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CONDOS

SOME ASPECTS OF THE THEORY OF ECONOMIC POLICY - LINEAR MODELS AND QUADRATIC OPTIMIZATION

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I The theory of economic policy is associated primarily with the names of Tinbergen and Theil [1,2]. Notable recent contributions in the same tradition include Fox, Sergupta, Thorbecke [3] and Brainard [4]. Spivey and Tamura [5] have discovered an elegant link in the formalisms of the Tinbergen and Theil approaches based on the concept of the generalized inverse of a matrix, not widely known among economists.

The purpose of this paper is to restate the elements of the Tinbergen and Theil theories and to discuss the formal relations between linear models and quadratic optimization in a more direct manner than that of Spivey-Tamura. The computational aspects for applications to econometric policy models are thus clarified and natural extensions involving different quadratic welfare criteria from those dealt with in [5] are made easy without appeal to Lagrangean methods.

II <u>Tinbergen's theory</u> The basic elements of Tinbergen's theory of economic policy are (1) a social preference function, $w = w(y_1,\ldots,y_I)$, (2) a model characterized by a set of mathematical relations linking together the following types of variables: y_i; $i = 1,\ldots,I$, target variables on which are defined the goals of economic policy; x_k, $k = 1,\ldots,k$, irrelevant variables (from the standpoint of the preference function); z_j, 1, ...,], instrument variables, namely those under the control of policy-making agents, and u_m, $m = 1,\ldots,M$, non-controlled exogenous variables or data.

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*My greatest debt is to Professor George Zyskind, Statistics, Iowa State University, for his course on Linear Models developed over many years. Of course, errors are mine.

Suppose now that the model is deterministic, static, linear and given by the following structural equations in matrix form

II-1 $$Ay + Bx + \Gamma z = u$$

where the symbols are matrices of appropriate order.

In typical economic analysis the principal distinction of variables is between endogenous (unknowns) and exogenous (knowns). Define vectors

$$w \equiv \begin{bmatrix} y \\ \cdots \\ x \end{bmatrix}, \qquad \xi \equiv \begin{bmatrix} z \\ \cdots \\ u \end{bmatrix}$$

and appropriate matrices Θ and Z, ($\Theta = [A:B]$, $\Gamma = [\Gamma:-I]$ so that system (II-1) is rewritten

II-2 $$\Theta w + Z\xi = 0.$$

If model (II-2) contains N = I+K independent equations we can exhibit the reduced form as

II-3 $$w = -\Theta^{-1}Z\xi.$$

Equation (II-3) exhibits the solution (equilibrium) set and if the parameters involved in matrix $-\Theta^{-1}Z$ and the exogenous variables ξ are numerically known, it yields the solution (equilibrium) values of the endogenous variables.

<u>The fixed target case</u> Assume that social preferences are expressed as a set of constants, $\bar{y} = (\bar{y}_1,\ldots,\bar{y}_I)$. The problem of economic policy at hand is to attain the thus specified targets by appropriate use of the instrument variables; that is, to find the z-values which satisfy system (II-1) when $y = \bar{y}$. Suppose there is a unique constellation of z-values which satisfy the requirements. We can then write

II-4 $$z = -\Gamma^{-1}A\bar{y} - \Gamma^{-1}Bx - \Gamma^{-}u.$$

Uniqueness in the policy solution above implies that the matrix of instrument coefficients is non-singular and of appropriate order, namely J x J. Therefore I must equal J. In other words, the number of instruments must be equal to the number of targets. Otherwise the product $\Gamma^{-1}A$ would not be defined. Further, consistency in the equational sense implies that $J + K = N$, the number of equations in the system, therefore also that $I + K = N$.

Assuming a mathematically consistent linear model, we speak of policy degrees of freedom if Γ is of column rank greater than I, which implies $J > I$. If $J < I$, the case is that of conflicting targets. If policy degrees of freedom are present, the fixed-target problem is solved by assigning arbitrary values to any independent subset of "excess" instruments.

The flexible target case While in the fixed-target case no apparent scope for optimization exists when social preferences are summarieed by an appropriate function, an optimal policy is that which results from maximizing that function subject to the economic model. Most generally then, the flexible-target case may assume any form of the "constrained extremum" problems, be it of the classical optimization, calculus of variations, or of the "programming" variety. No more need to be said of this case as it does not occupy a prominent position in Tinbergen's writings on the theory of economic policy.

III Theil's theory The core of Theil's contribution is contained in [2,VII;VIII]. We concentrate here on [2,8.1].

Consider the underlying economic model in reduced form

III-1 $$y = Rx + s.$$

y, x are the vectors of non-controlled and controlled variables respectively. s is a constant vector and R the multiplicative structure matrix of the system, the matrix of the impact multiplies. R, s are assumed functionally independent of x. They are all, of course, of appropriate order. The linear reduced form model (III-1) is the main type with which Theil works although certain important propositions can be derived with the more general model

(III-2) $$y = \psi(x) + u$$

where ψ is an arbitrary vector of functions $\psi_1, \ldots,$ and u a vector of random disturbances. In this case it is assumed that $Eu = 0$, Euu' is finite and stochastically independent of x.

It is postulated that the policy-maker can describe his preferences with respect to alternative states represented by different constellations of values of the controlled and non-controlled variables by a quadratic function. The principal justification of the quadratic form is that it is the simplest mathematical form which permits any two of its arguments to possess both "complementary" and "substitutional" domains with respect to the welfare index. Therefore, a function of the form

III-3 $$w(x,y) = a'x + b'y + \frac{1}{2}(x'Ax + y'By + x'Cy + y'C'x)$$

where a, b, A, B, C are vectors and matrices independent of x, y and of appropriate order (A, B, C symmetric) may describe adequately the preferences over a wide domain of variation of x and y.

One of the main results of Theil's theory is the "certainty equivalence" theorem. This theorem states that: under the assumptions: the underlying reduced-form model is as in (III-2); all vectors x and y (the latter real-valued) are completely ordered by a special quadratic welfare function

$$w(x,y) = A(x) + \sum_i A_i(x)y_i + \frac{1}{2}\sum_i \sum_j A_{ij}y_iy_j$$

where $A(x)$ and $A_i(x)$ are arbitrary functions of vector x and A_{ij} independent of both x and y; the policy-maker values according to the expected value of $w(x,y)$; then

$$Ew(x,y) = w(x,Ey) + \text{constant},$$

therefore the same x maximizes both sides provided a maximum exists. By postulating alternative welfare functions it is a relatively simple matter to identify the "certainty bias" as

$$Ew(x,y) - w(x,Ey) = B(x).$$

In such cases the maximizing x will be in general different for Ew and $w(x,Ey)$.

Much of Theil's static theory of policy is devoted to obtaining equations of "certainty bias" and to the investigation of the "relative extremum" policy problem by analogy with the Lagrangean analysis of consumer's equilibrium and the classical theory of production.

IV The Spivey-Tamura contribution Spivey and Tamura noticed that the Lagrangean method of maximizing a quadratic function subject to linear equality constraints yields a set of equations the solution of which is equivalent to certain linear transformations involving the generalized inverse of the appropriate matrix performed on the underlying linear model. They proceed to show the equivalence of three special cases of quadratic welfare criteria maximized under corresponding linear models by the Lagrangean method and the unique solutions of three linear systems of equations through the corresponding generalized inverse. Therefore, they show that the Tinbergen cases of excess or lack of policy degrees of freedom can be treated by linear methods equivalent to a Theil-type situation of quadratic optimization. In the next section we shall present the linear theory which leads directly to the Spivey-Tamura conclusions and then extend their results to some additional cases of interest. Incidentally, it will be shown that the concept of the generalized inverse is necessary only for the linear treatment of the case of policy degrees of freedom, while for the other cases a weaker concept suffices.

V Linear model and quadratic optimization In this section certain well-known results from the theory of linear equations will be presented for completeness together with some less well-known propositions. The objective is to lead directly to the equivalence of certain linear models and quadratic constrained optimization.

Consider system

V-1 $$Rx = q$$

where R is an n x m matrix, x, q an m-order column vector and an n-order column vestor, respectively. No particular stipulation about the rank of R is made at this point. Whether system (V-1) is compatible or not it is known that the transformed system

V-2 $$R'Rx = R'q$$

is always compatible for any vector q.

Proof: It is necessary and sufficient to show that $r(R'R) = r(R'R,R'q)$, where $r(A)$ denotes the rank of matrix A. Now suppose $r(R) = p$. This implies that p columns of matrix R are linearly independent, therefore, denoting the relevant n x p submatrix by Rp, there exists a non-zero vector x_p (of order p) such that

$$R_p x_p = b$$

where b is a non-zero vector of order n. It follows that the matrix $R_p'R_p$ is a positive definite matrix since

$$x_p'R_p'R_px_p = b'b = (R_px_p)'(R_px_p) > 0.$$

a sum of squares. This implies that R'pRp is of full rank, namely p. It follows consequently that

$$p = r(R) = r(R') = r(R'R) \leq r(R'R,R'q)$$

and

$$p \geq r[R'(R,q)] = r(R'R,R'q).$$

The two statements together imply r(R'R,R'q) = p = r(R'R) which proves the compatibility of normal-type equations, the first fundamental result needed in this exposition.

The second result needed is the uniqueness of a vector involved in the solution of systems like (V-2).

Let x_1 and x_2 be any two solutions of (V-2). Then we have

$$R'Rx_1 = R'q$$
$$R'Rx_2 = R'q$$

and

$$R'Rx_1 - R'Rx_2 = 0 = R'R(x_1-x_2).$$

Therefore also

$$(x_1-x_2)'R'R(x_1-x_2) = 0$$

and

$$[R(x_1-x_2)]'[R(x_1-x_2)] = 0.$$

Hence

$$R(x_1-x_2) = 0$$

therefore

$$Rx_1 = Rx_2.$$

Vector Rx is invariant for any x a solution to (V-2).

Now let q in (V-2) assume successively the values $\begin{pmatrix}1\\0\\ \vdots\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\ \vdots\\0\end{pmatrix}$, ..., $\begin{pmatrix}0\\0\\ \vdots\\1\end{pmatrix}$. For each q-value the solution of (V-2) produces a vector x with the property Rx unique. In addition for each q_j (a vector of zeros except a unity in jth row) the right hand side of (V-2) is simply the jth column of matrix R'. Bringing these results together and denoting the matrix formed by the set of solutions and by x we have

$$R'RX = R'.$$

From the preceding discussion clearly RX is a unique matrix. Two additional important properties of RX will now be proven.

Transposing the above matrix equation we obtain

$$X'R'R = R.$$

Postmultiplying by X we have

$$X'R'RX = RX.$$

Given that R'RX = R' we also obtain by substitution in the last expression above

$$X'R' = RX$$

therefore (RX)' = RX, which proves symmetricity. We also observe that (RX)'RX = RXRX = RX which proves idempotency. Therefore the unique matrix RX is symmetric and idempotent.

Proceeding to construct matrix RX we have incidentally proven that for any arbitrary matrix A there exists a matrix A* such that AA*A = A. For, on the basis of the immediately preceding discussion,

$$R'RX = R'$$

implies

$$(RX)'R = R$$

and $RXR = R$.

Here matrix A* is matrix X and its construction has been indicated. Given that for any q the solution to (V-2) is not in general unique, matrix X is not in general unique. X is known as a conditional inverse of matrix R.

The following is an important consequence of the concept of a conditional inverse: If X is a conditional inverse of R and if $Rx = q$ is a set of consistent equations then $\hat{x} = Xq$ is a solution.

The proof of this proposition is quite simple. Since $Rx=q$ are consistent there exists a vector $\hat{x}$ such that $R\hat{x} = q$ are satisfied. Now by premultiplying $\hat{x} = Xq$ by R, we obtain

$$R\hat{x} = RXq.$$

Since $q = R\tilde{x}$ we also have

$$R\hat{x} = RXR\tilde{x} = R\tilde{x} = q.$$ Which proves the statement.

The converse of the proven statement is also true. It can be stated as: For X matrix such that Xq is a solution to $Rx = q$ for every q for which there are consistent, X satisfies $RXR = R$. Note that

$$Rx = R_j$$

are always consistent (R_j the jth column of R). By assumption

$$RXR_j = R_j$$

are satisfied. For all j the above becomes $RXR = R$.

The consequence of the preceding statement and its converse for system (V-2) is that since (V-2) is always consistent

$$\hat{x} = (R'R)^*R'q$$

is a solution for some choice of conditional inverse of matrix R'R, denoted by $(R'R)^*$.

We already know that if $\tilde{x}$ is another solution corresponding to a different conditional inverse

$$R\hat{x} = R\tilde{x}.$$

Consequently, denoting the two conditional inverses now as $(R'R)^*_1$ and $(R'R)^*_2$, we have

$$R(R'R)^*_1R'q = R(R'R)^*_2R'q.$$

It follows that

$$R(R'R)^*_1R' = R(R'R)^*_2R'$$

which means that matrix $R(R'R)^*R'$ is invariant for any conditional inverse of $R'R$.

In addition the following are important properties of the invariant matrix $R(R'R)^*R'$: since

$$\hat{x} = (R'R)^*R'q$$

we have

$$R'R(R'R)^*R'q = R'q$$

hence

$$R'R(R'R)^*R' = R'.$$

Therefore matrix $(R'R)^*R'$ satisfies the property of matrix X, namely

$$R'RX = R'.$$

Consequently

$$RX = R(R'R)^*R'$$

is unique for all conditional inverses, symmetric and idempotent. It follows that for any P whose column space is that of R we have

$$P(P'P)^*P' = R(R'R)^*R'.$$

It is easy to prove that $(I - R(R'R)^*R')$ has the same properties of uniqueness, symmetricity and idempotency, and that the rank of $R(R'R)^*R'$ is $p = r(R)$ and $r(I - R(R'R)^*R') = n - p$ where n the number of rows of R.

At this point we introduce the concept of orthogonal projections. This concept is the key to the equivalence between certain linear systems and quadratic optimization.

In two dimensions it is a well-known fact that any vector can be decomposed into two vectors of which it is the sum in such a fashion that the one component vector has any preassigned direction and the other a direction perpendicular to that of the first. In many dimensions the above idea is expressed as follows:

Let q be any n x 1 vector and $\mathcal{R}$ any p-dimensional vector space. Then q admits a unique decomposition

$$q = q_1 + q_2,$$

$q_1 \in \mathcal{R}$ and $q_2 \in \perp\mathcal{R}$, where $\perp\mathcal{R}$ denotes the n-p dimensional vector space orthogonal to $\mathcal{R}$. Let R be a matrix whose column space is $\mathcal{R}$. Refer to (V-2). Define $q_1 \equiv R\hat{x}$ and $q_2 \equiv q - R\hat{x}$, where $\hat{x}$ is a solution to $R'Rx = R'q$. Clearly $R\hat{x}$ belongs to $\mathcal{R}$ and $q - R\hat{x}$ to $\perp\mathcal{R}$ since $R'(q-R\hat{x}) = 0 = R'q - R'R\hat{x}$ and $\hat{x}$ a solution. The uniqueness of the decomposition is based on the already proven fact of the uniqueness of $R\hat{x}$.

In the above $R\hat{x}$ is said to be the orthogonal projection of q on the column space of R and $q - R\hat{x}$ is said to be the projection of q on the vector space orthogonal to that of the columns of $\mathcal{R}$. It is then intuitively evident that in a Euclidean space the variable expression $(q-Rx)'(q-Rx)$ in x is a minimum if and only if $R\hat{x}$ is the orthogonnal projection of q on the column space of R.

We only need to know now how to obtain orthogonal projections to establish linear methods in solving quadratic optimization problems.

The following definitions and propositions accomplish this.

A square real matrix Q is said to be an orthogonal matrix projection operator on the column space of Q if for every vector y, Qy is the orthogonal projection of y on the column space of Q. (We have already shown that to every vector space there corresponds a unique orthogonal projection because of the uniqueness of a vector's decomposition as explained.)

A real square matrix Q is an orthogonal projection operator if and only if it is symmetric and idempotent. Proof: If Q symmetric and idempotent, for every y we have:

$$y'(I - Q')QZ = y'(I - Q)QZ = y'(Q - Q)Z = 0.$$

Thus $y = Qy + (I - Q)y$, which proves the projection implications of symmetricity and idempotency since Qy is shown to belong to the column space of Q and $(I - Q)y$ to the space orthogonal to that of Q.

Conversely suppose Q is an orthogonal projection matrix operator. If follows that

$$y'Q'(I - Q)y = 0.$$

So

$$Q'(I - Q) = 0$$

which implies

$$Q' = Q'Q.$$

But $Q'Q$ is symmetric so that $Q' = Q$ and thus

$$QQ = Q,$$

which proves idempotency.

In the context of system (V-2) we know that

$$R(R'R)^{*}R'$$

is unique for all matrices having the same column space as matrix R, and in addition it is summetric and idempotent. Thus we have identified the appropriate projection operators needed to minimize the distance of any vector from the column space of R.

VI Linear policy models and quadratic optimization

Let us now return to a reduced-form model of economic policy

VI-1 $$y = Rx + s.$$

Let us specify R as being of order n x m. As previously the vector of instrument variables is x while the target variables are y. If $M > N$ and we take without any essential loss of generality the matrix R to be of full column rank, the Tinbergen case is that of conflicting targets, if some constant y^* is specified as the desired state to be attained by the use of instruments x. If, on the other hand, $N < M$, and $r(R) = N$ we encounter policy degrees of freedom, since in principle an infinite of policy combinations can attain the target levels y^*.

Let us first concentrate on case $M < N$, $r(R) = M$.

Case One: Conflicting Targets

The policy model is:

VI-2 $$Rx = y^* - s.$$

Suppose we wish to approximate the desired level in such a fashion as to be as close as possible to it in the Euclidean sense. Every actual deviation from the ith desired target level contributes by assumption the same disutility per unit of deviation as any other. The policy solution is then on the basis of the orthogonal projection theory

VI-2 $$\hat{x} = (R'R)^{*}R'(y^{*} - s).$$

Since the transformed model

VI-3 $$R'Rx = R'(y^{*} - s)$$

is consistent for any $(y^{*} - s)$, if $(R'R)^{*}$ is the conditional inverse of $(R'R)$, $\hat{x} = (R'R)^{*}R'(y^{*} - s)$ is a solution.

Therefore

VI-4 $$R(R'R)^{*}R'(y^{*}-s) = y^{*} - s.$$

and $R\hat{x}$ is the orthogonal projection of $(y^{*} - s)$ the column space of R and consequently renders the variable expression in x, $(Rx-y^{*}-s)'(Rx-y^{*}-s)$ a minimum.

Given the particular assumption about the dimensionality of the column space of R we observe that $(R'R)^{*}$ is identical with the ordinary inverse $(R'R)^{-1}$ since $(R'R)$ is of rank m. The policy solution can then be rewritten as

VI-5 $$\hat{x} = (R'R)^{-1}R'(y^{*}-s).$$

Observe that matrix $(R'R)^{-1}R'$ satisfies the following properties:

(i) $R(R'R)^{-1}R'R = R$ (Conditional inverse)

(ii) $(R'R)^{-1}R'R(R'R)^{-1}R' = (R'R)^{-1}R'$

(iii) $[R(R'R)^{-1}R']' = R(R'R)^{-1}R'$

(iv) $[(R'R)^{-1}R'R]' = (R'R)^{-1}R'R.$

Any matrix G satisfying the above four properties with respect to a matrix A is called the generalized (Penrose) inverse of A. It is easy to show that a matrix G exists, and is unique for any arbitrary matrix A.

Case Two: Policy Degrees of Freedom

The model in this case has the following specifications:

R is of order $n \times m$, $n < m$, $r(R) = n$, again without essential loss of generality.

Writing

VI-6 $$Rx = y^* - s$$

an infinity of x vectors can satisfy the targets. It is a question of arbitrary choice the manner in which the policy degrees of freedom are closed.

One of the possible ways to proceed is to postulate that a given instrument vector x^* is the "best" among all possible ones. Then, of course,

VI-7 $$Rx^* = y^* - s$$

represents a system of incompatible equations. The introduction of an unknown vector $\bar{x}$ such that

VI-8 $$R(x^* + \bar{x}) = y^* - s$$

is satisfied, allows the formulation of the problem of finding that instrument vector $\hat{x}$ which is the closest approximation to the "best" vector x^* in the Euclidean sense and which in addition satisfies system (VI-8). Let us rewrite (VI-8) as

VI-9 $$R\bar{x} = y^* - s - Rx^* .$$

(VI-9) is a compatible system of equations. The formal pboblem consists in expressing $\bar{x}$ satisfying (VI-9) explicitly and then selecting in the set that which in addition minimizes $(\hat{x} - x^*)'(\hat{x} - x^*) = \bar{x}'\bar{x}$.

The explicit representation of the set of $\bar{x}$-solution of a compatible system necessitates the use of the concept of the (unique) generalized inverse of matrix R. It is known how to construct a conditional inverse by solving the equations

$$R'RX = R'.$$

Now define $R' = Q$ and solve the always compatible system

$$Q'QP = Q'.$$

It can easily be shown that matrix

$$P'RX = G$$

satisfies the four properties characterizing the generalized (Penrose) inverse, namely:

$$\text{(i) } RGR = R \qquad \text{(iii) } (RG)' = RG$$

$$\text{(ii) } GRG = G \qquad \text{(iv) } (GR)' = GR.$$

It is also easy to prove that any matrix satisfying the above four properties is unique and the fundamental rank relation $r(R) = r(RG)$.

From the theory of linear equations we know that the complete set of solutions to a system of compatible equations is characterized by the combination of any arbitrary solution with every solution to the auxiliary set of homogeneous equations, which in the case of (VI-9) is $R\bar{x} = 0$. Now all vectors belonging to the null space of R are members of the solution set and conversely, every member of the solution set belongs to the null space of R. In general, if $r(R) = N$ from the fundamental rank relation $r(R) = r(RG) = r(GR)$ (which itself is based on the more general rank relation involving a conditional inverse R^*, $r(R) = r(R^*R) = r(RR^*)$) it can be shown that

$$r(I_M - GR) = M - N,$$

namely, the column space of matrix (I - GR) is of dimension M - N. Since then

$$R(I_M - GR) = R - RGR = R - R \cdot 0,$$

the column space of I_M - GR is in fact the null space of R. It follows that we can exhibit the complete set of solutions to (VI-9) as

$$\text{(VI-10} \qquad \bar{x} = G(y^* - s - Rx^*) + (I_M - GR)\check{x}$$

where $\check{x}$ is an arbitrary vector. The first term on the right side is, of course, based on the result already proven that if a system of equations is compatible then the constant vector premultiplied by a conditional inverse satisfies the equations. But G is a conditional inverse so the first term follows.

System (VI-10) is the basic system proving the linear model equivalence to quadratic optimization of the special nature under discussion.

For any $\check{x}$ the resulting $\bar{x}$ satisfies (VI-9). Minimization of $\bar{x}'\bar{x}$ occurs however if and only if $\check{x} = 0$. To see this note that

$$(I_M - GR)' = (I_M - GR) \qquad \text{(symmetric)}$$

and

$$(I_M - GR)(I_M - GR) = I_M - GR - GR + GRGR = I_M - GR \qquad \text{(idempotent)}.$$

Now symmetric and idempotent matrices are always semidefinite positive. Hence for any $\check{x}$, the second term in (VI-10) is ≥ 0. It follows that the linear model corresponding to minimizing the Euclidean distance from the desired instrument vector x^* is

$$\text{VI-11} \qquad \bar{x} = G(y^* - s - Rx^*)$$

and consequently

VI-12 $$\hat{x} = x^* + \bar{x} = x^* + G(y^* - s - Rx^*).$$

It can easily be shown that for $r(R) = N$, $N < M$, the g-inverse $G = R'(RR')^{-1}$.

Case Three: Target and Instrument Variables in the Preference Function

We shall now seek the linear model which is equivalent to minimizing a preference function

$$w(x,y) = (x - x^*)'(x - x^*) + (y - y^*)'(y - y^*)$$

subject to a model

$$y = Rx + s.$$

Since y^* and x^* are given vectors representing "ideal" states, we may write

VI-13 $$y^* = Rx + s$$
$$x^* = x,$$

or

VI-14 $$\begin{pmatrix} R \\ I \end{pmatrix} x = \begin{bmatrix} y^* - s \\ x \end{bmatrix}$$

or

VI-15 $$\Theta x = q.$$

Proceeding as in case one we obtain

VI-16 $$\Theta'\Theta x = \Theta' q$$

and therefore

VI-17 $$\Theta(\Theta'\Theta)^*\Theta' q = q$$

since $x = (\Theta'\Theta)^*\Theta' q$ is a solution if $(\Theta'\Theta)^*$ is any conditional inverse of $\Theta'\Theta$. We also know that $\Theta(\Theta'\Theta)^*\Theta'$ is the orthogonal projection matrix operator associated with the vector space spanned by the columns of Θ, and consequently that

$$\Theta(\Theta'\Theta)^*\Theta' q$$

is the orthogonal projection of q on the column space of Θ. It therefore minimizes the variable expression in x, $(\Theta x - q)'(\Theta x - q)$.

It remains now to obtain the results in terms of the original matrices.

Now $(\Theta'\Theta)^* \equiv \left[\begin{pmatrix} R \\ I \end{pmatrix}' \begin{pmatrix} R \\ I \end{pmatrix}\right]^*$.

$$\left[\begin{pmatrix} R \\ I \end{pmatrix}' \begin{pmatrix} R \\ I \end{pmatrix}\right] = (R', I) \begin{pmatrix} R \\ I \end{pmatrix} = I + R'R.$$

It will be shown that for any matrix R, matrix $I + R'R$ is of full rank.

A matrix of the form $R'R$ is always positive semidefinite. Let R be of order $n \times m$, $n > m$ and of rank $p < m$. Then $r(R'R) = r(R) = p$. Let S be an orthogonal matrix such that

$$SR'RS' = \left[\begin{array}{c|c} \Lambda_p & 0 \\ \hline 0 & 0 \end{array}\right].$$

where Λ_p is a diagonal matrix of order p with positive diagonal elements, the non-zero characteristic roots of $R'R$. Then we have

$$S(I + R'R)S' = SIS' + SR'RS = I + \left[\begin{array}{c|c} \Lambda_p & 0 \\ \hline 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} I_p + \Lambda_p & 0 \\ \hline 0 & I_{M-p} \end{array}\right],$$

which proves the non-singularity of $(I + R'R)$.

It follows that

$$(\Theta'\Theta)^* = (I + R'R)^{-1}.$$

Therefore

$$\hat{x} = (I + R'R)^{-1}(R', I) \begin{pmatrix} y^* - s \\ x^* \end{pmatrix}.$$

It can be easily shown that matrix

$$(I + R'R)^{-1}(R', I)$$

satisfies the properties of the generalized inverse with respect to matrix

$\begin{pmatrix} R \\ I \end{pmatrix} \equiv \Theta$, so that an equivalent way of positing the solution is

(VI-18) $$\hat{x} = H \begin{pmatrix} y^* - s \\ x^* \end{pmatrix}$$

where H is the unique g-inverse of Θ.

VII <u>Certain Extensions</u> The linear approach to quadratic optimization as presented in VI can be extended in a straightforward manner to establish linear-model equivalents to optimization models involving more general quadratic welfare criteria. A natural situation emerges when each deviation from the desired level is weighted differently.

We could have then the following three situations as extensions of the three cases discussed where each deviation had the same weight, namely unity:

Subject to the model

VII-1 $$y = Rx + s$$

maximize

VII-2 $$W(Y) = -(y - y^*)'\Gamma(y - y^*)$$

VII-3 $$W(X) = -(x - x^*)'\Delta(x - x^*)$$

VII-4 $$W(X,Y) = -(x - x^*)'\Delta(x - x^*) - (y - y^*)'\Gamma(y - y^*).$$

Γ, Δ are diagonal matrices of appropriate order with positive diagonal elements.

Let us first seek the linear-model equivalent to situation (VII-1,2). Again we have the problem of lack of policy degrees of freedom, so $r(R) = M$, $M < N$. Each squared deviation

$$(y_i - y_i^*)^2$$

causes a welfare loss which can be compared with the welfare loss from the deviation of any other target variable from its desired level.

The weights are given, $(\Gamma_1,\ldots,\Gamma_n)$.

Define

$$\gamma_i \equiv \Gamma_i^{1/2}, \ \gamma \equiv \Gamma^{1/2}.$$

By premultiplying model (VII-1) by γ we obtain

VII-5 $\gamma y = \gamma Rx + \gamma s$, and VII-6 $\gamma y^* = \gamma Rx + \gamma s$

the incompatible system of equations.

Then

VII-7 $\hat{x} = K(\gamma y^* - \gamma s)$

is the policy solution with $K \equiv (R'\gamma\gamma R)^{-1}R'\gamma = (R'\Gamma R)^{-1}R'\gamma$, the generalized inverse of γR, since

$$\gamma R(R'\gamma\gamma R)^{-1}R'\gamma$$

is the orthogonal projection matrix operator and $\gamma RK\hat{x}$ renders the variable expression in x

$$(\gamma Rx - \gamma y^* - \gamma s)'(\gamma Rx - \gamma y^* - \gamma s)$$

a minimum.

Note that $(R'\gamma\gamma R)^{-1}$ is defined since γR is of rank m, γ being a full-rank diagonal matrix and any arbitrary vector u

$$u'R'\Gamma Ru = (\gamma Ru)'(\gamma Ru) > 0.$$

Situation (VII-1,3) is characterized by $r(R) = N$, $N < M$. The weights on the deviations $(x_j - x_j^*)^2$ are $(\Delta_1,\ldots,\Delta_m)$. The linear model is found as follows:

Define $\delta_j \equiv \Delta_j^{1/2}$, $\delta = \Delta^{1/2}$, $v = \delta x$. Then

VII-8 $y^* = Rx^* + s$ (the incompatible system)

or

VII-9 $y^* = R\delta^{-1}(v^* + \bar{v}) + s$

Therefore

VII-10 $$R\delta^{-1}\bar{u} = y^* - R\delta^{-1}u^* - s.$$

By analogy to (VI-10) we have

VII-11 $$\bar{u} - H(y^* - s - R\delta^{-1}u^*) + (I_M - HR\delta^{-1})u$$

where H is the generalized inverse of $R\delta^{-1}$ and defined

$$H \equiv \delta^{-1}R'(R\delta^{-1}\delta^{-1}R')^{-1} = \delta^{-1}R(R\Delta R')^{-1}.$$

Therefore

VII-12 $$\bar{x} = \delta^{-1}H(y^* - Rx^* - s) = \Delta R'(R\Delta R')^{-1}(y^* - Rx^* - s)$$

and the policy solution is

VII-13 $$\hat{x} = x^* + \Delta R'(R\Delta R')^{-1}(y^* - Rx^* - s).$$

Coming finally to (VII-14) the linear model is

VII-14 $$\begin{cases} \gamma y = \gamma Rx + \gamma s \\ u = \delta x. \end{cases}$$

Since $x = \delta^{-1}u$, by defining $\gamma y = z$, $\gamma R\delta^{-1} = B$, $\gamma s = c$ (VII-14) becomes

VII-15 $$\begin{cases} z^* = Bu + c \\ u^* = 0 \end{cases}$$

by replacing y by y* and x and x*. Hence

VII-16 $$\begin{pmatrix} B \\ I \end{pmatrix} u = \begin{pmatrix} z^* - c \\ u^* \end{pmatrix}$$

is the linear-model equivalent. The form of (VII-16) is exactly similar to that of (VI-14), so we may deduce that the generalized inverse S of $\begin{pmatrix} B \\ I \end{pmatrix}$ is

$$S \equiv (I + B'B)^{-1}(B',I) = [I + (\gamma R\delta^{-1})'(\gamma R\delta^{-1})][\delta R'\gamma, I].$$

It follows that

VII-17) $$u = S\begin{pmatrix} z^* & - & c \\ & u^* & \end{pmatrix}$$

and

VII-18 $$\hat{x} = \delta^{-1}S\begin{pmatrix} z^* & - & c \\ & u^* & \end{pmatrix}.$$

Bibliography

1. J. Tinbergen <u>On the Theory of Economic Policy</u>, North Holland, Amsterdam, 1963.

2. H. Theil <u>Economic Forecasts and Policy</u>, North Holland, Amsterday, 1961.

3. K. Fox, J. Sengupta, & E. Thorbecke <u>The Theory of Quantitative Economic Policy</u>, Rand McNally, 1966.

4. W. Brainard Uncertainty and the Effectiveness of Policy. AER, Papers and Proceedings, May 1967.

5. W. A. Spivey & H. Tamura Generalized Simultaneous Equation Models. (Mimeograph).

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INTERNATIONAL BANK FOR RECONSTRUCTION AND DEVELOPMENT

MACROECONOMIC PROJECTIONS

A SURVEY OF SOME ISSUES AND MODELS

Apostolos Condos
Creditworthiness Studies
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The paper contains three parts.

Part One discusses certain general characteristics of projection models in the form of an extended introduction.

Part Two deals with gap models in general and their limitations for projections and also presents a contrasting neoclassical growth policy model.

Part Three contains three appendices. Appendix A presents empirical results associated with the theoretical discussion of Part Two.

Appendix B reviews the theoretical models offering the intellectual backbone of the actual Five-Year Plans in India.

Appendix C reviews three models concerned with macroeconomic projections and planning in Pakistan.

Part One

General Observations on the Method and Use of Projection Models for Creditworthiness Analysis

Creditworthiness assessment concerns the prospects of external debt servicing. Therefore it deals with the factors affecting both the willingness and capacity of a country to meet its debt obligations. Although willingness and capacity may be interdependent to some extent, the former is viewed better as a political matter left to be treated with the flexible, imprecise and intuitive means appropriate to such a study. The problem of capacity for debt servicing is technically amenable to treatment by the known macroeconomic techniques yielding conditional predictions or projections. Capacity to service external debt signifies feasibility of conversion of domestic resources (normally current output) into foreign exchange over a specified time period to meet the terms of external loans. Without any constraints such a conversion is possible if current output valued at international prices exceeds the value of the amortization and interest payments for the same time period. Obviously, however, a multitude of constraints restrict the feasibility of conversion. The systematic study of these constraints involves no less than economy-wide models in which the interrelationships of a country's growth direction and potential and their dependence on foreign capital can be analyzed. The word "model" is used in this context to mean a set of independent and consistent statements thought to capture the essential characteristics of the empirical economy studied together with their correctly derived implications.

Since the answers sought of such a task are quantitative (numerical), the models must be capable of quantification, i.e. the variables involved must be directly or indirectly observable and the parameters, in whatever manner, estimable.

It is necessary to dispense at this point with the assertion that quantitative projections and quantitative analysis of alternatives is misleading and irrelevant, in particular in development economics, on the ground that:

(a) we do not know what motivates poor countries and people;

(b) economic structures change very quickly;

(c) for long-range analysis essential for creditworthiness assessment we reach so far into the future as to magnify grossly the problem of uncertainty;

(d) vast problems of data.

To the extent that quantitative answers are sought, there is no alternative to quantitative analysis. The alternative to a quantitative model judged to be misleading and irrelevant on any of the reasons above is a better quantitative model that satisfies the criticisms. If no alternative better quantitative model can be found and the available one is nevertheless judged to be misleading and irrelevant no quantitative questions should be asked.

Quantitative answers supplied by what passes for qualitative analysis are based necessarily on implicit quantitative models and it should be obvious that making the implicit explicit is always desirable in analysis.

These remarks do not imply that the problems raised in (a) - (d) are anything less than serious. Indeed, problems relating to data availability and reliability in under-developed countries have beplagued even modest quantitative tasks. The problem of uncertainty in its relation to economic structural change and social motivation may be insoluble, social change and transformation associated with development may not be arithmomorphic concepts. But the inescapable fix for the economist is that quantitative answers, if sought, must be predicated upon some form of tolerable simplification of reality so that the various phenomena of interest may unfold as implications of a few axiomatic propositions.

On the other hand the objections contained in (b) can be accounted for by and large by advances in the formulation of economic models.

In general, the constraints on the feasibility of the aforementioned conversion will be identified with the mathematical relations making up the model and will involve three classes of elements:

(a) The technical data of the economy expressed as the known or estimable structural coefficients together with their laws of change, if any;

(b) the economic objectives of the policy makers;

(c) the economic instruments under the control of the policy makers.

It is possible to produce a great variety of models by classifying the elements of (a) - (c) into special kinds. Particularly important for the methodology of macroeconomic projections for creditworthiness assessment are the varieties produced with special attention to the relation between balance of payments and economic growth. However no neat classification can be imposed on this subset any more than on economic models in general. Two aspects of the taxonomy, nevertheless deserve some comments. The first is the distinction between short, medium and long term models; the second is, what one may call, the rigid versus the flexible view of the economy.

The first general remark to be made about time horizon and economic models is that, with one exception, no distinct rules have been applied and, possible, in the present state of the art no general rules can be derived concerning the period of time for which a model can be considered valid a priori. This means that by looking at the form of a model, so to speak, it is not possible to determine whether it is a short, medium or long range model. The one exception is the class of models that can be called "aggregate demand" models generally recognized to be short-term in their predictions. The main objective in the construction of such models is the calculation of impact multipliers (although some authors derive long-term multipliers, as well, in those cases involving dynamic relationships, usually dynamic consumption functions, which turn out to be equal to the corresponding multipliers in static specifications). The questions that short-term models can answer concern the effect of fiscal, monetary and foreign trade policy measures. They are of interest for creditworthiness assessment to the extent that such measures bear on inflation, the balance of payments and the overall orientation of economic policy. Since the long run, however independently it may be viewed, is a succession of short runs, it appears important that current economic policies be assessed for their quantitative impact consistently and on a continuing basis. The "maintained", although continually revised, short-term model is an indispensable tool for systematic analysis of current economic policies. It is regrettable, as observed by L. Klein, that few model builders "stick" to their product after its completion. The fact is that although aggregate econometric models abound, few have been "maintained". Thus the quality of their performance is known, by and large, only in reference to the basic sample period. It would be an invaluable contribution to economic knowledge if some institution were to construct and "maintain" economy-wide models, with emphasis on "maintain", possibly in the tradition of the Dutch Central Planning Bureau.

Of more interest for creditworhtiness analysis are medium and long range models. While aggregate demand models concern themselves with economic stabilization, medium and long-term models are capable of exhibiting the dependence of economic growth on international trade and capital movements together with the associated policy alternatives. Accepting it as a postulate that developing countries lack the resources for satisfactory economic growth, international temporary transfer of capital is a necessary means of development. If the supply of capital were unlimited at some interest rate, each developing country would demand of it to the point where its social marginal productivity was equal to that interest rate. As a matter of fact the capital supply conditions facing each developing country, for both public and private capital, are imperfect and uncertain. In particular, the existing institutions and practices do not allow to speak meaningfully of an aggregate public capital supply facing the developing countries as a whole. Each country confronts special conditions and at most one can postulate a range of public capital availability at some average price. Although much remains to be done in the systematic study of supply, the main focus for creditworthiness assessment is the demand side of the problem for each country individually and the factors affecting it. This demand is expressed as foreign capital requirements to meet economic growth objectives embodied in medium term (usually 5-year) plans or long-range projections. Thus the demand for foreign capital is connected inextricably with a developing country's targets given its economic structure.

A distinction must be made at this point between official plans and projection models made either independently of official plans and meant as checks of the consistency, feasibility or optimality of the latter or "appended" to official plans providing guidelines for thinking on general questions.

The discussion will be devoted exclusively to these models. In this part of the paper I shall make certain general remarks about the formal characteristics of the medium and long-range models and their main uses deferring until later a more detailed examination of some central ideas.

A predominant feature of a number of important contributions (1, 6, 7, 8) is the two-gap analysis due primarily to Chenery and his collaborators. The two-gap models embody the view of structural disequilibrium at the factor level and depend on special assumptions concerning the technology and foreign trade price elasticities facing the developing countries. Gap analysis is closely related to linear programming formulations and the distinction therein of binding and non-binding constraint ranges. The gap analysis view and its implications for development policy will be discussed at some length later on. Here it should be noted that it can be incorporated in a number of formally different models - as it has been - characterizing the literature in this field. Thus with econometric models of projections as in the UNCTAD study, multisectoral models of the Feldman-Domar-Harrod variety, multi-sectoral models based on input-output techniques, be they optimizing or non-optimizing. The non-optimizing variety of gap models concern themselves principally with the necessary flow of foreign capital for a specified target of global growth rate or sectoral growth rates. Total investment requirements are estimated and sectoral allocation of investment achieves the pre-specified targets if total resource supply is adequate. Total resource supply is made of two parts: domestic savings and foreign capital necessary to purchase the imports associated with the global or sectoral growth rates as of a given structure of domestic production. The specification of import requirements presumes a fixed proportion with output levels and if total exports are insufficient to pay for total imports a trade gap exists. The usual adjustments are either a downward revision of growth targets or policies to improve the balance of payments. The latter consist of either introducing new trade creating activities or import substituting activities or both. The investigation is conducted by examining feasible ranges of the alternatives on the presumption that the policy makers will choose the socially preferred point.

In the optimizing or programming models the welfare criterion usually is an appropriately discounted consumption stream to be maximized subject to equality and inequality constraints expressing either technical conditions or additional welfare restrictions. A few examples of both classes of models will be reviewed in the appendices.

The most serious shortcoming from the medium or, worse, long-run point of view is the possible irrelevance of the input-output coefficient matrix derived as it is from past intersectoral flows which may not reflect at all correctly the range of technical possibilities.

This defect is absent from the partial programming models whose activity columns reflect engineering-like technical coefficients. It is difficult to imagine, unless disaggregation is carried to extreme, that intersectoral macroeconomic models will be free of this shortcoming. Of course, partial measures such as account for the time trend in the coefficient matrix may go a long way towards eliminating this source of error. See Tim (28), for example.

The second aspect of the taxonomy deserving some comments is the *rigid versus the flexible* view of the economy, in particular of the developing economy. In a deeper sense this is as much a philosophical issue as it is an economic one. At the economic level, the manifestation of the rigid view[1/] coincides with the absence of a price mechanism performing allocative functions on the available resources and equilibrating demands and supplies or with the presence of serious institutional and structural obstacles to this traditional role of market prices. All forms of gap models conform to this view although, of course, the extent may vary from model to model.

1/ Of course, it is only an expository simplification to contrast sharply the two views. More continuous gradations characterize the actual literature.

Extreme examples of the rigid view are model representations of the economy facing a given demanded quantity for its exports which price policies on its part can only affect adversely,[1/] strictly complementary technical import requirements for its domestic activities and exogenously given factor supplies, particularly labour, in a number of non-competing groups.

1/ The situation envisaged is different from that of price inelasticity of exports and more akin to Sweezy's model of oligopoly. Total demand for the exports of underdeveloped countries - classified into groups of competing producers - is given and international prices are associated with approximately fixed shares for each exporting under-developed country in the total. Price increases which in the case of a standard price inelasticity situation would increase export earnings, only cause loss or decrease of the market share and total revenue whilst price decreases lead to diminution of export earnings because competing producers in other underdeveloped countries following suit cause the downward segment of the individual country's demand schedule to be highly inelastic.

On the other hand, the extreme flexible view doubts systematically the presence of rigidities other than those imposed by wrong policies whose effects masquerade afterwards as "technical" discontinuities And, at any rate, if it is true that ex post much of economic reality is "clay" ex ante, the only relevant standpoint for medium and long range planning, it is "putty".

At the philosophical level, a few dissenting voices from the "conventional wisdom" of development theory and practice have decried the failure of the imagination which translates "packaged" wants in imitation of consumption patterns of the developed countries and their associated techniques into desired ends for the underdeveloped countries and, indeed, constituent elements in the definition of development. Thus, Ivan Illich (15) defines underdevelopment as the surrender of social consciousness to prepackaged solutions. Apart from the fact that the consumption and production patterns of the developed economies may be economically unsuited for the underdeveloped countries, they may not be socially optimal even in the developed countries as suggested by the transformation of the total environment - in particular in the USA - into a by-product of consuming habits. The following extended passage is from Illich's cited article.

> Rich nations now benevolently impose a straightjacket of traffic jams, hospital confinements, and classrooms on the poor nations, and by international agreement call this 'development'. The rich and schooled and old of the world try to share their dubious blessings by foisting their pre-packaged solutions on to the Third World. Traffic jams develop in Sao Paolo, while Brazilians flee the dought by walking 500 miles. Latin American doctors get training at the New York Hospital for Special

Surgery, which they apply to only a few, while amoebic dysentery remains endemic in slums where 90 percent of the population live. A tiny minority gets advanced education in basic science in North America - not infrequently paid for by their own governments. If they return at all to Bolivia, they become second-rate teachers of pretentious subjects at La Paz or Cochibamba. The rich export outdated versions of their standard models.

The Alliance for Progress is a good example of benevolent production for underdevelopment. Contrary to its slogans, it did succeed - as an alliance for the progress of the consuming classes, and for the domestication of the Latin American masses. The Alliance has been a major step in modernizing the consumption patterns of the middle classes in South America by integrating them with the dominant culture of the North American metropolis. At the same time, the Alliance has modernized the aspirations of the majority of citizens and fixed their demands on unavailable products.

Each car which Brazil puts on the road denies fifty people good transportation by bus. Each merchandised refrigerator reduces the chance of building a community freezer. Every dollar spent in Latin America on doctors and hospitals cost a hundred lives, to adopt a phrase of Jorge de Ahumada, the brilliant Chilean economist. Had each dollar been spent on providing safe drinking water, a hundred lives could have been saved.

I conclude these general observations about projection models with some remarks concerning their relevance and role in the process of actual planning.

Outside the socialist bloc, no country devoted more energy and thought -- assisted by some of the world's most eminent economists - to matters about appropriate model building for development planning than India. A vast literature already exists in part reviewed admirably in (3). The reviewers note:

"The formulation of the successive five-year plans in India has led to a steady evolution of economic thinking on questions relating to planning theory and technique. As we shall soon argue, however, the interplay between Plans and economic thinking has often been tenuous. At

> times there may even have been post-facto rationalization of investment decisions taken on political grounds by ingenious designing and suitable models. At other times, model-building and analysis have inevitably gone ahead of the Plans. However, it is possible to identify with each Plan certain basic model-types which have provided the intellectual backbone to that Plan and were the object of extensive economic debate..."

The same can be said about the experience of Pakistan although the extent of theoretical debate has been smaller and actual planning has had a shorter history. More specific comments will be made in the appendices where selected Indian and Pakistan models will be discussed.

Although little is known to me about this subject as it relates to other developing countries, it is highly unlikely that the interplay between models and policy making will be revealed to have been any closer.

Part Two

Concepts and Issues in Gap Analysis and a Contrasting View

Fundamentals of the "gap analysis".* The simplest possible model of an open economy by means of which the ideas involved in the gap analysis can be demonstrated is as follows: Assume (for simplicity) a constant capital-output ratio, average or marginal, describing the aggregate production function. Consider real domestic savings and imports to be positively-sloped functions of real output and exports given exogenously. If a rate of growth of real output is posited as a target of economic policy, savings and import projections are calculated through the respective functions. In general, investment requirements obtained by the target growth rate of output and the capital-output ratio will not coincide with projected savings. If investment requirements exceed forthcoming savings while the level of imports associated with the target growth rate of output does not exceed the projected level of exports, there exists a savings gap. If, with domestic savings adequate to meet the investment requirements, the imports corresponding to the target growth rate of output exceed the level of exports, there exists a trade gap (or foreign exchange gap). Of course, ex post or historically the excess of imports over exports is identically equal to the excess of investment over domestic savings, foreign capital inflows making up the difference between total use of real resources by an economy and total supply of real resources by it. Ex ante, the larger of the two gaps specifies the amount of foreign capital required if the target growth rate of output is to be met. Before discussing certain problems related with the outlined model, broadly, the economic assumptions implied for its validity, I shall present a simple formalization of it.

The standard identities below involve real variables:

1. $Y + M \equiv I + E + C$

2. $Y \equiv C + S$

---

* This part draws heavily on Chenery and Strout (8), Fei and Ranis (11), Landau (17) and Bruton (4).

3. $M \equiv E + F$

4. $I \equiv S + F$

5. $I \equiv \overset{o}{K} \equiv \frac{dK}{dt}$

Y = real output (income)

M = imports

I = investment

E = exports

F = foreign capital inflow

S = savings

C = consumption.

The following basic structural assumptions are made:

6. $\overset{o}{S} = \sigma\overset{o}{Y}$

That is, a constant marginal savings ratio, σ, is assumed implying a savings function.

6'. $S = S_o + \sigma(Y - Y_o)$

where S_o, Y_o are initial values.

Similarly, for imports it is assumed

7. $\overset{o}{M} = \mu\overset{o}{Y}$

implying an import function

7'. $M = M_o + \mu(Y - Y_o)$.

The production function appears in the form of a constant capital-output ratio

8. $K = kY$.

Exports are taken to grow exogenously at the rate ϵ.

9. $E = E_oe^{\epsilon t}$.

None of the specifications 6-9 is crucial to the gap model but they are very simple to deal with and Chenery et al adopt frequently the stated savings and import functions. What is crucial is a formal overdeterminacy introduced by positing a rate of growth of output, r, as a target of economic policy.

The desired (target) output path is then

10. $Y = Y_o e^{rt}$.

Given the production function, this entails investment requirements

11. $I \equiv \overset{o}{K} = \frac{d}{dt}\left[kY_o e^{rt}\right] = krY_o e^{rt}$.

Define $\sigma_o = \frac{S_o}{Y_o}$ and rewrite the savings function

12. $S = (\sigma_o - \sigma)Y_o + \sigma Y = (\sigma_o - \sigma)Y_o + \sigma Y_o e^{rt}$

Clearly, required investment and anticipated savings if the output path is realized will not, in general, coincide. Therefore, an <u>ex ante</u> balancing tem, required foreign capital, is derived as the difference of I and S. One has then:

13. $F^* = I - S = krY_o e^{rt} - (\sigma_o - \sigma)Y_o - \sigma Y_o e^{rt} = (kr - \sigma)Y_o e^{rt} - (\sigma_o - \sigma)Y_o$

On the other hand, foreign capital, <u>ex ante</u>, is the difference between anticipated imports and exports.

Defining $\mu_o = \frac{M_o}{Y_o}$, rewrite the import function

14. $M = (\mu_o - \mu)Y_o + \mu Y_o e^{rt}$.

The associated foreign capital requirements are given by

15. $F^{**} = (\mu_o - \mu)Y_o + \mu Y_o e^{rt} - E_o e^{rt}$.

Obviously, there is no reason why $F^* = F^{**}$. Generally, $F^* \neq F^{**}$ and this is the "gap between the gaps". The overdeterminacy introduced by the target rate of growth of output is reflected precisely in the two different values of one variable, F. The mechanism by which the two gaps become equated ex post will be discussed later on. It is obvious, however, that the larger of the two gaps must be filled with foreign capital or else the target growth rate is unattainable. And this, of course, implies that the smaller gap widens ex post. It must be noted also that the ex ante discrepancy between the two gaps is ascribed by Chenery to conditions of structural disequilibrium of the economy, a concept which will be examined in some detail. The remarks just made indicate that the model works, i.e. is made consistent, by relaxing the structural relationship corresponding to the smaller of the two gaps.

A concept crucial to the Chenery approach is the phase of the economy identified as the period during which one and the same structural relationship is associated with the larger gap. Economic development is thus viewed as a sequence of phases. Chenery does not commit himself to any particular sequence of phases as representing the typical experience of a generic developing economy, but rather he employs several empirical tests to identify the dominant gap. It may be the case that a trade gap becomes dominant, in general, after a savings gap but the matter should be decided according to each case.

Returning to equations 13 and 15 of the model, at t=o

$F_o^* = krY_o - S_o$

$F_o^{**} = M_o - E_o.$

Assume $F_o^* > F_o^{**}$, $F_o^* > 0$, so that the economy has a dominant savings gap. For any future t the time rate of change of foreign capital requirements is, respectively:

16. $\overset{o}{F}{}^* = r(kr - \sigma)Y_{oe}{}^{rt}$

17. $\overset{o}{F}{}^{**} = r\mu\ Y_{oe} \quad - \epsilon E_{oe}{}^{\epsilon t}$.

From inspection of equation 16 it is evident that the savings gap behaves monotonically:

$$\overset{o}{F}^{*} < 0 \qquad \text{if} \qquad r < \sigma/r$$

$$\overset{o}{F}^{*} > 0 \qquad \text{if} \qquad r > \sigma/r$$

The common sense of the inequalities is that the savings gap decreases over time only if the target growth rate of output is less than the growth rate that can be supported by domestic savings (= σ/r) whilst it increases in the opposite case. The trade gap path is monotonic only if either $r\mu Y_o > \epsilon E_o$ and $r > \epsilon$ or

$$r\mu Y_o < \epsilon E_o \quad \text{and} \quad r < \epsilon.$$

(The latter case describes, of course, a trade surplus path). If there is to be a sequence of phases, given the initial conditions, there must for some t $F^{*}=F^{**}$. Thereafter the dominant gap is due to foreign trade.

An outline of the simplest two-gap model has been presented thus far. More complex models retaining the fundamental notions - some of which will be surveyed below - can be constructed by any of the following types of specification:

a) Disaggregation of the basic structural relationships into sectors;

b) Use of the various structural parameters as policy instruments (or as variables changing over time); thus import substitution can be introduced, a changing capital-output ratio can be taken into account as a result of changes in the composition of investment and so on;

c) Introduce additional structural relationships (such as are based on the notion of absorptive capacity) increasing thereby the potential phases of development;

d) Convert the model into a mathematical programme by introducing a flexible target function (instead of a fixed target such as a given growth rate of output) and additional constraints, usually having to do with the economy's terminal profile (such as no dependence on foreign capital by a prespecified time); the structural relationships must then be interpreted as inequality constraints each referring to a particular resource.

Gap models, adjustment mechanisms and estimation of structural relationships *

The view of the economy described by a gap model departs from the view of neoclassical general equilibrium theory in that no recognition is made of a market price mechanism by which relative scarcities are equated at the margin. The absence of such an allocative mechanism is supposed to reflect a structural disequilibrium thought to prevail at least over the medium run. Thus a "gap" economy with no foreign capital available is supposed to adjust (in terms of output level) so as to close the larger of the two ex ante gaps. Since historically the relation "realized (I-S)≡realized (M-E)" always holds "excess" domestic savings cannot be converted into additional foreign exchange earnings either by increased export production or increased production of import substitutes or both; conversely, "excess" foreign exchange cannot mobilize domestic resources into capital formation. Domestic savings and foreign exchange are strictly complementary resources and either may be a free good (in the sense of zero marginal product) for a given level of output. This picture is consistent with an economy exporting mainly consumer goods and importing mainly capital goods and intermediate products necessary for domestic production.

It has been seen that ex post the smaller of the two gaps exceeds its ex ante value after foreign capital has filled the larger of the two gaps. For an economy dominated by a savings gap closed by foreign capital, the burden of adjustment falls entirely on either exports or imports or both.

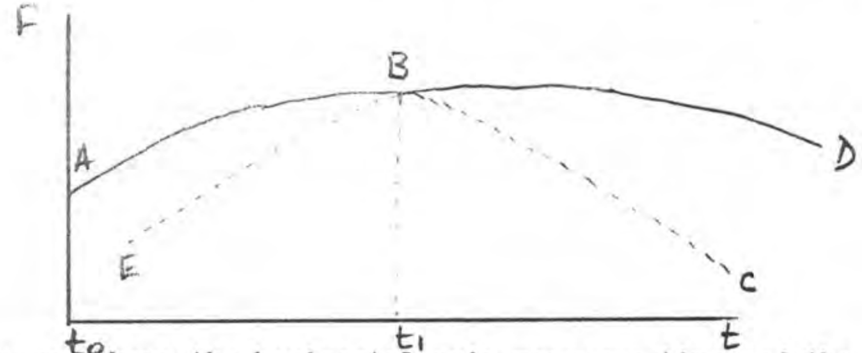
* For a criticism of standard econometric gap models for failure to satisfy basic assumptions of statistical theory see Shourie (27).

It is impossible to ascertain a priori the impact of foreign capital on exports in this case. If a portion of foreign capital is used for investment in the export sector and no significant gestation lags exist, exports will increase but in this case imports must increase necessarily by more. On the other hand, if factors of production are bid away from the export sector exports will decrease. It is also conceivable that they may not be affected at all. In two of the three possible cases imports must, whilst in the third they may, increase. It is therefore safe to assume in the absence of additional information that imports bear the brunt of adjustment in the short run. The economic explanation why realized imports exceed anticipated imports in the described case lies in the interpretation of the import function more in the sense of a technical relation associating output levels with minimum required imports than in the sense of a behavioral relation like the consumption function. The presence of additional foreign exchange through foreign capital inflows relaxes institutional constraints on the flow of imports allowing a behavioral import function (left unspecified) to be operative, and thus realized imports exceed the minimum required for the attainment of the target output level.

The gap model leaves also unspecified a possible effect on the price of foreign exchange in terms of domestic currency. An inflow of foreign capital would tend to lower the price of foreign exchange stimulating thereby imports. When foreign capital fills a dominating trade gap the ex post savings gap will exceed its ex ante value. This result is brought about in general by a positive effect on investment and a negative effect on domestic savings. These effects are not necessary separately in the sense that only the relation "realized investment - realized savings > anticipated investment - anticipated savings" is necessary. It remains nevertheless probable that realized investment will tend to exceed the minimum necessary for the target output growth as a consequence of the lower interest rate that will prevail with an inflow of foreign capital. Investigators have emphasized, however, more the negative impact of foreign capital on domestic savings. Among the various reasons mentioned are the disincentive to the growth of domestic financial institutions (13) and the greater laxity in the pursuit of domestic savings, in particular in the public sector that inflows of foreign capital entail (24).

The remarks made indicate immediately a difficulty in identifying the basic structural relationships of a gap model. A historical series of data, however long, represents the ex post magnitudes and therefore helps to identify the binding constraints. The standard statistical techniques are suited for equilibrium models and if used to estimate the structural relationships of an overdetermined (disequilibrium) model serious biases are introduced exactly because imports are above the minimum required in the time series when the savings constraint is binding and savings are represented below their potential level when the trade constraint is binding.

This point was raised by Fei and Ranis in their cited paper (11). Chenery and Strout in their reply (8) recognize the difficulty and suggest indirect methods for identifying the various phases, such as evidence of pressure on the limiting resource (falling foreign exchange reserves, import restrictions, rising interest rates) and existence of slack in the non-binding relation (rising foreign exchange reserves, import liberalization, falling interest rates). Once the phases have been identified through indirect methods, standard statistical techniques can be used to estimate an essentially equilibrium model (i.e. the binding structural relationships) and provided there has been a sequence of phases, the ex ante values of the non-binding factor can be found by interpolation and extrapolation.



In the diagram above the horizontal axis measures time and the vertical foreign capital. Suppose through indirect methods one can distinguish two phases of growth, t_0 to t_1 and thereafter. Assume that the first corresponds to a savings-binding constraint and the second to a trade-binding constraint. Time series data covering the period (t_0, t_1) can be used in principle to estimate the ex ante savings gap (≡ realized) (curve AB) and data from the following period similarly help to establish the ex ante trade gap (≡ realized) (curve BD). It is then a simple matter to obtain portions BC and EB and through them to estimate potential savings in the second phase and minimum imports in the first phase, respectively.

Landau in his doctoral dissertation (17) has proposed a straightforward specification by which the dominating gap can be found. The central ideas are as follows: Distinguish two types of saving and import functions, respectively, ex ante and ex post. In general, write

$$S = S(Y)$$
$$M = M(Y)$$

the ex ante functions and

$$S = S(Y) + \alpha_1 F$$

$$M = M(Y) + \alpha_2 F$$

the ex post functions. Assume exports and investment to be exogenously determined and no limitations on the availability of foreign capital. The assumption on exports and investment implies that the ex post adjustments are made exclusively by savings and imports, as already explained.

If a trade gap is binding over a particular period, it follows that statistical techniques should reveal α_2 to be not significantly different from zero, whilst α_1 should be significantly different from zero. Three cases can be distinguishes concerning the estimated value of α_1.

(1) If ex ante savings are equal to required investment ($\equiv$ realized), then $\alpha_1=-1$, and the ex post saving function is

$$S = S(Y) - F,$$

i.e. actual savings fall short of their potential level by the full amount of foreign capital inflow.

(2) If $I-S(Y)>0$ then α_1 should be revealed to lie between -1 and 0.

(3) If $I-S(Y)<0$, then $\alpha_1<-1$.

Whatever the case may be, given the statistical saving function

$$S = S(Y) + \alpha_1 F$$

dS/dY measures the potential marginal propensity to save.

By a symmetrical argument, if a savings gap is binding, α_1 should turn out to be zero and α_2 positive. If $-1 < \alpha_1 < 0$ and $\alpha_2>0$ the case may be either that of alternating binding constraints or a two-phase sequence in conformity with Chenery's ideas. Clearly, Landau's procedure is not a substitute for the indirect methods suggested by Chenery. It is by the latter that the different phases, if they exist, can be sorted out.

Landau reports empirical results for eighteen countries for the period 1950-66. For the coefficients $\partial S/\partial F=\alpha_1$, $\partial M/\partial F=\alpha_2$, α_2 is always positive and less than unity whilst α_1 lies in the range (-1, 0), with few insignificant exceptions, according to the author.

Programming Origin of Gap Models

So far the model used to illustrate the gap analysis was a structural macroeconomic system converted into a policy model by means of one fixed target, the global rate of growth, and one policy instrument, the inflow of foreign capital. No considerations of choice entered although it is relatively easy to increase the number of instruments by introducing import substitution and allowing the marginal saving ratio to be a variable subject to policy control. The core of gap analysis and the possibility of choice can be illustrated more directly by means of a programming model, following R.R. Nelson (21). Assume that an economy has available to it four activities:

I_p = domestic production of capital goods

I_m = imports of capital goods

C_p = domestic production of consumer

C_m = imports of consumer goods.

All activities require imports by assumption. Additionally, it is postulated - with wide empirical validity in the underdeveloped countries - that the domestic production of capital goods is more import intensive and less domestic input intensive than the domestic production of consumer goods. Let a_1 be the unit import requirements (intermediate imports) of I_p and similarly a_2 for C_p; also let b_1 be the domestic input coefficient of I_p and b_2 similarly of C_p. The stated assumptions are then: $a_1>a_2>0$, $0<b_1<b_2$. Let $\overline{M}$ represent a given capacity to import and $\overline{Y}$ a given level of output. The structural constraints are then:

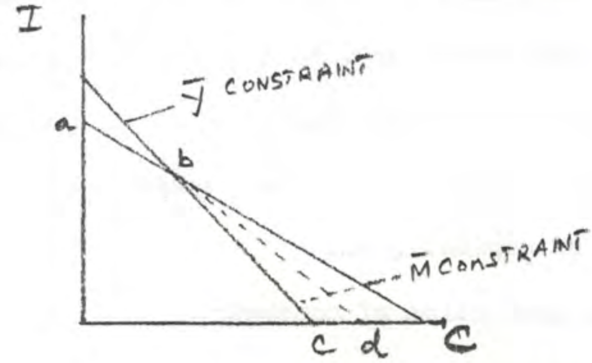
$$a_1 I_p + a_2 C_p + I_m + C_m \leqslant \overline{M}$$
$$b_1 I_p + b_2 C_p \leqslant \overline{Y}$$

The activity tableau corresponding to the constraints is:

Activities

		I_p	I_m	C_p	C_m
Output	I	1	1	0	0
	C	0	0	1	1
Input	M	a_1	1	a_2	1
	Y	b_1	0	b_2	0

The diagram below represents the constraints in (I-C) space.



The $\bar{Y}$-constraint represents the equation $b_1 I_p + b_2 C_p = \bar{Y}$ and the $\bar{M}$-constraint is drawn by neglecting direct imports (final good imports) $a_1 I_p + a_2 C_p = \bar{M}$.

The slope of the constraints incorporates the assumption about relative input intensities of the two activities I_p and C_p, i.e. for higher $^I/C$ ratios import availability is more constraining while for lower $^2/c$ ratios domestic factor availability (represented by $\bar{Y}$) is more limiting. Neglecting direct imports, the choice frontier is therefore abc, point b being defined $\left(\bar{C} = \frac{Ya_1 - Mb_1}{a_1 b_2 - a_2 b_1}, \bar{I} = \frac{Mb_2 - Ya_2}{a_1 b_2 - a_2 b_1}\right)$. To the left of b import availability is fully utilized but domestic resources remain unemployed. Strict complementarity exists between domestic and foreign resources.

To the right of b domestic resources are fully utilized but import potential remains unexploited. It is precisely this import potential that can be converted into either capital or consumer goods directly. Hence the actual choice frontier consists of segments ab and bd where point d is located as follows: For I=0, with no direct imports, maximum consumption is $\overline{Y}/b_2$ requiring imports $a_2\overline{Y}/b_2$. Therefore total consumption by exploiting entirely the availability of imports $\overline{M}$, at I=0, is $\overline{Y}/b_2+(\overline{M} - a_2\overline{Y}/b_2)$. The term in parenthesis is, of course, the length cd. (The slope of segment bd is -1 as import potential can be transformed into either type of goods one for one).

To translate the diagram in terms of the growth model of the gap analysis, the capital-output relation, K=kY, must be introduced. Then,

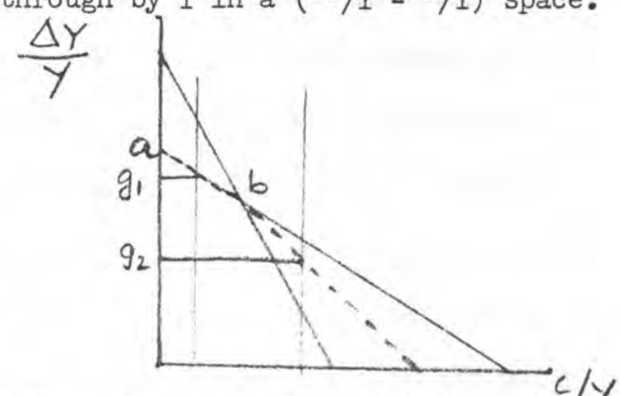
$$\Delta K = k\Delta Y = I$$

and the equations of the constraints can be rewritten

$$a_1 k\Delta Y + a_2 C = Y$$

$$b_1 k\Delta Y + b_2 C = M$$

For given Y and M one can either graph these equations in a (ΔY - C) space or by dividing through by Y in a ($\Delta Y/Y$ - C/Y) space.



The vertical axis measures the global growth rate. The lines labeled g_1 and g_2 correspond to two saving ratios.

It is observed that the change in the growth rate $\Delta Y/Y$ is greater for a unit displacement in the M-constraint than the change induced by a unit change in the saving ratio for all positions of the economy to the left of b. To see this algebraically notice that the global growth rate $\frac{\Delta Y}{Y} = g_1$ corresponding to saving ratio g_1 is found from the equation

$$b_1 k \frac{\Delta Y}{Y} + b_2 \frac{C}{Y} = \frac{M}{Y} ,$$

where C/Y is replaced by C_1/Y corresponding to σ_1. Thus

$$\frac{\Delta Y}{Y} = g_1 = \frac{1}{b_1 k Y} (M - b_2 C_1).$$

Then

$$\frac{\partial(\Delta Y/Y)}{\partial(-\frac{C_1}{Y})} \quad \frac{b_2}{b_1 k Y}$$

and

$$\frac{\partial(\Delta Y/Y)}{\partial M} = 1/b_1 k Y \,.$$

Since $b_2 < 1$, the assertion is proven.

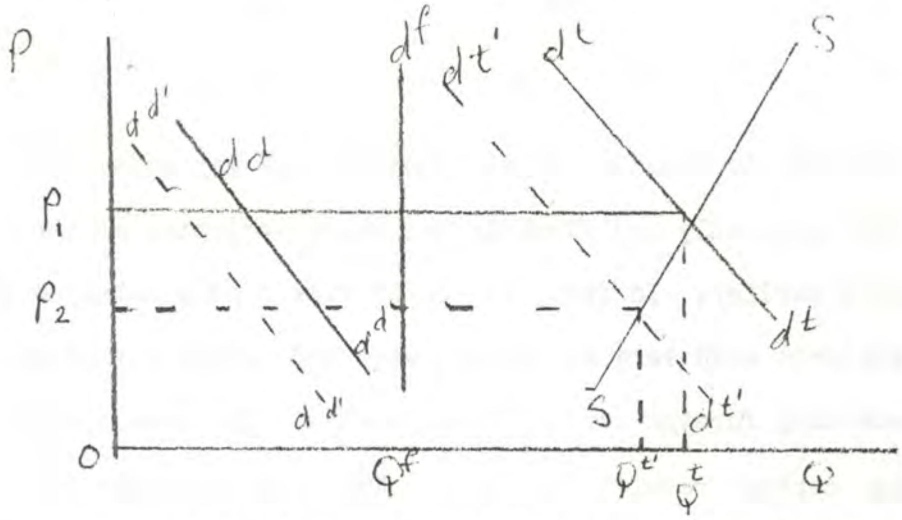
It is also evident from the diagram that a decrease in the consumption ratio in order to attain a higher growth rate releases imports from the consumption goods sector, all of which will be utilized in the production of capital goods (relative import intensity assumption) and domestic resources a portion of which will remain idle (by the converse assumption about relative domestic resource intensity of the two activities). For economies operating to the left of point b, it is thus seen that growth and employment objectives are in conflict provided the resources released from consumption do not serve to move the M-constraint upward. In this context (for all positions to the left of b) the trade gap is defined as the increase in import capacity, given the savings constraints necessary to achieve a particular growth objective.

The diagrammatic version of the "programming" gap model in $\left(\frac{\Delta Y}{Y} - \frac{C}{Y}\right)$ space displays more generally and flexibly the characteristics of the structural gap model presented earlier. In fact, it can be viewed as a detailed presentation of sensitivity analysis with varying growth rates and saving ratios and their implications concerning foreign capital requirements. The more specific problem of finding foreign capital requirements for a given target growth rate with a given saving ratio can be solved immediately by having the M-constraint pass through the intersection of a horizontal line indicating the target growth rate and a vertical line representing the constant saving ratio.

The Gap Model and the Specification of Export Functions

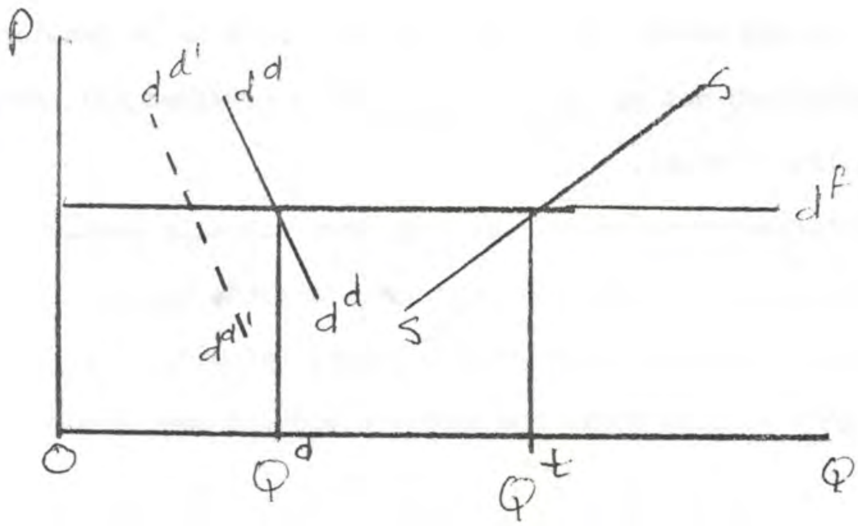
The most incisive criticism to date of the gap models is due to Burton (4). Bruton argues that actually Chenery's structural disequilibrium concept reflects rather unwillingness or incapacity of developing countries to pursue the right policies than characteristics of the economy that can be called structural. The theoretically genuine cases of structural disequilibrium are not empirically significant but they tend to be reinforced by wrong investment policies that take for granted the limited policy options open to developing economies. Underlying Bruton's position is his thesis that the prevalent obstacle to development is the inadequacy of savings and not of import availability provided all economic policy options are taken into account.

To demonstrate these ideas, the only theoretically genuine case of structural disequilibrium will be presented where a trade gap may be the limiting factor to growth. The economy is assumed to possess no capital goods production sector. All and only capital goods are imported and consumer goods are exported.



In the diagram, Q is the quantity of a typical exportable commodity, P its price, ss the supply curve, d^f the foreign demand curve assumed perfectly inelastic, d^d the domestic demand curve, and d^t the horizontal sum of d^f and d^d. In equilibrium, price is P_1 total production OQ^t and exports OQ^f. An increase in <u>ex</u> <u>ante</u> domestic savings corresponds to a shift to the left of d^d, say d^d. Total demand becomes d^t. Clearly, in this case, an increase in savings does not lead to an increase in the rate of capital accumulation (=imports=exports). In fact export earnings will decline and unemployment of resources will follow as the new equilibrium point (P_2, Q^{t1}) is reached.

Consider now the opposite case of export function specification.



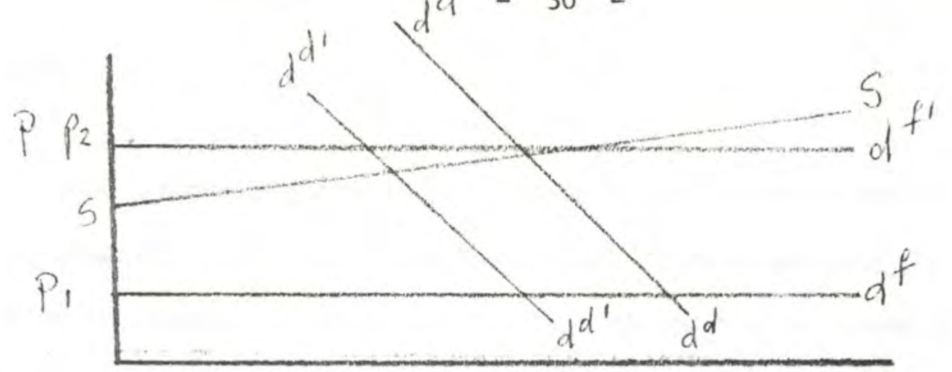
In the diagram above foreign demand is perfectly elastic, total production OQ^t, and the rate of capital accumulation Q^dQ^t. In this case however an increase in savings (a shift of the domestic demand curve to the left) increases automatically the rate of accumulation. That is, there can be no domestic savings which remain idle and no two gaps can emerge. If the rate of capital accumulation is insufficient to attain a desired growth rate of output, it is due to inadequate domestic savings.

In those cases where the elasticity of foreign demand is between zero and less than unity in absolute value the same qualitative results follow as in the case of perfect inelasticity. Thus, a shift in domestic demand to the left (an increase in saving) occasions a fall in price and thereby a reduction in foreign exchange earnings. Domestic savings are not converted into capital accumulation and two gaps may indeed emerge.

If foreign demand is elastic, but less than infinitely elastic, an increase in saving leads to an increase in capital formation but the relative cost of growth in terms of real resources devoted to export production increases. No trade gap can emerge as long as the relevant portion of the export function is elastic.

The conclusion is that for an economy such as described two gaps can emerge only as a consequence of an inelastic export function.

Consider now the case of a potentially exportable commodity. Currently it is not exported because of domestic supply conditions.



If P_1 is the international market price regardless of the saving rate exports and hence capital accumulation do not increase.

A currency devaluation however, with the effect of bringing foreign demand to point P_2 will permit increased savings to be converted into imports of capital goods. Thus, through devaluation the possibility of two gaps vanishes.

Bruton suggests that as a common feature the exports of the developing world fall into two categories: exports whose demand is perfectly inelastic and exports whose demand is perfectly elastic but at international prices below the supply capabilities of developing countries. The situation calls evidently for dual exchange rates, an appreciated rate to maximize foreign exchange earnings from the inelastic category and a low rate to allow domestic savings to increase capital formation.

The cases discussed refer to an economy without a domestic capital goods sector. Bruton examines the case where a domestic capital goods sector exists. Of course, it may be too small for the utilization of all investable resources, if it is defined as the sector where machines, plant and equipment and other forms of physical capital are produced. Recent literature, however, has emphasized the importance for economic development of education, research, health and other activities affecting the productive capacity of human capital. If the definition of the capital goods sector is broadened to include such activities, the notion of its fixity becomes untenable. Invest ble resources applied to education, research, health will have a positive social marginal product and if the rate of growth of total output falls short of the target rate, the problem is that domestic savings are insufficient. Thus, again, there exists only one gap.

The case where the structure of production "requires" imports as raw materials or spare parts may entail a minimum level of imports which must be supplied if there is to be no underutilization of domestic productive capacity. But even then, Bruton argues, available policy measures, i.e. correct pricing decisions, can eliminate a trade gap. As empirically prevalent cases of wrong pricing policies in economies experiencing a trade gap Bruton cites the subsidization of transportation and electricity prices. Both transportation and electricity are import intensive activities and their overexpansion induced by prices below costs has provided typically a strain on the balance of trade.

Conclusions from the criticism of gap models

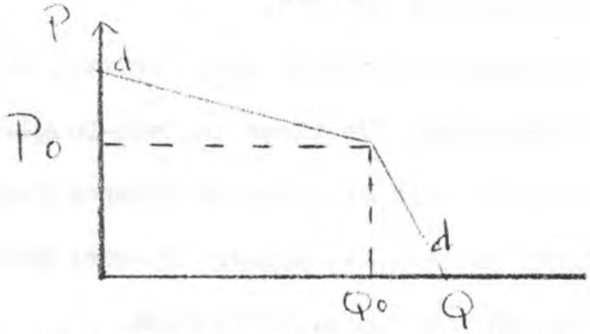
It is evident that the central criticism of the gap models is that they "give away" too much of available policy options by adopting the view that a structural disequilibrium prevails where only the correct policy is not exercised.

Multiple gaps emerge as a consequence of depicting the typical developing economy as a rigid mechanism unresponsive to economizing criteria, principally through the lack of intermarket links. Formal overdeterminacy of the mathematical systems employed to portray a developing economy and to derive gap estimates - more equations than unknowns - is typically produced by the absence of price variables which could produce equilibrium by serving as allocation signals. The mechanical extrapolation of an economy's profile resulting from past policy errors in order to estimate foreign capital needs may well serve to perpetuate these same errors.

In the short run, say for a period of two to four years, it may well be difficult, if not impossible, to bring about the changes which will establish a "correct" structure, i.e. an economy whose overall growth is limited by its own domestic resources available for capital formation. Planning over this period, and projections based conditionally on economic policy, must take account of equilibrium features which can be introduced as terminal conditions in programming models (typically no foreign capital inflows after some period and self-sustained growth) or else described in allocation models with prices playing a crucial role. The former has been done indicatively by Chenery and MacEwan (7) in the context of a sophisticated disequilibrium model applied empirically, the latter has been demonstrated most elegantly by Nelson (21) in a theoretical model of neoclassical persuasion with the Colombian economy in view.

Of course, a neoclassical model is based on a different perception of the economic system than that on which gap models are based, but few economists would disagree with the notion that it is more appropriate for a longer view than its disequilibrium competitors. The important relations and intermarket links suppressed in gap models occupy there the centre of the stage.

The only serious reservation I have concerning Bruton's view is that his only genuine case where two gaps are possible, namely the case of complete export price inelasticity, may well characterize the underdeveloped countries as a whole rather than individually. The distinction is important because - as alluded in a footnote of Part One - individually a developing country may confront an export situation described in the diagram familiar from oligopolistic theory.



The discontinuinty of the export demand function at (Q_o, P_o), the existing situation deprives the country of the possibility to increase its export earnings by raising the export price because its competitors do not raise theirs with the consequence that the demand function is upwards highly elastic, whilst for the opposite reason, downwards it is highly inelastic. If this is true, one should predict a tendency towards cartel formation among underdeveloped countries.

Nelson's neoclassical model of growth policy

Nelson's model will be presented in detail. Although theoretical, it can be estimated readily from observations - not necessarily by standard statistical techniques. It can also be disaggregated into additional sectors without undue complexities. The usefulness derives from its policy orientation and flexible handling of central policy options.

Assumptions: Constant returns to scale prevail in every sector and the specific form of the production functions is Cobb-Douglas. Competitive markets are also postulated for lack of a better alternative. For the sake of simplicity a multiplicative constant, θ, appears in most equations; it is to be interpreted as a different constant in each relation.

Another simplifying assumption is that the capital-labor ratio is the same in the production of both investment and consumption goods. There are three inputs: capital, K; labor, L; imports, M. Subscripts differentiate inputs in different uses in an obvious manner. Capital and labor produce alone a domestic input, value-added, V as follows:

(1) $V = AL^{\alpha}K^{1-\alpha}$

where A is a productivity index.

Consumption goods, C, are produced by:

(2) $C = \theta M_C^{\delta} V_C^{1-\delta}$

Investment goods, I, similarly:

(3) $I = \theta M_I^{b} V_I^{1-b}$

Relations (2, 3) can be expressed in terms of K, L, M as:

(2) $C = \theta M^{\delta} L^{\alpha(1-\delta)} K^{(1-\alpha)(1-\delta)}$

(3) $I = \theta M^{b} L^{\alpha(1-b)} K^{(1-\alpha)(1-b)}$

Define

W = going wage rate

P = price of domestic output v

i = equilibrium rate of return on capital

E = nominal exchange rate

P_I = price of capital goods

P_c = price of consumption goods

$r=P_I i$ = rental rate of a unit of capital.

Then, the derived demands for the inputs are obtained:

$$\frac{\partial V}{\partial L} = \alpha A L^{\alpha-1} K^{1-\alpha} = W/P; \quad \frac{\alpha L^{\alpha-1} K^{1-\alpha}}{L^{\alpha} K^{1-\alpha}} = \frac{W}{PV} \quad \text{and}$$

(4) $$\frac{L}{V} = \frac{\alpha P}{W}$$

Similarly,

(5) $$K/V = \frac{(1-\alpha)P}{r}$$

$$\frac{\partial C}{\partial M_c} = \delta\theta M_c^{\delta-1} V_c^{1-\delta} = \frac{E}{P_c}, \qquad \frac{\delta M_c^{\delta-1} V_c^{1-\delta}}{M_c^{\delta} V_c^{1-\delta}} = \frac{E}{P_c E},$$

(6) $$\frac{M_c}{C} = \frac{\delta P_c}{E}$$

Similarly,

(7) $$\frac{V_c}{C} = \frac{(1-\delta)P_c}{P}$$

(8) $$\frac{M_I}{I} = \frac{bP_I}{E}$$

(9) $$\frac{V_I}{I} = \frac{(1-b)P_I}{P}$$

Price equations are derived thus:

$$P = \frac{L}{V}W + \frac{K}{V}\,r \qquad \text{(from linear homogeneity of (1))}$$

$$\frac{V}{K} = A\left(\frac{L}{K}\right)^{\alpha}, \qquad {}^{K}/_{V} = \frac{\left({}^{L}/_{K}\right)^{-\alpha}}{A}$$

$${}^{L}/_{V} = \frac{L}{K}\,\frac{K}{V} = \frac{\left({}^{L}/_{K}\right)^{1-\alpha}}{A}$$

$${}^{L}/_{K} = \left(\frac{\alpha}{1-\alpha}\right)^{r}/_{W}$$

Hence,

$$P = \left[\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{r}{W}\right)^{1-\alpha} W + \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\left(\frac{r}{W}\right)^{-\alpha} r\right] A^{-1}$$

and

(10) $$P = \frac{\theta}{A}\,W^{\alpha} r^{1-\alpha}$$

Similarly,

(11) $$P_c = \theta E^{\delta} P^{1-\delta}$$

(12) $$P_I = \theta E^{b} P^{1-b}$$

The last three expressions can be recast in terms of the basic price variables, i, W, E. Remembering that $r = P_I i$, by substitution,

(13) $$P = \theta i^{\frac{(1-\alpha)}{\lambda}}\, W^{\alpha/\lambda}\, E^{b(1-\alpha)} A^{-\frac{1}{\lambda}}$$

(14) $$P_c = \theta i^{\frac{(1-\alpha)(1-\delta)}{\lambda}}\, W^{\frac{\alpha(1-\delta)}{\lambda}}\, E^{(1-\alpha)b+\alpha\delta} A^{-\frac{1-b}{\lambda}}$$

(15) $$P_I = \theta i^{\frac{(1-\alpha)(1-b)}{\lambda}}\, W^{\frac{\alpha(1-b)}{\lambda}}\, E^{b/\lambda} A^{-\frac{1-b}{\lambda}}$$

where

$$\lambda = \alpha + b - \alpha b.$$

In addition,

$$(16) \qquad r = \frac{\theta i^{1/\lambda} W^{\alpha(1-b)/\lambda} E^{b/\lambda}}{A^{1-b/\lambda}}$$

$$(17) \qquad \frac{W}{P_c} = W^{*} = \frac{\theta i^{-\frac{(1-\alpha)(1-\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{(1-\alpha)b+\alpha\delta}{\lambda}}}{A^{-\frac{(1-\delta)}{\lambda}}}$$

Equations (13)-(17) thus describe the equilibrium dependence of the domestic and foreign factor prices.

The consumption-investment choice presented in the form of activity analysis in the two-gap model can now be reexamined by remembering

$$\frac{M_I}{I} = \alpha_1$$

$$\frac{M_c}{C} = \alpha_2$$

$$\frac{V_I}{I} = b_1$$

$$\frac{V_c}{C} = b_2$$

and the special assumption

$$\frac{K}{V} = c_1$$

$$\frac{L}{V} = c_2$$

By ignoring the direct import activities, the constraints are rewritten:

$$M \geq d_1 I + a_2 C_2$$

$$K \geq C_1 b_1 I + C_1 b_2 C$$

$$L \geq C_2 b_1 I + C_2 b_2 C \ .$$

Also, if capital actually employed is K^*,

$$L \geq \frac{C_2}{C_1} K^*$$

$$K \geq K^*$$

Hence,

$$(18) \quad M \geq \frac{\theta i^{\frac{(1-\alpha)(1-b)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-b)}{\lambda}}}{A^{(1-b/\lambda)}} I + \theta i^{\frac{(1-\alpha)(1-\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-\delta)}{\lambda}} A^{-\frac{(1-\delta)}{\lambda}} C$$

$$(19) \quad K \geq (1-\alpha)(1-b)iI^{-1} + \theta(1-\alpha)(1-\delta)i^{-\frac{(\alpha+\delta-\alpha\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(b-\delta)}{\lambda}} A^{\frac{\delta-b}{\lambda}} C$$

$$(20) \quad L \geq \frac{\theta i^{1/\lambda} \left(\frac{W}{E}\right)^{-\frac{b}{\lambda}} K^*}{A^{1-b/\lambda}}$$

On the basis of equations (18) - (20) the following qualitative statements can be made:

For a given C-I bill, import requirements are negatively related to the exchange rate and positively to domestic factor prices W and i; this reflects the possibility of substitution between domestic and foreign resources;

Capital requirements depend positively on the wage rate and negatively on the exchange rate;

Labor employed per unit of capital depends negatively on the wage rate and positively on the exchange rate.

For given $W/_E$ and i the last three inequalities replicate in effect the two-gap diagram. It is evident that with given input prices in the above case full employment of all resources is impossible.

With flexible prices, however, it is possible to achieve full-employment equilibrium consistent with any target ratio $^{I}/_{C}$. From equations (18) and (19), given an amount of foreign exchange, one derives the locus

$$i = KM(^{W}/_{E}), \quad \frac{di}{d(^{W}/_{E})} < 0 \;;$$

from (19) and (20) one derives the locus

$$i = KL(^{W}/_{E}), \quad \frac{di}{d(^{K}/_{L})} > 0$$

The loci KM and KL comprise the set of pairs of i and $^{W}/_{E}$ for which capital and foreign exchange and capital and labor are fully utilized, respectively.

Given the slopes of the two equilibrium curves, they must intersect. There exists, therefore, a unique pair of rate of return on capital and wage-to-exchange rate ratio consistent with full employment of all factors and a given target ratio of investment to consumption. Incidentally, given the positivity of all marginal products entailed by the assumed production functions, it also follows that price pair corresponds to the maximal aggregate output.

Exportssensitive to exchange rate

Introducing the assumption that exports respond to price changes (and that only consumption goods are exported, for simplicity) import capacity is given by

$$(21) \quad M = B + \theta(^{Pc}/_{E})^{-\epsilon}$$

where B = net foreign borrowing.

Solving for $^{Pc}/_{E}$ and substituting, one obtains

$$(22) \quad M = B + \theta\left[A^{-\frac{1-\delta}{\lambda}} i^{\frac{(1-\alpha)(1-\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-\delta)}{\lambda}}\right]^{-\epsilon}$$

The main conclusions are that in the long run high investment rates need not imply underemployment of domestic resources, provided a sufficiently high $^{E}/_{W}$ is attained; and in general, regardless of the specification of the production functions, the choice frontier is smooth with the phenomenon of input redundancy of the two-gap models replaced by diminishing marginal productivity.

Some calculations of the effects of policy options

The following calculations are based on the considerations that:

(a) The exchange rate for exports and the exchange rate of imports can be considered separate policy instruments; focus will be placed on the import exchange rate;

(b) The policy targets are, first, the expansion of employment for a given capital stock and import capacity, and second, the expansion of investment at the expense of consumption for given capital stock, employment and import capacity.

In addition, total factor productivity is assumed constant (A constant and absorbed in θ) and all changes in $^{W}/_{E}$ result from changes in E.

From equation (20) one obtains

(23) $d \log L = \frac{b}{\lambda} d \log E + {}^{1}/_{\lambda} d \log i$

with given K

(24) $d \log V = \alpha\, d \log L.$

Instead of assuming proportionate expansion of C and I it is more convenient to assume that labor and value-added expand proportionately in the production of C and I.

(25) $d \log V - d \log V_I = d \log V_C.$

Total demand for imports is given by

$$(26)\qquad M = \left[\left(\frac{M}{V}\right)_c \frac{V_c}{V} + \left(\frac{M}{V}\right)_I \frac{V_I}{V}\right]V$$

and by using equations (6) - (9)

$$(27)\qquad \left(\frac{M}{V}\right)_c = \theta\left(\frac{\delta}{1-\delta}\right) i^{\frac{(1-\alpha)}{\lambda}} \left(W/_E\right)^{\alpha/\lambda}$$

$$(28')\qquad \left(\frac{M}{V}\right)_I = \theta\left(\frac{b}{1-b}\right) i^{\frac{(1-\alpha)}{\lambda}} \left(W/_E\right)^{\alpha/\lambda}$$

hence

$$(28)\qquad M = \theta\ i^{\frac{(1-\alpha)}{\lambda}} \left(W/_E\right)^{\alpha/\lambda}\left[\frac{b}{1-b}\left(\frac{V_I}{V}\right) + \frac{\delta}{1-\delta}\left(\frac{V_c}{V}\right)\right]V$$

The assumption of given import capacity entails

$$(29)\qquad d \log M = 0 = \frac{(1-\alpha)}{\lambda} d \log i - \frac{\alpha}{\lambda} d \log E + d \log V.$$

By substitution from equations (23, 24)

$$(30)\qquad d \log i = \alpha(1-b)\ d \log E.$$

Substituting back in (25) and (24), the simple expressions are obtained:

$$(31)\qquad d \log V = \alpha\ d \log E$$

$$(32)\qquad d \log L = d \log E.$$

Thus, according to the model a given percentage devaluation allows the same percentage expansion of employment with no increase in the demand for foreign exchange. The effects on investment and consumption are derived by recalling that

$$(33)\qquad I = \theta M_I^b V_I^{1-b} = \theta\left(\frac{M}{V}\right)_I^b V_I$$

$$(34)\qquad I = \theta\left[i^{\frac{(1-\alpha)}{\lambda}}\left(W/_E\right)^{\alpha/\lambda}\right]^b V_I$$

and using (25) and (30)

(35) $d \log I = \alpha(1-b)\ d \log E.$

Similarly,

(36) $d \log C = \alpha(1-\delta) d \log E.$

For Colombia the key parameter values are:

$\alpha = 0.40$

$b = 0.50$

$\delta = 0.20$

(α = labor's share in net value-added; b = direct + indirect imports in investment; δ = direct + indirect imports in consumption). With 10 percent urban unemployment and the expectation of continued migration into the cities a 15 percent increase in employment is a reasonable target. It would entail a 15 percent devaluation; a 3 percent increase in the long-run interest rate to avoid excess demand for capital [equ. 30: $d \log i = 0.40(1-0.50)0.15 = 0.03$]; a 3 percent increase in output of investment goods [equ. 35: $d \log I = 0.40(1-0.50)\ 0.15 = 0.03$]; a 5 percent increase in output of consumption goods [equ. 36: $d \log C = 0.40(1.020)0.15 = 0.05$].

Turning to the second policy objective, the reallocation of resources from C to I: In equation (28) the term $\left[\frac{b}{1-b}\left(V_I/V\right) + \frac{\delta}{1-\delta}\left(\frac{V_C}{V}\right)\right]$ is a variable; let it be denoted by $Q\left(\frac{V_I}{V}\right)$, $\left(\text{since } \frac{V_I}{V} = 1 - \frac{V_C}{V}\right)$. Then (29) is rewritten

$$(37)\quad 0 = \left[\frac{1-\alpha}{\lambda} d \log i - \frac{\alpha}{\lambda} d \log E\right] + \frac{d \log Q}{d \log \frac{V_I}{V}}\ d \log \frac{V_I}{V}$$

Equation (37) expresses the condition that the change in import demand resulting from a change in import intensity in C and I [term $\frac{1-\alpha}{\lambda} d \log i - \frac{\alpha}{\lambda} d \log E$] is just offset by internal reallocation from C to I. By assumption of a constant capital-labor ratio we have from equation (20)

(38) $i = (W/_E)^b.$

In consequence of this, the first bracketed term in (37) becomes - d log E. $\left[-\left(\frac{1-\alpha}{\lambda}\right)b \; d \log E - \frac{\alpha}{\lambda} \, d \log E = - \frac{\alpha+b-ba}{\lambda} \, d \log E; \quad \lambda \equiv \alpha+b-ba\right]$

Thus

$$(39) \quad d \log \frac{V_I}{V} = \left(1 \Bigg/ \frac{d \log Q}{d \log \frac{V_I}{V}}\right) d \log E$$

where

$$(40) \quad \frac{d \log Q}{d \log \frac{V_I}{V}} = \frac{\left[\frac{b}{1-b} - \frac{\delta}{1-\delta}\right] V_I/V}{\left(\frac{b}{1-b}\right) V_I/V + \left(\frac{\delta}{1-\delta}\right)\left(1 - \frac{V_I}{V}\right)}$$

Hence

$$(41) \quad d \log \frac{V_I}{V} = \left[\left(\frac{b}{1-b}\right)\frac{V_I}{V} + \left(\frac{\delta}{1-\delta}\right)\left(1 - \frac{V_I}{V}\right)\right] d \log E.$$

Recalling that

$$I = \theta\left(\frac{M}{V}\right)_I^b V_I$$
$$d \log \left(M/V\right)_I = - d \log E$$
$$d \log V_I = d \log \frac{V_I}{V} \qquad (V = \text{constant})$$

we have

$$(42) \quad d \log I = \left[-b\left(\frac{b}{1-b}\right)\frac{V_I}{V} + \left(\frac{\delta}{1-\delta}\right)\left(1 - \frac{V_I}{V}\right)\right] d \log E.$$

Analogously,

$$(43) \quad d \log C = \left[-\delta - \left(\frac{b}{1-b}\right)\frac{V_I}{V} - \left(\frac{\delta}{1-\delta}\right)\left(1 - \frac{V_I}{V}\right)\right] d \log E.$$

Since $\left(\frac{b}{1-b}\right)\frac{V_I}{V} + \left(\frac{\delta}{1-\delta}\right)\left(1 - \frac{V_I}{V}\right) > 1 > b$

I increases with E and C declines.

For Colombia the "actual" point on the transformation frontier is $\frac{V_I}{V} = 0.15$. This entails

$$\left(\frac{b}{1-b}\right)\frac{V_I}{V} + \left(\frac{\delta}{1-\delta}\right)\left(1- \frac{V_I}{V}\right) = 3.4.$$

Then it can be calculated that a 7 percent devaluation accompanied by 3.5 percent increase in the long-term interest rate will allow a 20 percent increase in I at the cost of a 5 percent decline in C. Capital, employment, import capacity remain unchanged.

Part Three

Appendix A

Colombian Export Demand Function

In Part Two, Bruton's hypothesis that the exports of the typical developing economy can be classified into two groups with significantly different demand elasticities was presented and its policy implications sketched. It was suggested that projection models incorporate empirical findings supporting this hypothesis as it increases the degrees of freedom of economic policy. Sheehan's and Clark's (26) empirical results concerning Colombia follow.

Colombian exports are usually classified into coffee, petroleum and "minor" exports. It is generally agreed that there is little that Colombia can do to influence its coffee exports. The world demand elasticity is low, Colombia's share in world supply is significant and there exists an international commodity agreement to support above equilibrium prices through controls.

Slightly different is the situation with respect to petroleum. Export earnings can be affected marginally by policy induced profit incentives.

The regression equations refer to 34 quarterly observations for the period 1958-66.

The variables are:

X_1 = Colombian exports other than coffee and oil, quarterly total in millions of dollars.

X_2 = Effective exchange rate: average of month-end rates for quarter, divided by average cost of living index for same period.

X_3 = World exports; annual rate in billion of dollars.

X_4 = time trend, in quarterly steps.

$Q_1 Q_2 Q_3$ = Quarterly variations in first three calendar quarters, the fourth serving as base of measurement.

X_5 = Ratio of Colombian exports (other than coffee and oil) to world exports

$X._1$ = first differences in variable $X._1$

1. $X_1 = -27.97 + 2.28X_2 + 0.23X_3$

$$(0.71) \quad (0.02)$$

$R^2 = .79$, $F = 59.42$

2. $\Delta X_1 = -32.84 + 2.48X_2 + 0.24X_3 + 2.55Q_1 + 4.40Q_2 + 3.70Q_3$

$$(0.64) \quad (0.02) \quad (1.37) \quad (1.36) \quad (1.42)$$

$R^2 = .82$, $F = 32.77$

3. $$X_1 = -32.78 + \underset{(0.67)}{2.52X_2} - \underset{(0.52)}{0.10X_4} + \underset{(1.57)}{2.68Q_1} + \underset{(1.43)}{4.47Q_2} + \underset{(1.81)}{3.91Q_3}$$

$$R^2 = .85, \quad F = 26.37$$

4. $$X_{1t} = -34.41 + \underset{(0.84)}{2.36X_{2t}} + \underset{(0.85)}{0.20X_{2t-1}} + \underset{(0.02)}{0.24X_{3t}}$$

$$+ \underset{(1.43)}{2.25Q_1} + \underset{(1.41)}{4.35} + \underset{(1.47)}{3.70Q_3}$$

$$R^2 = .85, \quad F = 25.38$$

5. $$X_5 = -1.21 + \underset{(0.05)}{0.16X_2} + \underset{(0.01)}{0.04X_4} + \underset{(0.10)}{0.16Q_1} + \underset{(0.10)}{0.29Q_2} + \underset{(.66)}{0.17Q_3}$$

$$R^2 = .66, \quad F = 10.98$$

6. $$\Delta X_1 = -4.94 + \underset{(1.00)}{1.78\Delta X_2} + \underset{(0.19)}{0.34\Delta X_3} + \underset{(4.06)}{7.89Q_1} + \underset{(2.23)}{6.02Q_2} + \underset{(3.70)}{4.87Q_3}$$

$$R^2 = .32, \quad F = 2.58$$

The three first equations exhibit a highly systematic relationship between effective exchange rates and the level of Colombian exports and afford a good basis for prediction. Equation (1) can easily be incorporated into an aggretate demand model, thus augmenting the policy choices that can be studied.

Appendix B

Planning and Models in India*

First Five-Year Plan and a Modified Harrod-Domar Aggregative Model

The relations of the model, in standard notation, are:

(1) $I_t = S_t$

(2) $S_t = ay_t - b$

(3) $y_t = \alpha K_t$

(4) $I_t = \dot{K}_t$

leading to a dynamic path for capital stock

(5) $K_t = (K_o - b/a\alpha)e^{a\alpha t} + b/a\alpha$

and income

(6) $y_t = (\alpha K_o - \frac{b}{a})e^{a\alpha t} + \frac{b}{a}$

Provided $a > \frac{S_o}{y_o}$, the rate growth increases with time, and the asymptotic relative growth rate in the system is $a\alpha$.

The relation of the model with the content of the Plan was tenuous. It served to give a longer-run perspective and to compute the necessary external assistance for supplementing domestic savings to sustain projected growth rates in output.

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* See (3).

Second Five-Year Plan and the Mahalanobis Models

The Second Plan was deeply influenced according to Bhagwati and Chakravarty by the two-sector Mahalanobis Model independently developed and similar to the Feldman-Domar model (9). Mahalanobis presented a similar four-sector model as well(18).

The Two-Sector Model Current investment flow I_t is divided into two parts, $\lambda_k I_t$ and $\lambda_c I_t$, λ_k being the proportion going into the capital goods sector, λ_c the proportion going into the consumption goods sector. Naturally, $\lambda_k + \lambda_c = 1$. Assuming constant output - capital ratios β_k, β_c, one has

$$I_t - I_{t-1} = \lambda_k \beta_k I_{t-1}$$
$$C_t - C_t - 1 = \lambda_c \beta_c I_{t-1}$$

Given $\lambda_t = C_t + I_t$, the solution paths are:

$$I_t = I_o (1+\lambda_k \beta_k)^t$$
$$C_t = C_o + \frac{\lambda_c \ \beta_c}{\lambda_k \ \beta_k} I_o \left\{ (1+\lambda_k \beta_k)^t - 1 \right\}$$
$$\gamma_t = \gamma_o \left[1+ \frac{I_o}{\gamma_o} \left(\frac{\lambda_c \ \beta_c + \lambda_k \ \beta_k}{\lambda_k \ \beta_k} \right) \left\{ (1+ \lambda_k \ \beta_k)^t - 1 \right\} \right]$$

By inspection of the solutions the following qualitative conclusions emerge: The asymptotic rate of growth of the system is given by $\lambda_k \beta_k$, while the relative growth rates of consumption and output are changing with time. λ_k is the crucial policy instrument which can be set to maximize, say, output for a specified time horizon $t = T$.

This is the most obvious policy use of the model. It is noted that in the choice of λ_k there corresponds an implicit choice of the consumption stream. The model provided a rationalization for India's shift of emphasis on heavy industry and minimized the role of foreign trade, although formally the latter can be accommodated quite easily.

The M-model lends itself readily to dynamic programming formulations. One of the possible variants is, for example, the following:

$$\text{Max}y_T = W_c C_T + W_I I_T, \; W_c, \; W_I \; 0$$

$$W_c + W_I = 1$$

subject to

$$I_{t-1} + \lambda_k \beta_k I_{t-1} \geq I_t$$

$$C_{t-1} + \lambda_c \beta_k I_{t-1} \geq C_t$$

$$\gamma_t + W_c C_t + W_I I_t$$

$$\lambda_k, \lambda_c = 1$$

$$I_t \geq 0, \; C_t \geq C_o > 0, \quad 0 \leq {}_k \leq 1, \quad \sum_{t=0}^{T} I_t \leq \bar{I}, \; t = 1, 2, . \; T$$

<u>The Four-Sector Model</u> This is a straightforward extension of the two-sector model by distinguishing the consumption sector into:

(1) factory production of consumer goods; (2) household production of consumer goods; (3) services. Four output-capital coefficients, β_i, i=1, ...4, and four labour-output coefficients θ_i, i=1, ... 4 provide

the parametric structure. Output growth and employment targets can be achieved by appropriate allocation coefficients, λ's, of total investment.

Third Plan and Multisectoral Models. The period of the third plan witnessed an increasing preoccupation with multisectoral models based on input-output with elements of intertemporal planning. Since most of the products came to fruition in the process of the fourth plan, they will be discussed below.

Fourth Plan and Multisectoral Models The first to be mentioned is Manne's and Rudra's consistency model (20). It is a straightforward multi-sectoral consistency model taking the consumption sector as a policy target and finding the gross production sector needed to satisfy terminal requirements.

Notation: γ = proportion of investment to be completed in terminal year

$\hat{x}$ = sector of production in 1970

$\hat{x}_o$ = initial conditions in 1960

M = imports

F = final demand

Then for each i sector

$$\hat{x}_i + M_i = \sum_j d_{ij} \hat{x}_j + F_i + \gamma \sum_j b_{ij} (\hat{x}_j - x_{o_j}).$$

The solution in matrix notation -M standing now for a diagonal matrix of import coefficients is

$$\hat{x} = (I + M - A - \gamma B)^{-1} [F - \gamma B x_o],$$

$$A = ((a_{ij})), \quad B = ((b_{ij})).$$

Clearly this model does not provide information about the time profile of the strategic variables as it relates the initial conditions to the terminal values only. Two interesting intertemporal consistency exercises are due to Chakravarty and Eckaus (5) and Manne and Bergsman (19). By far the most ambitious model, however, has been the Chakravarty - Eckaus - Lefeber - Parikh (CELP) intertemporal optimizing model appearing in its final form in (10).

The CELP Model The maximal is a linear function of discounted consumption over a finite period with fixed composition. Thus

$$\text{Max} \sum_{t=1}^{T} (1+r)^{-t} C(t)$$

$$C(t) \leq F(t) = \text{a given consumption sector}$$

$$(1+n) C(t) \leq C(t+1), \quad n \text{ a predetermined growth rate}$$

$$C(1) \geq \bar{C}(1) = \text{predetermined.}$$

The inequalities above comprise the consumption restrictions. Define in addition (standard notation is presented without explicit definitions):

$N(t)$ = capital accumulation

$H(t)$ = inventory accumulation

$G(t)$ = government expenditure.

Then

$$A\,X(t) + F(t) + N(t) + H(t) + G(t) + E(t) - M(t) \leq X(t)$$

are the standard interindustry flow relations.

The description of the capital accumulation process requires some new symbols:

b = diagonal matrix of capital- output coefficient

$Z(t)$ = gross new capacity

$R(t)$ = capital replacement

q^k = proportion of capacity increase that must be completed k periods in advance, k =1, 2, 3.

$$b\,X(t) - K(t) \leq 0$$

$$K(t) - K(t-1) - Z(t) + R(t-1) \leq 0$$

$$q^k Z(t) = I^t(t-k)$$

$$\sum_k q^k = 1$$

$$\sum_k Pij^k\, I_j^{t+k}(t) = Nij(t)$$

$$Ni(t) = \sum_j Nij(t)$$

For inventory accumulation

$$H(t) = S\,[X(t+1) - X(t)]$$

S = diagonal matrix of constants.

Foreign trade was introduced as follows:

Exports were assumed exogenous and imports were divided into a competitive and a non-competitive group. Non-competitive imports appear in a standard fashion as fixed proportions of gross sectoral outputs. Competitive imports were assigned <u>ceiling coefficients</u>. Thus let $M_i^2(t)$ denote competitive imports of the i^{th} kind. Then

$$M_i^2 (t) \leq m_i^2 \left[FA (t) + \Sigma E_i(t) - \Sigma M_i^1 (t) \right]$$

where m_i^2 the corresponding ceiling coefficient, FA foreign aid and M_t^1 non-competitive imports. The problem of import priorities can be handled therefore in this ingenious manner avoiding at the same time complete specialization, an undesirable feature of linear programming solutions.

The optimizing solution fixes the level of consumption - its composition being constant - whilst the terminal values for E, G and M were exogenously determined. The final capital stock value K(T) was determined, given the exogenous variables, as a function of post-plan growth rates and the length of the planning horizon T. The form of the function was determined by the optimization process.

Bhagwati and Chakravarty in their cited survey offer four main suggestions for improvements in the specification of the model which would increase its empirical reference, not only for India, but for developing economies in general. (a) Introduction of non-linearities in the welfare function of two kinds: the one-period utility function sort to reflect diminishing rates of substitution between commodities and the intertemporal kind to reflect diminishing marginal utility as consumption rises with time; (b) the foreign trade problem must be treated so as to capture changing comparative advantage positions through time. The reviewers are vague as to how this can be accomplished; (c) the specification of consumption and the production structure are too rigid; (d) the model should deal with the unemployment problem as well.

Despite these limitations the CELP model was used, on an experimental basis, to answer certain central policy questions concerning the Third Plan. The most important ones were:

(1) Accepting the Third Plan's terminal capacity targets (1965/66), together with the initial conditions and structural coefficients as in the Plan, was there a feasible timepath from initial conditions to targets? The answer to this question was positive.

(2) Was the implicit phasing of the Plan optimal with respect to a welfare function involving the discounted sum of consumption over a five-year period? The answer was negative.

(3) Relaxing the exogenous (policy) determination of terminal targets and positing postplan growth rates of consumption groups, optimal paths were derived for the Plan period which were significantly different from those of the official plan. The overall optimality of the latter was, therefore, questioned.

Appendix C

Three Models for Pakistan

The Chenery-MacEwan Model (7)

The model by Chenery and MacEwan - hereafter the C-M model - belongs to the class of gap models of the programming variety. Its objective is to explore the properties of optimal growth strategies of an open economy receiving foreign capital when the total amount and time pattern of the foreign resource inflow can be varied within limits. The analytical framework, although applied to Pakistan, is general enough to be applicable to those developing countries which must search for possibilities of transforming domestic resources into foreign exchange.

The C-M model introduces parameters to reflect political and institutional constraints on choice which may vary from case to case, a welfare function where certain parameters can be fixed so as to reflect the standpoint of either the recipient or the donor country - when foreign capital inflows are mainly aid - and also a parameter representing "absorptive capacity", namely a maximal rate at which either domestic or foreign resources can be transformed into capital formation. The model recognizes two sectors, a trade-improving sector, i.e. production for increased exports and of import substitutes, and all other production; also two scarce factors, capital and foreign exchange.

The definitional aspects of the model

(1) $V_t = V_t^o + V_t^1$

GNP is the sum of the net outputs of the non-trade-improving sector, V^o, and of the trade-improving sector, V^1.

Similarly, investment I is the sum of investment in the two sectors:

(2) $I_t = I_t^o + I_t^1$.

Also, investment is equal to domestic savings, S and foreign capital, F:

(3) $I_t = S_t + F_t$.

The trade gap is defined as the difference between traditional imports, M and exports, E - namely imports and exports that would be anticipated with unchanged economic structure - less the output of the trade-improving sector. Thus foreign capital,

(4) $F_t = (M_t - E_t) - V_t^1$.

Finally, GNP is defined

(5) $V_t = C_t + I_t + E_t - M_t + V_t^1$

where C is consumption.

Assumption about exports. An exogenous trend is assumed,

(6) $E_t = E_o(1 + e)^t$.

The welfare function. The objective is to maximize the welfare function, W, subject to the various constraints below (and the definitional relationships);

$$W = \sum_{t=1}^{T} \frac{C_t}{(1+i)^t} + \theta V_T - \gamma \sum_{t=1}^{T} \frac{F_t}{(1+i)^t} ,$$

where T is the terminal year of the plan, i a rate of discount, γ the cost of foreign capital specified exogenously and θ a variable weight for terminal year income defined as

$$\theta = \delta(1 - \alpha) \sum_{t=1}^{\infty} \frac{(1+p)^t}{(1+r)^{T+t}}$$

where δ is a weight on postplan consumption, $(1-\alpha)$ marginal propensity to consume, p postplan sustained growth rate of output (whose derivation depends on conditions about which later), and r a discount rate on postplan consumption. The welfare function contains thus two positive terms, the discounted stream of consumption over the planning period and the welfare value of terminal output - which involves a discount procedure of perpetual postplan consumption; and one negative term reflecting the cost of foreign capital. By various i, r and δ constellations the policy makers can express different attitudes toward proximate versus remote future. Typically, r>i reflecting diminishing marginal utility of consumption between the two "periods". A given γ - together with certain constraints - specifies perfectly elastic supply conditions of foreign capital within limits.

Structural and behavioral constraints

The limit to "regular" production is given by

$$(7) \quad V_t^o \leq V_o + \frac{1}{K_o} \sum_{0}^{t-1} I_t^o .$$

Trade-improving production begins after the operation of the plan has begun:

$$(8) \quad V_t^1 \leq \frac{1}{K_1} \sum_{1}^{t-1} I_t^1 .$$

By assumption $K_1 > K_o$.

The domestic savings constraint is

$$(9) \quad S_t \leq S_o + \alpha(V_t - V_o),$$

where α is taken as a fixed parameter.

The constraint on traditional imports is to be interpreted as a technical constraint. Thus the coefficients are taken as fixed.

$$(10) \quad M_t \geq M_o + M_o(V_t - V_o) + M_1(I_t - I_o).$$

The "absorptive capacity" limit of capital formation is expressed by

(11) $I_t \leq (1 + \beta) I_{t-1}$

where β, a constant, is the maximal growth rate of investment.

A lower bound is also placed on capital formation,

(12) $I_t \geq I_{t-1}$.

Policy constraints

Consumption is constrained to grow at least as fast as the population;

(13) $C_t \geq C_{t-1}(1 + p)$

and foreign capital inflows must terminate within n years from the beginning of the plan:

(14) $F_t \leq 0, \qquad t = T-n, \; T.$

Since the model imposes that foreign capital inflow should cease after a predetermined period, it is possible to calculate iteratively the long-run sustained rate of growth, p, from the long-run average propensity to save ($\approx\alpha$) and a weighted average of the capital-output coefficients K_o and K_1.

Constraint (14) is an alternative specification to γ of foreign capital supply conditions. A third form is introduced by

(15) $$\sum_{t=1}^{T} \frac{F_t}{(1+i)^t} \leq \overline{F}$$

where $\overline{F}$ is total foreign capital available over the planning period. The solution of the C-M model is based on a combined form of a minimum price γ and a maximal termination date of foreign capital inflows.

An additional constraint reflecting institutional rationing of foreign capital - of aid in particular - from the supply side completes the set of inequalities:

(16) $F_t \leq qV_t$.

The application of the model

The empirical considerations for the application of the C-M model conform more or less to the two alternative 20-year projections (1965-85) of the Pakistan Planning Commission which served as a basis for the Third Five-Year Plan (1965-70). Foreign assistance was supposed to terminate by 1985 in both versions.

The basic solution of the model takes account of relations (1-14) and is based on the following parametric values:

Structural parameters	Non-structural parameters
α = .24	i = .08
M_o = .10	r = .10
M_1 = .35	ρ = .073
K_o = 3	γ = 2
K_1 = 4.5	θ = 3.4
p = 2.5	δ = 1
β = .13	T = 23 years
e = .049	n = 20 years

According to which constraints were alternately binding over the 23-year planning period, three phases of development are distinguished (throughout the period limits on capacity, savings and trade were binding):

Phase I: Maximum investment and growth; binding constraint absorptive capacity; 1963-76

Phase II: Trade improvement; binding constraint lower bound on investment ($I_t \geq I_{t-1}$); 1977-81

Phase III: Balanced growth; binding constraint no foreign capital; 1982-85.

The emerging profile of the economy is summarized in the following figures:

Years	Growth Rate	I_n/V_n	S_n/V_n	F_n/V_n	I_n^1/I_n
1965-70	5.9	.21	.14	.07	.05
1970-75	7.7	.27	.17	.10	.07
1975-80	8.0	.23	.19	.03	.39
1980-85	6.3	.21	.21	0	.21

Thus the first 13-year phase is characterized by a maximum rate of capital formation aided by sharply increasing inflows of foreign capital reaching 10 percent of GNP over the period 1970-75. Very little of total investment goes to the trade-improving sector in the first phase. The second phase is characterized by a fall in the overall rate of growth of investment and a sharp increase in the proportion going to the trade-improving sector. By the end of the second phase lasting four years, the trade gap is closed consistent with the requirement that foreign assistance terminate by 1982. The third phase is that of balanced growth in which capital formation in the two sectors occurs in fixed proportion sufficient to maintain self-sustained output growth of about 6.3 percent.

C-M have conducted additional experiments after obtaining the described basic solution. In one, they use the total amount of foreign capital obtained in the basic solution in order to calculate its shadow price by solving the dual problem. That value turned out to be 7.4 instead of γ=2, so that the original problem involved undervalued foreign capital supply. With a preassigned date for termination of foreign capital inflow and any price of it below 7.4, the same sequence of phases will emerge as the basic solution.

In another set of interesting exercises, C-M calculated the impact of different foreign capital supply conditions by varying (1) γ, (2) the total discounted amount of F, (3) the termination date with no price or quantity limitations, (4) by imposing annual rationing, e.g. $F_t \leq qV_t$. In relation to (4) it was found that with q = 5%, more F was needed to attain the same growth target as in the basic solution, the termination date now postponed to 1992 and the consumption stream was lower, as long as the discount rates were not greater than 9 percent. Another qualitative result of great interest is the insensitivity of the optimal growth path, as long as the termination requirement is maintained, to variations in the relative weights of consumption during and after the plan. With the initial specifications, the same basic solution results as long as the terminal income welfare weight θ exceeds unity.

Limitations of the C-M model

The virtues of the C-M model for purposes of projection of foreign capital requirements over simple methods of extrapolation are obvious. Thus foreign capital needs are wedded explicitly to a development criterion expressed as a robust welfare function. The case would have been otherwise if the optimal development path were sensitive to small differences in parametric specification of the maximand. The two sector disaggregation recognizing trade-improving production is crucial in making the projections of foreign capital needs endogenous to the development process. An equally obvious limitation is the fact that disaggregation in that sector is absent, so it is by means of arbitrary assumptions that investment there is allocated between further (non-traditional) export production and production of import substitutes.

The absence of price variables from either the import or the export relations does limit artifically the development policy only to real investment allocation decisions between two sectors but it may be inevitable if the linear programming framework is to be preserved.

Another disagreeable consequence of the linear programming framework is the sharp succession of phases and the drastic reallocation of investment prescribed in phase two.

In the initial phase the productivity of foreign capital is limited by the parameter β denoting "absorptive capacity".

Two objections can be raised against introducing a parameter limiting the rate of capital formation and remaining fixed for a long period: (a) If there exists such a limit it is likely to reflect technical limitations in producing fixed capital. By defining capital formation broadly enough to include investment in human capital (which is unlikely to entail zero social marginal productivity at any conceivable rate) a "technical" parameter β is no longer tenable. Instead, a realized or planned rate of capital formation broadly defined will reflect a balance between the rate of return to investment and the social rate of time preference. (b) A typical cause for the existence of a technically maximum rate of fixed capital formation is the lack of complementary factors such as managers, technicians, and other types of skills which are the object of investment in education, research, i.e. in human capital.

It may be important therefore to distinguish a third investment sector and to establish the causal link with the rest of the economy. Foreign capital can be used easily to promote the expansion of that sector and thus raise the absorptive capacity in the narrow sense.

Tim's Model (32)

Tim's model was developed in connection with Pakistan's official Third Plan (1965-70) and sought to provide a consistent framework for the investigation of the questions related to the selection of a feasible growth target for the economy as a whole and the sectoral implications of such a target. In particular, the interrelations between the agricultural and the non-agricultural sectors were made explicit.

It belongs to the variety of interindustry models designed to handle "gap analysis" problems. A subset of the relations in the model generates estimates of investment requirements and domestic savings; another subset generates estimates of import requirements and future exports. The "gap between the gaps" is closed by import substitution which first affects the trade gap and second the savings gap as a consequence of relatively costly investment in import-substituting activities.

The Structure of the Model. Seven sectors are distinguished:

1. Agriculture;
2. Consumer goods manufacturing;
3. Intermediate goods production;
4. Investment goods industries;
5. Construction;
6. Transport and communications;
7. All other services.

The intersectoral relations are all of the form:

(1) $xij = a_i X_j + b_i$, $i,j + 1,\ldots,7$ where xij is the ith input into sector j, X_j the level of gross output of sector j, and a_i, b_i are constants. The unconventional form of equations (1) was estimated from two input-output tables, one for the beginning (1959/60) and one for the end (1964/65). Thus, the model incorporates constant marginal and variable average input-output coefficients. Estimates of input elasticities with respect to gross outputs

can be obtained for any year $\{\varepsilon_{ij} = \dfrac{a_i}{a_i - \dfrac{b_i}{x_j}}\}$, and structural changes are incorporated.

The final demand relations consist of three subsets, consumption expenditures, investment by sector of origin and stock formation.

(2) $c_i = d_i C + f_i,$ $\qquad i = 1,\ldots,7$

where C is total consumption ($C = \sum_1^7 c_i$), and d_i, f_i constants.

The investment relations are more complicated:

(3.1) $i_4 = k_1^{(4)}(\lambda_1 V_1) + k_2^{(4)}(\lambda_2 \sum_{j=2}^{5} V_j + \theta E_s) + k_3^{(4)}(\lambda_3 V_6) + \text{constant}.$

(3.2) $i_5 = k_1^{(5)}(\lambda_1 V_1) + k_2^{(5)}(\lambda_2 \sum_{j=2}^{5} V_j + \theta E_s) + k_3^{(5)}(\lambda_3 V_6) + k_4^{(5)}(\lambda_4 V_7)$

(3.3) $i_6 = k_1^{(6)}(\lambda_1 V_1) + k_2^{(6)}(\lambda_2 \sum_{j=2}^{5} V_j + \theta E_s) + k_3^{(6)}(\lambda_3 V_6)$

(3.4) $i_7 = k_2^{(7)}(\lambda_1 V_1) + k_2^{(7)}(\lambda_2 \sum_{j=2}^{5} V_j + \theta E_s) + k_3^{(7)}(\lambda_3 V_6)$

The k's, λ's, θ, are parametric constants, V_j is value added in sector j and E_s represents import substitution.

The stock formation relations are specified as:

(4.1) $n_i = 1_i X_i$, $\qquad i = 1,\ldots,4$

(4.2) $n_m = m_n M$ where M is total imports.

A group of nine equations deals with imports:

(5.1) $m_j = \mu_j X_j + \text{constant},$ $\qquad j = 1,\ldots, 7$

(5.2) $m_c = \mu_c C + \text{constant}$

(5.3) $m_I = \sigma_1(\lambda_1 V_1) + \sigma_2(\lambda_2 \sum_{j=2}^{5} V_j + \theta E_s) + \sigma_3(\lambda_3 V_6) + \sigma_4(\lambda_4 V_7) + \text{constant}.$

The m_j's are intermediate imports; m_c, m_I, direct imports of consumption and investment goods, respectively.

The next group refers to indirect taxes net of subsidies:

(6.1) $t_i = \tau_i X_i + \text{constant}$, $i = 1,\ldots, 7$

(6.2) $t_c = \tau_c C + \text{constant}$

(6.3) $t_I = \tau_1^I i_1' + \tau_2^I i_{2-5}' + \tau_3^I i_6'$

(6.4) $t_e = \tau^e (E - E_s) + \text{constant}$

E is exports, and the i''s are defined immediately below:

Investment-output relations

(7.1) $i_1' = \lambda_1 V_1 = 0.1044V1$

(7.2) $i_{2-5}' = \lambda_2 \sum_{j=2}^{5} V_j + \theta E_s = 0.3405 \sum_{j=2}^{5} V_j + .0213E_s$

(7.3) $i_6' = \lambda_3 V_6 = 0.4515V6$

(7.4) $i_7' = \lambda_4 V_7 = 0.1610V7$

Definitions

(8) Production

$X_1 = \sum^{4} x_{1j} + x_{17} + c_1 + n_1 + e_1$

$X_2 = \sum^{5} x_{2j} + x_{27} + c_2 + n_2 + e_2 + .35\ E_s$

$X_3 = \sum^{7} x_{3j} + c_3 + n_3 + e_3 + .65\ E_s$

$X_4 = \sum^{6} x_{4j} + c_4 + n_4 + i_4 + e_4$

$$X_5 = x_{57} + c_5 + i_5$$

$$X_6 = \overset{7}{\Sigma} x_{6j} + c_6 + i_6$$

$$X_7 = \overset{7}{\Sigma} x_{7j} + c_7 + i_7 + e_7$$

(9) Value added

$$V_j = X_j - \sum_{1=1}^{7} x_{ij} - m_i - t_i$$

(10) Export earnings

$$E = \overset{7}{\Sigma} e_i + t_e + E_x$$

(11) Imports

$$M = \overset{7}{\Sigma} m_j + m_c + m_I + n_m$$

(12) Balance of Payments

$$B = E - M$$

(13) Gross National Product

$$Y = \overset{7}{\Sigma} v_j$$

(14) Investment

$$I = i_1' + i_{2-5}' + i_6' + i_7'$$

(15) Savings

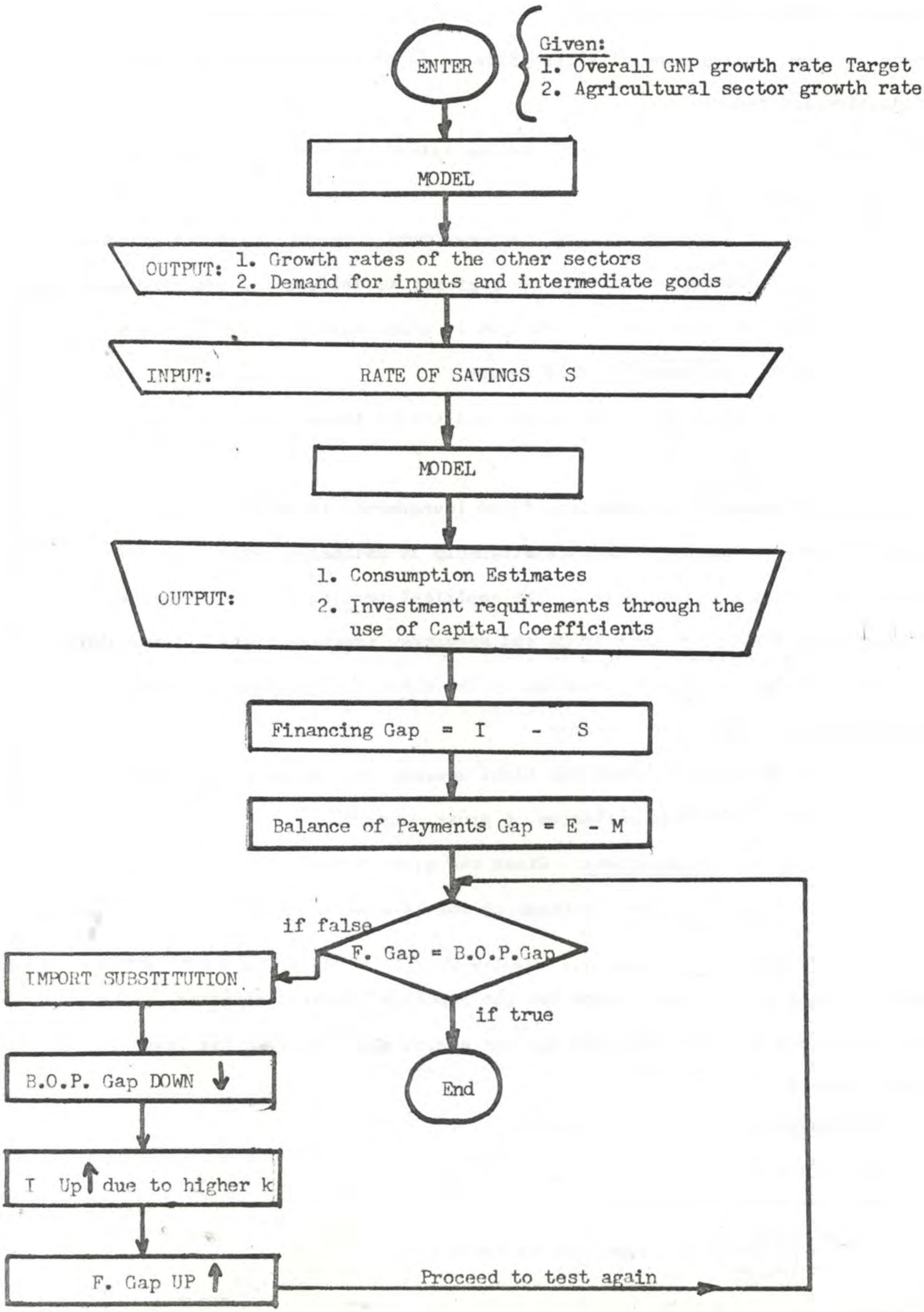
$$S = I + B + \overset{7}{\Sigma} n_i + n_m$$

In all there are 98 endogenous variables and 98 equations and seven exogenous variables: Y, X_1, S (or the marginal savings rate), e_2, e_3, e_4.

The operation of the model and certain empirical exercises. The overall operation of the model is presented schematically below. In contrast with the C - M model, the emphasis here is on the impact of the agricultural sector on the economy and inversely. The first experiment concerned the finding of the minimum rate of growth of agricultural production compatible with an annual global growth rate of 6%, given a marginal savings rate of 0.23 and a minimum of agricultural exports. It was found that a 4.3% per annum agricultural growth was required. A higher rate of growth of agricultural production (around 5%) would be compatible with a 6.4% total growth. Since still higher rates of agricultural production were deemed possible, the guidance of agricultural surpluses with consequent price decreases required higher overall growth rates.

These higher rates required a rate of import substitution -- in order to keep the trade deficit within acceptable limits -- whose upper feasible limit was placed at 15% of additional production in the consumer and intermediate-goods sectors. The ultimate choice of goals and strategy for the Third Plan with the help of the model was a question of taste within a relatively narrow range of feasibility. For the five-year period, a 37 percent overall growth was posited with a marginal savings rate of 22 percent implying an increase in agricultural exports by Rs. 1200 million, an improvement in the balance of payments by almost Rs.200 million and an 11% rate of import substitution.

SEQUENCE OF OPERATION, TIM'S MODEL



Kresge's Model (16)

(a) Compare several alternative economic policies for the period 1965-69;

(b) Trace out the implications of alternative rates of substitution of domestic production for imports;

(c) Investigate the relationships among saving rates, trade deficits and rates of growth of output.

The model used is a non-optimizing, macro-economic scheme combined with inter-industry relations. Although it recognizes regional variables, the reported empirical results are aggregative, therefore, subscripts by which regional variables are distinguished will be suppressed in this review.

The Structure of the Model. The model consists of three blocks of equations giving estimates at time t of:

A. Final demand (personal consumption; fixed investment; inventory investment; government purchases; exports). Each relationship is estimated separately for every recognized group of commodities. The empirical results are based on the four groups: agriculture, consumer goods and services, investment and intermediate goods, transport. The exogenous variables in the block are government policy variables and passed outputs and incomes.

B. Interindustry relations. Given the final demands from group A, and input-output scheme is used to obtain estimates of gross outputs.

C. Distribution of income relations. Given the gross outputs obtained in group B, wages and profits are estimated in terms of the level of outputs, and revenues and costs, respectively.

Blocks A and B are the basic ones for the empirical results reported.

Price relations are also adjoined but not empirically utilized (at least in the paper under review).

The Equations

A. Final Demand

(a) Consumption functions

notation
C = personal consumption in current prices
Y = income

C_i^* = consumption of commodity i in constant prices.

P_i = price of i.

C_{im}^* = consumption of imported commodity i.

P_{im} = import price of i.

1. $C_t = \alpha Y_{t-1}$

2. $C_i^* = \alpha_i \, C/P_i$

3. $C_{im}^* = \alpha_{im} \, C/P_{im}$

$$\sum_i (\alpha_i + \alpha_{im}) = 1$$

(b) Investment functions

Notation

I_i = gross fixed investment outlays by sector i.

X_i = output of i.

$\hat{X}_i$ = capacity output of i.

R_i = retained earnings plus depreciation in sector i.

$\overline{I}$ = exogenous investment.

H_i = investment demand of commodity i.

H_{im} = demand for imported investment goods i.

V_i = inventory demand of i.

B_i = acceleration coefficient.

γ_i = amount of investment required to increase capacity by one unit.

b_{ij} = proportion of investment demand by industry j for commodity i.

b_{ijm} = analogously for imported i.

θ_i = target inventory-output ratio for i.

4. $I_i = B_i \gamma_i \, (X_{it-1} - \hat{X}_{it-1}) + \delta_i R_{it-1} + e_i X_{it-1} + \zeta_i I_{it-1} + \overline{I}_i$

5. $H_i = \sum_j b_{ij} I_j$

6. $H_{im} = \sum_j b_{ijm} I_j$

7. $V_{it} = \theta_i X_{it} - V_{it-1}$

$$\sum_i (b_{ij} + b_{ijm}) = 1$$

(c) Aggregate relations

Notation

Q_i = total final demand for i

G_i = government expenditure on currently produced i

E_i^* = exports of i

8. $Q \equiv \sum_i Q_i = \sum_i (C_i^* + H_i + V_i + G_i + E_i^*)$

B. Interindustry Relations

Notation

Z_i = gross output of industry i

a_{ij} = input-output coefficient

a_{ij}^* = element of $\{I - ((a_{ij}))\}^{-1}$

9. $Z_i = \sum_j a_{ij} Z_j + Q_i$

10. $Z_i = \sum_j a_{ij}^* Q_j$

C. Distribution of Income

Notation

W_o = unit labour cost for outputs less than capacity.

ω_i = marginal wage elasticity with respect to over-capacity production in industry i.

U = revenue

S = sales

τ = indirect tax rate

ψ = production cost

π = profit

ρ = proportion of distributed earnings

λ = personal income tax rate

Y_d = disposable personal income

a_{ijm} = quantity of imports of i required for unit production of j

11. $\delta W_i = W_{oi} \left| 1 + \omega_i \left(\frac{X_i}{\hat{X}_i} - 1 \right) \right|$) Overcapacity

12. $W_i = W_{oi} X_i$) Undercapacity

13. $U_i = (P_i - \tau_i) S_i$

14. $\psi_i = \left| \sum_j (a_{iji} P_j + a_{jim}) P_{jm} \right| X_i + W_i$

15. $\pi_i = U_i - 4_i$

16. $R_i = (1 - \rho_i)\, \pi_i$

17. $Y = \sum_i (W_i + (1 - \rho_i)\, \pi_i)$

18. $Y_d = (1 - \lambda)\, Y$

The following equations give aggregate results:

19. $GNP = \sum_i Q_i - \sum_i \sum_j a_{ijm} Z_j$

20. $C^* = \sum_i (C_i^* + \overset{*}{C}_{im})$

21. $I = \sum_i I_i$

22. $\Sigma E_i = E_i^* P_{iw}$ = value of exports (P_{iw} = price of i in world market) = E

23. $\Sigma M_i^* = \sum_i (C_{im}^* + G_{im} + I_{im} + \sum_j a_{jim} X_j)$

where

G_{im} = government imports of i

I_{im} = investment imports of i

Simulation Results. The model is based on data of the period 1960-64 and is used for predictions for the period 1965-69.

The parameter estimates were derived either directly from the data or iteratively until a good fit resulted. The basic simulation run, the "null hypothesis" of the experiment was conducted by assuming continuation but not acceleration of trends of the exogenous variables and parameters and served as a basis for comparisons with alternative policies.

The alternative runs assumed combinations of the following separate conditions:

(a) Predetermined investment at levels of the Third 5-year Plan;

(b) Increased savings; (decline of APC from .922 to .883);

(c) Import substitution up to 25%; (decrease in import coefficients by 25%);

(d) Increased savings as in (b) and decrease of import coefficients as in(c);

(e) Same saving rate and import substitution as in (d), investment equal to projected amounts; (e) is the "official" hypothesis;

(f) Import substitution as in (c), higher reinvestment rate, lower increase in saving rate (APC=.906);

(g) APC=.922, increase in acceleration coefficient, import substitution same as in basic run.

Basic Run (Run 2).

The rate of growth is smaller than the officially projected in the Third Plan.

Run 3

More growth (clue to officially projected investment and incorporated) but appreciably lower than the officially projected.

Run 4 Incorporates (b)

Straight-forward example of Keynesian depression due to insufficient demand.

Run 5 Incorporates (d).

All measures of economic activity show an increase over those in run 4 but still fall short of those in the "null hypothesis". It is accompanied by reduced cumulative trade deficit.

Run 6 Incorporates (e).

In addition the input-output tables, the capital-output ratios, the composition of consumption and investment demand are those specified by the Third Plan. The experiment shows that these assumptions are not sufficient to generate the high growth rate and the low trade deficit projected. (Projected growth rate 6.6%; run 6 rate 5.8%). In addition it shows that, unless there is a great increase in capital-output ratios, the investment level, specified by the Plan will generate high excess capacity in the capital goods sector.

Run 7

Incorporates much of the official plan but assumes a higher reinvestment ratio from profits, and an APC=.906. It shows comparable growth to that of the official plan but at the cost of higher investment levels and increased trade deficit. Kresge interprets these results as the model's estimates of what it would take to achieve a planned growth rate of 6.6%

Run 8

Emphasis on consumer goods industries. APC=.922, import substitution at less than 12.5%. Acceleration coefficient increase.

Growth achieved comparable to officially planned at much lower investment levels and with a reduction of trade deficit compared to run7 . Per capita consumption would increase by 21% rather than by the planned 13%.

The results of the Tim and Rresge models are not directly comparable in detail both on account of the different specifications of the structural relations and the differences in the conducted exercises. Yet from the listed principal results the two models are close enough in their estimates, roughly speaking, and both describe the Pakistani economy with fidelity. Although the economy is not greatly integrated (as shown by the fact that in 1964/65 the ratio of inputs from other sectors to total inputs was only 0.357 for the economy as a whole and that if agricultural deliveries are left out that ratio becomes 0.243), the use of input-output relations has helped undoubtedly to increase the perception of the interdependence of the problems of overall growth, agricultural growth and the balance of payments.

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REFERENCES

1. Adelman, I. and Chenery, H., "Foreigh Aid and Economic Development: The Case of Greece", Rev. Econ. Stat., February 1966, 48.

2. Adelman, I. and Sparrow, Federick T., "Experiments with Linear and Piece-wise Linear Dynamic Programming Models." In Adelman and Thorbecke, The Theory and Design of Economic Development, John Hopkins, 1966.

3. J. Bhagwati and Chakravarty, "Contributions to Indian Economic Analysis: A Survey", AER, Supplement, Sept. 1969.

4. Bruton, H., "The Two-Gap Approach to Aid and Development: Comment", AER, June 1969, pp. 439-446.

5. Chakravarty, S. and Eckaus, R.S., "An Approach to a Multi-Sectoral Intertemporal Planning Model in Rosenstein-Rodan, ed., Capital Formation and Economic Development" London 1964.

6. Chenery, H.B. and Bruno, M., "Development Alternatives in an Open Economy: "The Case of Israel", Economic Journal, March 1962, 72.

7. Chenery, H.B., and MacEwan, A., "Optimal Patterns of Growth and Aid: The Case of Pakistan", Pakistan Development Review, Summer 1966.

8. Chenery, H.B., and Strout, A., "Foreign Assistance and Economic Development", AID Discussion Paper No. 7, June 1965.

9. Domar, E., "A Soviet Model of Growth" in Essays in the Theory of Growth, New York, 1957.

10. Eckaus, R.S. and Parikh, K.S., "Planning for Growth", Cambridge, Mass. 1968.

11. Fei, John C., "An Analysis of the Long-Run Prospect of Economic Development in Pakistan", Institute of Development Economics, Monograph in the Economics of Development, No. 9, Karachi, April 1962.

12. Ghosh, A., "Planning, Programming and Input-Output Models", Cambridge, England, 1968.

13. Gurley, J. G., "Hacia una Teoria de las Estructuras Financieras y el Desarrollo Economico", Di Tella, Buenos Aires, 1968.

14. Haq, Mahbub, Ul "The Strategy of Economic Planning", Pakistan Branch of the Oxford University Press, Karachi.

15. Illich, Ivan, "Outwitting the 'Developed' Countries", New York Review of Books, November 6, 1969.

16. Kresge, D., "Simulation Model for Economic Planning: A Pakistan Example", Economic Development Report No. 81, D.A.S. Harvard University.

17. Landau, L., "Differences in Savings Ratios among Latin American Countries", Harvard University, 1969.

18. Mahalanobis, P.C., "Some Observations on the Process of Growth of National Income", Sankhya, September 1953.

"The Approach of Operational Research to Planning to India, Sankhya, December, 1955.

19. Manne, A.S. and Bergsman, J., "An Almost Consistent Intertemporal Model for India's Fourth and Fifth Plan" in Adelman and Thorbecke, ed., The Theory and Design of Economic Development", John Hopkins U. Press, 1966.

20. Manne, A.S. and Rudra, A., "A Consistency Model of India's Fourth Plan", Sankhya, February 1965.

21. Nelson, R.R., "The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrainted Economy", Memorandum RM 5680-AID, November 1968, Rand.

22. ______________ "A Study of Industrialization in Colombia: Part I, Analysis", Rand Corporation (Memorandum RM 5412 - AID, December 1967).

23. Rahman, Habibur, "Growth Models and Pakistan", The Allies Book Corporation, Karachi, 1962.

24. Rahman, M. Anisur, "The Welfare Economics of Foreign Aid", Pakistan Development Review, Summer 1967.

25. Rosenstein-Rodan, edit. "Capital Formation and Economic Development", MIT, 1964.

26. Sheehan, John and Clark, Sara, "The Response of Colombian Exports to Variations in Effective Exchange Rates", Research Memorandum No. 11, Center for Development, Economics Williams College, Williamstown, Mass., June 1967.

27. Shourie, Arun, "The Relevance of Econometric Models for Medium- and Longer- Term Forecasts and Policy Prescription, Quantitative Techniques and Analysis Division, IBRD, December 1969.

28. Tims, W., "Growth Model for the Pakistan Economy", Pakistan Planning Commission, Karachi 1965 (Mimeograph).

29. United Nations Study, "Trade Prospects and Capital Needs of Developing Countries", UNCTAD Secretariat, UN, New York, 1968.

30. Vanek, T., "Estimating Foreign Resource Needs for Economic Development: Method and a Case Study of Colombia", McGraw Hill, New York, 1967.