Linking Regions in ICP

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Outline

Existing methods for linking regions:

Basic heading level - Region-Product Dummy (RPD) – Diewert (2004)

Aggregate level - Country Approach with Redistribution (CAR) – Kravis, Heston and Summers (1982)

Here we consider an alternative method that can be used at either basic heading level or at the aggregate level.

The method has been referred to sometimes as CAR-PPP (as noted by Sergey). I referred to it as the Least-squares Fixity method in Hill (2016). A more informative name might be Geometric Average linking.

See also the accompanying paper:
Hill, R. J. (2016), A least squares approach to imposing within-region fixity in the International Comparisons Program, *Journal of Econometrics* 191, 407-413.

Note: Hill (2016) is based on an earlier paper I presented to the TAG in 2010. Sergey Sergeev also discusses the properties of this method in a paper in 2011.

Regionalization and Within-Region Fixity

ICP is constructed from separate regional comparisons. The main regions are:

Africa	Asia Pacific
CIS	Eurostat-OECD
Latin America	Western Asia

All regions use the same list of basic headings.

But each region has its own product list within each basic heading. Within-region basic heading price indices are computed using the country-product-dummy (CPD) method.

For each basic heading there is also a core list of products priced by all regions. The RPD method uses this core list to link the regions at basic heading level.

Linking at the Basic Heading Level – RPD

The RPD method estimates the following hedonic model for each basic heading:

$$y = Z\beta + R\gamma + \varepsilon,$$

where y is a vector of log prices (converted for each region into region-specific base country prices using the within-region CPD comparisons). Z is a matrix of product dummies and R is a matrix of region dummies.

Importance weights can be used when estimating the RPD hedonic model (and the CPD models) if desired.

The RPD between-region linking factors are obtained as follows:

$$LF_r = \exp(\hat{\gamma}_r).$$

The overall global comparison hence takes the form:

$$P_{rj}^{Global} = LF_r \times P_{rj}^{Region},$$

where j denotes a country in region r.

Linking at the Basic Heading Level - Geometric Averaging

The Geometric-Averaging method makes separate CPD comparisons for each basic heading for each region and an overall CPD comparison for each basic heading for the whole world based on the core list. Again, importance weights can be used in the CPD comparisons if desired.

Each region has a base in the within-region CPD comparisons, but there is only one base for the whole world in the core CPD comparison.

The Geometric-Averaging between-region linking factors are obtained as follows:

$$LF_r = \prod_{k=1}^{K_r} \left(\frac{P_{rk}^{Core}}{P_{rk}^{Region}} \right)^{1/K_r},$$

where k indexes the countries in region r. Again, the overall global comparison takes the form:

$$P_{rj}^{Global} = LF_r \times P_{rj}^{Region}$$

Alternatively, we can write the Geometric-Averaging method as follows:

$$P_{rj}^{Global} = P_{rj}^{Region} \left[\prod_{k=1}^{K_r} \left(\frac{P_{rk}^{Core}}{P_{rk}^{Region}} \right)^{1/K_r} \right].$$

Geometric Averaging is a natural extension of GEKS.

GEKS alters the Fisher price indices by the logarithmic least squares amount necessary to make them transitive.

Geometric Averaging alters the core price indices by the logarithmic least squares amount necessary to make them satisfy within-region fixity (Hill, 2016).

Advantages of Geometric Averaging at basic heading level:

- It treats all regions symmetrically.
- Its logarithmic least squares property makes it a natural extension of GEKS.

If the core price indices are computed using CPD, then some asymmetry in the treatment of regions will already be present. The use of geometric average linking will not undo any prior asymmetry in the core indices. (Sergey pointed this out)

The main concern with RPD is that regions containing more countries will have a larger impact in determining the shadow prices on the product dummies. In this sense RPD does not treat all regions symmetrically when determining the linking factors.

A version of CPD that gives more weight to regions with less countries could be used in the core comparison.

Linking at the Aggregate Level – The CAR Method

The CAR price indices are calculated implicitly from the CAR quantity indices as follows:

$$\tilde{P}_{rj}^{Global} = \frac{\sum_{n=1}^{N} p_{rj,n} q_{rj,n}}{Q_{rj}^{Global}},$$

where $p_{rj,n}$ is the price index for basic heading n in country j in region r (obtained from the global basic-heading comparison) and $q_{rj,n}$ is the corresponding quantity obtained by dividing expenditure by price.

The CAR quantity indices are calculated as follows:

$$Q_{rj}^{Global} = Q_{rj}^{Region} \left(\frac{\sum_{k=1}^{K_r} Q_{rk}^{Unfixed}}{\sum_{k=1}^{K_r} Q_{rk}^{Region}} \right),$$

where $Q_{rk}^{Unfixed}$ denotes a global GEKS quantity index that does not satisfy within-region fixity.

Summing across all countries in region r, we obtain that

$$\sum_{k=1}^{K_r} Q_{rk}^{global} = \sum_{k=1}^{K_r} Q_{rk}^{unfixed}.$$

Hence, CAR preserves the regional expenditure shares from the unfixed global comparison.

The CAR price indices are derived implicitly as follows:

$$P_{rj}^{Global} = \left(\frac{\sum_{n=1}^{N} p_{rj,n} q_{rj,n}}{Q_{rj}^{Region}} \right) \left(\frac{\sum_{k=1}^{K_r} Q_{rk}^{Region}}{\sum_{k=1}^{K_r} Q_{rk}^{Unfixed}} \right),$$

or

$$P_{rj}^{Global} = LF_r \times \tilde{P}_{rj}^{Region},$$

where

$$LF_r = \left(\frac{\sum_{k=1}^{K_r} Q_{rk}^{Region}}{\sum_{k=1}^{K_r} Q_{rk}^{Unfixed}} \right), \quad \tilde{P}_{rj}^{Region} = \left(\frac{\sum_{n=1}^{N} p_{rj,n} q_{rj,n}}{Q_{rj}^{Region}} \right).$$

Linking at the Aggregate Level – Geometric Averaging

Like GEKS, Geometric Averaging satisfies the strong factor reversal test.

$$P_{rj}^{Global} = P_{rj}^{Region}\left[\prod_{k=1}^{K_r}\left(\frac{P_{rk}^{Unfixed}}{P_{rk}^{Region}}\right)^{1/K_r}\right]$$

$$Q_{rj}^{Global} = Q_{rj}^{Region}\left[\prod_{k=1}^{K_r}\left(\frac{Q_{rk}^{Unfixed}}{Q_{rk}^{Region}}\right)^{1/K_r}\right]$$

Hence the linking factor for the price indices is as follows:

$$LF_r = \left[\prod_{k=1}^{K_r}\left(\frac{P_{rk}^{Unfixed}}{P_{rk}^{Region}}\right)^{1/K_r}\right].$$

An asymmetric version of this method is used by OECD to impose within-region fixity in its comparisons.

Advantages of Geometric Averaging

- It treats all regions symmetrically.
- Its logarithmic least squares property (for both price and quantity indices) makes it a natural extension of GEKS (see Hill, 2016). It alters the unfixed GEKS indices by the logarithmic least squares amount necessary to make them satisfy within-region fixity.
- It satisfies the strong factor reversal test. This implies that it treats price and quantity indices symmetrically.

Advantages of CAR

- It treats all regions symmetrically.
- The expenditure shares of the regions in the CAR global comparison are the same as in the unfixed global comparison.

A Topic for Future Consideration: Spatial Chaining to Improve on GEKS

Hajargasht, Hill, Rao and Shankar have a current paper that shows how spatial chaining methods can be used to improve on GEKS.

The problem with GEKS is that it uses bilateral Fishers between all possible pairs of countries. We can get a better comparison by deleting some of the weakest bilaterals or replacing them with chained indices.

Going all the way to a minimum-spanning tree (MST) may be too extreme. We prefer something in between GEKS and an MST.

We provide criteria to show that our spatial chaining approach outperforms GEKS.