu}{L}\right] \tag{21}$$

Last applying proposition 1, this situation is an equilibrium if and only if capital flows L_i and L_{-i} are such that (20) holds. ■

The optimal interest rates under Nash or cooperation have three important properties. First, contrary to the optimal interest rate θ_n in the Nash equilibrium which depends only on the characteristics of *one* region (at least locally), the optimal interest rate θ_c in the cooperative equilibrium depends on the characteristics of *both* regions. This difference is important because it has implications for how the pledgeability parameters φ_i and φ_{-i} affect the optimal interest rate, and thereby how macro-prudential policy affects financing conditions. Specifically, in the Nash equilibrium, assuming the optimal interest rate θ_n depends on region i characteristics, an increase in the fraction φ_i raises the amount of claims L_i issued by banks of region i and hence reduces the optimal interest rate θ_n. Conversely, an increase in the fraction φ_{-i} cuts the amount of claims L_i issued by banks of region i and therefore raises the optimal interest rate θ_n:

$$\frac{\partial \theta_n}{\partial \varphi_i} \leq 0 \leq \frac{\partial \theta_n}{\partial \varphi_{-i}} \tag{22}$$

But in the case case of the cooperative equilibrium, both an increase in the fraction φ_i and/or in the fraction φ_{-i} has a negative effect on the global interest rate θ_c^s because under cooperation, the optimal interest rate depends (negatively) on global capital flows which tend to increase in response to a positive change in either φ_i or φ_{-i}.

$$\frac{\partial \theta_c}{\partial \varphi_i} \leq 0 \text{ and } \frac{\partial \theta_c}{\partial \varphi_{-i}} \leq 0 \tag{23}$$

Second, the optimal interest rates set in the cooperative equilibrium are strictly lower than those set in the non-cooperative equilibrium. To see that, let us recall that i denotes the region setting the global interest rate in the Nash equilibrium. Then the optimal interest rate is higher under the Nash equilibrium if and only if $\frac{\nu_i}{L_i} \geq \frac{\nu}{L}$, which is equivalent to $\frac{\nu_i}{L_i} \geq \frac{\nu_{-i}}{L_{-i}}$, which holds by definition if i is the region setting the interest rate in the Nash equilibrium. Monetary authorities therefore set too high interest rates in the Nash equilibrium, this being due to the fact that monetary authorities in the interest rate setting region do not internalize the negative effect of their own decision on expected profits of banks in the other region.

Third, given the difference in optimal interest rates under cooperation vs. Nash, we can compute the welfare gain stemming from cooperation. Denoting respectively π_c and π_n global expected profits under the cooperative and the Nash equilibrium and following on previous notation (i is the region setting the global interest rate in the Nash equilibrium), the gain in global welfare stemming from cooperation writes as

$$\pi_c - \pi_n = (\theta_n - \theta_c)\left[\left[\frac{\theta_c + \theta_n}{\beta} - 1\right] L - \nu\right] = \frac{\beta}{4}\left[\left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right] \frac{L_{-i}}{L_i + L_{-i}}\right]^2 [L_i + L_{-i}] \tag{24}$$

From this expression, we can see that gains to monetary cooperation are decreasing in L_i but increasing in L_{-i}. This is because higher capital inflows L_i into region i tend to reduce the difference between the optimal interest rates under Nash and under cooperation $\theta_n - \theta_c$. Conversely, higher capital inflows L_{-i} into region $-i$ tend to increase the difference between the optimal interest rates under Nash and under cooperation $\theta_n - \theta_c$. Next we investigate how gains to monetary policy cooperation are distributed among regions.

Proposition 4 *Banks from the region whose monetary authorities set the global interest rate in the Nash equilibrium, enjoy lower welfare under cooperation.*

Proof. Expected profits for banks of region k under cooperative interest rate setting $\pi_{k,c}$ and under Nash interest rate setting $\pi_{k,n}$ write as

$$\pi_{k,s} = [1 + (1 - \theta_s) L_k] \theta_c + (1 - \theta_s)(1 - \nu_k) \text{ for } s = \{n; c\} \tag{25}$$

The change in expected profits for banks of region k, when comparing cooperation to Nash writes as

$$\pi_{k,c} - \pi_{k,n} = \left[\left[\frac{\theta_c + \theta_n}{\beta} - 1\right] L_k - \nu_k\right] (\theta_n - \theta_c)$$

Using the expressions for θ_c and θ_n, this expression writes as

$$\pi_{k,c} - \pi_{k,n} = \frac{\beta}{4}\left[\left(\frac{\nu_{-k}}{L_{-k}} - \frac{\nu_k}{L_k}\right)\frac{L_{-k}}{L_k + L_{-k}} + \left(\frac{\nu_i}{L_i} - \frac{\nu_k}{L_k}\right)\right]\left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right]\frac{L_{-i}}{L_i + L_{-i}} L_k$$

Hence, considering banks from region $-i$, this quantity writes as

$$\pi_{-i,c} - \pi_{-i,n} = \frac{\beta}{4}\left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right]^2 \left[\frac{L_{-i}}{L_i + L_{-i}}\right]^2 [2L_i + L_{-i}]$$

Banks from region $-i$ therefore always benefit from monetary cooperation. However for banks of region i , the change in expected profits writes as

$$\pi_{i,c} - \pi_{i,n} = -\frac{\beta}{4}\left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right]^2 \left[\frac{L_{-i}}{L_i + L_{-i}}\right]^2 L_i$$

Banks from the interest rate setting region therefore always lose when switching to the cooperative equilibrium. ■

The intuition for the asymmetry in the gains to monetary cooperation and for why the region setting the interest rate in the Nash equilibrium looses to cooperation is as follows. The Nash equilibrium is inefficient at the global level for two reasons: First competition on the market for ex post funding drives up the global interest rate to the maximum level of the two interest rates. Second, policy makers in the interest setting region do not internalize the negative effect of high interest rates on banks from the other region, which eventually leads to interest rates being too high. As a result, as was noted above, the global interest rate while optimal for banks from the interest rate setting region, happens to be too high for banks from the non-interest rate setting region. Moving then to cooperation with a lower interest rate, the interest rate setting region is bound to lose while the other country should win by definition.

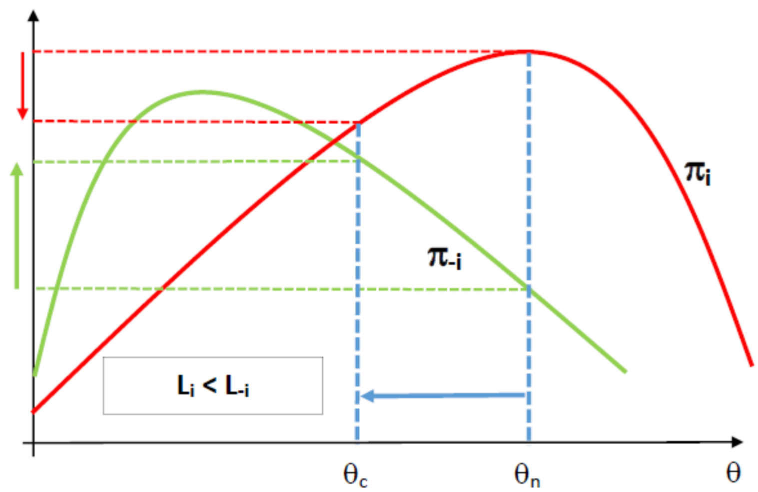

Figure 3: Gains and losses from monetary policy cooperation with $\nu_i = \nu_{-i}$.

Last, two properties of the global interest rate under the cooperative equilibrium are worth mentioning. First, larger holdings of cross-border assets imply a lower global interest rate. This is because when banks issue a large amount of liabilities for risk sharing purposes, then a marginally higher global interest rate has a major negative impact on banks' expected profits, as this raises the yield banks have to pay on such liabilities issued at date 0 for risk sharing purposes. Hence, as the amount of liabilities issued at date 0 gets larger, then the global planner favours a relatively lower global interest rate. For example, considering the case where $\nu_i = \nu_{-i} = 1/2$ and $\beta = 0.8$, Figure 4 plots the change in expected profits stemming from the move from Nash to cooperative monetary policy for different combinations of cross-border risk sharing, $(L_i; L_{-i})$. In this case, monetary policy cooperation is unambiguously welfare improving at the global level and this welfare improvement is increasing in L_{-i} and decreasing in L_i.

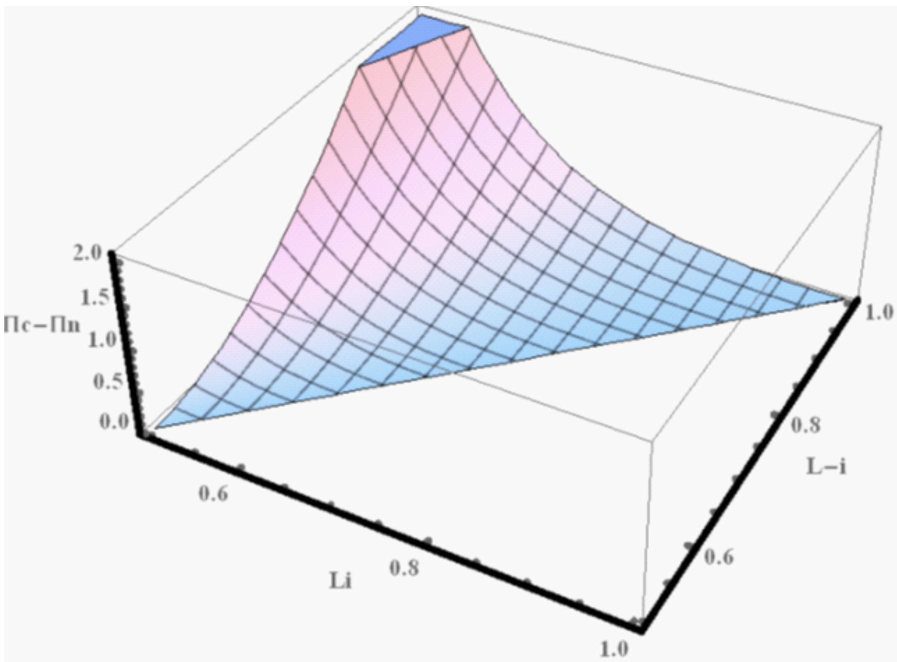

Figure 4: Global welfare gains from monetary policy cooperation.

This is because the inefficiency stemming from the global interest rate under the Nash equilibrium being too high increases with capital inflows L_{-i} into region $-i$. Moving from Nash to cooperation reduces the global interest rate and the more so the larger the capital flows L_{-i} into region $-i$. Conversely, when capital flows L_i into region i are larger, the inefficiency stemming from the global interest rate under the Nash equilibrium being too high is reduced because the global interest rate decreases with capital inflows L_{-i}. Hence the benefits of cooperation also tend to be lower. Next, Figure 5 plots the change in expected profits for banks of each region stemming from the move from Nash to cooperative monetary policy, once again assuming $\nu_i = \nu_{-i} = 1/2$ and $\beta = 0.8$. The graph confirms that the interest rate setting region tends to lose when monetary policy is set cooperatively relative to the case of Nash monetary policies. Conversely, the region which does not set the global interest rate tends to gain. In addition under this specific calibration ($\nu_i = \nu_{-i} = 1/2$ and $\beta = 0.9$), the graph still shows that for any combination of capital flows $(L_i; L_{-i})$, losses for the interest rate setting region tend to be one order of magnitude smaller than the gains for the region which does not set the global interest rate.

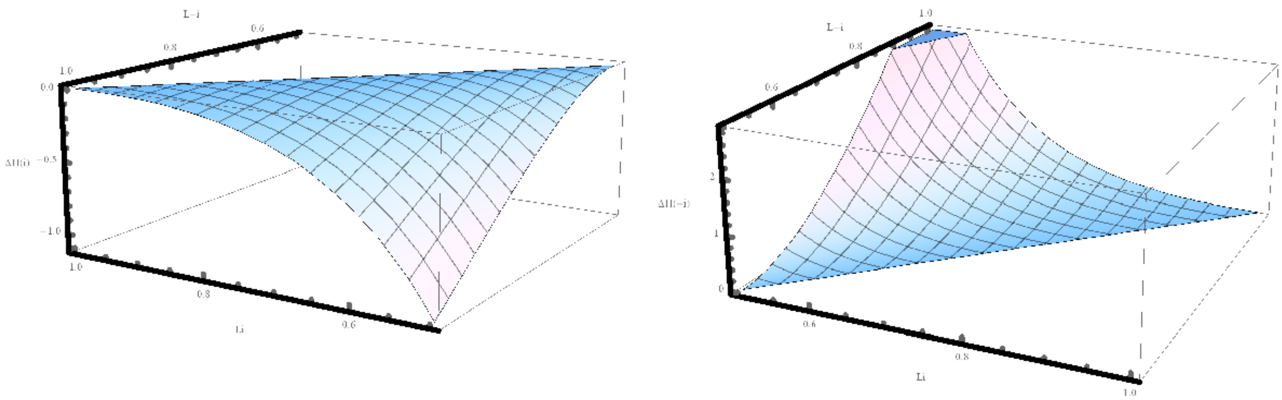

Figure 5: Regional gains and losses from monetary policy cooperation

5 Optimal macroprudential policy.

Up to now, we have studied the problem of monetary authorities in each region choosing the interest rates θ_i and θ_{-i} at which they can offer banks to park their funds. We now turn to the problem faced by macro-prudential authorities. As stated above, macro-prudential authorities can affect the amount of liabilities L_i and L_{-i} banks issue and thereby the amount of cross-border risk sharing. Macro-prudential authorities in each region can do so because they can indeed choose the maximum fraction ϕ_i of the initial investment in risky assets, domestic banks are allowed to pledge to outside investors.[10] To simplify further computations, we assume that each macro-prudential authority can pick up the parameter $\varphi_i = \frac{\phi_i}{1-\phi_i}$ in each region.

5.1 Optimal risk sharing with non-cooperative interest rate setting.

In this section, we assume that interest rates are set in a Nash equilibrium. Under this assumption, we first look at the case of optimal macro-prudential policies in the non-cooperative equilibrium and then turn to determining optimal macro-prudential policies under cooperation.

[10] To simplify further computations, we assume each macro-prudential authority can pick up the parameter $\varphi_i = \frac{\phi_i}{1-\phi_i}$ in each region.

5.1.1 Non-cooperative risk sharing.

When monetary and macro-prudential policies are determined in a Nash equilibrium, then the problem for macro-prudential authorities in region k consists in solving

$$\begin{aligned} &\max_{\varphi_k} \pi_k\left(\theta_n; L_k\right) = 1 - \nu_k + \theta_n \left[\nu_k + \left(1 - \tfrac{1}{\beta}\theta_n\right) L_k\right] \\ &\text{s.t.} \begin{cases} L_k = \frac{\varphi_k\left(1-\varphi_{-k}\right)}{1-\varphi_{-k}\varphi_k} \text{ and } \theta_n = \frac{\beta}{2}\left(1 + \frac{\nu_i}{L_i}\right) \text{ and } \nu_i \leq L_i \leq \frac{\nu_i}{\nu_{-i}} L_{-i} \\ L_{-i} \leq \nu_{-i} + \left(1 - \theta_n\right) L_i \text{ and } L_i \leq \nu_i + \left(1 - \theta_n\right) L_{-i} \end{cases} \end{aligned} \tag{26}$$

In this problem, macro-prudential policy makers in each region control the amount of capital inflows and outflows through the parameter φ_k that they can choose. Moreover, the expression for the global interest rate θ_n depends on capital flows L_i into region i, hence the constraints $\nu_i \leq L_i \leq \frac{\nu_i}{\nu_{-i}} L_{-i}$. Last the decentralized equilibrium requires that capital flows $(L_i; L_{-i})$ satisfy the constraints $L_{-i} \leq \nu_{-i} + (1 - \theta_n) L_i$ and $L_i \leq \nu_i + (1 - \theta_n) L_{-i}$.

Proposition 5 *Denoting i the region such that $\nu_i \geq \nu_{-i}$, when interest rates and ex ante capital flows are determined non-cooperatively then macro-prudential authorities choose capital flows L_i and L_{-i} such that*

$$L_i = \nu_i + \left(1 - \theta_n\right) L_{-i} \textit{ and } L_{-i} = \min\left\{\nu_{-i} + \left(1 - \theta_n\right) L_i; L_n\right\} \tag{27}$$

where

$$L_n = \frac{2\nu_{-i}\nu_i L_i + L_i^2 - \nu_i^2}{L_i^2 + \nu_i^2}$$

Proof. Let us start with the case of banks of region i, i.e. the region whose monetary authority sets the global interest rates. Then, banks expected profits are strictly increasing in φ_i: Allowing banks to pledge a larger fraction of their initial investment in risky assets raises capital inflows into region i which is positive for banks' profits. Moreover this reduces the global interest rate θ_n, but this has no effects on banks' expected profits given that θ_n is by definition the profit maximizing interest rate:

$$\frac{\partial \pi_i}{\partial \varphi_i} = \frac{\partial \pi_i}{\partial L_i}\frac{\partial L_i}{\partial \varphi_i} + \underbrace{\frac{\partial \pi_i}{\partial \theta_n}}_{=0}\frac{\partial \theta_n}{\partial L_i}\frac{\partial L_i}{\partial \varphi_i} > 0 \tag{28}$$

Macro-prudential authorities in region i therefore choose to maximize capital inflows L_i. Given the upper bounds on capital flows L_i, we have

$$L_i = \min\left\{\nu_i + (1-\theta_n) L_{-i}; \frac{\nu_i}{\nu_{-i}} L_{-i}\right\} \tag{29}$$

Turning to the case of banks of region $-i$, i.e. the region whose monetary authority does *not* set the global interest rate, expected profits π_{-i} depend on the fraction φ_{-i} in two opposite ways. On the one hand, it increases capital inflows which raises banks' expected profits. However it also also reduces capital outflows from region $-i$ -or equivalently capital inflows into region i, which raises the global interest rate and thereby reduces banks' expected profits as the global interest rate is, by construction, too high for the region not setting the global interest rate:

$$\frac{\partial \pi_{-i}}{\partial \varphi_{-i}} = \left(1 - \frac{\theta_n}{\beta}\right)\theta_n \frac{\partial L_{-i}}{\partial \varphi_{-i}} + \left[\nu_{-i} + \left(1 - \frac{2}{\beta}\theta_n\right) L_{-i}\right]\frac{\partial \theta_n}{\partial L_i}\frac{\partial L_i}{\partial \varphi_{-i}} \tag{30}$$

Moreover one can easily check that banks expected profits π_{-i} are concave in the macro-prudential policy φ_{-i}. As a result, macro-prudential policy makers in region $-i$ choose to set φ_{-i} such that $\frac{\partial \pi_{-i}}{\partial \varphi_{-i}} = 0$. And making use of the property that $L_i = \varphi_i (1 - L_{-i})$, the first order condition for region $-i$ implies that capital flows L_i and L_{-i} should satisfy

$$\frac{\beta}{4}\left[1 - \left(\frac{\nu_i}{L_i}\right)^2 - 2\left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right]\frac{L_{-i}}{L_i}\frac{\nu_i}{L_i}\frac{L_i}{1 - L_{-i}}\right]\frac{\partial L_{-i}}{\partial \varphi_{-i}} = 0 \tag{31}$$

When the condition $\frac{\nu_i}{\nu_{-i}} L_{-i} < \nu_i + (1-\theta_n) L_{-i}$ holds, then capital inflows into region i satisfy $L_i = \frac{\nu_i}{\nu_{-i}} L_{-i}$ and expected profits for banks of region $-i$ are by implication strictly increasing in domestic banks ex ante leverage, $\frac{\partial \pi_{-i}}{\partial \varphi_{-i}} > 0$. Optimal capital inflows into region $-i$ therefore write as $L_{-i} = \nu_{-i} + (1-\theta_n) L_i$. And with these expressions for optimal capital inflows, the condition $\frac{\nu_i}{\nu_{-i}} L_{-i} < \nu_i + (1-\theta_n) L_{-i}$ simplifies as $\nu_i < \nu_{-i}$, which by assumption does not hold since we assumed $\nu_i > \nu_{-i}$. The case where optimal capital inflows into region i satisfy $L_i = \frac{\nu_i}{\nu_{-i}} L_{-i}$ is therefore not possible in the Nash equilibrium. We are hence left with case $L_i = \nu_i + (1-\theta_n) L_{-i} \leq \frac{\nu_i}{\nu_{-i}} L_{-i}$. Then, the first-order condition which determines optimal capital inflows L_{-i} into region $-i$ writes as

$$\left(\frac{L_i}{\nu_i}\right)^2 = 1 + 2\frac{L_{-i} - \frac{\nu_{-i}}{\nu_i} L_i}{1 - L_{-i}} \tag{32}$$

Hence denoting L_n the value of L_{-i} which satisfies (32), optimal capital flows under the Nash equilibrium therefore satisfy

$$L_i = \nu_i + (1-\theta_n) L_{-i} \text{ and } L_{-i} = \min\left\{\nu_{-i} + (1-\theta_n) L_i; L_n\right\} \tag{33}$$

■

Consistent with what was described above, macro-prudential authorities in the region which does *not* set the global interest rate may refrain from maximizing capital inflows. For such authorities, the trade-off is that on the one hand larger capital inflows tend to increase expected profits through the direct effect. But on the other hand, larger capital inflows reduce expected profits indirectly through their effect on the global interest rate. Indeed when capital inflows into the region which *does not* set the global interest rate increase, this leads to lower capital inflows into the region which *does* set the global interest rate, which raises the global interest rate. However, we have seen that when monetary policy is determined in a Nash equilibrium, the global interest rate is too high for the region which does not set the global interest rate. As a result, raising it further always reduces banks expected profits. This is why macro-prudential authorities in the region which does not set the global interest rate actually limit the amount of capital inflows as this contributes to reduce the global interest rate and hence improve domestic welfare.

5.1.2 Cooperative risk sharing.

Let us now turn to optimal macroprudential policies under cooperation. Assuming as in the previous section that monetary policy is conducted in a Nash game, the global planner chooses φ_i and φ_{-i} to solve

$$\max_{\varphi_i;\varphi_{-i}} \pi_c(\theta_n; L_i; L_{-i}) = \pi_i(\theta_n; L_i) + \pi_{-i}(\theta_n; L_{-i})$$
$$\text{s.t.} \begin{cases} L_k = \frac{\varphi_k(1-\varphi_{-k})}{1-\varphi_{-k}\varphi_k} \text{ and } \theta_n = \frac{\beta}{2}\left(1+\frac{\nu_i}{L_i}\right) \text{ and } \nu_i \leq L_i \leq \frac{\nu_i}{\nu_{-i}} L_{-i} \\ L_{-i} \leq \nu_{-i} + (1-\theta_n) L_i \text{ and } L_i \leq \nu_i + (1-\theta_n) L_{-i} \end{cases} \tag{34}$$

We can derive the following proposition.

Proposition 6 *Denoting i the region such that $\nu_i \geq \nu_{-i}$, when interest rates are set non-cooperatively but ex ante capital flows are set cooperatively, then macro-prudential authorities choose capital flows L_i and L_{-i} to satisfy:*

$$L_i = \nu_i + (1-\theta_n) L_{-i} \text{ and } L_{-i} = \nu_{-i} + (1-\theta_n) L_i \tag{35}$$

Proof. Given that region i set the global interest rate θ_n, expected π_c are strictly increasing in φ_i since we have

$$\frac{\partial \pi_c}{\partial \varphi_i} = \frac{\beta}{4}\left[\left(1-\left[\frac{\nu_i}{L_i}\right]^2\right)(1-\varphi_{-i}) + 2\left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right]\frac{L_{-i}}{L_i}\frac{\nu_i}{L_i}\right]\frac{\partial L_i}{\partial \varphi_i} > 0 \tag{36}$$

When macro-prudential policy makers in region i allow domestic banks to issue more claims for risk sharing, global capital flows increase and so do global expected profits. Moreover given that monetary policy is conducted in a Nash game, global funding conditions are too tight. Hence larger capital inflows into region i as they contribute to ease funding conditions on the market for ex post funding also contribute to increase global expected profits. Optimal capital inflows into region i therefore still satisfy $L_i = \min\left\{\frac{\nu_i}{\nu_{-i}} L_{-i}; \nu_i + (1-\theta_n) L_{-i}\right\}$. Yet, when macro-prudential policy in region i is such that $L_i = \frac{\nu_i}{\nu_{-i}} L_{-i}$, then global expected profits π_c are strictly increasing in φ_{-i} and optimal macro-prudential policy in region $-i$ satisfies $L_{-i} = \nu_{-i} + (1-\theta_n) L_i$. However, as was mentioned above, this case is not possible because it requires that $\nu_i < \nu_{-i}$ and the assumption we are working with is $\nu_i > \nu_{-i}$. We are hence left with the case where optimal macro-prudential policy in region i is such that capital inflows L_i satisfy

$$L_i = \nu_i + (1-\theta_n) L_{-i} \tag{37}$$

We then have two possible cases. The first case is the one where optimal macro-prudential policy in region $-i$ is an interior solution and it satisfies

$$\frac{\partial \pi_c}{\partial \varphi_{-i}} + \frac{d\varphi_i}{d\varphi_{-i}} \frac{\partial \pi_c}{\partial \varphi_i} = 0 \text{ with } \frac{d\varphi_i}{d\varphi_{-i}} \frac{\partial L_i}{\partial \varphi_i} = \frac{\varphi_i + (1-\theta_n) + \varphi_{-i} \frac{\partial \theta_n}{\partial L_i} L_{-i}}{1 + \varphi_{-i}(1-\theta_n) + \frac{\partial \theta_n}{\partial L_i} L_{-i}} \frac{\partial L_{-i}}{\partial \varphi_{-i}} \tag{38}$$

the second expression measuring how macro-prudential policy makers in region i respond to a change in macro-prudential policy in region $-i$ along the optimality condition $L_i = \nu_i + (1-\theta_n) L_{-i}$. Alternatively optimal macro-prudential policy in region $-i$ can be a corner solution and capital inflows into region $-i$ satisfy $L_{-i} = \nu_i + (1-\theta_n) L_i$. To solve for the case of the interior solution, we can use the expressions for the first derivatives of global expected profits $\frac{\partial \pi_c}{\partial \varphi_i}$ and $\frac{\partial \pi_c}{\partial \varphi_{-i}}$. Given that the latter writes as

$$\frac{\partial \pi_c}{\partial \varphi_{-i}} = \frac{\beta}{4} \left[\left(1 - \left[\frac{\nu_i}{L_i} \right]^2 \right) (1-\varphi_i) - 2 \frac{L_{-i}}{\nu_i} \left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}} \right] \left[\frac{\nu_i}{L_i} \right]^2 \varphi_i \right] \frac{\partial L_{-i}}{\partial \varphi_{-i}} \tag{39}$$

And denoting A the positive scalar such that $\frac{d\varphi_i}{d\varphi_{-i}} \frac{\partial L_i}{\partial \varphi_i} = A \frac{\partial L_{-i}}{\partial \varphi_{-i}}$, the expression for the first order condition (38) simplifies as

$$1 - \left[\frac{\nu_i}{L_i} \right]^2 = 2 \left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}} \right] \frac{L_{-i}}{L_i} \frac{\nu_i}{L_i} \frac{\varphi_i - A}{(1-\varphi_i) + (1-\varphi_{-i}) A}$$

Yet given that $A > \varphi_i$ this condition would imply that $L_i < \nu_i$, which is not possible given that we have $L_i = \nu_i + (1-\theta_n) L_{-i} > \nu_i$. There is hence no φ_{-i} which maximizes global expected profits π_c and is an interior solution. As a result, we a left with a single possibility: capital flows which maximizes global expected profits satisfy

$$L_i = \nu_i + (1-\theta_n) L_{-i} \text{ and } L_{-i} = \nu_{-i} + (1-\theta_n) L_i \tag{40}$$

And one can check that under such a solution we have $\frac{\partial \pi_c}{\partial \varphi_{-i}} + \frac{d\varphi_i}{d\varphi_{-i}} \frac{\partial \pi_c}{\partial \varphi_i} > 0$ so that the constraint $L_{-i} \leq \nu_{-i} + (1-\theta_n) L_i$ is effectively binding. ■

Under cooperative macro-prudential policies, capital flows tend to be larger and the global interest rate tends to be lower than under the non-cooperative equilibrium. The intuition for these results is pretty straightforward. When interest rates are determined in the Nash equilibrium, the equilibrium interest rate is optimal for banks from the interest rate setting region, but too high for those from the region which does not set the global interest rate. As a result, macro-prudential authorities in the former region just aim at maximizing capital inflows while macro-prudential authorities in the latter region use capital inflows to fulfill two opposite goals: increasing profits by allowing larger capital inflows on the one hand vs. increasing profits by reducing the global interest rate on the other hand. And as was noted above, these two goals are not compatible because reducing the global interest rate implies reducing capital inflows. As a result the region which does not set the global interest rate trades off the benefits of capital inflows against the cost of suboptimal funding conditions, which leads it to limit capital inflows.

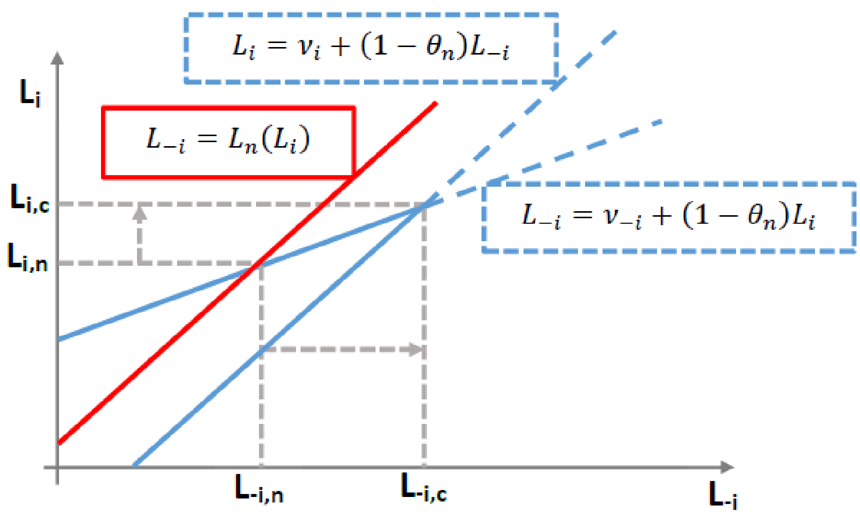

Figure 6: Optimal macro-prudential policies under Nash monetary policies.

Now under cooperative macro-prudential policies, the previous arguments still hold with one key difference: macro-prudential policy makers now internalize how their own decisions affect those taken by macro-prudential policy makers of the other region. In other words it may be optimal to consider a policy that on its own looks "sub-optimal" because doing so allows macro-prudential policy in the other region to take actions that can yield sizeable gains to global expected profits that outweigh any possible loss. This is what happens in the cooperative equilibrium: Increasing capital inflows into the interest rate setting region L_i always contributes to increase global expected profits, because it increases global capital flows and reduces the Nash interest rate that is too high from a global perspective. As result, macro-prudential policy in the interest rate setting region allows as much capital L_i to flow in as possible. And given the cross-border externality -$L_i \leq \nu_i + (1-\theta_n) L_{-i}$- which states that capital inflows L_i are limited by the amount of capital L_{-i} flowing in the other region, macro-prudential policy makers in region $-i$ -the region which does not set global funding conditions- is willing to deviate from strict maximization, i.e. to set φ_{-i} such that $\partial \pi_c / \partial \varphi_{-i} < 0$, because of the positive spill-over this has, through macro-prudential policy φ_i in the interest rate setting region. In practise, the non-interest rate setting region allows as large capital inflows L_{-i} as possible, i.e. $L_{-i} = \nu_{-i} + (1-\theta_n) L_i$, even if at the margin this is reducing global expected profits ($\left.\frac{\partial \pi_c}{\partial \varphi_{-i}}\right|_{L_{-i}=\nu_{-i}+(1-\theta_n)L_i} < 0$) because this allows region i to attract more capital inflows L_i. And the positive effect on global expected profits of such policy outweighs any loss related to region $-i$ deviating from strict global profit maximization.

5.1.3 Gains to macro-prudential policy cooperation under Nash interest rate setting.

Comparing expected profits under cooperative macro-prudential policies with expected profits under Nash macro-prudential policies, we can easily compute the welfare gains each region enjoys as a result from cooperation. Denoting $\pi_{k,c}$ (resp. $\pi_{k,n}$) expected profits of banks of region k under cooperative macro-prudential policies (resp. under Nash macro-prudential policies) when monetary policies are conducted in a Nash game, we have

$$\pi_{k,s} = 1 - \nu_k + \left[\nu_k + \left(1 - \frac{1}{\beta}\theta_{n,s}\right) L_{k,s}\right] \theta_{n,s} \text{ for } s = \{n; c\} \tag{41}$$

where $L_{k,n}$ (resp. $L_{k,c}$) denotes capital inflows into region k when macro-prudential policy is Nash (resp. cooperative) under the assumption that monetary policy is Nash. Similarly, the equilibrium interest rate $\theta_{n,s}$ for when macro-prudential policy is Nash ($s = n$) (resp. cooperative ($s = c$)) assuming monetary policy is Nash writes as

$$\theta_{n,s} = \min\left\{\beta; \frac{\beta}{2}\left(1 + \frac{\nu_i}{L_{i,s}}\right)\right\} \tag{42}$$

Then computing the difference in expected profits for banks of region k under alternative macro-prudential policies we have

$$\pi_{k,n} - \pi_{k,c} = \left[\nu_k + \left[1 - \frac{\theta_{n,n} + \theta_{n,c}}{\beta}\right] L_{k,n}\right] (\theta_{n,n} - \theta_{n,c}) - \left(1 - \frac{1}{\beta}\theta_{n,c}\right) \theta_{n,c} (L_{k,c} - L_{k,n}) \tag{43}$$

Assuming $L_{k,s} \geq \nu_k$ for $k = \{i; -i\}$ and $s = \{n; c\}$, this expression simplifies in the case of the interest rate setting region, as

$$\pi_{i,c} - \pi_{i,n} = \frac{\beta}{4}\left[1 - \frac{\nu_i}{L_{i,n}} \frac{\nu_i}{L_{i,c}}\right] [L_{i,c} - L_{i,n}] \tag{44}$$

As is clear, the increase in expected profits for banks if region i is proportional to the increase in capital inflows $[L_{i,c} - L_{i,n}]$, which is consistent with the fact that the global interest rate $\theta_{n,s}$ being optimal for such banks, changes in this interest rate have no effect on expected profits in region i. Turning to banks of region $-i$, the change in expected profits when macro-prudential policies move from Nash to cooperation writes as:

$$\pi_{-i,c} - \pi_{-i,n} = \frac{\beta}{4}\left[\left[1 - \frac{\nu_i}{L_{i,c}} \frac{\nu_i}{L_{i,n}}\right] (L_{-i,c} - L_{-i,n}) + \nu_i \frac{L_{i,c} - L_{i,n}}{L_{i,n}} \left[\left(\frac{\nu_i}{L_{i,n}} - \frac{\nu_{-i}}{L_{-i,n}}\right) L_{-i,n} + \left(\frac{\nu_i}{L_{i,c}} - \frac{\nu_{-i}}{L_{-i,c}}\right) L_{-i,c}\right]\right] \tag{45}$$

This expression has two terms. The first term represents the increase in expected profits for banks of region $-i$ due to the increase in capital inflows under cooperative macro-prudential policies relative to the Nash equilibrium. It is similar to the expression for the change in expected profits for banks of region i in the sense that it is equally proportional to the increase in domestic capital inflows. The second term on the right hand side of expression (45) represents the increase in expected profits for banks of region $-i$ due to the drop in the global interest rate under cooperation since we have $\theta_{n,c} \leq \theta_{n,n}$. As was noted above, when monetary policies are conducted non-cooperatively, the global interest rate is too high for banks of region $-i$. Hence a lower interest rate is always positive for such banks. This is why banks from region $-i$ benefit from macro-prudential policy coordination along two margins: larger capital inflows and a lower global interest rate. So, unlike the case of monetary policy, coordinating on macro-prudential policy is a Pareto improvement as both regions enjoy positive net gains, because of the presence of a positive (general equilibrium) externality in cross-border capital flows.

5.2 Optimal risk sharing with cooperative interest rate setting.

We now move to the case of optimal macro-prudential policy when interest rates are set cooperatively. Under this assumption, we first look at the case of optimal macro-prudential policies in the cooperative equilibrium and then turn to determining optimal macro-prudential policies under Nash.

5.2.1 Cooperative risk sharing.

Looking at symmetric equilibria, i.e. equilibria where funding costs and interest rates are equalized across regions, $R_{i,2} = R_{-i,2}$ and $\theta_i = \theta_{-i} = \theta_c$, when macro-prudential authorities play a cooperative game, the global planner chooses φ_i and φ_{-i} to solve

$$\max_{\varphi_i;\varphi_{-i}} \pi_c\left(\theta_c; L_i; L_{-i}\right) = \pi_i\left(\theta_c; L_i\right) + \pi_{-i}\left(\theta_c; L_{-i}\right)$$
$$\text{s.t.} \begin{cases} \theta_c = \frac{\beta}{2}\left(1 + \frac{\nu}{L}\right) \text{ and } L = L_i + L_{-i} \geq \nu \text{ and } L_i = \frac{\varphi_i\left(1-\varphi_{-i}\right)}{1-\varphi_{-i}\varphi_i} \\ L_i \leq \nu_i + \left(1-\theta_c\right) L_{-i} \text{ and } L_{-i} \leq \nu_{-i} + \left(1-\theta_c\right) L_i \end{cases} \tag{46}$$

Proposition 7 *Denoting $\underline{\theta} = \frac{\beta}{2-\beta}$, when interest rates and ex ante capital flows are determined cooperatively then macro-prudential authorities choose capital flows L_i and L_{-i} to satisfy:*

$$L_i = \nu_i + \left(1-\underline{\theta}\right) L_{-i} \textit{ and } L_{-i} = \nu_{-i} + \left(1-\underline{\theta}\right) L_i \tag{47}$$

Proof. Using the expression for θ_c, it is straightforward to note that global expected profits are increasing in both φ_i and φ_{-i}. As a result, both macro-prudential policy makers in region i and region $-i$ set capital flows L_i and L_{-i} such that $L_i = \nu_i + \left(1-\theta_c\right) L_{-i}$ and $L_{-i} = \nu_i + \left(1-\theta_c\right) L_i$. Solving from these two equations for L_i and L_{-i} yields expressions in (47). ■

Macro-prudential policy makers choose to maximize capital inflows when monetary and macro-prudential policies are set cooperatively because there is no trade-off between managing capital flows and managing the global funding cost:by definition, the latter is set at the level which maximizes global expected profits, i.e. $\frac{\partial \pi_c}{\partial \theta_c} = 0$. It is therefore always welfare improving to increase capital inflows as the global interest rate by construction adjusts so that it does not affect, at the margin, global expected profits. Macro-prudential policy makers therefore choose to maximize capital flows L_i and L_{-i}, i.e. set them such that the conditions under which the decentralized equilibrium holds bind, i.e. $L_{-i} = \nu_{-i} + (1-\theta_c) L_i$ and $L_i = \nu_i + (1-\theta_c) L_{-i}$, which yields expression (47). Let us now look at optimal macro-prudential policies under the Nash equilibrium.

5.2.2 Non-cooperative risk sharing.

Let us now turn to optimal macroprudential policies under Nash. Assuming as in the previous section that monetary policy is conducted in a cooperative game, the macroprudential policy maker in region k chooses φ_k to solve

$$\max_{\varphi_k} \pi_k (\theta_c; L_k)$$
$$\text{s.t.} \begin{cases} \theta_c = \frac{\beta}{2}\left(1+\frac{\nu}{L}\right) \text{ and } L = L_i + L_{-i} \geq \nu \text{ and } L_k = \frac{\varphi_k (1-\varphi_{-k})}{1-\varphi_{-k}\varphi_k} \\ L_{-i} \leq \nu_{-i} + (1-\theta_c) L_i \text{ and } L_i \leq \nu_i + (1-\theta_c) L_{-i} \end{cases} \tag{48}$$

We can now derive the following proposition.

Proposition 8 *Denoting $\underline{\theta} = \frac{\beta}{2-\beta}$, when interest rates are determined cooperatively but and ex ante capital flows are determined non-cooperatively then macro-prudential authorities choose*

$$L_i = \nu_i + (1-\underline{\theta}) L_{-i} \text{ and } L_{-i} = \nu_{-i} + (1-\underline{\theta}) L_i \tag{49}$$

Proof. When monetary policy makers set interest rates cooperatively, expected profits for banks of region k write as $\pi_k (\theta_c; L_k) = 1 - \nu_k + \left[\nu_k + \left(1-\frac{1}{\beta}\theta_c\right) L_k\right] \theta_c$. As a result, when macro-prudential policy makers in region k decide to allow banks to increase ex ante borrowing from abroad, the change in expected profits writes as

$$\frac{\partial \pi_k}{\partial \varphi_k} = \theta_c \left(1 - \frac{1}{\beta}\theta_c\right) \frac{\partial L_k}{\partial \varphi_k} + \frac{\partial \theta_c}{\partial L} \frac{\partial L}{\partial \varphi_k} \left[\nu_k + \left(1 - \frac{2}{\beta}\theta_c\right) L_k\right] \tag{50}$$

As is clear, the first term of the RHS expression $\theta_c \left(1 - \frac{1}{\beta}\theta_c\right) \frac{\partial L_k}{\partial \varphi_k}$ is always positive because allowing banks to issue more claims ex ante always contributes to raise expected profits. But the second term, which represents the effect on expected profits of a change in the interest rate θ_c stemming from an increase in claims issued ex ante by banks in region k, could be either positive or negative. For instance, denoting i the region such that $\frac{\nu_i}{L_i} \geq \frac{\nu_{-i}}{L_{-i}}$, this term is positive for banks of region $-i$ but negative for banks of region i. For the former, the equilibrium interest rate under cooperation θ_c is too high relative to the domestically optimal interest rate. Hence reducing the interest rate θ_c by allowing larger capital inflows is positive for expected profits:

$$\frac{\partial \pi_{-i}}{\partial \varphi_{-i}} = \frac{\beta}{4} \left[1 - \left[\frac{\nu}{L}\right]^2 + 2\left[\frac{\nu}{L}\right]^2 (1 - \varphi_i) \left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right] \frac{L_{-i} L_i}{\nu L}\right] \frac{\partial L_{-i}}{\partial \varphi_{-i}} > 0 \tag{51}$$

Macro-prudential authorities in region $-i$ therefore choose to set φ_{-i} such that

$$L_{-i} = \nu_{-i} + (1 - \theta_c) L_i \tag{52}$$

Conversely macro-prudential authorities in region i face a trade-off when setting the optimal level of capital flows since on the one hand increasing capital inflows is positive for expected profits but on the other doing so reduces the equilibrium interest rate θ_c which is negative for expected profits because, under the assumption that $\frac{\nu_i}{L_i} \geq \frac{\nu_{-i}}{L_{-i}}$, the interest rate θ_c is too low from a domestic perspective:

$$\frac{\partial \pi_i}{\partial \varphi_i} = \frac{\beta}{4} \left[1 - \left[\frac{\nu}{L}\right]^2 - 2\left[\frac{\nu}{L}\right]^2 (1 - \varphi_{-i}) \left[\frac{\nu_i}{L_i} - \frac{\nu_{-i}}{L_{-i}}\right] \frac{L_{-i} L_i}{\nu L}\right] \frac{\partial L_i}{\partial \varphi_i} \tag{53}$$

Now given that $\frac{\partial \pi_i}{\partial \varphi_i}$ is increasing in φ_i for $\frac{\partial \pi_i}{\partial \varphi_i} = 0$, i.e. $\pi_i(\theta_c)$ is convex in φ_i, macro-prudential authorities therefore choose either to minimize φ_i and set L_i such that $L_i = \nu_i + \nu_{-i} - L_{-i}$ or they choose to maximize φ_i and set $L_i = \min\left\{\nu_i + (1 - \theta_c) L_{-i}; \frac{\nu_i}{\nu_{-i}} L_{-i}\right\}$. Comparing expected profits under this two different options shows that maximizing φ_i is always the policy choice which maximizes expected profits. Hence macro-prudential policy makers in region i choose

$$L_i = \min\left\{\nu_i + (1-\theta_c)\, L_{-i}; \frac{\nu_i}{\nu_{-i}} L_{-i}\right\} \tag{54}$$

Last if macro-prudential policy makers in region i set $L_i = \nu_i + (1-\theta_c)\, L_{-i}$, then the validity condition $\nu_i + (1-\theta_c)\, L_{-i} \leq \frac{\nu_i}{\nu_{-i}} L_{-i}$ turns into $\nu_{-i} \leq \nu_i$ which always holds by definition. Optimal macro-prudential policies hence satisfy

$$L_i = \nu_i + (1-\underline{\theta})\, L_{-i} \text{ and } L_{-i} = \nu_{-i} + (1-\underline{\theta})\, L_i \tag{55}$$

■

The intuition for these results is pretty straightforward. When interest rates are determined cooperatively, the optimal interest rate is too high for banks in one region but too low for banks from the other region. It is typically too low for banks located in the region which would have set the global interest rate had such interest rate been determined in a Nash equilibrium and it is typically too high for banks located in the region which would have been able to set the global interest rate had such interest rate been determined in a Nash equilibrium. For macro-prudential policy makers of this latter region, there is no trade-off between allowing more capital inflows and controlling financial conditions. When more capital flows in, banks enjoy larger expected profits and the global interest rate falls which further contributes to increase expected profits given that it was too high in the first place. But turning to macro-prudential policy makers of the other region, they do face a meaningful trade-off. Allowing more capital inflows has a positive direct effect on banks' profits but it also carries a negative indirect effect as larger capital inflows reduce the global interest rate which cuts domestic banks profits given that the global interest rate was already too low. Yet, macro-prudential policy makers always choose to maximize capital inflows, in spite of the presence of this trade-off. Why ? The reason is this result is very simple: In the cooperative game , the equilibrium interest rate depends on global capital flows, i.e. the sum of capital inflows and outflows, which each macro-prudential policy maker has only limited influence on, given that larger capital inflows always imply lower capital outflows. As a result, the macro-prudential policy maker, even when it faces a meaningful trade-off, always prefers to maximize capital inflows, the cost of setting financial conditions in line with domestic needs being too large relative to the benefits of simply allowing more capital to flow in. The conclusion from this last section is therefore that when monetary policy makers play a cooperative game, optimal macro-prudential policy always consists in maximizing capital inflows irrespective of whether it is determined in a cooperative or non-cooperative game. In other words, there are no gains not macro-prudential policy cooperation, when monetary policy is itself cooperative.

6 Quantifying the gains to policy coordination.

In this section, we aim at quantifying the model's findings to determine how large gains from policy cooperation can be. To do so, we focus on three parameters of the model. The first one is the β parameter which scales the friction on the market risk sharing. The second and the third are respectively the parameters ν_i and ν_{-i}, each of which is an inverse measure of ex post reinvestment and therefore of ex post leverage in each region. For each of these parameters we consider a range of possible values as follows. For the β parameter scaling the friction on the market for risk sharing, we assume it ranges from 0.55 to 0.95. This means that between 5% and 45% of the return on assets traded on the market risk sharing is paid by the issuer without being earned by the buyer. Turning to the parameters ν_i and ν_{-i}, we assume ν_i ranges between 0.4 and 0.8, while the parameter ν_{-i} ranges between 0.0 and 0.4. Given that we always have $\nu_i \geq \nu_{-i}$, region i will be the interest rate setting region and hence labelled in what follows as the core region while region $-i$ will be refered to as the periphery region. For each combination of parameters $(\beta; \nu_i; \nu_{-i})$, we compute two different measures. We first derive welfare gains computed as the relative change in expected profits (global or regional) under alternative scenarios for monetary and macro-prudential policies.

First we compare global welfare under cooperative (monetary and macro-prudential) policies with global welfare under Nash (monetary and macro-prudential) policies. Figure 7 plots the distribution of these welfare gains for all possible combinations of the parameters $(\beta; \nu_i; \nu_{-i})$ with the ranges of values described above. What is already visible from Figure 7 is that the mass of global welfare gains is situated between 0.5 and

2%, even if there are non negligeable parts of the distribution for which welfare gains are either below 0.2% or above 3%. Welfare gains at the global level therefore tend to exhibit a relatively wide dispersion.

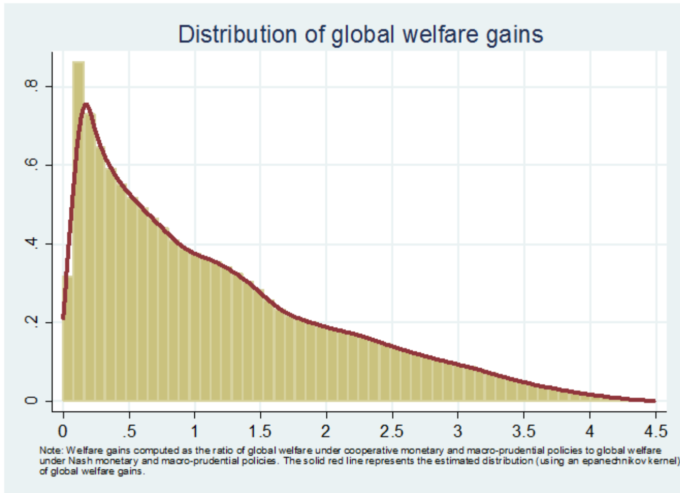

Figure 7

With these first findings in mind, we can decompose welfare gains at the global level into two different ways. First we can look at the contribution of coordinating each policy separetely. To do so we compute welfare gains due to macro-prudential policy coordination assuming non-cooperative monetary policies and welfare gains due to monetary policy coordination assuming cooperative macro-prudential policies. The left hand panel in Figure 8 shows the distribution of total welfare gains as well as those of each policy separately.Global welfare gains from monetary policy coordination typically tend to outweigh those from macro-prudential policy coordination. For example the average gain in global welfare is about 1.30% (median gain is 1.06%). But the contribution of monetary policy coordination is 0.87%, i.e. about two thirds of the total gain, while that of macro-prudential policy coordination is only 0.43%, i.e. about one third of the total gain.[11] This observation is not surprising: under coordinated monetary policy, funding conditions on the market for ex post funding are optimal from a global perspective and how macro-prudential policy is conducted is then irrelevant. By contrast coordinating macro-prudential policy under Nash monetary policies reduces the inefficiency from sub-optimal ex post funding conditions but is never able to eliminate

[11] Looking at medians, the ratio of the median gain from macro-prudential policy coordination to the median gain from monetary policy coordination is even more biased towards monetary policy (around one to five).

it. And sometimes (with roughly a 25% probability), coordinating macro-prudential policy does not bring any welfare benefit. This is why global welfare gains from monetary policy coordination tend to outpace those from macro-prudential policy coordination. Secondly we split global welfare gains by region. And consistenly with findings from the split by policy, the periphery region tends to grab a significantly larger part of coordination gains. On average, welfare increases by 2.26% in the periphery (median at 1.83%) but only by 0.16% in the core region (median at 0.10%). Moreover the core region can be worse-off under coordinated monetary and macro-prudential policies. This typically happens in 37-38% of the cases with a an average welfare loss of -0.28% (median at -0.19%), something that never happens to the periphery which is always better-off under coordinated policies. As was highlighted above, this perfectly makes sense: For the core region, funding conditions are optimal under Nash monetary policy. Hence moving to cooperative macro-prudential or monetary policy policies is unlikely to deliver large benefits. By contrast, funding conditions are sub-optimal under Nash monetary policy for the periphery. As a consequence, shifting to cooperative macro-prudential and/ monetary policies delivers relatively larger gains.

Last, we look at correlations between coordination gains across policies and regions. Focusing first on the periphery region, the correlation matrix shows that all three coordination gains (total gains, gains from macro-prudential policy coordination, and gains from monetary policy coordination) are highly correlated with each other. In other words, larger gains from coordinating one policy imply larger gains from coordinating the other policy and also imply larger gains from coordinating both policies. For instance, the correlation between gains from macro-prudential policy coordination and gains from monetary policy coordination is about 0.72 in the periphery region.

By contrast, the correlation between coordination gains for the core region tend to be weaker and turns even negative between gains from macro-prudential policy cooperation and gains from monetary policy cooperation. Hence when the core region derives larger benefits from macro-prudential policy cooperation, it tends to enjoy lower benefits from monetary policy cooperation. Last looking at the cross-region correlation in gains from policy cooperation shows that coordination gains that accrue to the periphery tend to exhibit relatively low correlation with the total gains that accrue to the core region. And these low correlations actually hide two sets of opposite correlations: on the one hand, a positive and large correlation between gains from macro-prudential policy cooperation that accrue to the core region and cooperation gains that accrue to the periphery. But on the other hand a negative correlation between gains from monetary policy cooperation that accrue to the core region and cooperation gains that accrue to the periphery. These findings reinforce the general point that incentives to cooperate on macro-prudential policy are more aligned across regions than those to cooperate on monetary policy.

Periphery \ Core	**Total coordination gains**	**Gains from MaP coordination**	**Gains from MP coordination**
Total coordination gains	**0.174**	**0.766**	**-0.322**
Gains from MaP coordination	**0.212**	**0.919**	**-0.382**
Gains from MP coordination	**0.130**	**0.579**	**-0.245**

Figure 10: Cross-region correlation matrix in welfare gains.

7 Conclusions

This paper provides a theoretical model which investigates the question of international coordination of monetary and macro-prudential policies in the context of a general equilibrium model with two regions where agents choose the optimal mix between ex ante and ex post liquidity. Its main conclusions are threefold. First, there are gains to coordinating monetary policy across regions even though gains tend to be asymmetric. Redistribution tools are therefore needed to sustain monetary policy cooperation. Second, under non-cooperative monetary policies, macro-prudential policy coordination delivers welfare gains that tend to accrue to both regions. Coordinating on macro-prudential policy is therefore "easier" than coordinating on monetary policy. Last when monetary policy is cooperative, Nash and cooperative macro-prudential policies are similar to each other. Cooperation on monetary policy hence annihilates any gain from macro-prudential policy coordination. In sketching these results, the model highlights that macro-prudential policy can be a useful complement for policy makers because it allows them to affect domestic funding conditions, particularly when such conditions are not fully in line with domestic needs.

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Appendix: Allowing central banks to set state-contingent interest rates.

The model as presented, assumes that central banks need to announce at date $t = 0$ the interest rate, they propose on their respective deposit facilities between date $t = 1$ and date $t = 2$. This assumption therefore precludes central banks from setting state-contingent interest rates, which is at odds with reality. This section proposes to relax this assumption and show that the main intuitions of the model still go through. In particular, we will show that the trade-off for macro-prudential policy makers between allowing for larger cross-border risk sharing and controlling domestic financial conditions is still valid.

To do so, let us turn to the market for ex post funding. On this market they are two types of banks: those with variable-scale investment (facing a financial constraint) and those with fixed-scale reinvestment (facing a size constraint). Let us now assume that banks of the latter type can shirk and claim to be of the former type. Indeed, when distressed banks have large risk sharing assets, then claiming to be financially constrained instead of being size-constrained may be a profitable option. This is because borrowing decreases with the amount of risk sharing assets for size-constrained banks while it increases with the amount of risk sharing assets for financially-constrained banks. Hence banks have a larger amount of risk sharing assets have incentives to claim that they are financially and not size constrained as they can borrow more. Yet for banks which are genuinely size-constrained, claiming to be financially constrained has a cost because they then reap a return ρ_s only with some probability p_s.

Assuming banks from region i are distressed, and hold $\beta r_{-i,1} L_{-i}$ as the product of their risk sharing assets, the incentive constraint which ensures that distressed banks disclose their true type then writes as

$$(\beta r_{-i,1} L_{-i} + D_j)\,\rho - D_j r_{i,2} \geq [(\beta r_{-i,1} L_{-i} + D)\,\rho_s - D r_{i,2}]\,p_s \tag{56}$$

Here, borrowing D_j for size-constrained banks is just the difference between reinvestment and initial funds $\overline{j_i}\rho - \beta r_{-i,1} L_{-i}$ while borrowing D for financially constrained banks is proportional to initial funds $\frac{\phi}{1-\phi}\beta r_{-i,1} L_{-i}$. Assuming the probability p_s is sufficiently small, this incentive constraint translates into an upper bound the funding cost $R_{,i.2}$:

$$R_{i,2} \leq \frac{\overline{j_i} - \frac{1}{1-\phi}\beta R_{-i,1} L_{-i} \frac{\rho_s p_s}{\rho}}{\overline{j_i} - \frac{1}{1-\phi}\beta R_{-i,1} L_{-i} \left[1-\phi+\phi p_s\right]} \tag{57}$$

As is clear for this constraint to be relevant, we need that $\left[1-\phi+\phi p_s\right]\rho < \rho_s p_s$. Otherwise, it would always be satisfied since the numerator would exceed the denominator and by construction we have $R_{i,2} \leq 1$. Note that when reinvestment and the shirking alternative have the same expected returns $\rho = \rho_s p_s$, then the condition $\left[1-\phi+\phi p_s\right]\rho < \rho_s p_s$ which ensures the incentive constraint is relevant is always met since it simplifies as $p_s < 1$. Note also that under the same condition, i.e. $\left[1-\phi+\phi p_s\right]\rho < \rho_s p_s$, **the upper bound on the funding cost $R_{i,2}$ is decreasing** in the initial funds $\beta R_{-i,1} L_{-i}$. When initial funds $\beta R_{-i,1} L_{-i}$ are larger, then size-constrained banks can borrow a lot more by claiming to be financially constrained. And on top of this, they only pay liabilities back with some probability. Hence they are willing to shirk even if the expected return under shirking is lower $\rho_s p_s < \rho$. The only way to preclude this possibility is to cap the funding cost and the more so the larger the initial funds $\beta R_{-i,1} L_{-i}$.

Then, using the property that $R_{k,2} = \beta R_{k,1}$ for $k = \{i; -i\}$ and using the fact that the wedge β between the return paid on risk sharing liabilities and the return earned on risk sharing assets implies that the cost of ex post funding is equal to the interest rate set by the central bank, i.e. $R_{k,2} = \theta_k$ for $k = \{i; -i\}$, expected profits for banks of region k $(k = \{i; -i\})$ write as

$$\pi_k = \left(1 + \left(1 - \frac{1}{\beta}\theta_{-k}\right) L_k\right)\theta_k + \left(1-\theta_{-k}\right)\left(1-\nu_k\right) \tag{58}$$

Last denoting $\lambda_k = \frac{\rho_s p_s}{\rho(1-\phi)\overline{j_{-k}}}$ and $\mu_k = \frac{1-\phi+\phi p_s}{(1-\phi)\overline{j_{-k}}}$, $(k = \{i; -i\})$ the problem for monetary policy makers in region k in the non-cooperative game writes as

$$\begin{gathered} \max_{\theta_k} \pi_k \left(\theta_k; \theta_{-k}\right) \\ \text{s.t. } \theta_k \leq \frac{1-\theta_{-k} L_k \lambda_k}{1-\theta_{-k} L_k \mu_k} \text{ and } \theta_{-k} \leq \frac{1-\theta_k L_{-k}\lambda_{-k}}{1-\theta_k L_{-k}\mu_{-k}} \end{gathered} \tag{59}$$

As is clear, expected profits of banks π_k of region k are strictly increasing in the interest rate set by the domestic central bank θ_k and strictly decreasing in the interest rate set by the central bank from the other region. Central banks therefore choose in the Nash equilibrium to maximize their respective interest rates θ_i and θ_{-i}, which then satisfy

$$\theta_i = \frac{1 - (\lambda_i L_i)\,\theta_{-i}}{1 - (\mu_i L_i)\,\theta_{-i}} \text{ and } \theta_{-i} = \frac{1 - (\lambda_{-i} L_{-i})\,\theta_i}{1 - \left(\mu_{-i} L_{-i}\right)\theta_i} \tag{60}$$

The solution is therefore the fixed point to this system and writes as

$$\theta_k = \theta_k\left(L_k; L_{-k}\right) \text{ with } \frac{\partial \theta_k\left(L_k; L_{-k}\right)}{\partial L_k} \leq 0 \leq \frac{\partial \theta_k\left(L_k; L_{-k}\right)}{\partial L_{-k}} \tag{61}$$

for $k = \{i; -i\}$. For instance, when $\mu_i = \mu_{-i} = 0$ the solution writes as $\theta_i = \frac{1-\alpha_i L_i}{1-\alpha_i \alpha_{-i} L_i L_{-i}}$ and $\theta_{-i} = \frac{1-\alpha_{-i} L_{-i}}{1-\alpha_i \alpha_{-i} L_i L_{-i}}$, and one can easily check that we indeed have $\frac{\partial \theta_k}{\partial L_k} \leq 0 \leq \frac{\partial \theta_k}{\partial L_{-k}}$ for $k = \{i; -i\}$. As a result, macro-prudential policy makers still face the same type of trade-off when determining φ_k, i.e. how tight should the constraint be on domestic bank ex ante leverage. On the one hand, allowing banks to leverage up by more and allowing larger cross-border capital inflows improves banks expected profits, while on the other hand, this changes funding conditions for domestic banks in a way that is detrimental to expected profits as larger inflows tend to reduce the interest rate on the market for ex post funding when domestic banks acts as lenders and increase it when domestic banks acts as borrowers.