Preferred and Non-Preferred Creditors*

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May 28, 2019

Abstract

International financial institutions (IFIs) generally enjoy preferred creditors treatment (PCT). While PCT rarely appears in legal contracts, sovereigns often restructure bilateral or commercial debts while paying IFIs in full. This paper presents a model where a creditor, such as an IFI, that can commit to lend limited amounts at the risk-free rate and to refrain from lending into arrears is always repaid and adds value. The analysis suggests that IFIs should not mimic commercial lenders, but exploit their complementarity, even if banning commercial borrowing can sometimes be optimal. IFIs should also focus on countries with limited market access and should not be forced into debt restructurings.

JEL Classification Numbers: F34, H63, O19, P33.
Keywords: Preferred Creditor Treatment, Preferred Creditor Status, Sovereign Debt, Sovereign Defaults, International Financially Institutions, Emergency Financing.

*We would like to thank Aitor Erce, Paolo Garella, Bernardo Guimarães, Alessandro Missale, Andrew Neumeyer, Leonardo Martinez, Ugo Panizza, Anushka Thewarapperuma, Harold Uhlig as well as participants at a Seminar at the University of Milan, at the 2018 ESM Workshop on "Debt sustainability: Current Practices and Future Perspectives," and at the DebtCon3 conference at Georgetown University for constructive comments and suggestions. The usual disclaimers apply.

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1 Introduction

The role of International Financial Institutions (IFIs) in the global financial architecture has been widely analyzed both by academics and by the international policy community.[1] Much has been said about their role as providers of emergency funding but a long-standing central puzzle remain. Namely, the International Monetary Fund (IMF) and the main Multilateral Development Banks (MDBs) enjoy Preferred Creditor Treatment (PCT) in relation to their sovereign lending, meaning that they expect to be paid even if the borrower restructures private or bilateral debt. And yet, while PCT is critical for the operating model of IFIs, and the Paris Club Agreed Minutes exonerate IFIs from a "comparability of treatment" clause, their preferred standing is not strongly backed in international law.[2]

At times, the preferred treatment of IFIs has been called into question. Perhaps, the most emblematic of such discussions to date has been when Greece fell into arrears with the IMF in July 2015.[3] On the other hand, during the prolonged "trial of the century" following Argentina's 2002 default, IMF liabilities were paid early and in full. While there were many attempts from hold-outs to disrupt payments to creditors that accepted the 2005 restructuring and subsequent offers, there was virtually no mention of Argentina's preferred lenders in those cases, nor any serious attempt to crowd them in.[4] Similarly, in many recent bond restructurings, private creditors suffered changes in contracts and present-value haircuts but IFIs were eventually paid in full.[5]

The persistence of IFIs' preferred-standing is intriguing, especially given that it is a market practice, which is not backed by any contractual clause. The resilience of PCT, together with the lack of any strong legal backing, suggests that it should be understood as an "equilibrium outcome." And yet we know of no economic model to date that shows this to be the case. Sovereign debt models that focus on "willingness to pay" do not consider seniority,[6] while those that focus on seniority[7] assume it, without explaining its origin. Despite the vast literature on international financial architecture and sovereign debt restructuring, and despite the critical nature of PCT to the workings of the main IFIs, to our knowledge there is no model that explains why sovereign borrowers treat such lenders as preferred. This paper attempts to fill this gap. Our contribution is to develop a model that endogenizes the repayment decision of both commercial and IFI creditors,[8] to show how such decisions are interdependent, and to describe the potential advantages and trade-offs faced by country that may borrow from both the market (commercial lenders) and IFIs.

In what follows, we develop a relatively simple model of emergency sovereign lending in the aftermath of

[1] We review relevant academic literature in the next section. As examples of the policy discussion, see Council of Foreign Relations (2018) and the section on the global financial safety net in G20 Eminent Persons Group on Global Financial Governance (2018).

[2] "The Paris Club Agreed Minutes "comparability of treatment" clause aims to ensure balanced treatment of the debtor country's debt by all external creditors. In accordance with this clause, the debtor country undertakes to seek from non-multilateral creditors, in particular other official bilateral creditor countries that are not members of the Paris Club and private creditors (mainly banks, bondholders and suppliers), a treatment on comparable terms to those granted in the Agreed Minutes." See: http://www.clubdeparis.org/en/communications/page/what-does-comparability-of-treatment-mean On the other hand, see Martha (1990) and Schadler (2014) on the lack of strong legal standing for PCT in international law.

[3] See, for example, "Defaulting on the IMF: a stupid idea whose time has come," *Financial Times,* Alphaville, July 1st, 2015.

[4] For example in Cruces and Samples (2016) account of Argentina's "trial of the century" there is virtually no mention of Argentina's senior creditors.

[5] Such cases include Belize, the Dominican Republic, Jamaica and Uruguay. As reviewed further below, there have been cases of arrears with IFIs. Still, with just three countries in long-standing arrears to the IMF Oeking and Simlinski (2016) argued that persistent arrears to that organization may be a thing of the past. At the time of writing, Venezuela is accumulating arrears with the Inter-American Development Bank (IDB). In keeping with the expectation of being paid both capital and interest in full, the IDB is making provisions solely on interest on interest payments, which is not charged. Five countries are currently in arrears with the World Bank: Eritrea, Somalia, Sudan, Syria and Zimbabwe.

[6] See, for instance, the classic papers by Eaton and Gersovitz (1981), Bulow and Rogoff (1989), Kletzer and Wright (2000) or, for a more up-to-date discussion, Aguiar and Amador (2014).

[7] See, e.g., Gonçalves and Guimaraes (2015), Bolton and Jeanne (2009), Chaterterjee and Eyigungor (2015), Boz (2011), and Hatcheondo et al (2017).

[8] Throughout the paper we use IFIs and multilateral lenders/debt interchangeably. We thus ignore the fact that some multilateral lenders such as the IFC (the private sector arm of the World Bank Group) do not generally claim preferred status.

a major negative shock,[9,10] which allows us to obtain a set of analytical results. This contrasts with much of the recent literature, which tends to rely on numerical simulations. To keep things straightforward, our model abstracts from several important real-world features such as liquidity and creditor coordination issues as well as reforms and conditionality. Our aim is to understand the fundamental differences between IFIs and private lenders, focussing on the underlying incentives for a country to borrow and to repay each type of creditor. We show that IFIs are preferred because their bylaws allow them to commit to (i) lend limited amounts at close to the risk-free rate under most circumstances, and (ii) refrain from lending until any unpaid arrears are cleared. This avoids the possibility of debt dilution, sets IFI lending aside from private lenders, and explains why, in many instances, the presence of IFIs adds value. However, we also find that preferred lending may be constrained by a country's willingness to honor the commitments made, with the constraint depending on the probability and the severity of future shocks, on repayment costs and on the county's access to private lenders, which, in turn, depends on similar parameters.

In a similar vein to the common statement in corporate finance that if all financing is debt then none is (it turns into equity), we may quip that if all lending is preferred then no lending is. Preferred lending cannot be unlimited, otherwise it may not be considered as preferred. From a normative standpoint, our analysis suggests that if preferred lenders' capital is restricted, they may wish to concentrate their activities where they can obtain the biggest "bang for the buck," and that, in general, is in countries that are prone to larger but not very frequent shocks. When shocks are either extremely rare, or when they happen regularly and their costs are not particularly high, market solutions may be just as good and may even dominate IFIs. In addition, we find that there are specific situations in which a country is better off without the existence of market lenders, providing a potential justification for restrictions on commercial lending in some circumstances.

In the next section, we review the literature on different aspects of PCT. In section 3, we introduce the basic model and first study the case in which the country borrows from an IFI. In the fourth section, following the standard sovereign debt literature, we assume that the country relies on private lenders. In the fifth section, we allow for the simultaneous presence of multilateral and private lenders. Section 6 discusses how a capital constrained IFI should optimally allocate its resources, while section 6 looks at some of the key assumptions of the model and discusses the robustness of the results. Finally, section 7 provides a set of policy implications and concludes.

2 On preferred creditor treatment, a brief review

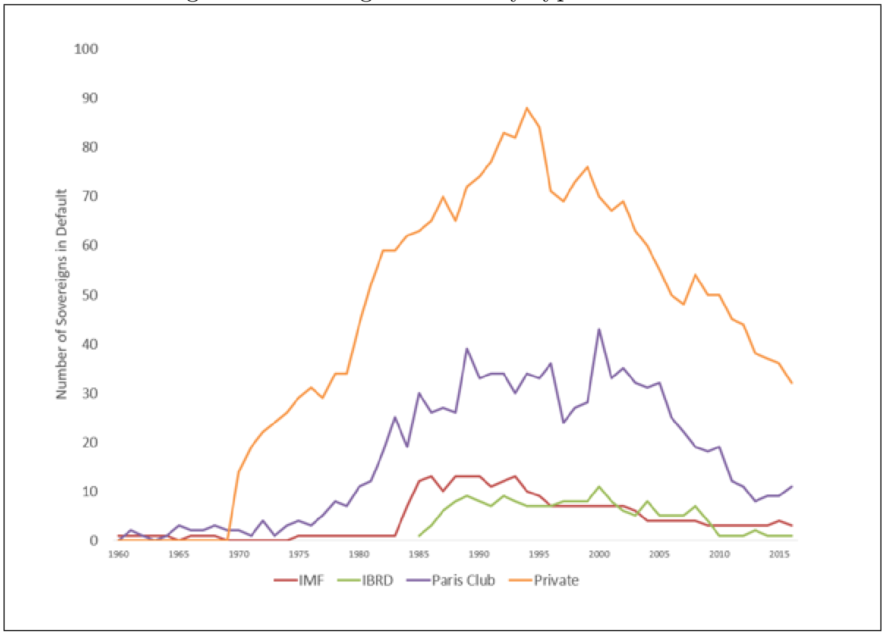
Our paper borrows from several strands of literature on IFIs and PCT. First, a number of papers have discussed potential explanations for why IFIs may enjoy PCT, although none contains a model illustrating how it can be supported as an equilibrium outcome. Among these, Buiter and Fries (2002) suggest countries may confer PCT to IFIs in return for competitive lending rates. Levy Yeyati (2009) argues that PCT is related to an insurance motive–namely the expectation that IFIs will extend credit during a crisis. Humphrey (2015) stresses IFIs mutual ownership structure. Risk Control (2017) collects statistics related to these three potential "drivers" of PCT (favorable rates, countercyclical lending, cooperative nature of the institutions) and finds evidence that IFIs do indeed extend funds at lower rates than the market, that they are countercyclical, and that they lend in bad times. The paper also compares the degree of "mutuality" of each institution and discusses how that may affect preferential treatment.

Second, within the large empirical literature on sovereign defaults, a subset of papers consider defaults (and the building up of arrears) on IFIs. Schlegl et al (2015) and (2019) claim a hierarchy in seniority with the IMF and MDBs at the top. Their analysis is based on World Bank data and covers 127 countries from 1980 to 2006. Considering arrears, bonds appear below MDBs in the pecking order, and then come bilateral lenders, banks and trade credit. The analysis also shows that not all multilateral lenders are

[9] Focussing on emergency lending (and abstracting from lending for consumption-smoothing motives in normal times) provides a clean way to illustrate how a preferred lender could exist as an equilibrium outcome.

[10] In the analysis shocks are always negative, hence, the "negative" is omitted hereafter.

Figure 1: Sovereign defaults by type of creditors



treated equally; for instance, during the European crisis the EFSF/ESM was not as preferred as other IFIs. Steinkamp and Westermann (2014) also analyze the European crisis and present survey evidence on market participants' perception of seniority levels and report that the IMF was perceived as the most senior creditor among official lenders. Finally, since MDBs regularly issue bonds on international markets, another way of looking at their seniority is through the lens of credit ratings and the methodologies adopted by credit rating agencies. The IBRD and the four main regional MDBs (ADB, AfDB, EBRD and IDB) all maintain AAA ratings. Moody's and Standard and Poor's both suggest these five organizations enjoy PCT although they have slightly different methodologies for how much of a bonus this provides in terms of formulating ratings.[11]

To motivate our work, Figure 1 shows the number of countries in "default" in each year from 1960 to 2016 classified by the type of creditor.[12] In default here means with outstanding arrears, as noted above, in the end, IFIs are virtually always repaid. Note that very few countries are in arrears with the IMF or with the IBRD. The maximum number of countries in arrears with the IMF was 13 during the 1980's and this figure falls to 3 in recent years. The IBRD had between 7 and 9 countries in arrears during the 1990's and a peak of 11 in 2000.[13] Arrears are much more frequent with Paris Club (bilateral) lenders with a peak of 43 countries in arrears in the year 2000, but falling to 11 in 2016. Defaults on private creditors reached a peak of 88 countries in arrears with at least some private creditors in 1994. This number falls to 32 by 2016.

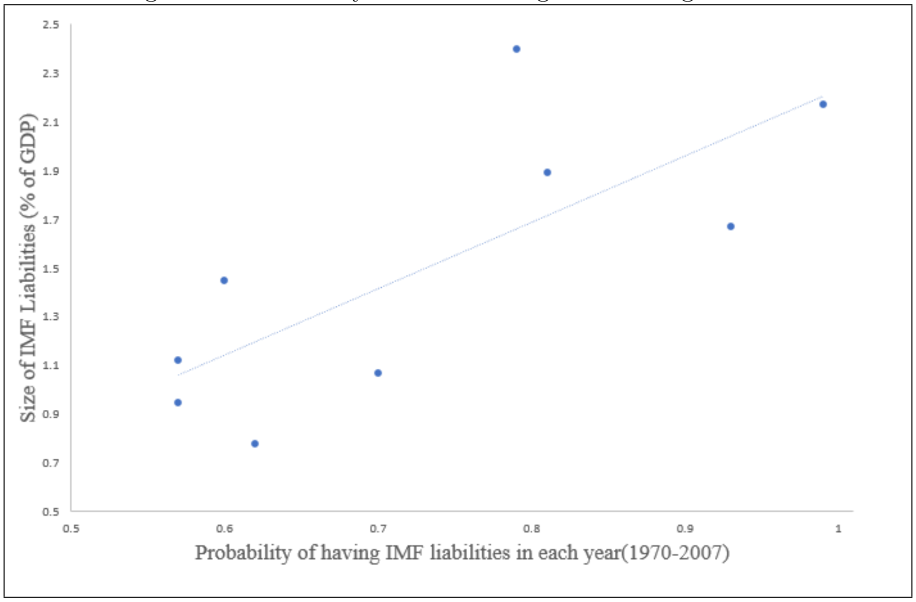
As reviewed above, much of the literature considers emergency lending as a potential driver of PCT. During emergencies, default probabilities are likely considerably higher and private creditors anticipating greater risks will require considerably higher interest rates as compensation. If there is an expectation that IFIs will be repaid, then they can lend in these difficult times at more competitive rates. If the default probability is close to zero, then IFIs can charge rates very close to the riskless rate of interest and still make enough returns to support their operating costs. For a country that may suffer emergencies in the future

[11] See Humphrey (2015), Perraudin et al (2016), and Risk Control (2017) for relevant commentary. Moody's (2017) and Standard and Poor's (2017) contain information on the ratings methodologies, the latter discusses recent changes to the S&P approach.

[12] The data comes from the Bank of Canada sovereign default database, Beers and Mavalwalla (2017).

[13] The Bank of Canada dataset does not separate other MDBs normally considered as being senior from other official creditors that are not considered as senior.

Figure 2: Probability of IMF lending and lending volumes



this relationship may be very valuable. In the model developed below, it is the value of this relationship that then provides the incentives for repayment.

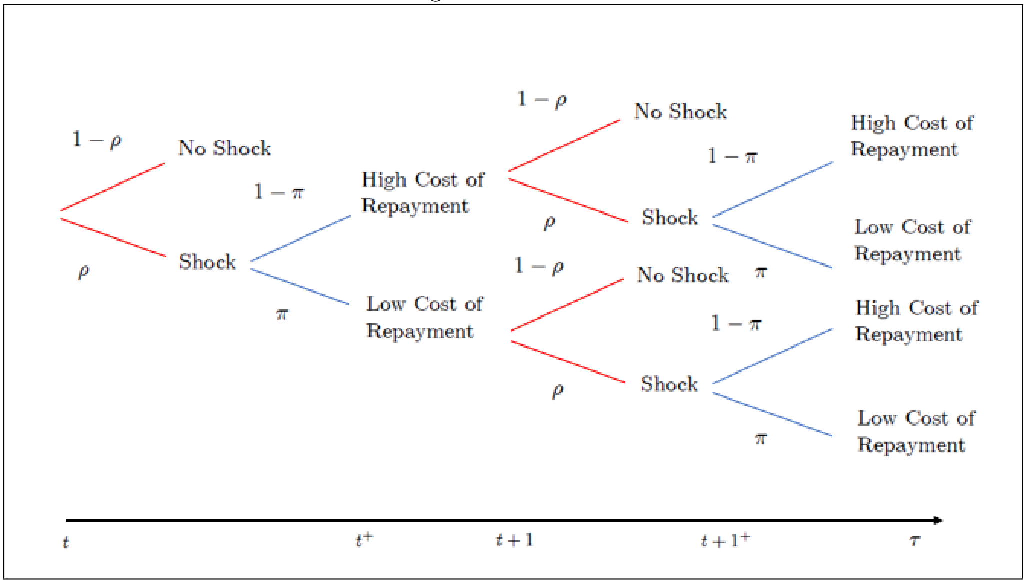
This implies that the amount that IFIs can safely lend is related to the probability of the occurrence of shocks. Assuming that preferred lenders can credibly commit not to lend if a country has defaulted on its loans, this suggests that a greater amount of preferred lending may be supported as the probability of shocks rises. This suggests a positive relationship between the probability of stress periods and the amount of preferred lending. Figure 2 employs data presented in Boz (2011) regarding the probability of there being IMF lending outstanding (a signal of a negative shock/crisis) and the size of IMF lending as a percentage of GDP for the selection of emerging countries covered in that paper from 1970 to 2007. As can be seen, there is indeed a positive relationship between crisis probability and IMF lending, providing some empirical support to our arguments.

3 The model

In this section, we present a stylized model, where a country hit by a shock has the opportunity to borrow to alleviate the associated consumption costs. In our set up, time, τ, is discrete and runs from the current period, t, to infinity. We further assume that, in the initial period t, the country is hit by a shock with probability ρ; if no shock occurs in t, then no shock will ever occur thereafter. If, instead, a shock occurs, then with the same probability ρ a shock occurs in $t+1$, and so forth. These assumptions simplify the analysis, allowing for a very simple Markovian structure. All borrowing in the model is short term: the full amount of the loan plus the interest should be repaid at the end of each period, which we refer to as τ^+. Figure 3 illustrates the timing of the model.

For the sake of simplicity, the country's discount factor and the (gross) risk-free interest rate are both set equal to 1, and we also normalize the utility in all non-shock states to zero; while in shock states, absent lending, we assume it is equal to $-C$. However, by borrowing an amount L, the utility loss is reduced by $aL - \frac{L^2}{2}$. Thus, a is a measure of the value of emergency lending, which exhibit decreasing returns. We assume that C is large enough so that the utility during an emergency periods is never higher than the

Figure 3: Timeline



utility in non-emergency periods.[14]

We further assume that there is some utility cost for the country to repay debt. This cost may vary depending on political, economic, or other considerations. To capture this, while keeping things simple, we assume that there are just two states, so this cost may be either high or low. We refer to these states as the high- and the low-repayment-cost states. We label the probability of the low-repayment-cost state as π, and we normalize the cost of repayment in that low-repayment state to 1. We denote by k the cost of repayment in the high-repayment-cost state and we assume that $a > k > \frac{1}{\pi}$, that is, the marginal benefit from borrowing is higher than its marginal cost in the high-repayment-cost state and that the expected cost of servicing the debt in the high-repayment-cost state is higher than the cost of servicing it in the low-repayment-cost's.

Given that we want to investigate cases where the country can borrow both the market and an IFI, it is useful to keep the model as simple as possible, ruling out a number of special cases. For example, and as we discuss further below, the country might default on the official sector in the high repayment-cost state and then wait until a low-repayment-cost state materializes to clear the arrears and regain access to borrowing. It turns out that we can rule out that possibility assuming that the cost of servicing the debt in the high-repayment-cost state is not too elevated, that is, $k < 1 + \frac{1}{\rho(2\pi-1)}$. But, at the same time, it cannot be too low if we want to consider the interesting case in which borrowing from the market and defaulting in the high-repayment-cost states yields higher utility than borrowing from the official sector at the risk-free rate and always repaying, this is the case when $k > \frac{a-\pi}{(a-1)\pi} > \frac{1}{\pi}$. To avoid abusing the reader's patience with too many cases and sub-cases, we thus assume that k is neither too big or too small and that

$$Min[a, 1 + \frac{1}{\rho\,|2\pi - 1|}] > k > \frac{a-\pi}{a\pi - 1}. \tag{A1}$$

It is easy to show that such inequalities are verified for "reasonable" parameter values. For instance, in the numerical simulations we present below, where we assume $a = 8$, $\pi = .6$, (A1) is verified for all $\rho \in [0, 1]$, both in the case of a "low" k, $k = 2$, and "high" k, $k = 3$.

[14] This is always the case if $C > \frac{a^2}{2}$, which we assume holds true.

3.1 IFI lending

We start our analysis by considering the case in which the country only has access to an IFI.[15] IFIs are different from commercial lenders since, according to their internal rules, they normally lend at rates well below market anticipating that they will receive preferential treatment and be repaid during periods of debt distress. Indeed, some MDBs' internal risk models essentially assume a zero probability of default on the loan principal and significant adjustments are required to standard risk models applied by rating agencies to analyze MDB risks.[16] While some IFIs have different facilities with different interest rates, to a first approximation these rates are close to the risk-free rate.[17] However, as we investigate below, this means that, in equilibrium, countries should only borrow amounts that they are willing to repay under all possible circumstances. For this risk-free lending to be sustainable in equilibrium,[18] we assume that, in contrast to the market, IFIs can commit to specific lending volumes. We also assume that if a country defaults on an IFI it is excluded from borrowing from any IFI until it fully repays the arrears. This reflects IFIs actual rules and working practices.[19]

At time t, the value function for the country, assuming that, if faced with a shock, borrows L_{I_t} from an IFI at the risk-free interest rate, and it repays both in the high- and in the low-repayment-cost state, is given by:

$$V_{I_t} = \rho(-C + aL_I - \frac{L_{I_t}^2}{2} - L_{I_t}((1-\pi)k + \pi) + V_{I_{t+1}}), \tag{1}$$

where subscript I denotes IFI. Equating V_{I_t} with $V_{I_{t+1}}$, we can rewrite the value function as

$$V_I = \frac{\rho}{1-\rho}(-C + aL_I - \frac{L_I^2}{2} - L_I((1-\pi)k + \pi)). \tag{2}$$

For the loan to be risk free and support the specification of this value function, the country must be willing to service the loan in the high-repayment-cost state. This is the case if

$$V_I - kL_I \geq -\frac{\rho C}{1-\rho}, \tag{3}$$

that is, when the continuation value of the official lending relation V_I, net of the repayment cost kL_I, is greater than the continuation value of defaulting, which is given by $-\sum_{\tau=t}^{\infty} \rho^{(t-\tau)}C = -\frac{\rho C}{1-\rho}$. Also, as we mentioned above, we rule out the possibility for the country to default on the official sector if the high-repayment-cost state materializes and then to clear its arrears in the next draw of a low-repayment-cost state. This is the case when

$$(k-1)L_I \leq \frac{\rho}{1-(1-\pi)\rho}(aL_I - \frac{1}{2}L_I^2), \tag{4}$$

where $(k-1)L_I$ is the amount the country "saves" by repaying the creditors in the low-repayment-cost states, and $\frac{\rho}{1-(1-\pi)\rho}(aL_I - \frac{1}{2}L_I^2)$ the expected cost of the lack of access to credit.[20] When this is the case, we have that

[15] While we assume one IFI lender, our analysis would carry through if there were more IFIs assuming that they coordinate, which is usually the case during emergencies.

[16] As discussed above, arrears and delays to repayment have occurred but in virtually all cases, principal and interest have eventually been repaid in full. Some MDBs do not charge interest on delayed interest.

[17] Actually, IFIs lend slightly above the risk-free rate to cover their own operational costs.

[18] As will become clearer, we are here referring to the case in which risk-free lending is not a market equilibrium.

[19] For instance, the IMF's ability to lend to a country is (at least in principle) a multiple of the country's IMF quota. The IBRD lending envelope to any particular country is approved by the World Bank board every 4 years in the Country Partnership Strategy and is a function of each country's size, needs and ability to repay. As per the no-lending into arrears clause, there is indeed a mutual understanding between IFIs that arrears to different institutions have to be cleared simultaneously. For, instance according to IMF (2013) "The Fund maintains a policy of non-toleration of arrears to official creditors. Fund supported programs required the elimination of existing arrears and the non-accumulation of new arrears during the program period with respect to official creditors," (p.48).

[20] The expected cost of the lack of access to credit is the cost of lacking access to credit in the next period should a crisis

Proposition 1 *The optimal amount of IFI lending is given by*

$$L_I^* = \begin{cases} 0, & \rho \le \overline{\rho}_I \equiv \frac{k}{a+\pi(k-1)}; \\ 2(a-\pi-k(\frac{1}{\rho}-\pi)), & \overline{\rho}_I < \rho < \widehat{\rho}_I; \\ a-\pi-k(1-\pi), & \rho \ge \widehat{\rho}_I \equiv \frac{2k}{a+\pi(k-1)+k}; \end{cases} \tag{5}$$

and the associated utility by

$$V_I^* = \begin{cases} -\frac{\rho C}{1-\rho}, & \rho \le \overline{\rho}_I; \\ -\frac{\rho C}{1-\rho} + \frac{2k(\rho(\alpha-\pi)-(1-\pi\rho)k)}{\rho}, & \overline{\rho}_I < \rho < \widehat{\rho}_I; \\ -\frac{\rho C}{1-\rho} + \frac{\rho(\alpha-\pi-(1-\pi)k)^2}{2(1-\rho)}, & \rho \ge \widehat{\rho}_I. \end{cases} \tag{6}$$

Proof: In Appendix.

According to Proposition 1, if the probability of a shock is sufficiently low, $\rho \le \overline{\rho}_I$, then risk-free IFI lending is not supported in equilibrium and the country cannot improve on $-C$ if faced with a shock. The reason is that, given the low probability of a shock, the borrower will default in the bad state when repayment costs are high as the value of the relationship with the official lender is insufficient to outweigh such costs. On the other hand, when the probability of a shock is sufficiently large, $\rho \ge \widehat{\rho}_I$, the lending relation is valuable enough to allow the IFI to lend the optimal unconstrained amount. For intermediate values of ρ ($\widehat{\rho}_I > \rho > \overline{\rho}_I$), IFI lending is supported in equilibrium, but the participation constraint (ensuring that the IFI is repaid in the bad state) binds. This constraint is then what determines the amount of IFI lending.

The optimal amount of IFI lending, as a function of ρ, is illustrated in Figure 4. In Proposition 1, we have expressed the three regimes as a function of ρ, the probability of a shock. However, the critical values $\overline{\rho}_I$ and $\widehat{\rho}_I$, also depend on the other parameters. More precisely, $\overline{\rho}_I$ and $\widehat{\rho}_I$ both increase in a and in π, and decrease in k. As discussed above, a is a measure of the value of emergency lending and thus, the higher is a, the higher is the opportunity cost of defaulting. So higher values of a support larger amounts of IFI lending. Similarly, the higher is π and the lower is k, the lower is the expected cost associated with servicing the debt in the high-repayment-cost state, which may be written as $(1-\pi)k$, and thus the higher is the value of maintaining the relationship with the IFI and being able to borrow in the future. This, in turn, implies that higher values of π, or lower values of k, also support larger volumes of IFI lending.

4 Market lending

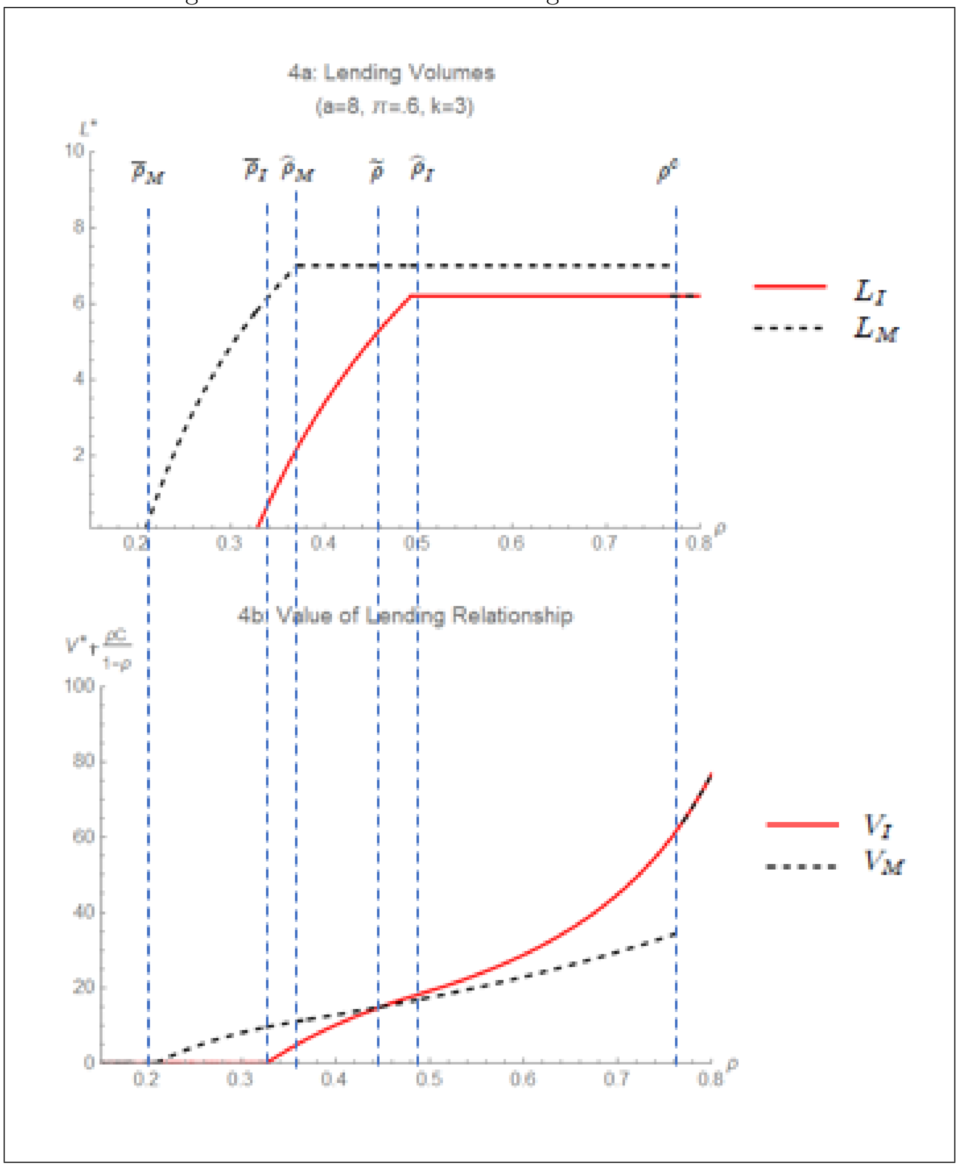
Consider now the case in which the country only borrows from private lenders, which we refer to as the market. The key differences between market and IFI lenders are that private lenders are willing to extend credit even in presence of default risks and that, being unable to commit to a maximum amount of lending, they face a dilution problem. Private lenders are competitive and risk neutral and to break even they must charge an interest rate commensurate with the risk-free interest rate adjusted for the probability of default. Loans are short term as before. As mentioned above, we assume that if the country defaults it can never regain access to credit from the market.[21]

To solve for the optimum amount of market lending, we start by assuming that the borrower always defaults in the high-repayment-cost state so that the gross interest rate is $1/\pi$. Again, we assume that in each period τ, loans are short term and must be repaid at the end of the period. In the initial period t, the value function for the country–assuming it borrows L_{MD_t} from commercial lenders at the risk-adjusted

occur (with probability ρ), and in the period after should a crisis again occur and the country again faces a high-repayment-cost state(with probability $\rho^2(1-\pi)$) and so forth. Hence it is equal to $(\rho + (1-\pi)\rho^2 + (1-\pi)^2\rho^3 + ...)(aL_I - \frac{1}{2}L_I^2) = \frac{\rho}{1-(1-\pi)\rho}(aL_I - \frac{1}{2}L_I^2)$.

[21] Had we assumed that, after a default, the borrower could borrow again with a certain probability in the subsequent period, the qualitative results of the paper would remain the same.

Figure 4: IFI and Market Lending: Volumes and Value



interest rate and pays the total due amount $\frac{L_{MD_t}}{\pi}$ in the low-repayment-cost state, which occurs with probability π, is given by

$$V_{MD_t} = \rho(-C + aL_{MD_t} - \frac{L^2_{MD_t}}{2} + \pi(-\frac{L_{MD_t}}{\pi} + V_{MD_{t+1}}) + (1-\pi)\frac{-\rho C}{1-\rho}), \tag{7}$$

where superscript MD denotes market (M) lending when repayment occurs in the low-repayment-cost state and default (D) in high-cost-repayment state. Again, $-\frac{\rho C}{1-\rho}$ is the continuation value assuming default in the bad state and no lending. Equating V_{MD_t} with $V_{MD_{t+1}}$, the value function can be written as

$$V_{MD} = \frac{\rho}{1-\pi\rho}(-C + aL_{MD} - \frac{L^2_{MD}}{2} - L_{MD} - (1-\pi)\frac{\rho C}{1-\rho}). \tag{8}$$

For the market to be willing to offer loans at this interest rate, the country must be willing to repay the debt in the low-repayment-cost state. This condition can be written as

$$V_{MD} - \frac{L_{MD}}{\pi} \geq -\frac{\rho C}{1-\rho}. \tag{9}$$

The optimal amount of borrowing is then found by maximizing (8) subject to the constraint (9). The results are summarized in Lemma 1.

Lemma 1 *If the country defaults in the high-repayment-cost state, and it repays its debt in the low-repayment-cost state, the optimal amount of borrowing is given by*

$$L^*_{MD} = \begin{cases} 0, & \rho \leq \overline{\rho}_M \equiv \frac{1}{a\pi}; \\ \frac{2(a\rho\pi-1)}{\pi\rho}, & \overline{\rho}_M < \rho < \widehat{\rho}_M; \\ a-1, & \rho \geq \widehat{\rho}_M \equiv \frac{2}{\pi(1+a)}; \end{cases} \tag{10}$$

and the associated utility is given by

$$V^*_{MD} = \begin{cases} V^*_{MD1} = -\frac{C}{1-\rho}, & \rho \leq \overline{\rho}_M; \\ V^*_{MD2} = -\frac{\rho}{1-\rho}C + \frac{2(a\pi\rho-1)}{\pi^2\rho}, & \overline{\rho}_M < \rho < \widehat{\rho}_M; \\ V^*_{MD3} = -\frac{\rho}{1-\rho}C + \frac{(a-1)^2\rho}{2(1-\rho\pi)}, & \rho > \widehat{\rho}_M. \end{cases} \tag{11}$$

Proof: In Appendix

Thus, also in the case in which the country borrows from the market (and defaults in the high-repayment-cost state) there are three different regimes. If the probability of a shock is low, $\rho \leq \overline{\rho}_M$, the market cannot lend as it will not be repaid: the value of the borrowing relationship is so low that the country always finds it in its interest to default (even in the low-repayment-cost state). On the other hand, if the probability of a shock is high, $\rho \geq \widehat{\rho}_M$, then maintaining the relation with private lenders is very valuable and the unconstrained optimal amount of market lending can be supported in equilibrium. In intermediate cases, $\overline{\rho}_M < \rho < \widehat{\rho}_M$, the private sector is willing to lend, but the constraint that the lender must be repaid in the low-repayment-cost state binds. This repayment constraint is then what determines the amount of market lending.

In the above, we derived the optimal amount a country can borrow from the market when there is default in the high-repayment-cost state and repayment in the low-repayment-cost state. For this to be an equilibrium, it should indeed be the case that it is in the country's interest to default in the high-repayment-cost state, that is,

$$V_{ND} - k\frac{L^*_{MD}}{\pi} \leq -\frac{\rho C}{1-\rho}, \tag{12}$$

where V_{ND} is the value of borrowing from the market L^*_{MD} when the market charges the risk adjusted rate, but the country never defaults. Of course, this condition is always verified if $\rho \leq \widehat{\rho}_M$: if the non-default constraint is binding in the low-repayment-cost state, it is *a fortiori* binding in the high-repayment-cost state. In the Appendix, we show that, when $\rho > \widehat{\rho}_M$ the country has no incentives in deviating from the MD strategy if

$$\rho \leq \frac{2k}{(a+2k-1)\pi} \equiv \rho^c, \tag{13}$$

with

$$\widehat{\rho}_I < \rho^c < 1. \tag{14}$$

If, instead, $\rho > \rho^c$, then the country will not default in any state and the market is thus willing to offer the very same loan the IFI offers at the risk-free rate.[22] Hence, it follows that

Proposition 2 *If $\overline{\rho}_M \leq \rho \leq \rho^c$, $L^*_M = L^*_{MD}$, is the equilibrium level of market borrowing, and the country defaults in the high-repayment-cost state, while if $\rho > \rho^c$, $L^*_M = L^*_I$ is the equilibrium, the market charges the riskless rate of interest, and there is no default in either state.*

Proposition 2 confirms the intuition that, when shocks are rare, the incentives of a country to repay are lower and market lending cannot be risk free. In contrast, if shocks are more frequent, there is a high likelihood the country would like to borrow from the market in the future. This, in turn, implies that the value of keeping the borrowing relation in good standing may be high enough that the market can mimic the official sector and also lend risk free.[23]

4.1 Comparison between market and IFI lending

Looking at Figure 4, which compares market and IFI lending, it is evident that, for low values of ρ, no lending, IFI or market, can be supported in equilibrium. When ρ is too small, the value of the lending relationship is low so that the country will have the incentive to default, even if the cost of repayment is low. At higher values of $\rho, (\rho > \overline{\rho}_M)$, market lending is supported, the country will repay in the low-repayment-cost state and default in the high-repayment-cost state. At somewhat higher values of $\rho, (\rho > \overline{\rho}_I)$ the IFI can lend and, as the value of the relationship is higher, it can expect to be repaid even in the high repayment cost state.[24] At still higher values of ρ, the market can replicate the IFI and also lend risk free. The comparison between market lending and IFI lending is most interesting for intermediate values or ρ, when ρ is high enough to support both types of lending but not so high to eliminate the difference between the two. We can then show that

Proposition 3 *For low values of ρ, $\rho < \widetilde{\rho}$, the utility level associated with market lending is higher than that associated with IFI lending. For intermediate values of ρ, $\rho \in [\widetilde{\rho}, \rho^c]$, the utility associated with IFI lending is higher than that associated with market lending. For sufficiently high values of ρ, $\rho > \rho^c > \widehat{\rho}_I$, the utility value of market and IFI lending is the same.*

Proof: In Appendix.

This implies that, as shown in Figure 4, when the probability of a shock is low, market lending dominates IFI lending. The intuition is that, given the existence of a high-repayment-cost state, market lending *de*

[22] This follows directly from the fact that if the country has no incentive in defaulting on L^*_{MD} in the high-repayment-cost state, *a fortiori it* will have no incentives in defaulting on the optimal amount of borrowing L^*_I. The argument is so straightforward that we omit the formal proof.

[23] Notice that the equilibrium characterized above, is not necessarily the only market equilibrium in the interval $(\widetilde{\rho}, \rho^c)$–where $\widetilde{\rho}$ is the value of ρ, such that $\rho > \widetilde{\rho} \Longleftrightarrow V^*_I > V^*_{MD}$. There can be another equilibrium in this interval where the market believes that the country always repays creditors and thus charges the risk-free rate. However, we can show that Proposition (2), and (3) below hold true if we had selected the other equilibrium.

[24] Condition (A1) implies that $\overline{\rho}_M < \overline{\rho}_I < \widehat{\rho}_M < \widehat{\rho}_I$.

facto offers a state-contingent contract. Given that a new shock will only occur with a low probability, the costs associated with losing market access are limited, when compared with the gains associated with state-contingent repayments/defaults. Hence, for low values of ρ, the utility associated with market lending is higher than that associated with IFI lending. When the probability of a shock is large, the market and the IFI offer the very same volume of lending, at the same price, and thus they bring the same utility levels. For intermediate values of ρ, however, the utility associated with IFI lending is higher. It is more valuable for the country to obtain a lower rate of interest and to be assured continued access than to pay a higher rate, default and lose access. IFIs are able to offer this contract (and maintain preferred creditor status), as they can limit lending to volumes that are consistent with repayment in all states. In our model, preferred creditors do not price risk, by definition they lend risk free. Instead, they manage risk by restricting credit volumes, if, and when, it is necessary to do so.

5 Market and IFI lending-the blended case

In the previous section, we discussed IFI and market borrowing separately and independently of each other. However, in general, countries may borrow both from the market and from IFIs; in such a situation, the volume of market lending will affect optimal IFI lending and vice versa. In addition, even if the market does not lend, the very possibility that it can lend may affect the volume of lending extended by a preferred creditor. In this section, we thus allow the country to borrow both from a set of competitive private lenders and from an IFI. Once again we will investigate feasible and optimal lending allocations imposing the restriction that the preferred lenders should be repaid in all states. We refer to this as the blended case, denoted by the subscript B.

An important assumption is that there are no cross-default clauses between IFIs and private creditors so that, if the country defaults on the market, it can continue to borrow from the IFI and vice versa. Indeed, we work under the premise that IFIs offer the same terms when the country is in good standing with the market and when it is in default. A symmetric assumption also applies to the market, which also offers similar terms whether the country defaulted on IFIs or not. While one may question the realism of such assumptions, without them it would be difficult to understand why, absent any repayment enforcement mechanism, IFIs can still enjoy PCT. Assuming that the country always repays the IFIs, that it repays the market in the low-repayment-cost state and defaults in the high-repayment-cost state, the value function can be written as

$$V_{B_t} = V_{IB_t} + V_{MB_t}, \tag{15}$$

where

$$V_{IB_t} = \rho(-C + aL_{IB_t} - \frac{{L_{IB_t}}^2}{2} - L_{IB_t}(\pi + (1-\pi)k) + V_{IB_{t+1}}), \tag{16}$$

$$V_{MB_t} = \rho(aL_{MB_t} - \frac{(L_{IB_t} + L_{MB_t})^2 - L_{IB_t}^2}{2} - L_{MB_t} + \pi V_{MB_{t+1}}), \tag{17}$$

where (16) denotes the value of the relation with the IFI, same as (1), and (17) the additional utility associated with borrowing L_{MB} at an interest rate $1/\pi$ from the market.

As discussed previously, IFIs differs from the market in that they can commit to the amounts they lend, while there can be no such commitment from commercial lenders. It is natural therefore to solve the blended case assuming that the IFI moves first (as a Stackelberg leader) and decides how much to lend anticipating the volume of loans that the country would then choose to take from the market, with the interest rate charged by private lenders reflecting the default risk. When we solve for the optimal amount of market lending for a given level of IFI lending, we obtain

Lemma 2 *If the country defaults on the market in the high-repayment-cost state, and honors its debt in the low-repayment-cost state, for any given amount of risk-free IFI lending L_{IB}, the optimal amount of market* *borrowing is given by*

$$L_{MB}^{*}(L_{IB})\begin{cases} a-1-L_{IB}, & \text{if} \quad 0<L_{IB}<\widehat{L}\equiv a+1-\frac{2}{\pi\rho};\\ \frac{2(\rho(a\pi-\pi L_{IB})-1)}{\pi\rho}, & \text{if} \quad \widehat{L}<L_{IB}<\overline{L}\equiv a-\frac{1}{\pi\rho};\\ 0, & \text{if} \quad L_{IB}>\overline{L}; \end{cases} \tag{18}$$

and the associated utility by

$$V_B(L_{IB})=\begin{cases} V_{B1}=-\frac{\rho C}{(1-\rho)}+\frac{L_{IB}\rho(2a-L_{IB}-2((1-\pi)k+\pi)}{2(1-\rho)}+\frac{(1-a+L_{IB})^2\rho}{2(1-\pi\rho)}, & \text{if} \quad 0<L_{IB}<\widehat{L};\\ V_{B2}=-\frac{\rho C}{(1-\rho)}+\frac{+L_{IB}\rho(2a-L_{IB}-2((1-\pi)k+\pi)}{2(1-\rho)}+\frac{2((a-L_{IB})\pi\rho-1)}{\rho\pi^2}, & \text{if} \quad \widehat{L}<L_{IB}<\overline{L};\\ V_{B3}=-\frac{\rho C}{(1-\rho)}+\frac{L_{IB}\rho(2a-L_{IB}-2((1-\pi)k+\pi)}{2(1-\rho)}, & \text{if} \quad L_{IB}>\overline{L}. \end{cases} \tag{19}$$

Proof: In Appendix.

Once again there are three different regimes and, in this case, we choose to differentiate them by the amount of IFI lending offered, so that (18) can be thought of as a reaction function–how market lending responds to the chosen volume of lending by the IFI.[25] Such a reaction function has a different specification in each regime. In the first regime, the volume of IFI lending is low and the volume of market lending is unconstrained. This means that, for each additional dollar of official lending, market lending is reduced one to one. In the second regime, the volume of IFI lending is higher, and the market-repayment constraint in the low-repayment-cost state is now binding. Here, for each additional dollar of IFI lending, market lending must thus fall more steeply. In the third regime, IFI lending is higher still and no market lending is supported.

The constraint that the private sector must be repaid in the low-repayment-cost state is embedded in these reaction functions but, so far, the fact that preferred creditors must be repaid in all states has been ignored. A necessary and sufficient condition for the IFI to always be repaid is that, in the high-repayment-cost state, the country is better off repaying, rather than defaulting and relying henceforth solely on the market.[26] Formally:

Definition 1 *L_I is risk free iff*

$$V_B(L_{IB})-kL_{IB}>V_M, \tag{20}$$

and the set $\mathbf{L}_{IB}^{*}$ where IFI lending is risk-free is

$$\mathbf{L}_{IB}^{*}=\{L_{IB}\mid V_B(L_{IB})-kL_{IB}>V_M\}. \tag{21}$$

Let us now start investigating under which conditions IFI lending is risk free and how the different variables affect the set $\mathbf{L}_{IB}^{*}$. We begin by proving that

Lemma 3 *If $k^a \equiv \frac{a-\pi}{1-\pi} > k > \frac{\pi(\pi(a-1)-2)}{4(1-\pi)} \equiv k^b$, for sufficiently large values of the shock probability ($\rho>\rho^a>\overline{\rho}_I$) the set $\mathbf{L}_{IB}^{*}$ where official lending is risk free is non empty.*

Proof: In Appendix

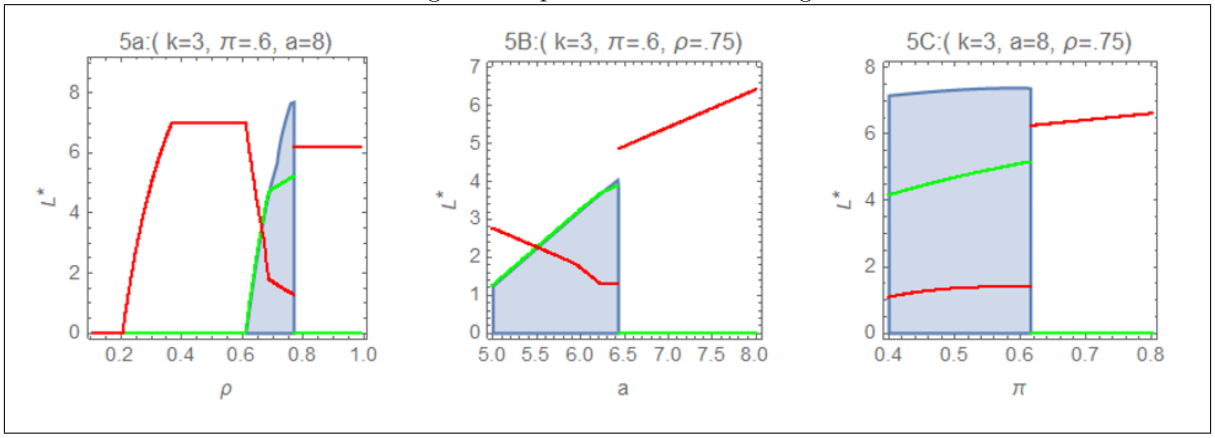
The blue/gray shaded area in Figure 5 depicts the set of feasible risk-free IFI lending as a function of the different parameters of the model.[27] At very low values of ρ (the probability of a negative shock leading

[25] Note that we do not allow IFIs to re-optimize should the country default on the private sector. A justification for this is that countries' lending envelopes with IFIs tend to be relatively fixed. Moreover, re-optimization would imply greater lending from IFIs but this is generally frowned upon as it may be seen as rewarding a country that had defaulted on the market. Technically, the continuation value of only being able to borrow from IFIs is like a constant outside option and so this assumption has little bearing on the overall nature of most of the results, see also the discussion below.

[26] To simplify the analysis and to focus on the policy relevant cases, we rule out the possibility for the country to default on the IFI, to borrow from the market, and use the lending proceeds to repays its IFI arrears and resume IFI borrowing.

[27] Note that the feasible set for the IFI is a function of the volume of market lending.

Figure 5: Optimal and safe lending



to emergency lending) no preferred lending (the green line) nor market lending (the red line) is supported. As ρ rises market lending becomes feasible and then at still higher values of ρ preferred lending becomes feasible and the amount of IFI preferred lending increases as ρ increases. Those countries where such large negative shocks are more frequent are able to borrow more as they have greater incentives repay. Note that the volume of feasible IFI lending also increases with a, the value of lending, and with π, that is, when the probability of the high-repayment-cost state declines. However, as ρ, a and π increase at a certain point preferred lending becomes infeasible. This happens when the market is willing to offer the very same loan the IFI offers, in which case there is no (or very little) cost[28] in defaulting on the IFI and thus no risk-free IFI lending can be supported in equilibrium.[29]

5.1 Optimal lending

Let us now switch our attention to the optimal amount of IFI lending. To compute the optimal lending levels, we not only take into account the reaction functions (18)–computed assuming that the country defaults on the market in the high-repayment-cost state–but we also consider the case in which the market can mimic IFIs–lending the same amount risk free and charging the risk-free rate. In perhaps the most interesting cases, the country will choose to borrow both from the IFI, which offers the risk-free rate, and from the market, which offers a more expensive contract, anticipating default in the high-repayment-cost state.

Figure 5 also plots the optimal volume of IFI (preferred) and private (defaultable) lending. The optimal lending volume by the market is depicted by the red line and that of the preferred IFI lending by the green line. At low values of ρ, lending is infeasible then, as ρ increases, there is a region where market lending becomes feasible and the optimum consists of solely borrowing from the market. At first the market is constrained by the repayment constraint in the low-repayment-cost state. In this region, the optimal amount of market lending rises with ρ. As the probability of negative shocks rise, the value of the relationship with private lenders also rises and so does optimal market lending. As ρ rises further, the constraint becomes irrelevant–this is the part of the red line that is horizontal, and market lending is at the optimal level and independent

[28] To be precise, if $\rho > \rho^c$, for low values of k, $k < k^T \equiv \frac{1+\pi}{\pi}$, there are no gains for the country to borrow defaultable debt at the risk adjusted rate and thus there is no cost in defaulting from the IFI. If $k < k^T$ it would instead be optimal for the country to borrow both from the IFI and from the market. However such gains are not large enough to induce the country to repay the IFI.

[29] For this particular result to hold, the assumption that IFIs do not re-optimize the volume of lending when the country default on the market is critical. Were not this the case, we could end up in a situation where, nothwithstanding the fact the risk free lending is feasible, neither the market nor IFIs will be willing to lend because it would be always in the country interest to default on one type of lenders and relying on the other from then on.

of ρ. As ρ continues to rise, risk-free IFI lending becomes feasible and becomes part of the optimal allocation. As noted above, as IFI lending rises, with higher ρ, market lending falls in the blended optimum.

To understand the interaction between the market and IFI schedules fully (the red and the green lines) it is useful to note (comparing Figure 5 with Figure 4 or considering Figure 6 below) that when both the market and IFI lenders are present, preferred IFI lending is feasible only for values of ρ that are higher than those for which, absent market borrowing, IFI lending is feasible. The reason is that the very presence of a market alternative increases the incentives to default on IFI loans. Intuitively, when the V_I and the V_{MB} schedules cross, at $\rho = \widetilde{\rho}$, the value of market and IFI lending are the same. Hence, it would be in the country's interest to default on the IFIs (saving on debt service) and borrow from the market; this makes preferred lending infeasible. For higher values of ρ, the advantages of IFI lending *vis-à-vis* market lending are greater and a positive volume of both IFI and market lending may be sustained in equilibrium. But then, at a higher value of ρ ($\rho = \rho^c$), the market is able to also offer risk-free loans and this undermines IFIs' ability to lend risk free and being repaid in equilibrium. Formally, we can prove that

Proposition 4 *If $k \in [k^b, k^a]$ and $\rho \in (\rho^a, \rho^c)$, $L^*_{IB} > 0$ and the presence of IFIs strictly improves welfare.*

Proof: In Appendix

As IFI safe lending becomes feasible, it is at first constrained (the optimum is at the frontier of the feasible set) and as ρ rises, in the optimum IFI lending increases and market lending falls. At higher values of ρ, IFI lending becomes unconstrained and the market becomes constrained–this is where the optimum for IFI lending leaves the frontier of the feasible set. In this region, the IFI offers loans at lower interest rates, but which may become onerous to repay in the high-repayment-cost state; in contrast, the market offers loans at higher interest rates but will face default should the high-repayment-cost state materialize. Of course, the higher the probability of a shock ρ, the higher is the appeal of relying on IFI *vis-à-vis* market lending so that, as ρ rises, optimal IFI lending also rises, while optimal market lending falls.

In order to better understand the welfare implications of the three different types of lending, in Figure 6, we plot the value of the different lending relation (6a), the lending volumes (6b), and the welfare associated with the blended case with that of only the market and only IFI lending–to assess the welfare contribution of IFI lending (6c and 6d).

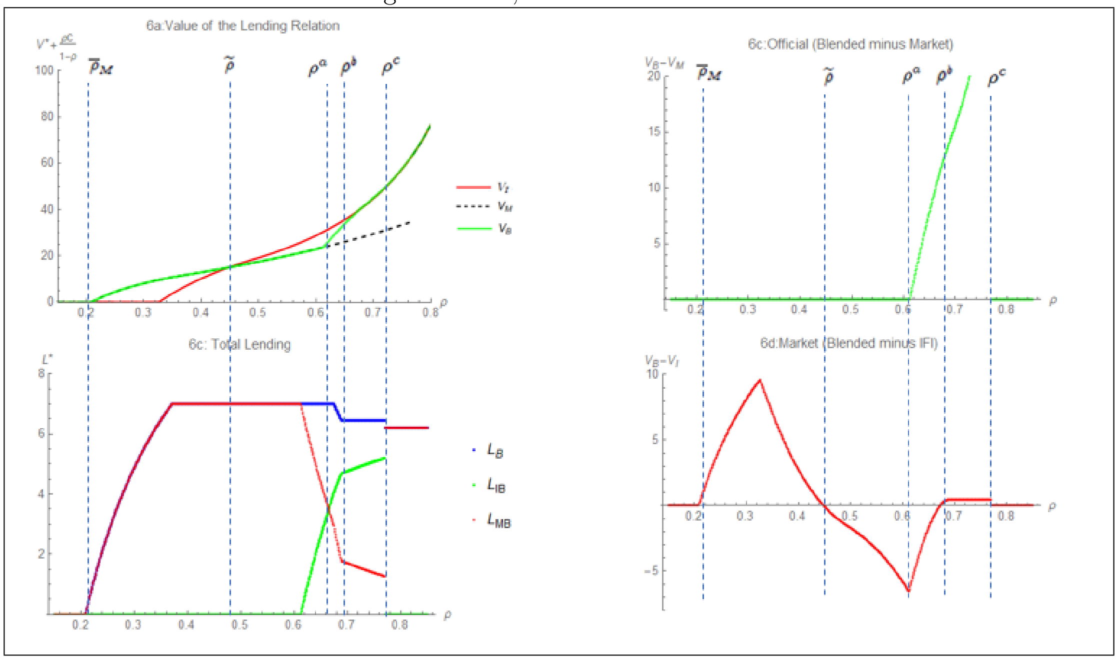
As discussed, for low values of ρ ($\rho < \widetilde{\rho}$), market lending is preferred. Things become more interesting in the interval $[\widetilde{\rho}, \rho^b]$ where IFI lending is optimal but not feasible in the blended case. This is because the very presence of market lenders would induce the country to default on IFIs and thus to be able borrow (solely) from the market. In this region, it would be in the country's self interest to be barred from borrowing from private creditors and only borrow from IFIs. The existence of the market reduces welfare in this region and so if private lending can be barred, in the interval $[\widetilde{\rho}, \rho^b]$ the country can borrow larger amounts from IFIs and this strictly improves welfare–see Figure 6d. Note that having the possibility of blended lending adds most value (relative to just market) in the intermediate region for ρ–see Figure 6c.

6 Optimal "capital allocation"

In the above, we have assumed that IFIs always have the necessary resources to lend the optimal amount. However, there may well be quantitative restrictions on the amounts IFIs can lend. Indeed, in the real world, greater lending implies larger capital and overhead costs, and while the IMF and MDBs have different financial models, in the end lending is limited by the amount of resources available, which, in turn, is a function of the resources given or promised to these institutions by they shareholders.

As noted above, there are circumstances where IFIs should not lend at all–at least not for the type of shocks we have in mind here. These includes situations where it is not feasible to lend and expect to be repaid in all states, as well as situations in which ρ and the market can mimic IFIs, which are thus unable to bring anything to the table. Our first normative result is then that IFIs can avoid lending to such countries at no cost.

Figure 6: IFI, market and blended



However, there is a more general question, namely if IFIs are constrained in the amount they can lend, how should they allocate loans across a set of heterogeneous countries? To answer such a question, we run numerical simulations considering a continuum of countries,[30] differing only with respect to the value of the probability of a shock ρ, which we assume to be uniformly distributed. To make progress we have to make an assumption regarding the representative IFI's objective function.[31] We assume that the official lender weighs each country's utility equally, and that it maximizes the sum of the utilities in the blended case, net of the utilities associated with optimal market lending if it abstained from offering any loan. We then compare the globally unconstrained optimum against situations in which the preferred creditor's ability to lend is limited by increasingly tighter global lending limits. These limits may arise because of resource or capital constraints. The results are summarized in Figure 7.

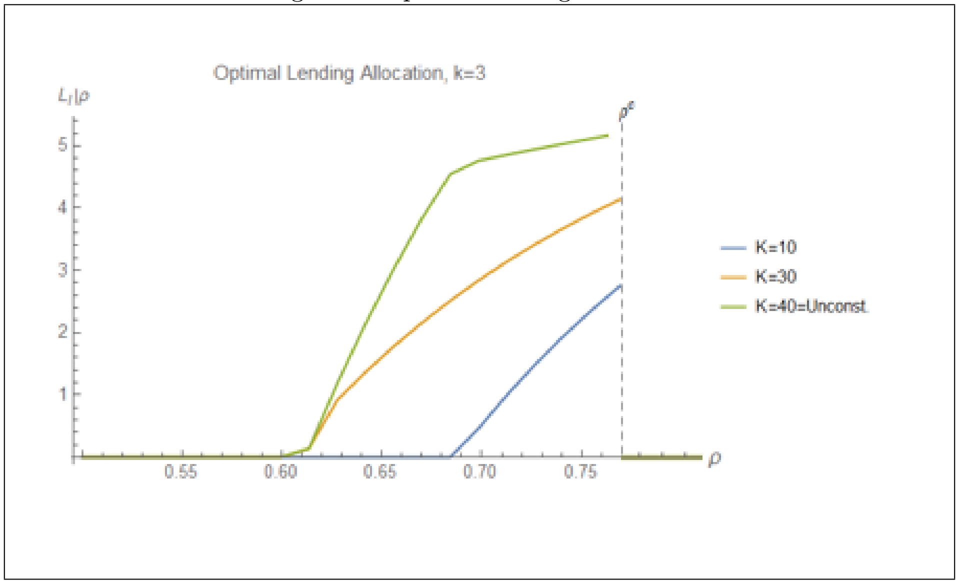
The green line represents the case where the preferred lender has adequate resources to fund the total amount of optimal lending, which is exactly equal to the green line in Figure 5a. When the preferred lender becomes constrained, it will start allocating resources where they are most valuable. Considering Figure 6c, preferred lending adds most value at intermediate values of ρ where the relative advantage of IFI *vis-à-vis* market lending is the highest. Of course, for higher values of ρ ($\rho > \rho^c$), the market can mimic IFIs, and thus there is no reason for IFIs to lend in that region.

These results suggest where IFIs should focus their lending firepower and the results are quite intuitive. IFI lending should be focussed on countries where the probability of a major shock is relatively high–but not too high. It is in this intermediate range that the additional value of preferred lending over market lending is greatest. For countries where the probability of a shock is low, IFI lending may not be feasible. In the intermediate range, countries will in general wish to borrow from both IFIs and the market and countries benefit from having the two types of contract–a low-cost loan that is always repaid, and a higher cost loan, which is only paid in some, but not in all states. At very high values of ρ, preferred lenders add nothing

[30] More precisely we run the numerical simulations for 20 different values of ρ, using the same parameter values as in the previous simulations.

[31] In our analysis, we assume that only one IFI exists. But the problem is equivalent to one in which the IFI community has to allocate its limited resources across countries. Of course, here we have to abstract from differences in preferences among IFIs.

Figure 7: Optimal lending allocation



compared to the market.

7 Interpretation and robustness of the results

In our modeling strategy, we stripped the problem down to a set of core elements. This allowed us to obtain a set of closed-form results, something relatively rare in the current sovereign debt literature, which tends to rely on numerical solutions. However, questions may arise on how we should interpret the different parameters of the model and on whether our results would hold true in a more general set up. We now try to answer these questions.

To present the different cases, we have mostly focused on the parameter ρ, the probability of a large negative shock that prompts the need for emergency lending. Our main finding is that IFI lending is more valuable for intermediate values of ρ. The intuition is that if ρ is low, no lending would be supported because the country will always default and, if ρ is large, the value of the lending relationship is so high that the country will always repay and can thus borrow risk free from the market. The most important assumption behind these results is that the probability of the negative shocks and their intensity are orthogonal, which in reality may not be the case. Had we assumed that the frequency and intensity were correlated, perhaps negatively, both components would affect the value of the lending relationship.[32] In addition, in the case of frequent and relatively small shocks, the result that for large enough values of ρ the market can lend risk free, would be a pretty accurate description of business cycle lending in advanced economies, rather than simply a theoretical possibility.

As per the intensity of the shocks, we assumed that it is equal to a fixed parameter C. We did not vary that parameter in our analysis. Instead, we paid more attention to the parameter a, the marginal value of lending. If we think that the value of lending is more valuable if shocks are more severe, then the parameter a can be thought of as a measure of the severity of these emergencies. Following the above discussion, the value of the lending relation should depend not only on the values of a and ρ, but also on their correlation.

In our analysis, the key parameter that differentiates the value of IFI and market lending is k, the cost of repayment in the high-repayment-cost state. The higher is k, the more valuable is market lending because

[32]It is worth noting that in a more general model the relative weight of the two would also depend on risk aversion.

of the state contingency. Notice that having normalized the cost of repayment in the low-repayment-cost state to one, k can be thought of as a "measure" of the volatility of the cost of servicing the debt, and hence as a measure of the volatility of fiscal revenues or of fiscal needs.

With the aim of simplifying the analysis, we assumed that k could only assume two value, "high" and "low." When this is the case, it is necessarily optimal for IFIs to limit their lending to be repaid in both states, letting the market offer contracts that will in general not be paid in the high-repayment-cost state. Notice, however, that if we had a continuum of values of k, IFIs may not wish to be constrained to only lend expecting to be repaid in absolutely all states. Consider, for example, the aftermath of a major financial crisis, a conflict, or a large natural catastrophe. Probably, in such states, the country will either enter into arrears with IFIs, or its obligations be "evergreened." Strictly speaking, totally risk-free lending may not exist. Hence, in a more general set-up it may be optimal for IFIs to offer loans that are repaid in full in almost all states, while the market will offer loans that are repaid less often. Our qualitative results would carry through. Countries may fall into arrears with IFIs in such states and may repay capital and interest at a later date.

In our analysis, we also assumed that if a country defaulted on one type of lender (market or IFI), this would not affect its ability to borrow from the other type. This, however, is generally not necessarily the case. The credit rating of a country would normally be severely affected if arrears were built up with IFIs, which would then curtail the ability to borrow from the market. Had we incorporated this into the model, the value of the IFI lending relation would have increased and as would the ability of IFIs to lend risk free.

8 Conclusion and policy discussion

One of the most persistent puzzles regarding the global financial architecture has been in relation to the preferred status of the main IFIs. While PCT is critical to the operations of the IMF and the major MDBs, it is not generally stated in legal contracts, and it has been normally described as a market custom. This suggests that it should be understood as an endogenous outcome of the relation of a country with its creditors rather than something that is imposed. And yet we know of no paper to date that provides an explanation of why this may be the case. Our contribution is to provide a relatively simple model that illustrates how IFIs can lend at close to the riskless interest rate and expect to be repaid in all states of nature (as it is always in the country's interest to repay) and how the existence of preferred creditors, in addition to private lenders (which may suffer default), may be valuable. Existing papers that consider sovereigns' willingness to pay do not include seniority and those models that include seniority do not appear to consider willingness to pay. We believe that this is the first paper that attempts to provide a theoretical justification for both the existence of preferred lenders and how they may improve welfare.

From the standpoint of the country, borrowing from a private creditor is more expensive but repayments are ex-post state contingent. In contrast, preferred lenders are in a position to offer cheaper more competitive financing but countries must be prepared to repay in all states. In our model, preferred lenders may limit the amount of financing that is offered. The potentially binding constraint for the preferred lender is that it must be repaid in bad states of nature, where the cost of repayment is high, while the potentially binding constraint for private lenders is that they must be repaid in the good state where that cost is low. Hence, while both lenders offer loans with the same fair ex-ante returns, ex post outcomes may differ.

Our results may also be considered in a more normative fashion as IFIs consider their role in sovereign lending versus that of the market. As the two types of lenders play different roles, there is no reason to think that IFIs should mirror the behavior of private lenders. Indeed IFIs add value precisely because they behave differently. If IFIs priced loans to risk, as private lenders do, then this would be tantamount to admitting that their lending is risky and that borrowers may then default. Taking this logic to the limit, IFIs would then become just one more (defaultable) lender among many, and lose their PCT. Rather, we suggest that IFI should consider where they may add most value given that they can offer loans at low rates but subject to the restriction that they expect to be repaid in all states.

We conclude that IFIs should focus their firepower where the probability of shock recurrence is in an intermediate zone. This result holds if preferred lenders are unconstrained in terms of capital and is strengthened if preferred lenders are constrained. Indeed, if the overall lending constraint is very tight, then preferred lenders should refrain from lending to countries with a high probability of crisis recurrence, even if fiscal volatility is quite high (and leave this to the market), and concentrate their activity even more on the group of intermediate shock-prone countries where there is the greatest bang for each buck of lending.

We also discussed a number of caveats to our results given our relatively simple model. In addition to those points, it is worth noting that throughout our analysis, as it is common in the sovereign debt literature, we assumed that the market and IFIs cannot offer state-contingent contracts, where repayment depends on the realization of the state of nature. Were this the case, it might be possible to improve on the current allocation and countries might be able to borrow more and avoid costly defaults. There are various reasons why such contracts do not exist. Some argue that a finance minister is generally not blamed if there is a slump in the global market for an important commodity export but that if a costly hedge is purchased and not needed then there may be an ex-post inquiry as to why resources were "wasted." There have been various proposals for GDP-indexed debt but here there are concerns that statistics might be manipulated and/or such contracts may be difficult to price. A further intriguing reason, more related to our model, is that, if a state-contingent contract takes the form of requiring lower payments in bad states but higher repayments in good states, then the willingness to repay in the good state may be threatened. In the comparison of the continuing value of the lending relationship versus the cost of repayment, this may actually restrict lending.[33]

We also assumed that loans should be repaid at the end of each period and hence we ruled out the possibility that countries might roll-over debt with IFIs and borrow more to avoid repayment when it is most costly to do so. This discussion is closely related to the restriction maintained by the main IFIs that they should not "evergeen"–extend new loans to repay old loans. Had we allowed this, the official sector could lend to help countries repay market. However, this would go against the principle of "private sector involvement" in periods of debt distress or, in other words, against the principle that the private sector should suffer losses as, presumably, it had lent at rates precisely reflecting such risks. If "evergreening" of official loans and additional borrowing more to "bail out" private creditors were allowed, then this would add even greater value to official lending (from the standpoint of the country) and make it easier and cheaper for the country to borrow from the market. However it is unlikely that such practices would be consistent with maintaining PCT, a AAA credit rating, and a low cost of financing for those multilaterals that borrow on global capital markets.

A further interesting result is that, for some parameter values, it would be in a country's own best interest to not be able to borrow from private markets. This is an important issue that is discussed at length in IFIs and even merits an acronym, NCBP, which stands for non-concessional borrowing policy, and applies to countries that received debt relief under the Multilateral Debt Relief Initiative (MDRI). More precisely, IFIs sought to restrict the amount of commercial debt such countries could contract, both to avoid the repeat of the build up of unsustainable levels of debt and in order to preserve PCT. IFIs have tried to implement NCBPs with the threat that future concessional resources might be curtailed if countries did not abide. But it is not easy to enforce such a rule. Countries have many ways to contract debt. For example, public companies may provide an indirect way of borrowing that is hard to monitor. Another is that investment needs in the developing world are large so that, in many instances, IFIs may be tempted to offer NCPBs' waivers. Indeed, despite the NCBP, public debt in low income countries that obtained debt relief has increased in the last decade and is now close pre-MDRI levels. Still, this is an issue that deserves further thought and consideration.

To conclude, our fundamental result, is that creditors that expect to be paid in all states of nature are very different animals to commercial lenders. While commercial lenders price loans according to risk and should therefore expect restructuring if unfavorable states materialize, IFIs who expect loans to be risk free play by different rules. But in order to play by those rules their behavior must be consistent. Considering

[33] Anderson, Gilbert and Powell (1989) suggest that the World Bank may wish to guarantee such state contingent contracts. The argument is that markets can manage price risks while multilaterals may have a comparative advantage in controlling "performance risks." This argument depends on the multilateral having some other power of persuasion on the country perhaps coming from its governance structure.

carefully the constraints and ensuring that preferred lending is complementary to market lending should help maximize the bang for each buck of preferred creditors' capital.

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9 Appendix

Proof of Proposition 1:

From (4) we have that a country will not default in high-repayment-cost states and repay in low-repayment-cost state if

$$L_I \leq 2(a - \frac{(1-(1-\pi)\rho)(k-1)}{\rho}) \equiv \widetilde{L}_I.$$

Substituting (2) into (3), the maximum amount of official lending that will be repaid in both states is given by

$$\overline{L}_I = 2(a - \pi + (\pi - \frac{1}{\rho})k). \tag{22}$$

It follows that $\widetilde{L}_I - \overline{L}_I > 0 \iff \pi < \frac{1}{2} + \frac{1}{2\rho(k-1)} \equiv \pi^k$, which is always the case because of (A1), so that (3) implies (4). We thus have that

$$\overline{L}_I > 0 \iff \rho > \frac{k}{a + \pi(k-1)} \equiv \overline{\rho}_I. \tag{23}$$

If $\rho \leq \overline{\rho}_I$, no official lending occurs. Consider the case $\rho > \overline{\rho}_I$. Absent default constraints, in period t, the country would choose

$$\widehat{L}_I \equiv \arg\max_{L_I} - C + \ aL - \frac{L_I^2}{2} - ((1-\pi)k + \pi)L_I, \tag{24}$$

that is,

$$\widehat{L}_I = a - \pi - (1-\pi)k. \tag{25}$$

However, this ignores the default constraints. The solution will be constrained, if $\widehat{L}_I - \overline{L}_I = \frac{2k - \rho(a+k-\pi(k-1)}{\rho} > 0 \iff \rho < \frac{2k}{a+\pi(k-1)-\pi} \equiv \widehat{\rho}_I$. So the optimum lending will be the constrained solution, $L_I^* = \overline{L}_I$ if $\rho < \widehat{\rho}_I$, and the unconstrained one, $L_I^* = \widehat{L}_I$ if $\rho \geq \widehat{\rho}_I$. Substituting L_I^* into (2) we can then solve for the optimal value of the value function V_I^* for each case. □

Proof of Lemma 1:

Substituting (8) into (9), the maximum amount of lending that will be repaid in the high-repayment-cost state is given by

$$\overline{L}_{MD} = \frac{2(\rho a \pi - 1)}{\pi \rho}, \tag{26}$$

and

$$\overline{L}_{MD} > 0 \iff \rho > \frac{1}{a\pi} \equiv \overline{\rho}_M. \tag{27}$$

Hence, if $\rho \leq \overline{\rho}_M$, no market lending occurs. Consider the case $\rho > \overline{\rho}_M$. Absent default constraints, should a shock occur, the country would choose

$$\widehat{L}_{MD} \equiv \arg\max_{L} - C + \alpha L_{MD} - \frac{L_{MD}^2}{2} - L_{MD}, \tag{28}$$

that is,

$$\widehat{L}_{MD} = a - 1. \tag{29}$$

However, as we just said, this ignores the default constraints. The solution will be constrained, if $\widehat{L}_{MD} - \overline{L}_{MD} = \frac{2-\pi\rho(a+1)}{\pi\rho} > 0 \iff \rho < \frac{2}{\pi(a+1)} \equiv \widehat{\rho}_M$. So the optimum lending will be the constrained solution, $L_{MD}^* = \overline{L}_{MD}$ if $\rho < \widehat{\rho}_M$, and the unconstrained solution will be, $L_{MD}^* = \widehat{L}_{MD}$, if $\rho \geq \widehat{\rho}_M$. Substituting L_{MD}^* into (8), we can then solve for the optimal value of the value function V_{MD}^* for each case.□

Derivation of Condition 13:

The value function for the country, assuming that, if faced with a shock, borrows $\widehat{L}_{MD}$ from an IFI at the risk-free interest rate, and it repays both in the high- and in the low-repayment-cost state, is given by:

$$V_{ND} = \rho(-C + a\widehat{L}_{MD} - \frac{\widehat{L}_{MD}^2}{2} - \frac{\widehat{L}_{MD}}{\pi}((1-\pi)k + \pi) + V_{ND}). \tag{30}$$

Solving for V_{ND}, and using (29), we have that

$$V_{ND} = \frac{((2K - (a-1)^2)\pi - 2(a-1)(1-\pi)k)\rho}{2\pi(1-\rho)}, \tag{31}$$

and substituting this value into that condition (12), and using (29), we obtain

$$\rho \leq \frac{2k}{(a+2k-1)\pi} \equiv \rho^c \Longleftrightarrow V_{ND} - k\frac{L_{MD}^*}{\pi} \leq -\frac{\rho C}{1-\rho}.$$

Proof of Proposition 3:

To prove Proposition 3, it is useful to start by proving two Lemmas.

Lemma 4 *If $\rho \in [\overline{\rho}_I, 1)$, $V_I - V_{MD}$ increases with ρ.*

Proof: Under Assumption A1, we have that $\overline{\rho}_M < \overline{\rho}_I < \widehat{\rho}_M < \widehat{\rho}_I$. We show that $\frac{\partial(V_I - V_{MD})}{\partial\rho}$ is positive in all the different sub-intervals of ρ.
(i) In the interval $\rho \in (\overline{\rho}_I < \widehat{\rho}_M]$, $\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{2(k^2\pi^2-1)}{\pi^2\rho^2} > 0$, because of (A1).
(ii) In the interval $\rho \in (\widehat{\rho}_M, \widehat{\rho}_I]$, $\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{2k^2}{\rho^2} - \frac{(a-1)^2}{2(1-\pi\rho)^2}$. We further have that $\underset{\rho\to\widehat{\rho}_M^+}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{(a+1)^2(k\pi^2-1)}{2} > 0$, and $\underset{\rho\to\widehat{\rho}_I^-}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{(k+1)(1-\pi)(2a-1+k(1-\pi)-\pi)(a+k+(k-1)\pi)^2}{2(a+k-(1+k)\pi)^2} > 0$. Notice further that $\frac{\partial(V_I - V_{MD})}{\partial\rho} = 0$ iff $k = \pm\frac{\rho}{2}\frac{a-1}{1-\pi\rho}$. This means that there is at most one value of $k > 0$ in the interval for which the expression can change sign. Hence the expression is positive in the whole interval.
(iii) In the interval $[\widehat{\rho}_I, 1)$, $\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{1}{2}(\frac{(a-k-\pi+k\pi)^2}{(1-\rho)^2} - \frac{(a-1)^2}{(1-\pi\rho)^2})$. We further have that $\underset{\rho\to\widehat{\rho}_I}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{(k+1)(1-\pi)(2a-1+k(1-\pi)-\pi)(a+k+(k-1)\pi)^2}{2(a+k-(1+k)\pi)^2} > 0$ and that $\underset{\rho\to1}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} > 0$. Notice further that $\frac{\partial(V_I - V_{MD})}{\partial\rho} = 0$ iff $\rho = \frac{k-1}{a-1+(k-1)\pi} \equiv \rho^d$ or $\rho = \frac{2a-1-k+\pi(k-1)}{a(1+\pi)-1-\pi(k-\pi(k-1)} \equiv \rho^e$. Since $k < a \Longleftrightarrow \rho^f > 1$ then there is at most one value of $\rho \in (\widehat{\rho}_I, 1]$ for which the expression can change sign. Hence $\frac{\partial(V_I - V_{MD})}{\partial\rho}$ is positive in the interval. This, together with the fact that $V_I - V_{MD}$ is a continuous function proves the Lemma.□

Lemma 5 *There is a $\widetilde{\rho} \in [\overline{\rho}_I, 1)$, such that for $\rho > \widetilde{\rho} \Longleftrightarrow V_I > V_{MD}$.*

Proof: In the interval $[\overline{\rho}_M, \overline{\rho}_I]$, $V_M > V_I$ follows directly for the fact that $L_M > L_I = 0$. We further have that in the interval $\rho \in (\widehat{\rho}_I, 1)$, $V_I - V_{MD} = \frac{\rho}{2}\left(\frac{(\alpha-\pi-(1-\pi\rho)k)^2}{(1-\rho)} - \frac{(a-1)^2}{(1-\rho\pi)}\right)$, so that $\underset{\rho\to1}{Lim}(V_I - V_{MD}) > 0$. This together with the fact that $V_M > V_I$ at $\rho = \overline{\rho}_I$, and that $V_I - V_M$ is increasing in ρ in the interval $\rho \in [\overline{\rho}_I, 1)$, because of Lemma 2, proves the Lemma.□

Now, to prove Proposition 3, first, notice that, if $\rho < \widetilde{\rho}$, $V_{MD} > V_I$, and hence $L_M^* = L_I^*$ cannot be an equilibrium, so that $L_M^* = L_{MD}^*$. For sufficiently high values of ρ, $\rho > \rho^c$, we have that $L_M^* = L_I^*$ and thus $V_M^* = V_I^*$. It remains to prove that that the interval $[\widetilde{\rho}, \rho^c]$ is non-empty so that the interval in which $V_I^* > V_M^*$ is also non empty. Since, $\rho^c > \widehat{\rho}_I$, it is enough to show that

$$\rho^c > \rho : V_{MD}^* \big|_{\rho > \widehat{\rho}_M} = V_I^* \big|_{\rho > \widehat{\rho}_I} \equiv \rho^f$$

Using (6) and (11), we have that

$$\rho^f = \frac{(a-1)^2 - (a-k(1-\pi)-\pi)^2}{(a-1)^2 - \pi(a-k(1-\pi)-\pi)^2},$$

and (after some algebra) $a > 1 \Longrightarrow \rho^c > \rho^f$.$\square$

Proof of Lemma 2:

For a given L_{IB}, the additional utility associated with borrowing L_{MB} at an interest rate $1/\pi$ from the market V_{MB_t} is given by (17). Equating V_{MB_t} with $V_{MB_{t}+1}$ the value of market borrowing (on top of official borrowing) can be written as:

$$V_{MB} = \frac{L_{MB}(2(a-1) - L_{MB} - 2L_{IB})\rho}{2(1-\pi\rho)}. \tag{32}$$

For the market to be willing to offer risky loans, the country must be willing to service this debt in the low repayment-cost state. This condition can be written as

$$V_{MB} \geq \frac{L_{MB}}{\pi}. \tag{33}$$

Substituting (32) into (33) at equality, the maximum amount of lending that will be repaid in the low repayment-cost state is given by:

$$\overline{L}_{MB} = \frac{2(\rho\pi(a - L_{IB}) - 1)}{\pi\rho}. \tag{34}$$

We further have that

$$\overline{L}_{MB} > 0 \Longleftrightarrow L_I < a - \frac{1}{\pi\rho} \equiv \overline{L}.$$

Absent default constraints, in period t, the country would choose

$$\widehat{L}_{MB} \equiv \underset{L_{MB}}{\arg\max} V_{MB_t} = a - 1 - L_{IB}. \tag{35}$$

However, as we just said, this ignores the default constraints. The solution will be constrained, if $\widehat{L}_{MB} - \overline{L}_{MB} = a + 1 - L_I - \frac{2}{\pi\rho} > 0 \Longleftrightarrow L_I < a + 1 - \frac{2}{\pi\rho} \equiv \widehat{L}$. We thus have that

$$L^*_{MB}(L_{IB}) = \begin{cases} a - 1 - L_{IB}, & \text{if } \ 0 < L_{IB} < \widehat{L}; \\ \frac{2(\pi\rho(a-L_{IB})-1)}{\pi\rho}, & \text{if } \ \widehat{L} < L_{IB} < \overline{L}; \\ 0, & \text{if } \ L_{IB} > \overline{L}. \end{cases} \tag{36}$$

Substituting these values into (32)

$$V_{MB} \begin{cases} \frac{(1-a+L_{IB})^2\rho}{2(1-\pi\rho)}, & \text{if } \ 0 < L_{IB} < \widehat{L}; \\ \frac{2(\pi\rho(a-L_{IB})-1)}{\rho\pi^2}, & \text{if } \ \widehat{L} < L_{IB} < \overline{L}; \\ 0, & \text{if } \ L_{IB} > \overline{L}. \end{cases} \tag{37}$$

We further have that the value of the relation with official lenders in the blended case, V_{IB_t}, is given by

$$V_{IB_t} = \rho(-C + aL_{IB_t} - \frac{{L_{IB_t}}^2}{2} - (1-\pi)kL_{IB_t} - \pi L_{IBt} + V_{IB_{t+1}}), \tag{38}$$

and equalizing V_{IB_t} and $V_{IB_{t}+1}$ we have that

$$V_{IB}(L_{IB}) = \frac{-2\rho C + L_{IB}\rho(2a - L_{IB} - 2((1-\pi)k + \pi)}{2(1-\rho)}. \tag{39}$$

Finally, the overall value function (official and market lenders) is then given by $V_B = V_{IB}(L_{IB}) + V_{MB}(L_{IB})$. Using (37), and (39), and assuming that the country is always willing to repay any $L_{IB} \in [0, \overline{L}]$, we obtain (19).This proves the Lemma.□

Proof of Lemma 3:

First notice that if $\rho \leq \overline{\rho}_I$ there is no risk-free lending in the IFI lending scenario and thus, a fortiori, there can be no risk-free IFI lending in the blended case. Consider now the interval $\rho > \widehat{\rho} > \overline{\rho}_I$. A sufficient condition for the existence of risk-free IFI lending if the market does not offer the risk-free loan is

$$k < \underset{L_{IB}\to 0}{Lim} \frac{\partial (V_{B1} - V^*_{MD3})}{\partial L_{IB}} = \underset{L_{IB}\to 0}{Lim} \frac{\partial V_{B1}}{\partial L_{IB}} \Longleftrightarrow \rho > \rho^a \equiv \frac{2k}{2\pi k + (1-\pi) + \sqrt{(1-\pi)(4(a-1)k + (1-\pi)}}, \tag{40}$$

and

$$\rho^a < 1 \Longleftrightarrow k < \frac{a-\pi}{1-\pi} \equiv k^a.$$

It remains to verify that the market does not offer the risk-free loan, this is the case if

$$\rho^c - \rho^a = \frac{2k\left(\sqrt{(1-p)(4k(a-1)+1-p)} + 1 - ap\right)}{p(a+2k-1)\left(\sqrt{(p-1)(-(4ak-4k-p+1))} + 2kp + 1 - p\right)} > 0$$

and

$$\rho^a > \rho^c \Longleftrightarrow k > \frac{\pi(\pi(a+1)-2)}{1-\pi} \equiv k^b.$$

The fact that $k^a > k^b$ completes the proof.□

Proof of Proposition 4:

From Lemma 3, we know that official lending is feasible if $\rho > \rho^a$ and that $\underset{L_{IB}\to 0}{Lim} \frac{\partial V_{B1}}{\partial L_{IB}} > 0$. Hence, in the interval $k \in [k^a, k^b]$ some level of official lending strictly improves welfare.□