Basic Heading PPPs and the Lack of Matching Problem

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Meeting of the World Bank

Technical Advisory Group

March 7-8, 2025

Ease 1345 6th Ave, New York, NY 10105

The Basic Problem

- **The Basic Problem is: how can we determine Basic Heading PPPs between the 6 Regions taking part in the World Bank's International Comparison Program?**
- **Answer: we apply a Multilateral Index Number methodology.**
- **In these notes, we will consider 3 alternative multilateral methods:**

  (i) The Country Product Dummy Method;

  (ii) The Jevons "Star" Country Method and

  (iii) The Jevons-GEKS Method.
- **We will provide a brief introduction to each of the above methods and then show how the various methods performed using some actual data from the 2017 ICP comparisons.**

The Basic Data

- **Suppose that there are K Regions in a regional international comparison of prices with N products in scope. In our 2017 examples, K will equal 5.**
- **The products in scope are the products on the interregional product list for a Basic Heading; e,g., N rice products.**
- **If product n is priced in Region k for the time period under consideration, we denote its annual average price by p_{kn} for k = 1,…,K and $n \in S(k)$ where S(k) denotes the set of products that are priced in Region k.**
- **The prices are expressed in units of Region k's numeraire country's currency.**
- **Each product is priced in a common unit of measurement across countries.**

The Country Product Dummy (CPD) Method

- **The basic assumption made in the CPD model is that the observed country prices satisfy the following equations (approximately):**

(1) $p_{kn} \approx \pi_k \alpha_n$; $k = 1,\ldots,K$; $n \in S(k)$;

- **where α_n is a quality adjustment parameter for product n and π_k is the overall level of prices (for the N product prices in scope) in Region k relative to the level of prices in other Regions; it is the Regional Purchasing Power Parity for Region k that we want to estimate.**
- **Take logarithms of both sides of equations (1) and add error terms to obtain the following linear regression model:**

(2) $\ln p_{kn} = \rho_k + \beta_n + \varepsilon_{kn}$; $k = 1,\ldots,K$; $n \in S(k)$;

- **where ε_{kn} is an error term and ρ_k and β_n are the logarithms of π_k and α_n; i.e.,**

(3) $\rho_k \equiv \ln \pi_k$ for $k = 1,\ldots,K$ and $\beta_n \equiv \ln \alpha_n$ for $n = 1,\ldots,N$.

The CPD Method (continued)

- **Estimates for the parameters ρ_k and β_n in equations (2) can be found by solving the following least squares minimization problem:**

(4) $\min_{\rho,\beta} \sum_{k=1}^{K} \sum_{n \in S(k)} [\ln p_{kn} - \rho_k - \beta_n]^2$

$= \min_{\rho,\beta} \sum_{n=1}^{N} \sum_{k \in S^*(n)} [\ln p_{kn} - \rho_k - \beta_n]^2$

- **where $\rho \equiv [\rho_1,\ldots,\rho_K]$ and $\beta \equiv [\beta_1,\ldots,\beta_N]$ and $S^*(n)$ is the set of Regions k that have priced product n for n = 1,…,N.]**
- **When we exponentiate the ρ and β vectors to obtain the optimal π_k and α_n solutions to equations (1), we obtain the following solutions:**

(7) $\pi_k = \prod_{n \in S(k)} [p_{kn}/\alpha_n]^{1/N(k)}$; $\quad k = 1,\ldots,K;$

(8) $\alpha_n = \prod_{k \in S^*(n)} [p_{kn}/\pi_k]^{1/N^*(n)}$; $\quad n = 1,\ldots,N;$

- **where $N(k)$ is the number of products that are priced in Region k for k = 1,…,K and $N^*(n)$ is the number of Regions that price product n for n = 1,…,N.**

The CPD Method (continued)

- **The sets S(k) and $S^*(n)$ are assumed to be nonempty for k = 1,…,K and n = 1,…,N.**
- **For more details on the algebra in this section, see Rao (1995) (2005) and Diewert (2004) (2023).**
- **Using (7), the ratio of the PPP for Region k to the PPP for Region m is given by:**

(9) $\pi_k/\pi_m = \Pi_{n \in S(k)} [p_{kn}/\alpha_n]^{1/N(k)} / \Pi_{n \in S(m)} [p_{mn}/\alpha_n]^{1/N(m)}$.

- **But the two Regions may not have many products in common so the product prices in the numerator of the right hand side of (9) may be totally different from the product prices in the denominator of the right hand side of (9).**
- **Thus the CPD multilateral method with missing prices will in general have a *lack of matching problem*.**
- **Note that if Regions k and m price exactly the same products, then π_k/π_m collapses down to the ordinary Jevons index.**

Jevons Star Indexes

- **Define S(k,m) as the set of products that are priced in Regions k and m and define N(k,m) as the number of common products that are priced in Regions k and m for k = 1,…,K and m = 1,…,K.**
- **The *maximum overlap Jevons index* which compares the common product prices in Region m to the corresponding product prices in Region k, $P_J(m/k)$, is defined as the geometric mean of the common product price ratios:**

(13) $P_J(m/k) \equiv [\Pi_{n \in S(k.m)} (p_{mn}/p_{kn})]^{1/N(k,m)}$; **m = 1,…,K; k = 1,…,K.**

- **The K vectors of K *maximum overlap Jevons star indexes*, $P_J(k)$, are defined as follows:**

(14) $P_J(k) \equiv [P_J(1/k), P_J(2/k), \ldots, P_J(K/k)]$; **k = 1,…,K.**

- **Note that the prices of Region k are in the denominators of the matched product Jevons indexes $P_J(1/k), P_J(2/k), \ldots, P_J(K/k)]$ while the numerators in these indexes go through all K regions.**

Jevons Star Indexes (continued)

- **Advantages of a Jevons star index over a CPD index are its simplicity, ease of computation and relevance.**
- **A Jevons bilateral star index simply takes an average of price ratios for identical products across regions rather than on estimating preferences of purchasers across countries.**
- **The focus is on obtaining product matches across regions; the greater the number of matches, the more reliable the index will be.**
- **If every Region priced all products in scope, every Jevons Star index would be proportional to the vector of geometric means of product prices across the K regions; after normalization, all Jevons Star indexes would be identical.**
- **A disadvantage of the Jevons star methodology is that in the case of missing observations, there are K possible choices for the base region and the different Jevons star PPPs will in general generate different PPPs.**

The GEKS-Jevons Multilateral Index

- **Following the example of Gini (1931) and others, the vector of *GEKS-Jevons indexes*, P_{GEKS-J}, is defined (up to a proportional factor) by taking the geometric mean of the Jevons star indexes. The resulting indexes turn out to be proportional to the following vector:**

(15) $$P_{GEKS-J} \equiv [\Pi_{k=1}^{K} P_J(1/k)^{1/K}, \Pi_{k=1}^{K} P_J(2/k)^{1/K}, \ldots, \Pi_{k=1}^{K} P_J(K/k)^{1/K}].$$

- **As noted above, if there are missing prices, then the resulting Jevons star indexes can differ substantially from each other. The GEKS Jevons index simply takes an average of the K star indexes.**
- **In the case where the individual star indexes are far from being proportional, then the reliability of the GEKS Jevons index is questionable**

The 2017 ICP Methodology for Linking the Regions

- **The world was divided up into 5 regions:**
- **Region 1 was Africa with 50 countries;**
- **Region 2 was Asia with 22 countries;**
- **Region 3 was the OECD with 49 countries;**
- **Region 4 was Latin America with 13 countries and**
- **Region 5 was West Asia with 12 member countries.**
- **Roughly speaking, there were 86 Basic Heading consumption categories of product that were used to compare prices across the 5 regions.**
- **Within each of these Basic Heading categories, the ICP, in cooperation with the regions, assembled a core list of 631 specific products were priced (in the same units of measurement) across the regions so that the prices of the regions could be linked to each other.**
- **The number of specific products within each BH category varied between 1 and 57.**
- **There were two categories with only 2 specific products. The BH category "Eggs and Egg Based Products" has two specific products: (i) Chicken eggs, caged hen, large size and (ii) Chicken eggs, caged hen, medium size. The BH category "Sugar" has two specific products: (i) White sugar and (ii) Brown sugar.**

The 2017 ICP Methodology for Linking the Regions (continued)

- **There were two categories with only 2 specific products. The BH category "Eggs and Egg Based Products" has two specific products: (i) Chicken eggs, caged hen, large size and (ii) Chicken eggs, caged hen, medium size. The BH category "Sugar" has two specific products: (i) White sugar and (ii) Brown sugar.**
- **The median number of specific products in each BH category was 5.**
- **Once the country prices were collected, they were divided by the regional PPPs for the respective Basic Heading categories.**
- **The regionally deflated prices were then used to link the regional PPPs into World PPPs using a variant of the Country Product Dummy model.**

Some Examples

- **In order to illustrate potential lack of matching problems when making international comparisons, we will look at some examples drawn from the ICP's 2017 data.**
- **There were 631 products on the Global Core List in 2017; however some items were not priced in any region and therefore should be excluded from the calculations.**
- **On top of that, some additional items were effectively excluded from computing inter-regional PPPs as they were only priced in one region. Altogether, there were 50 items like that, so the actual number of items used in calculating the inter-regional PPPs was 581 and the overall fill rate was 72.2%, ranging from 67.8% (OECD) to 77.1% (Asia):**
- **While the average fill rates are relatively high, they are not uniform, and some Basic Headings have much lower rates. Thus there is a significant variance in fill rates between regions in the same Basic Heading, where the fill rates can range from 25% to 100%.**

Example 1: Basic Heading Category is Rice

- There are 11 products in the Rice BH category of products. Here is a listing of the individual products along with their average prices by region:

Table 4: Regional Geometric Average Prices for the BH Category Rice

Product title	Region 1	Region 2	Region 3	Region 4	Region 5
Long-grain rice, parboiled, WKB	15.48997	11.90368	2.50671	4.69858	0.88093
Long-grain rice, not parboiled, WKB	16.37012	12.49387	1.14681	4.17688	0.94589
Long grain rice, family pack, WKB	17.16246	0	0	0	0
Jasmine rice, WKB	0	0	0	0	0
Basmati rice, WKB	33.32396	22.13316	4.01875	20.28345	1.13046
Broken rice, 25%, BNR	11.29647	7.02469	0	2.91915	0
Medium-grain rice, BNR	12.44030	8.84751	0	3.65276	0
Brown rice, family pack, BL	0	13.38436	0	0	0
Short-grain rice, BNR	11.78141	8.19426	0	0	0
Sticky rice, WKB	0	13.23967	4.95208	0	0
Long-grain rice, UNCLE BEN'S ORIGINAL	0	0	0	0	0

Example 1: Basic Heading Category is Rice (cont)

- **Products 4 and 11 were not priced in any country (or at least, average regional prices for these products were not calculated). Thus they are dropped from the list of products that are used in the CPD regression model.**
- **Products 3 and 8 were only priced in one country so these products can only have a very limited contribution to the calculation of interregional price indexes. The maximum overlap Jevons indexes does not use these isolated prices.**
- **If products 3 and 8 are dropped from the CPD regression, the resulting PPPs are the same as the PPPs generated by the full set of 9 products.**
- **The interregional CPD, GEKS-Jevons (GEKS-J) and Jevons maximum overlap Star indexes, J1-J5, are listed in Table 5 below.**
- **These indexes have been normalized so that the OECD level is equal to 1 for all 7 indexes.**
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Example 1: Basic Heading Category is Rice (cont)

- **The interregional CPD, GEKS-Jevons (GEKS-J) and Jevons maximum overlap star indexes, J1-J5, are listed in Table 5 below. These indexes have been normalized so that the OECD level is equal to 1 for all 7 indexes.**
- **Table 5: Alternative Interregional PPPs for the BH Rice**

Region	P_{CPD}	P_{GEKS-J}	P_{J1}	P_{J2}	P_{J3}	P_{J4}	P_{J5}
1	8.229	8.872	9.010	7.482	9.010	10.041	9.010
2	5.445	6.052	6.326	5.254	5.254	7.064	6.580
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	2.680	3.001	2.920	2.420	3.254	3.254	3.254
5	0.385	0.415	0.434	0.346	0.434	0.434	0.434

Example 1: Basic Heading Category is Rice (cont)

- It can be seen that the 5 Jevons star indexes differ considerably and the CPD and GEKS-Jevons indexes also exhibit a considerable amount of variation.
- This variation is caused by the missing prices: if each region priced all 7 overlapping products, all 7 of the indexes listed in Table 5 would be identical.

Table 5: Alternative Interregional PPPs for the BH Rice

Region	P_{CPD}	P_{GEKS-J}	P_{J1}	P_{J2}	P_{J3}	P_{J4}	P_{J5}
1	8.229	8.872	9.010	7.482	9.010	10.041	9.010
2	5.445	6.052	6.326	5.254	5.254	7.064	6.580
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	2.680	3.001	2.920	2.420	3.254	3.254	3.254
5	0.385	0.415	0.434	0.346	0.434	0.434	0.434

Example 2: Basic Heading Category is Other Cereals, Flour and Other Cereal Products

- There are 5 products in the Cereals BH category of products. Here is a listing of the individual products along with their average prices by region:
- Table 6: Regional Average Prices for the BH Other Cereals

Product title	Region 1	Region 2	Region 3	Region 4	Region 5
Cornflakes, KELLOGG'S	99.31355	117.92527	6.36323	37.28978	2.46250
Wheat flour, not self-rising, BL	11.32623	11.89930	0.53268	4.49298	0.34202
Wheat semolina (suji), WKB	0	15.47009	1.65450	0	0.56756
Oats, rolled, WKB	0	0	2.55555	13.72639	0
Corn (maize) flour, white, WKB	12.10363	22.88166	0	0	0.53977

Example 2: Basic Heading Category is Other Cereals, Flour and Other Cereal Products

- **Only 18 of the 25 possible regional average prices are positive. This means that we will have only 18 degrees of freedom for the CPD model, which has 5 + 5 – 1 = 9 parameters. Thus we have only 2 degrees of freedom for each parameter.**
- **The interregional CPD, GEKS-Jevons (GEKS-J) and Jevons maximum overlap star indexes, J1-J5, are listed in Table 7 below.**

Table 7: Alternative Interregional PPPs for the BH Other Cereals

Region	P_{CPD}	P_{GEKS-J}	P_{J1}	P_{J2}	P_{J3}	P_{J4}	P_{J5}
1	13.66883	15.48717	18.21698	11.79602	18.21698	16.65341	13.66686
2	16.49907	17.87091	24.24814	15.70138	15.70138	18.60031	16.39303
3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4	5.86518	6.28162	7.03059	5.42545	6.42715	6.42715	6.20715
5	0.44294	0.46536	0.58661	0.42152	0.44009	0.45569	0.44009

Example 3: Basic Heading Category is Eggs and Egg-Based Products

- There are 2 products in the Eggs BH category of products.
- Only 1 price out of 10 possible regional average prices is missing.
- There are 9 degrees of freedom and 6 CPD parameters to estimate so the number of degrees of freedom per parameter is 1.5.

Product title	Region 1	Region 2	Region 3	Region 4	Region 5
Chicken eggs, caged hen, large size	13.44271	11.95967	1.133313	3.419534	0.396964
Chicken eggs, caged hen, medium size	0	10.30738	1.001608	3.028731	0.378172

Example 3: Basic Heading Category is Eggs and Egg-Based Products (continued)

- The interregional CPD, GEKS-Jevons (GEKS-J) and Jevons maximum overlap star indexes, J1-J5, are listed in Table 9 below.
- Note that the star Jevons indexes P_{J2}-P_{J5} are exactly the same for regions 2-5. This follows from our earlier algebra and the fact that these 4 regions price the same products.
- For this example, CPD and GEKS-J are close but the problem is that the number of degrees of freedom per parameter is only 1.5 so the resulting PPPs cannot be very accurate.

Table 9: Alternative Interregional PPPs for the BH Eggs

Region	P_{CPD}	P_{GEKS-J}	P_{J1}	P_{J2}	P_{J3}	P_{J4}	P_{J5}
1	11.9389	11.9234	11.8614	11.7133	11.8614	11.8744	12.3149
2	10.4210	10.4473	10.5529	10.4210	10.4210	10.4210	10.4210
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	3.0206	3.0199	3.0173	3.0206	3.0206	3.0206	3.0206
5	0.3637	0.3609	0.3503	0.3637	0.3637	0.3637	0.3637

Example 4: Basic Heading Category is Wine

- **No region priced product 2 and only 1 region priced products 5, 6 and 7 so we are down to only 3 useful product categories: 1, 3 and 4. Thus we have 13 positive regional product prices out of a possible 15 prices for the 3 useful wine categories. We use these 13 degrees of freedom to estimate 7 CPD parameters.**

Product title	Region 1	Region 2	Region 3	Region 4	Region 5
Red wine, table wine, WKB	43.40769	66.44665	2.838234	24.46784	6.647364
Red wine, European, WKB	0	0	0	0	0
White wine, table wine, WKB	49.10292	65.22707	2.761142	26.1614	5.308037
Sparkling wine, WKB	61.43441	0	8.649288	46.4729	0
Red wine, Australian, WKB	0	96.41996	0	0	0
White wine, South African, WKB	55.42196	0	0	0	0
Red wine, Chilean, WKB	0	0	0	31.50088	0

Example 4: Basic Heading Category is Wine (cont)

- **It can be seen that the 5 Jevons star indexes differ considerably and the CPD and GEKS-Jevons indexes also exhibit a considerable amount of variation. As usual, this variation is caused by just two missing prices: if each region priced all 3 overlapping products, all 7 of the indexes listed in Table 12 would be identical.**
- **This example shows that having a small number of missing prices can sometimes lead to very uncertain PPPs.**

Region	P_{CPD}	P_{GEKS-J}	P_{J1}	P_{J2}	P_{J3}	P_{J4}	P_{J5}
1	12.4544	13.9348	12.4544	16.4918	12.4544	12.4544	16.4918
2	20.2134	21.4752	17.7597	23.5170	23.5170	19.7743	23.5170
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	7.5994	8.1450	7.5994	9.0377	7.5994	7.5994	9.0377
5	1.8238	1.9377	1.6024	2.1219	2.1219	1.7842	2.1219

Example 5: Basic Heading Category is Jewelry, Clocks and Watches

- **Only one region priced product 1 so this product is dropped from the list of products. Only 12 of the remaining 20 possible regional average prices are positive. Thus the CPD regression has 12 degrees of freedom to allocate to the estimation of 8 parameters.**
- **The CPD interregional PPPs turned out to be 13.8588, 9.1675, 1.0000, 3.1523 and 0.3099 for the 5 regions.**

Product title	Region 1	Region 2	Region 3	Region 4	Region 5
Wrist-watch, children's, SWATCH Flik Flak	0	0	52.89648	0	0
Wrist-watch, men's, CITIZEN Eco-Drive BM6060	2947.241	1428.628	0	407.2255	50.24698
Analog travel alarm, quartz, BL	152.5293	0	0	0	2.718193
Wedding ring, 14 Karat gold, BNR	827.9919	0	75.06806	297.3313	0
Wall clock, SEIKO	295.5763	237.8775	0	0	8.658026

Example 5: Basic Heading Category is Jewelry, Clocks and Watches (cont)

- **It can be seen that regions 2 and 3 do not have any common products and regions 3 and 5 also do not have any common products.**
- **Thus a complete set of maximum overlap bilateral Jevons indexes cannot be calculated and hence the GEKS-Jevons index cannot be calculated either.**
- **It is possible to calculate maximum overlap Jevons indexes using Regions 1 and 4 as the base region but the resulting PPPs are not reliable.**
- **The products in this Basic Heading group of products are very heterogeneous and so it is extremely important that all regions price the 5 detailed products.**
- **The CPD index and the two Jevons indexes that are possible are not reliable. The missing prices in this product category have led to more or less meaningless PPPs.**

Points for Discussion

- **The above examples show that the lack of matching of annual average Basic Heading prices across the Regions can lead to very inaccurate interregional BH PPPs.**
- **The practical problem is: how can more matching be achieved?**
- **When I wrote these notes two years ago, I thought that similarity linking might be the way forward. The main task of the ICP is to link the 6 Regions. All that is required to complete this task (given the regional fixity constraints) is the production of 5 interregional links between 5 pairs of countries where the 2 countries in each pair come from different regions. These 5 "optimal" pairs would be determined by the pairs that had the the lowest measure of relative price dissimilarity. Once these optimal pairs have been determined, then these countries (with the possible help from the World Bank) could concentrate on getting more accurate bilateral matching of products.**

Points for Discussion (continued)

- **The problem with the above strategy is that the country expenditures and annual average prices are not necessarily very accurate and this could lead to "optimal" matches which are far from being optimal.**
- **For example, we could have a pair of countries from 2 different regions that did not have the resources to construct a vector of annual average prices by Basic Heading category and simply used exchange rates to fill in the BH PPP price vectors. The resulting measure of relative price dissimilarity would be zero and these two countries would determine one of the interregional links.**
- **Thus in order to eliminate "crazy" bilateral interregional links, we would have to exclude some countries from being eligible for the purpose of linking the regions. This becomes a political problem which would be very difficult to solve.**

Points for Discussion (continued)

- **My work with Yuri Dikhanov on Similarity Linking has convinced me that the problem of "crazy links" is a real one. In order to solve this problem, we would have to agree to some generally accepted rules for including countries in the similarity linking procedure. As mentioned above, this is a difficult task.**

Other Problems that need Discussion

- **It might be best to give up on doing comparisons of GDP.**
- **The treatment of the government sector is not satisfactory; indirect taxes should not be a separate category (they lead to negative items in the product list (see Sergeev on this point).**
- **Different countries do very different treatments of the government sector with some attempting to directly measure outputs but most measuring output prices by input prices, but they do not use user costs to value durable inputs.**

Other Problems that Need Discussion

- **The treatment of international trade and the use of exchange rates as export and import prices will add a lot of bias to the overall PPP if the country's trade balance is large and positive or large in magnitude and negative. If the trade balance is 0, there will be no bias. (See Sergeev and Feenstra)**
- **Thus G, X and M could be dropped from the ICP. Investment I is also problematic. Machinery and Equipment is in principle conceptually satisfactory but this category covers a vast range of products.**
- **Construction price indexes are problematic for sure. This sector could also be dropped.**
- **It might be best if our primary focus was on a comparison of household consumption.**

Other Problems that Need Discussion

Problems within the Household Sector

- **Insurance and FISIM are also problematic categories where there is little agreement on an appropriate methodology.**
- **Many countries are unable to fill in many household BH categories. Some categories are very narrow and some are extremely broad. Are we happy with the current list of Consumption Basic Headings?**
- **The treatment of Net Purchases Abroad as a household category is not satisfactory; Purchases abroad are in scope for the domestic household sector but the purchases of foreigners are not in scope. This again leads us to question what perspective we want to take in making international comparisons: a household perspective or a producer perspective?**

Addendum on Two Stage GEKS/CCDI

- **How should Two Stage GEKS be done?**
- **Single stage GEKS for each of the 6 regions is straightforward.**
- **In order to be able to Chart the results, we divided each BH price for each country by its US dollar price and we also divided the corresponding expenditures by the country's US dollar price**
- **Do GEKS for each region and normalize the resulting regional PPPs so that the PPP for the numeraire country in each region is equal to 1.**
- **How should we link the regions using GEKS type principles?**
- **Take Region 1 (Africa) as the numeraire region to start off. There are 52 countries in Africa and 21 countries in Region 2 (Asia) so there are 52x21 = 1092 possible ways of linking Regions 1 and 2 using bilateral Fisher indexes between the two Regions.**
- **Take each of these 1092 bilateral links and use the first stage Regional GEKS PPPs to adjust each bilateral link so that it is a comparison between the numeraire countries in the two regions.**

Addendum on Two Stage GEKS/CCDI (cont)

- **Then take the geometric mean of these 1092 adjusted bilateral links. This gives us the number 1.58919 in the Link 1 column of the matrix below.**
- **The Tables below use only the BH consumption data for 2021 for 109 BH categories.**

TABLE 1	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6
1.	1.00000	0.62925	0.53340	1.13809	0.96221	1.48325
2.	1.58919	1.00000	0.85771	1.77975	1.49810	2.36566
3.	1.87475	1.16589	1.00000	2.14927	1.84980	2.70722
4.	0.87867	0.56188	0.46527	1.00000	0.84639	1.30898
5.	1.03927	0.66751	0.54060	1.18149	1.00000	1.56250
6.	0.67419	0.42272	0.36938	0.76395	0.64000	1.0000

Now compare each country's prices in Region 1 with each country's prices in Region 3 (OECD). There are 52x48 = 2496 possible ways of linking Regions 1 and 3 via bilateral Fisher indexes between the two Regions. Take each of these 2496 bilateral links and use the first stage Regional GEKS PPPs to adjust each bilateral link so that it is a comparison between the numeraire countries in the two regions. Then take the geometric mean of these 2496 adjusted bilateral links. This gives us the number 1.87475 in the Link 1 column of the matrix above.

Addendum on Two Stage GEKS/CCDI (cont)

- **Now compare prices in Region 1 with prices in Region 4 in the same way, get the number 0.87867 and so on.**
- **The Link 1 column above gives us the Regional Link factors which are multiplied by the Regional GEKS PPPs to give as set of World PPPs with Region 1 acting as the "star" region.**
- **The Link 2 column does the same sort of operations, but now Region 2 is the numeraire region. And so for each Region. The above Table 1 matrix with entries m(i,j) has the following symmetry property:**

  m(i,j) = 1/m(j,i) for all i not equal to j.
- **The above property acts as a check on our computations.**
- **The next step is to "harmonize" the above regional link factors so that the row 3 entries in the adjusted link factors all equal 1 because the OECD region has the global numeraire country (the USA) .**

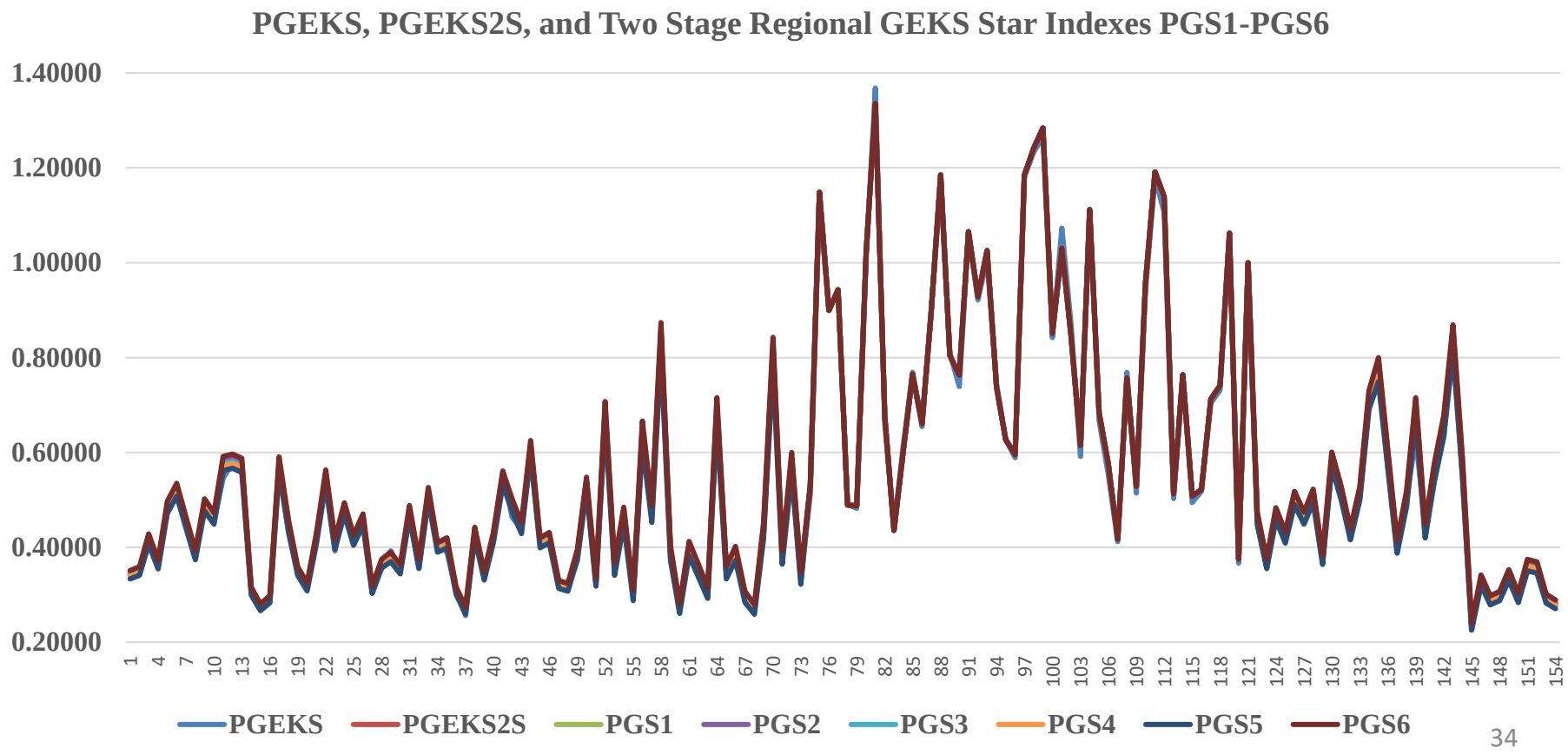
Addendum on Two Stage GEKS/CCDI (cont)

- **Thus divide the entries in the Link 1 column by 1.87475, divide the Link 2 column by 1.16589, divide the Link 4 column by 2.14927, divide the Link 5 column by 1.84980 and divide the Link 6 column by 2.70722 to obtain the adjusted Link 1-Link6 columns, LGEKS1-LGEKS6 , listed below.**
- **Finally take the geometric mean of the 6 regional star linking factors LGEKS#. This is our final estimate for the 2 stage GEKS Interregional Linking factors, LGEKS.**
- **What is interesting is that the Regional Star Linking factors are all fairly similar, with more variability in LGEKS4-LGEKS6 (the smaller regions, Latin America, West Asia and the CIS).** **TABLE 2**

LGEKS	LGEKS1	LGEKS2	LGEKS3	LGEKS4	LGEKS5	LGEKS6
1.00000	0.53395	0.53340	0.53972	0.53340	0.52952	0.52017	0.54789
2.00000	0.84555	0.84768	0.85771	0.85771	0.82807	0.80987	0.87383
3.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.00000	0.47028	0.46868	0.48193	0.46527	0.46527	0.45756	0.48352
5.00000	0.55564	0.55435	0.57253	0.54060	0.54972	0.54060	0.57716
6.00000	0.36030	0.35962	0.36257	0.36938	0.35545	0.34598	0.36938

Addendum on Two Stage GEKS/CCDI (cont)

- **The above Regional Linking factors were used to adjust the Regional GEKS indexes into World (Two Stage) GEKS indexes.**
- **They are listed and plotted (along with the single stage World GEKS index) in the first attachment below. These series are labelled as PGEKS, PGEKS2S, and the Regional Star World indexes PGS1-PGS6.**



Addendum on Two Stage GEKS/CCDI (cont)

- **On the previous Chart, it can be seen that the 8 World indexes cannot really be distinguished. Remember, the PPPs are US Dollar PPP. To get the "true" county PPPs, multiply the 154 PPPs on the Chart by the corresponding country US dollar exchange rates.**
- **Now use bilateral Törnqvist indexes instead of bilateral Fisher indexes as the basic building blocks and repeat the above steps.**
- **The Törnqvist counterpart to Table 1 above is the following Table:**

TABLE 3

	Link 1	Link 2	Link 3	Link 4	Link 5	Link6
1	1.00000	0.62834	0.53590	1.14883	0.92445	1.48621
2	1.59150	1.00000	0.84765	1.79848	1.48772	2.36512
3	1.86600	1.17973	1.00000	2.14680	1.80212	2.71221
4	0.87045	0.55603	0.46581	1.00000	0.83390	1.28951
5	1.08172	0.67217	0.55490	1.19919	1.00000	1.57437
6	0.67285	0.42281	0.36870	0.77549	0.63518	1.00000

Addendum on Two Stage GEKS/CCDI (cont)

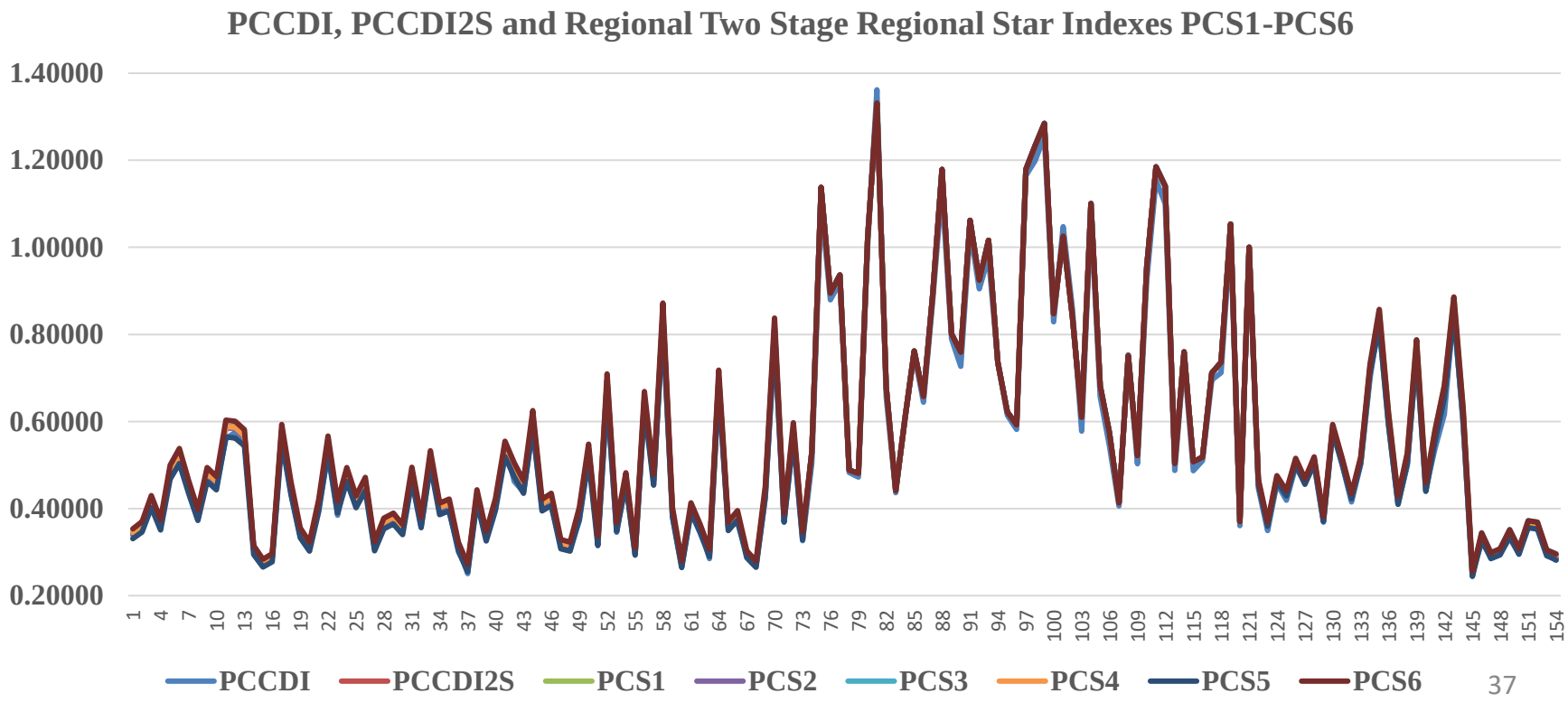
- **The Törnqvist counterpart to Table 2 listed above is the Table 4 below.**
- **Compare Table 4 to Table 2; there is very little difference. It does not seem to matter much whether we use bilateral Fisher indexes or bilateral Törnqvist indexes as basic building blocks for the multilateral method.**

TABLE 4 Single Stage CCDI and Regional Star Törnqvist Links

	CCDI	LCCDI1	LCCDI2	LCCDI3	LCCDI4	LCCDI5	LCCDI6
1	0.53332	0.53590	0.53261	0.53590	0.53513	0.51298	0.54797
2	0.84713	0.85289	0.84765	0.84765	0.83775	0.82554	0.87203
3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4	0.46791	0.46648	0.47132	0.46581	0.46581	0.46273	0.47544
5	0.56628	0.57970	0.56977	0.55490	0.55859	0.55490	0.58047
6	0.36163	0.36059	0.35840	0.36870	0.36123	0.35246	0.36870

Addendum on Two Stage GEKS/CCDI (cont)

- **The Regional Linking factors LCCDI and LCCD1-LCCD6 can be used to form Two Stage World indexes and the resulting world indexes can be compared to the single stage CCDI index.**
- **These comparisons the above Regional Linking factors were used to adjust the Regional CCDI indexes into World (Two Stage) CCDI indexes. These series are labelled as PCCDI, PCCDI2S, and the Regional Star World indexes PCS1-PCS6.**
- **It can be seen that the 8 World indexes cannot really be distinguished.**



Addendum on Two Stage GEKS/CCDI (cont)

- **Finally the Chart below compares PGEKS, PCCDI PGEKS2S and PCCDI2S.**
- **It is difficult to distinguish the four series.**
- **Two Conclusions:**

(i) We have an alternative to the CAR method to linking the regions. This alternative method producesn Two Stage World PPPs that are very close to the counterpart Single Stage World PPPs. (At least for 2021 data!)

(ii) For the 2021 data, Two Stage GEKS was very close to Two State CCDI.