Financial Frictions and Firm Informality: A General Equilibrium Perspective*

Luis Franjo[a], Nathalie Pouokam[b], and Francesco Turino[a]

[a]Fundamentos del Análisis Económico (FAE), Universidad de Alicante

[b]International Monetary Fund

September 21, 2021

Abstract

This paper assesses the extent to which financial development and informality are related, and how this relation translates into differences in GDP and TFP across countries. To this end, we develop a quantitative life-cycle general equilibrium model of occupational choice with imperfect tax enforcement, in which informal entrepreneurs have no access to credit and face an endogenous probability of being caught for tax evasion. Our quantitative analysis shows that the degree of financial frictions of a country is crucial in shaping the firm's incentives to evade taxation, a feature that, in the aggregate, results into a non-linear relationship between financial development and both size of informality and GDP per capita. We test these model's predictions with cross-country data and find supporting evidence in favour of both non-linearities.

Keywords: informality, financial frictions, taxation, entrepreneurship, productivity, misallocation.
JEL Classification: E21, E22, E26, E44, H26, O10, O40.

*We would like to thank Lian Allub, M. Dolores Collado, Francesco Serti, Selene Perazzini, Fidel Pérez Sebástian, Perdo Albarrán, Iñigo Iturbe-Ormaetxe, Pedro Maia Gomes, Carlos Alberto De Resende and seminar participants at the Institute for Capacity Development of International Monetary Fund in Washington D.C. Turino acknowledges financial support from MINECO/FEDER (ECO2015- 70540-P). Luis Franjo acknowledges financial support from the Cátedra Germán Bernácer.

1 Introduction

Developing countries have less developed financial markets (Abiad, Detragiache and Tressel (2010)) and larger informal sectors (Antunes and Cavalcanti (2007), Quintin (2008)). A less developed financial market is associated with lower productivity and GDP per capita (Buera, Kaboski and Shin (2011), Allub and Erosa (2019)), while a larger informal sector may imply lower aggregate productivity (D'Erasmo and Boedo (2012), Ulyssea (2018)) and a higher rate of tax evasion (Orsi, Raggi and Turino (2014)). At the same time, advanced economies with more developed financial markets may still host large shares of informality (Kuehn (2014), Hassan and Schneider (2016)).

Motivated by the above evidence, this paper quantitatively assesses the extent to which informality and development in financial markets are related. We are interested in understanding how the incentive of firms to evade taxation are shaped by the degree of financial frictions of a country, and in how this relationship translates, at the aggregate level, into adjustments in GDP and TFP across the stages of financial development. To this end, we build a life-cycle general equilibrium model of entrepreneurship with credit market imperfections and limited tax enforcement. The model is similar in spirit to Buera (2009) and Buera and Shin (2013), with individuals that are heterogeneous in terms of managerial abilities and face a discrete occupational choice: whether to be entrepreneurs or workers. In our framework, entrepreneurs must also decide whether to operate in the formal or informal sector. By opting for the former, entrepreneurs have access to credit up to a proportional amount of their savings, but they have to pay progressive taxes on personal income. By contrast, informal firms may be able to avoid taxation by concealing production activities from the government, but they have to operate in a financial autarky regime. In addition, they also face an expected cost of detection (penalty), with a probability of being caught for tax evasion that increases with the firm's size. This last property prevents wealthy entrepreneurs from running large informal businesses, and allows the model to capture a prominent feature of data in developing countries: a large share of output is produced by informal firms that are predominantly small-scale enterprises (La Porta and Shleifer (2014)).

The structure of the proposed model is sufficiently rich to trigger an interesting heterogeneity in informality. In particular, we identify three different types of informal firms that coexist in equilibrium: (i) those sufficiently productive to operate in the formal sector but established in the informal sector due to the lack of access to credit; (ii) firms productive enough to profitably operate in the formal sector, but which nevertheless choose to hide their productions in order to avoid taxation (*parasite* firms), and (iii) firms owned by low-skilled agents, who are too unproductive to ever become formal entrepreneurs (*survival* firms). We show that the degree of financial frictions is crucial in shaping the distribution of informal firms across the three different types, a feature that in the aggregate translates into non-trivial responses in both GDP and TFP.

We calibrate our model to match key features of the Brazilian economy, including the size distribution of both formal and informal firms. To address the main question of the paper, we first evaluate the long-run effects of a financial market reform in Brazil that brings the credit-to-GDP ratio to the level of more advanced economies. We find that easing financial frictions in this country triggers a significant drop in the size of the informal economy (from 37.9% to 20.7% of GDP) and in tax evasion (-35.6%), together with an important increase

in the official GDP (+28%), in fiscal revenues (+13.4%) and in measured TFP (+11.2%). We then evaluate the potential gains from financial market reforms in economies with any degree of financial development (ranging from financial autarky to perfect credit). Our model predicts that informality and financial development are negatively correlated and interact in a non-linear fashion. More specifically, we find that easing financial frictions has a significant impact on the size of the informal economy (and on tax evasion) for countries with credit-to-GDP ratios below 60%, while a virtually negligible effect is found above this threshold. The mechanism behind this finding triggers a reallocation of resources from the informal sector to the formal sector, which magnifies the impact of removing financial frictions on official GDP in countries where the credit-to-GDP ratio is below the threshold. In other words, our model also predicts a non-linear relationship between official GDP and financial development.

We test the implications of our model with cross-country data, reporting evidence in support of the non-linearities described above. These findings suggest that accounting for the informal economy is crucial to understanding the relationship between financial and economic development. As far as the non-linear pattern observed is concerned, our model offers a clear theoretical explanation. With the three different types of informal firms described above, easing credit market imperfections produces two countervailing effects on informal production. On the one hand, fewer financial frictions bring firms out of informality as highly productive entrepreneurs have greater access to credit. At the same time, the reform also results in higher production costs, thereby boosting the relative gains from tax evasion and bringing more *parasite* firms to the informal sector. The former effect prevails over the latter at a high level of financial frictions, where an increase in credit availability boosts the gains from formality of talented entrepreneurs. By moving to the formal sector, these entrepreneurs operate on a larger scale, inducing, in the aggregate, a sizeable increase in the official GDP and a rapid decline in the size of the informal sector, together with a substantial growth in TFP. The strength of this effect, however, vanishes as the credit-to-GDP ratio becomes larger. With more developed financial markets, in fact, highly productive entrepreneurs become less financially constrained. Hence, informal production is mostly carried out by small *parasite* and *survival* firms, which, optimally, decide to self-finance their business to take advantage of tax evasion. In those economies, therefore, further relaxing collateral constraints has a little impact on the size of the informal sector.

In terms of policy, the main message of our findings is clear: the role played by lowering credit market frictions in reducing informality and tax evasion is only relevant at high levels of credit market imperfections. In these circumstances, the gains in terms of GDP per capita and TFP of financial market reforms are amplified by adjustments due to firms moving from the informal to the formal sector. These findings contribute to the literature of economic development (Buera et al. (2011) and Allub and Erosa (2019)) by providing an additional argument in favour of easing credit marker imperfections in low-income countries. The results of our analysis also complement and extend the literature that relies on heterogeneous agents' models with financial frictions to study the sources and implications of firm informality. In this respect, Amaral and Quintin (2006) introduce informality in a model of occupational choice to analyse cross-country differences in labour market regulation and its interaction with credit market imperfections. Antunes and Cavalcanti (2007) assess the role of contract enforcement and regulation costs in explaining differences in income between the US, some

Mediterranean countries, and Peru, whereas Quintin (2008), in a very similar environment, focuses on assessing the extent to which the size of the informal sector is determined by the enforceability of credit contract obligations. D'Erasmo and Boedo (2012) also develop a model with informal entrepreneurs, but focus on evaluating the costs of informality in terms of aggregate TFP, while D'Erasmo (2013) studies the linkage between credit conditions and formalization in Brazil. We extend these models to an environment with overlapping generations, endogenous probability of detection and progressiveness in personal income taxation.[1] Our results provide a more comprehensive picture of how the interaction between financial frictions and informality affects economic outcomes, both at the aggregate and firms level. In this regard, we contribute to the existing literature by showing, both theoretically and empirically, that informality and financial development are related in a non-linear fashion.

The rest of the paper is organized as follows. Section 2 describes the theoretical background, while Section 3 provides details on the calibration of the model to the Brazilian economy. The long-run implications of financial frictions in Brazil are evaluated in Section 4, where we also discuss the predictions of the model regarding the relationship between the informality of firms and financial development. This section also contains empirical evidence in favour of the implications of our model, as well as an analysis of the effects of a flat tax policy with different degree of financial frictions. Section 5 contains concluding remarks. Details on the solution, data and additional results of the model are available in the online technical appendix.

2 Model

Building on Erosa (2001) and Buera (2009), we consider an overlapping generations model with imperfect financial markets and distortionary taxation.[2] Each generation consists of heterogeneous individuals that live for $J < \infty$ periods, each of them endowed with one unit of time until retirement, which occurs at the mandatory age $J_R < J$. Heterogeneity takes the form of individual-specific endowments of managerial ability and endogenously idiosyncratic wealth profiles, the latter resulting from individual savings decisions over the life-cycle. During her working life, an individual decides between becoming a worker or becoming an entrepreneur, on the basis of her managerial ability and her financial wealth. Workers supply their time-endowment inelastically and receive a gross wage, while entrepreneurs decide how

[1] Our framework is similar in spirit to the ones developed in Di Nola, Kocharkov, Scholl and Tkhir (2020) and Fernandez-Bastidas (2018), even though they do not explicitly include informal firms in their analysis. Moreover, unlike from ours, both these papers assess the fiscal policy implications of tax evasion in the U.S. economy.

[2] The overlapping generations structure of our model economy is motivated by the empirical evidence available, which shows that the stage of the life-cycle for firms and entrepreneurship is one of the determinants of informality rates among enterprises in developing countries. In particular, there is robust evidence in support of the fact that *formalization is more likely for older business owners than for younger ones* (see, for example, Perry et al. (2007), de Paula and Scheinkman (2010), de Paula and Scheinkman (2011) and Williams, Shahid and Martnez (2016)). In the same line, the empirical evidence available also shows that informality among firms tends to diminish with the time in business (e.g., Levenson and Maloney (1998), Perry et al. (2007) and Diaz et al. (2018)).

much to produce by combining labour and capital with a technology that exhibits decreasing returns to scale.

As in Buera and Shin (2013), imperfections in credit markets take the form of collateral requirements on the capital rental proportional to the individual's savings. Tax enforcement is also imperfect in that informal transactions − i.e. those carried out by unregistered firms − are detectable by the government only after a monitoring process. Entrepreneurs can then escape taxation by running their businesses informally. However, tax avoidance comes at a cost. First, in line with Dabla-Norris and Feltenstein (2005), access to credit is precluded to informal entrepreneurs, who therefore operate in financial autarky. Second, as in Allingham and Sandmo (1972) and Orsi et al. (2014), a number of individuals are audited by the government in each period, so informal entrepreneurs face a probability of being discovered evading, convicted of tax evasion, and forced to pay taxes augmented by a penalty surcharge.

2.1 Households

In each period, a new generation of individuals is born. Time is discrete and each agent discounts the future exponentially with a common discount factor $\beta \in (0,1)$. Preferences over consumption of a newly born individual over her life-cycle are represented by the following inter-temporal utility function

$$\sum_{j=0}^{J} \beta^j u(c_j)$$

where c_j denotes consumption at age j, while $u(c_j) = \frac{c_j^{1-\sigma}}{1-\sigma}$ is a standard CRRA instantaneous utility function with relative risk aversion coefficient equal to $\sigma > 0$. At birth (i.e., age $j = 0$), each individual is endowed with zero assets and idiosyncratic managerial ability $e \in \Theta$ that is drawn from an invariant distribution with cumulative distribution function $\Phi(e)$ and remains unchanged throughout the life-cycle.

2.2 Firms

There is a homogeneous consumption good in the economy that is produced by two distinct sectors: the entrepreneurial sector, which consists of small-scale enterprises, each owned by a specific household engaging in entrepreneurship; and the corporate sector, which consists of large-scale impersonal firms (in the spirit of Quadrini (2000), Meh (2005), Cagetti and De Nardi (2006) and Cavalcanti and Santos (2020), among others). In this paper, the main characteristics that differentiate small firms in the entrepreneurial sector from large corporations are the possibility to run the business informally and the strictness of the financial constraints.[3]

[3]As discussed in technical Appendix C, the presence of the corporate sector in our model is important to guarantee the existence of the general equilibrium. In short, financial frictions depress aggregate demand for capital by entrepreneurs in a way that may prevent the clearing of the capital market if borrowing constraints are sufficiently tight. With respect to standard models, moreover, this issue is more serious in our framework as informal entrepreneurs produce under a financial autarky regime, a feature that further depresses the

2.2.1 Entrepreneurial sector

Entrepreneurs combine their managerial abilities, e, with capital, k, and labour, n, to produce output via the following technology

$$e^{\eta}\left(k^{\alpha}n^{(1-\alpha)}\right)^{1-\eta} \tag{1}$$

where $\alpha, \eta \in (0,1)$. This production function exhibits decreasing returns to scale, capturing the idea of *span of managerial control* popularized by Lucas (1978).

Workers are hired from a perfectly competitive labour market at the wage rate w, while capital is financed by the entrepreneurs, by using their own assets, a, and by borrowing from financial intermediaries. We assume that to have access to credit, entrepreneurs need to comply with business regulations, including registration with the tax authorities, making their production activities observable by the government. Entrepreneurs who choose to comply with business regulations are referred to as formal entrepreneurs. They demand quantities k_f and n_f of (formal) capital and (formal) labour, respectively. Entrepreneurs who choose not to register with tax authorities are referred to as informal entrepreneurs. These agents run their businesses informally by hiring n_i (informal) workers from the labour market, and by self-financing (informal) capital, k_i. As they are not registered with tax authorities, informal entrepreneurs may escape taxation by concealing their production activities, which would then be detected by the government only after a monitoring process, as in Orsi et al. (2014).

2.2.2 Corporate sector

The corporate sector consists of a large number of perfectly competitive firms, which are heterogeneous in productivity and produce goods with the same technology adopted by firms in the entrepreneurial sector. In addition, we assume that corporations (i) pay an operational fixed cost; (ii) cannot engage in informal activities, and (iii) are not subject to borrowing constraints.[4] As shown in technical Appendix C, these assumptions imply that the net output of the sector − namely Y_c − can be represented as

$$Y_c = A\left(K_c^{\alpha}N_c^{(1-\alpha)}\right)^{1-\eta} - \phi_f \tag{2}$$

where $A > 0$; K_c and N_c respectively denote aggregate capital and aggregate labour in the corporate sector, while ϕ_f stands for the operational fixed cost. Profits maximization of price-takers firms implies that the aggregate demand of labour and capital in the sector respectively satisfy

$$(1-\eta)(1-\alpha)\left(Y_c + \phi_f\right)/N_c = w \tag{3}$$

demand for capital. By modelling corporations that are not subject to borrowing constraints, we introduce an additional component of capital absorption into the model, which mitigates the problem and guarantees the clearing of the capital market for any parameterization of the model.

[4]The assumption on the operational fixed cost has been introduced to guarantee that corporate profits are zero in equilibrium.

$$(1-\eta)\alpha\left(Y_c+\phi_f\right)/K_c = r_k \tag{4}$$

where r_k is the capital rental rate.

2.3 Financial intermediaries

Perfectly competitive financial intermediaries receive deposits from households at a risk-free interest rate, r, and rent capital to firms at rental rate r_k. In equilibrium, a zero-profit condition requires that

$$r_k = r + \delta \tag{5}$$

where $\delta \in (0,1)$ is the capital depreciation rate. As in Buera and Shin (2013), we assume that there is limited contract enforceability for small businesses so that the demand for rented capital by a formal entrepreneur who has accumulated wealth a is subject to the following collateral constraint

$$k_f \leq \lambda a \tag{6}$$

where $\lambda \geq 1$ measures the degree of financial imperfections, with $\lambda = \infty$ corresponding to perfect capital markets, and with $\lambda = 1$ denoting financial autarky.[5] The latter case corresponds to the situation of an informal entrepreneur, who has no access to credit and therefore can only self-finance capital with her accumulated wealth $\tilde{a}$, i.e.,

$$k_i \leq \tilde{a} \tag{7}$$

2.4 Government

The government collects taxes to finance wasteful public expenditure. We assume that there are two sources of fiscal revenues: a consumption flat tax τ_c and a progressive tax on personal income. As in Heathcote, Storesletten and Violante (2017), the latter is specified with the following tax function

$$T(y) = y - \lambda_y y^{1-\tau} \tag{8}$$

[5]This formulation of credit market imperfections is analytically convenient. In technical Appendix E.1, we show that the main results of the paper generalize to different specifications of financial frictions. In particular, we consider an alternative version of the model, in which borrowing limits are individual-specific as a result of the imperfect enforceability of credit obligations (in the spirit of Antunes, Cavalcanti and Villamil (2008) and Buera et al. (2011)). The results we find by means of this version of the model are reassuring that the main conclusions of the paper are robust with respect to alternative specifications of financial frictions.

where y denotes gross personal income and $T(y)$ stands for taxes.[6] Parameter $\tau \in (0,1)$ controls for the degree of progressivity in income taxation with $\tau = 0$ corresponding to a flat tax rate. For a given τ, parameter $\lambda_y \in (0,1)$ determines the individual's average tax rate (see Holter, Krueger and Stepanchuk (2019)).

To discourage tax evasion, the government periodically conducts audits. Following an audit, entrepreneurs found concealing production are convicted of tax evasion and forced to pay the taxes due, increased by a penalty surcharged factor $s > 1$. We assume that the probability of being audited in a given period, namely $p(k_i)$, is an increasing function of the amount of capital used by an informal firm. This assumption is common in tax evasion literature and can be rationalized by the fact that large establishments are more visible to tax authorities and therefore find it harder to conceal production than smaller ones (see, for example, Leal Ordoñez (2014)).

2.5 Individual's decisions problems

2.5.1 Working age population

During her working life (i.e., for $j = 0, 1, ..., J_R - 1$), an individual decides whether to be a worker or an entrepreneur at the beginning of each period, knowing her managerial ability, e, and given the amount of assets a accumulated in the previous period. Thereafter, workers choose how much to consume, c, and save, a', while entrepreneurs instead decide first whether to comply with business regulations, and on the basis of that choice, how much to produce by hiring workers and renting capital (taking collateral constraints into account). After production decisions have been made, random audits are conducted and fines enforced by the government. Once informal entrepreneurs have realized whether or not they have been caught concealing production, they decide how much to consume and save.

The decision problem of an individual with state variables e and a can be written in a recursive formulation, with the beginning-of-period value function given as follows

$$V(a,e) = max\{V^W(a,e), V_f^E(a,e), V_i^E(a,e)\}$$

The function $V^W(a,e)$ denotes the value function for the agent who chooses to be a worker in the current period, i.e.

$$V^W(a,e) = \max_{c,a'}\{u(c) + \beta V(a',e)\}$$

[6] According to the Federal tax regulations, the individual income tax in Brazil (IRPF - *Imposto de Renda Pessoa Física*) is applied according to a progressive schedule, with five rates ranging from 0% (for yearly incomes of up to 21453,24 Brazilian reals) to 27.5%. The corporate income taxes (IRPJ - *Imposto de Renda Pessoa Jurídica and CSLL - Contribuição Social sobre o Lucro Líquido*) are also progressive with marginal tax rates of 24% (for monthly profits of up to 20000 Brazilian reals) and 34%. The tax function specification borrowed from Heathcote et al. (2017) accommodates the progressivity of Brazilian the Federal tax system in the model.

subject to

$$(1+\tau_c)c + a' = y^w + a - T(y^w)$$

$$a' \geq 0 \tag{9}$$

where $y^w = w + ra$ denotes the worker's personal income, while constraint (9) captures that workers cannot borrow. Functions $V_f^E(a,e)$ and $V_i^E(a,e)$ respectively stand for the value of being a formal entrepreneur and the value of being an informal entrepreneur. The decision problem of a formal entrepreneur takes the following form

$$V_f^E(a,e) = \max_{k_f, n_f, c, a'} \{u(c) + \beta V(a', e)\}$$

subject to (6), (9) and

$$y^E = e^\eta \left(k_f^\alpha n_f^{(1-\alpha)}\right)^{1-\eta} - wn_f - (r+\delta)k_f + ra$$

$$(1+\tau_c)c + a' = y^E + a - T(y^E)$$

In the above equation, y^E denotes the formal entrepreneur's declared income, which amounts to her actual earnings.

By contrast, informal entrepreneurs escape taxation by concealing their production activities and reporting only their capital incomes, but they face a probability of detection $p(k_i)$. So, let $V_d^E(a,e)$ and $V_{nd}^E(a,e)$ denote the informal entrepreneur's value functions corresponding to the cases of detection and non-detection, respectively. The expected value of being an informal entrepreneur can then be written as follows

$$V_i^E(a,e) = \max_{k_i, n_i} \{p(k_i)V_d^E(a,e) + (1-p(k_i))V_{nd}^E(a,e)\}$$

subject to (7), which describes the collateral constraint in financial autarky. The value function in the case of non-detention is given by

$$V_{nd}^E(a,e) = \max_{c,a'} \{u(c) + \beta V(a', e)\}$$

subject to (9) and

$$y^E = ra$$

$$(1+\tau_c)c + a' = y^E + \pi + a - T(y^E)$$

where π represents profits from business activities, i.e.

$$\pi = e^\eta \left(k_i^\alpha n_i^{(1-\alpha)}\right)^{1-\eta} - wn_i - (r+\delta)k_i. \tag{10}$$

Accordingly, concealing production allows the informal entrepreneur to hide profit income π from the tax authorities. However, in the event of detection, the government would force the informal entrepreneur to pay the taxes due on the unreported income (i.e., $T(y^E + \pi) - T(y^E)$) scaled up by a penalty surcharge factor s. Consequently, the value function of an informal entrepreneur that has been detected by the government is given by

$$V_d^E(a, e) = \max_{c,a'} \{u(c) + \beta V(a', e)\}$$

subject to (9) and

$$y^E = ra$$

$$(1 + \tau_c)c + a' = y^E + \pi + a - (1 + s)\left[T(y^E + \pi) - T(y^E)\right]$$

where π is defined as in Equation (10).

2.5.2 Retired agents

During retirement (i.e., for $j = J_R, J_R + 1, ..., J$), an individual consumes and saves on the basis of the financial wealth accumulated during her working life. Hence, the value function of a retired individual is given as follows

$$V(a, e) = \max_{c,a'} \{u(c) + \beta V(a', e)\}$$

subject to (9) and

$$(1 + \tau_c)c + a' = y^R + a - T(y^R)$$

where $y^R = ra$ is the retired individual's declared income.

2.6 Equilibrium

For each agent in the economy, let us define with $\omega = \{e, a, b(e, a)\}$ the vector containing the individual state variables, e and a, and the occupational status $b(e, a)$ (i.e. retired, workers, formal entrepreneurs and informal entrepreneurs (detected and undetected)). A stationary equilibrium is given by a price vector $\{r, w\}$, allocations $\{c(\omega), a(\omega)\}$, occupational choices $b(e, a)$, formal and informal workers $\{n_f(\omega), n_i(\omega)\}$, investment in formal and informal capital $\{k_f(\omega), k_i(\omega)\}$, labour and capital in the corporate sector $\{L_c, K_c\}$ and a distribution of individuals over ω, $\xi(\omega)$, such that given the free-risk interest rate r, the wage rate w and the tax system (i.e., s, $p(\cdot)$, τ_c and $T(\cdot)$):

- The policy functions $\{c(\omega), a(\omega), k_f(\omega), k_i(\omega), n_f(\omega), n_i(\omega), b(e, a)\}$ solve the agents' decision problems described in Section 2.5.
- Labour and capital in the corporate sector, $\{L_c, K_c\}$, solve optimality conditions (3) and (4).

- Capital and labour market clear:

$$\int \left(k_f(\omega) + k_i(\omega)\right) d\xi(\omega) + K_c = \int a(\omega) d\xi(\omega)$$

$$\int \left(n_f(\omega) + n_i(\omega)\right) d\xi(\omega) + L_c = \int \mathbb{1}_W d\xi(\omega)$$

  where $\mathbb{1}_W$ is an indicator function taking value 1 if the agent is a worker, and 0 otherwise.

- The government budget constraint is balanced, i.e.

$$G = \int \left\{\tau_c c(\omega) + T(y(\omega)) + \mathbb{1}_D(1+s)\left[T(y(\omega) + \pi(\omega)) - T(y(\omega))\right]\right\} d\xi(\omega)$$

  where G are public expenditures, $y(\omega)$ is the agent's declared income and $\mathbb{1}_D$ stands for an indicator function that takes a value of 1 if the individual is an informal entrepreneur that has been audited, and 0 otherwise.

- Financial intermediaries earn zero profit, i.e. Equation (5) is satisfied.

- The distribution $\xi(\omega)$ is the invariant distribution for the economy.

To close the model, we need to introduce a proper definition of official − or measured − GDP. Given that informal production activities are concealed from the government, the latter does not necessarily coincide with the total output of the economy. This paper assumes that the official GDP is given by total formal output, i.e.

$$GDP = \int \mathbb{1}_f y(\omega) d\xi(\omega) + Y_c$$

where $\mathbb{1}_f$ is an indicator function taking value 1 if the agent is a formal entrepreneur and 0 otherwise.[7] We will likewise refer to measured TFP as total factor productivity in the formal sector.[8]

[7]The Brazilian statistical institute adjusts the official GDP by incorporating estimates for the so-called *non-observed economy*, which includes underground production, home production and illegal activities. The definition of informal sector in the model corresponds to the underground component. As discussed in more detail in technical Appendix F, the quantitative importance of underground production may be largely underestimated in the adjustment procedure implemented by the Brazilian statistical institute. For this reason, we do not include informal production in the definition of measured GDP in the model.

[8]TFP in the model is defined as $TFP_s = Y_s \left(K_s^{\alpha(1-\eta)} N_s^{(1-\alpha)(1-\eta)}\right)^{-1}$, where $s \in \{f, i\}$ refers to formal (f) or informal (i) sector, and Y_s, K_s and N_s to, respectively, total output, total capital and total labour in sector s.

3 Calibration

We calibrate the model to the Brazilian economy. Parameter values are assigned by relying either on external sources or by targeting key macro and micro statistics, including among them estimates for the size of the informal economy, distribution of firms in the formal and informal sector, elements of the tax system and features of the life-cycle profile of earnings and informality of firms. To compute micro statistics, we make use of 2 data sets: (i) the ECINF survey (*Pesquisa de Economia Informal Urbana*), which covers detailed information on formal and informal firms; and (ii) the PNAD survey (*Pesquisa Nacional por Amostra de Domicílios*) which contains general characteristics of population such as occupational structure and earnings profiles. Both surveys are compiled by the Brazilian Institute of Geography and Statistics (IBGE). A description on how parameters are identified is provided below.

3.1 Identifying restrictions

Time period, life-cycle duration and retirement age. The time period in the model is one year. We assume that agents live for $J = 55$ periods and retire at $J_R = 46$. The model can then be interpreted as one in which individuals are born economically at age 20 and live up to 74 years. According to World Bank estimates, this last number corresponds to the average value of life expectancy in Brazil over the 2010-2018 period. The value assigned to J_R in turn implies that individuals retire at age 65 in conformity with the Brazilian pension system.

Parameters in the utility function. There are two preference parameters (β, σ). The subjective discount factor β is chosen so that the capital to GDP ratio in the steady state equilibrium is equal to 2.10. This number is taken from Allub and Erosa (2019) and corresponds to the average capital-output ratio in Brazil over the 2004-2010 period. The relative risk aversion coefficient σ is set equal to 1.5, in line with the bulk of literature on occupational choice models with financial frictions (see, among others, Buera et al. (2011) and Buera and Shin (2013)). This value is also consistent with the available empirical estimates of the relative risk aversion coefficient for Brazil, implying a range of values for σ from 1 to 3 (see e.g., Fajardo, Ornelas and Farias (2012)).

Distribution of managerial abilities. We assume that the managerial ability is drawn from a generalized Pareto distribution with CDF

$$\Phi(e) = 1 - \left(1 + \frac{\nu(e-\mu)}{\kappa}\right)^{-1/\nu}$$

where parameters μ, κ and ν determine location, scale and shape of the distribution, respectively. As in Buera and Shin (2013), the support of the ability distribution is discretized into 40 grid points, and the first grid point, namely e_{min}, is set to the one with probability mass

$\Phi(e_{min})$.[9] We chose to calibrate $\Phi(e)$ so it is consistent with the data on firm size distribution in the formal sector. In particular, we need 4 moments to identify the 4 parameters $(\mu, \kappa, \nu, \Phi(e_{min}))$, and we therefore target the share of formal firms with up to 5, 5 to 10, 11 to 20, and 21 to 50 employees. Estimates for the size distribution of formal establishments in Brazil are taken from Ulyssea (2018).

Penalty surcharge factor and probability of detection. We set the surcharge factor s to 0.75, which corresponds to the default penalty applied by the Brazilian tax authorities to a taxpayer issued with an infraction notice. To calibrate the probability of detection $p(k_i)$, we note that a characteristic of the Brazilian data is that a large share of output is produced informally by firms that are predominantly small-scale enterprises.[10] The estimates provided by Medina and Schneider (2018) document an average size of the Brazilian informal output over the 1991-2015 period that is equivalent to a 37.6% of GDP, and at the same time − according to the ECIFN 2003 survey − firms with 2 or fewer employees constitute around 96% of the total number of informal enterprises. Our parameterization of $p(k_i)$ is designed to capture these characteristics of the data.

Specifically, as in Di Nola et al. (2020), we assume that the detection process follows a logistic distribution with parameters $(p_1, p_2) \in \mathbb{R}^2_+$, i.e.

$$p(k_i) = \frac{1}{1 + p_1 exp(-p_2 k_i)}$$

Accordingly, for any given level of informal capital, the probability of detection is decreasing in p_1. Hence, the higher p_1, the higher the gains will be from tax evasion and, consequently, the larger the amount of output produced in the informal sector. This parameter is therefore assigned to match data on the size of the Brazilian informal economy. On the other hand, the logistic specification implies that $p(k_i)$ increases quickly as the capital becomes larger, so that the benefits from tax evasion decline substantially with the size of the firm. Depending on the value taken by p_2, this property implies that informal firms tend to stay small in equilibrium, as a larger size would make informality less convenient. Parameter p_2 is then chosen so that share of informal firms with up to 2 workers in the steady state matches its counterpart in the Brazilian data.

Production technologies and borrowing constrains. The production function in the entrepreneurial sector is standard and, therefore, we rely on external sources to calibrate parameters η and α. In particular, we follow Allub and Erosa (2019) and set $\eta = 0.198$ and $\alpha = 0.406$. The production function in the corporate sector, in turn, introduces two additional technological parameters: A and ϕ_f. The former is assigned to match data on the share of capital that in employed in the corporate sector.[11] An estimate of this statistic

[9] The remaining grid points are determined in a way that makes them equidistant in probability space.

[10] As emphasized by La Porta and Shleifer (2014), this feature of the data is common across developing countries.

[11] We want our model to be consistent with a measure of the size of the corporate sector in the data. The standard practice in the literature is to use the share of total capital that is employed in the corporate

Table 1: Calibration Results: Parameter Values

Parameters	Description	Source/ Targeted Moment	Value
(A) Externally calibrated			
σ	Relative risk aversion coefficient	Standard	1.500
s	Surcharge factor	Brazilian law	0.750
δ	Capital depreciation rate	Cavalcanti and Santos (2020)	0.060
α	Production function parameter	Allub and Erosa (2019)	0.406
η	Span of control	Allub and Erosa (2019)	0.198
τ_c	Consumption tax	Jung and Tran (2012)	0.150
(B) Internally calibrated			
β	Subjective discount factor	Capital-Output ratio	0.935
μ	Location Pareto distribution	Size distribution formal firms	4.251
κ	Scale Pareto distribution	Size distribution formal firms	0.505
ν	Shape Pareto distribution	Size distribution formal firms	0.778
$\Phi(e_{min})$	Probability mass (Pareto)	Size distribution formal firms	0.425
p_1	Probability of detection parameter	Informal output to GDP	1766
p_2	Probability of detection parameter	Distribution of informal firms	1.147
A	TFP in the corporate sector	% of K used by corporations	2.486
ϕ_f	Operational fixed cost	Zero-profits condition	51.57
λ	Tightness of borrowing constraints	Credit to GDP ratio	1.378
τ	Income tax parameter	Average income 43-65/21-42	0.137
λ_y	Income tax parameter	Total fiscal revenues to GDP	0.834

for Brazil is provided by Antunes, Cavalcanti and Villamil (2015), who report that around 30% of total capital is absorbed by the corporate sector in this country.[12] The fixed cost parameter, ϕ_f, is determined in equilibrium by a zero-profit condition in the corporate sector. Finally, the capital depreciation rate, δ, is set to 0.06 as in Cavalcanti and Santos (2020), while the parameter controlling for the tightness of borrowing constraints, λ, is chosen to match a credit-to-GDP ratio of 0.42. According to the financial structure database of the World Bank, this last number corresponds to the average value of the private credit-to-GDP ratio in Brazil over the 1991-2015 period.[13]

Taxes. There are 3 tax-related parameters in the model: the consumption tax rate τ_c and the parameters in the income tax function, i.e. τ and λ_y. We set $\tau_c = 0.15$ as in Jung and

sector as a measure of the size of this sector (see e.g., Quadrini (2000)). This approach fits naturally with our modelling choices given that we introduce the corporate sector in our framework precisely to provide an additional source of capital absorption that guarantees the existence of a general equilibrium.

[12]This statistic is computed using data on firms listed on the Brazilian stock market.

[13]We use the 2019 release of the database. The average credit-to-GDP ratio is computed using the variable *private credit by deposit money banks and other financial institutions* (code: pcrdbofgdp).

Table 2: Calibration Results: Moments

Moments	Source	Data	Model
(A) Targeted moments			
Capital-output ratio	Allub and Erosa (2019)	2.100	2.100
Credit-to-GDP ratio	World Bank Database	0.420	0.407
Share of capital in the corporate sector	Antunes et al. (2015)	0.300	0.293
Ratio of informal to formal output	Medina and Schneider (2018)	0.376	0.379
Total fiscal revenues to GDP	OECD revenues statistics	0.324	0.334
Average income 43-65/21-42	PNAD survey	1.299	1.290
Size distribution: informal firms			
$\leq$ 2 workers	ECIFN 2003	0.957	0.958
Size distribution: formal firms			
$\leq$ 5 workers	Ulyssea (2018)	0.701	0.706
$\leq$ 6-10 workers	Ulyssea (2018)	0.141	0.152
$\leq$ 11-20 workers	Ulyssea (2018)	0.083	0.082
$\leq$ 21-50 workers	Ulyssea (2018)	0.048	0.059
(B) Non-Targeted Moments			
Ratio of informal to formal workers	Ulyssea (2018)	0.354	0.369
Share of informal firms	Ulyssea (2018)	0.698	0.651
Lifetime of formal business (years)	Demography of Enterprises	11.20	16.20
Share of entrepreneurs in population	PNAD 2015	0.267	0.220
Labor income share	Penn World Table 9.1	0.526	0.490
Effective income tax rate	Paes and Bugarin (2006)	0.165	0.165
Distribution of wealth (Gini)	Davies et al. (2011)	0.784	0.767
Average age of formal entrepreneurs	ECIFN 2003	43.22	53.80
Average age of informal entrepreneurs	ECIFN 2003	42.24	50.42

Notes: The share of entrepreneurs refers to working-age population. The lifetime of formal business is computed using the Demography of Enterprises and Statistics of Entrepreneurship data set compiled by the IBGE. The reported number is the average over the 2016-2018 period. The effective income tax rate is computed as the average between the effective tax rate on labour income and the effective tax rate on capital income.

Tran (2012), who also calibrate an OLG model to the Brazilian economy. The specification of the tax function $T(y)$ in turn provides restrictions that are useful to identify parameter τ. In particular, according to Equation (8), we note that for any arbitrary levels of income y_j and y_{j+k} of two individuals at different ages in the life-cycle, the corresponding ratio of

disposable incomes, i.e.

$$\begin{aligned}\frac{\tilde{y}_{j+k}}{\tilde{y}_j} &= \frac{\lambda_y y_{j+k}^{1-\tau}}{\lambda_y y_j^{1-\tau}} \\ &= \left(\frac{y_{j+k}}{y_j}\right)^{1-\tau},\end{aligned}$$

only depends on τ. We thus take advantage of this property, and identify parameter τ by matching a measure representing a stylized characterization of the life-cycle profile of Brazilian incomes. To this end, we follow Castaneda, Diaz-Gimenez and Rios-Rull (2003) by targeting the ratio of the average incomes of households aged between 65 and 43 to that of households aged between 42 and 21.[14] We compute this statistic with PNAD data, according to which the average value over the 2011-2014 period is 1.30.[15] Finally, parameter λ_y is chosen so that the ratio of total fiscal revenues to GDP in the steady state of the model is equal to 32.4%. According to OECD revenue statistics for Latin America, this last number corresponds to the average value of the total tax revenues-to-GDP ratio in Brazil over the 2000-2018 period.

To summarize, our calibration strategy partitions the model's parameters into to 2 sub-vectors: one containing those that are fixed according to external sources, i.e. (σ, s, δ α, η, τ_c), and another one that contains the internally calibrated parameters, i.e. $(\beta, \mu, \kappa, \nu, \Phi(e_{min}), p_1, p_2, A, \phi_f, \lambda, \tau, \lambda_y)$. Values for the latter are jointly assigned by minimising a loss function that computes the distance between the targeted moments from the data and their counterparts in the model. Results are provided in Table 1, which reports calibrated parameter values, and in Table 2, which compares model with data for both targeted and non-targeted moments.[16]

3.2 Goodness of fit and external validation of the model

As Panel A of Table 2 illustrates, the model does an extremely good job of replicating targeted moments. Exactly as in the data, the steady state of the economy is characterized by an informal sector producing an amount of output equivalent to 37.9% of the official GDP, with around 95% of informal firms producing with a maximum of 2 employees. The model also accurately replicates the size distribution of formal firms, thereby capturing the fact

[14] Further support in favour of this identification strategy is provided in technical Appendix E.3.

[15] For each individual in the survey, we use the variable income from any sources − code V4720 − as a measure of disposable income.

[16] We have also tested whether the calibration restrictions described above allow for the identification of parameters. Details on the procedure and results are provided in technical Appendix B. In short, we follow Adda, Dustmann and Stevens (2017), and numerically check whether the loss function is flat around the vector of internally calibrated parameters. The basic idea here is to test whether the predicted moments are sensitive to changes in parameters so that the empirical ones are effectively informative for identification. The results we find show that the loss function is very sensitive to small changes in the parameter's values, thereby providing reassurance regarding the identification of the model.

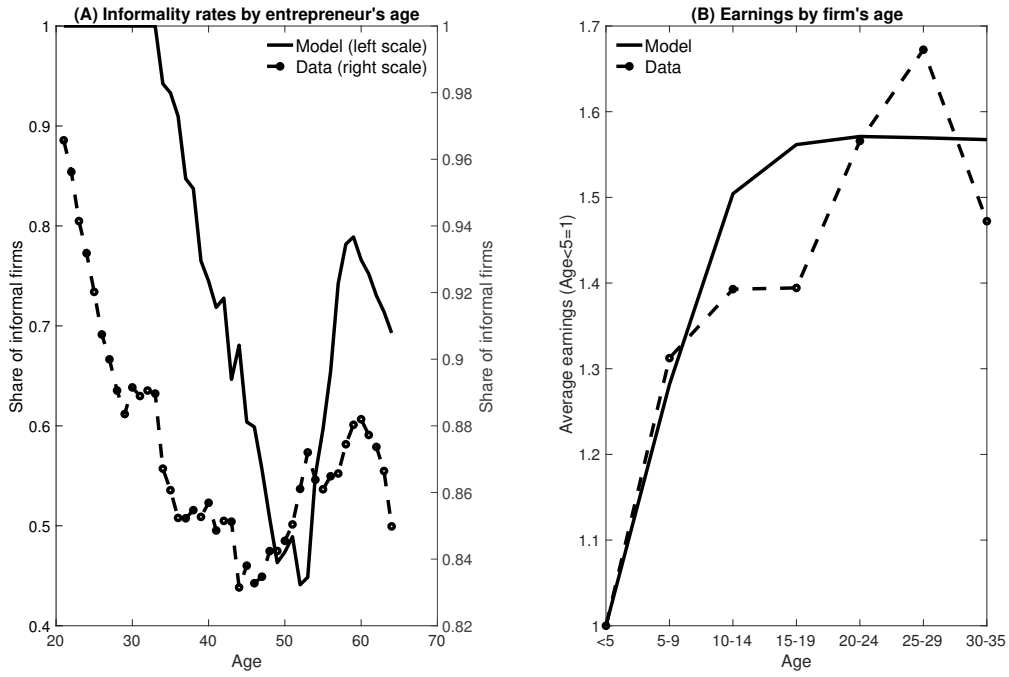

Figure 1: Informality rates by entrepreneurs' age and life-cycle of informal firms

Notes: Informality rates in the data are computed by discarding observation where the age of entrepreneurs is lower than 15 and higher than 65. The resulting figures reported in panel A of the picture (dashed line) have been smoothed using a moving average filter. Average earnings reported in panel B are normalized to be 1 in the youngest group (i.e. Age<5).

that in the formal sector firms are relatively large in comparison with the informal firms.[17] As for the macro targets, both *great ratios* and life-cycle statistics are matched very well, with moments in the model that are quite close to what is observed in the data.

We also assess how the model performs compared to selected non-targeted moments. Results are reported in panel B of Table 2, which shows that the model accurately replicates all of the considered dimensions of the data. The Gini index for wealth distribution, the share of informal workers, the labour income share and the effective tax rate on personal income in the model match the data almost perfectly. The model is also consistent with the data in predicting a large degree of informality among Brazilian entrepreneurs, as indicated by the share of informal firms, even though the latter statistic in the model is slightly lower than its empirical counterpart. Furthermore, the overlapping generations structure allows us to test the predictions of the model with regard to life-cycle characteristics of entrepreneurship and firms informality. To this end, we perform two experiments. First, we use the ECIFN 2003 survey to compute the average age of formal and informal entrepreneurs and the share of informal firms by entrepreneurs' age (depicted with a dashed line in panel A of Figure 1). The reported results show that, although the average age of formal and informal

[17]Given that the distribution of managerial abilities is *ex ante* the same across formal and informal entrepreneurs, the endogenous probability of detection is crucial for the model to capture this important feature of the data. Further details on this property of the model are provided in Section 4.1.

entrepreneurs is virtually the same (see Table 2), the rate of informality is disproportionately high among younger entrepreneurs and tends to decline − with a U-shaped pattern − as the entrepreneur's age increases. The comparison with the predicted results (reported with a continuous line in Panel A of Figure 1) highlights that the model accurately reproduces the pattern observed in the data, albeit with important differences in quantitative terms.[18] As a second experiment, we test whether the model is able to replicate the age-size profile of informal firms observed in the data (reported with a dashed line in panel B of Figure 1). The latter is again computed by means of the ECIFN 2003 survey, using earnings as a measure of firm size.[19] The reported results suggest that the Brazilian informal firms tend to grow relatively little with age, as shown by the fact that, on average, the 35-year-old establishments are less than 50% larger than their 5-year-old counterparts in term of earnings. As the picture illustrates, the model's predictions about the life-cycle dynamics of firms (reported with a continuous line in panel B of Figure 1) are fully consistent with the data.

4 Results

This section first characterizes the properties of the general equilibrium in the calibrated model (the benchmark economy), with a specific focus on occupational choices and informality. We then discuss the implications of relaxing financial frictions in Brazil. The section concludes with a general discussion on the potential gains of financial market reforms for developing countries, stressing in particular the effects on the size of the informal economy and tax evasion.

4.1 The benchmark equilibrium

Figure 2 graphs how savings and capital decisions (first row) and occupational choices (second row) evolve over the age of individuals with the lowest (first column), middle (second column) and the highest (third row) managerial abilities.[20] Age is a key dimension for occupational

[18]Intuitively, given the borrowing constraints, it takes time in the model before an individual accumulates enough assets to make formal entrepreneurship a profitable option. For low levels of wealth, informality provides a valuable alternative for agents to enter into entrepreneurship in an earlier stage of the life-cycle and, thanks to tax evasion, to accumulate the assets required for formal entrepreneurship faster. As firms grow, however, the benefit of informality declines, because the probability of detection increases, thereby strengthening even further the incentive of entrepreneurs to enter into formality. Furthermore, note that the model effectively captures the observed U-shaped pattern, with the informality rate that tends to increase again with older entrepreneurs. This feature is driven by workers with low managerial ability that enter into informal entrepreneurship late in the life-cycle, i.e. once they have accumulated enough assets to make the occupational switch a profitable option.

[19]In this experiment, we use earnings instead of employment as a measure of firm size because the latter in the ECIFN data is truncated at 10 employees by the survey design.

[20]Given that the optimal consumption and saving choice of informal entrepreneurs is contingent on detection, we compute the stationary distribution of the economy by carrying out Monte Carlo simulations with regard to a random variable describing audits by the government. In each simulation, we keep track of

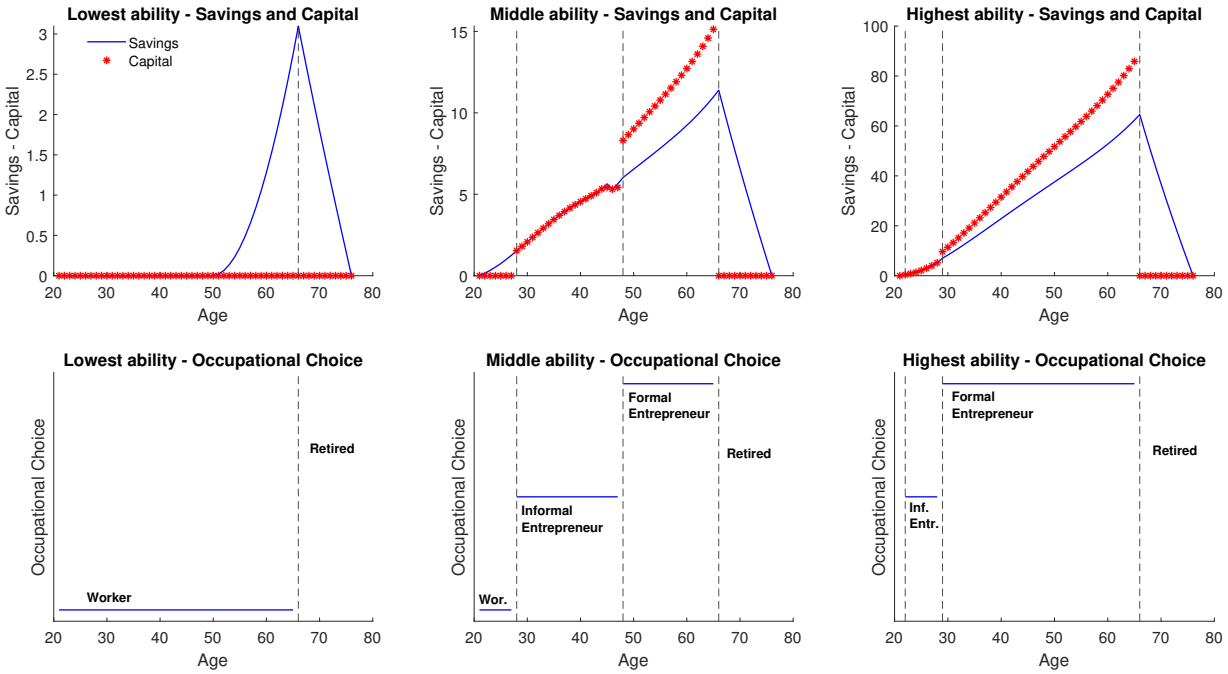

Figure 2: Inspecting the mechanism

Notes: The picture shows optimal decisions for agents with the lowest (first column), middle (second column) and highest (third column) abilities. The first row shows the evolution across ages of capital (discontinuous red line) and savings (continuous blue lines). Occupational choices are reported in the second row.

decisions, as it determines wealth accumulation and, therefore, borrowing limits, which are characteristic of our credit-constrained economy. In this regard, note that all agents are endowed with zero assets at birth and, due to the collateral requirements on the capital rental (Equation (6)), the only option they have is to become workers in the first period of their life. The picture illustrates that individuals in the lowest-ability group will remain workers throughout their entire working life. These agents have the unique feature that their ability level is too low to make the operation of a business viable, even at a sufficiently low scale that tax evasion could be afforded with a virtually negligible probability of being detected. These individuals therefore save for retirement only, and never engage in entrepreneurship, even informally.

By contrast, individuals in the middle and in the upper range of the ability distribution switch occupation as soon as they have accumulated enough assets to self-finance informal businesses. These agents have managerial abilities that are sufficiently high to make entrepreneurship a viable option but, because of the collateral constraints, they have not

whether the individual has been audited or not and, therefore, compute the actual pattern of savings and capital over the life-cycle. The results shown in Figures 2 and 3 refer to a specific simulation that has been selected randomly among those we performed to compute the invariant distribution of the economy. Hence, any twist in the depicted policy functions is due to the fact that in that specific simulation, the selected individual running her business informally has been caught by the government and forced to pay the taxes due, increased by the fine.

accumulated enough assets to make operating a formal business immediately profitable.[21] Hence, as an alternative to remaining as workers, they choose initially to become informal entrepreneurs in order to take advantage of the additional resources coming from tax evasion. In this respect, Figure 2 also illustrates that individuals with a higher managerial ability save more during their working life, since they have, in general, more incentives to become entrepreneurs, and hence, in comparison with individuals in the middle-ability group, start informal business at a younger age.

An important feature highlighted by the capital patterns shown in Figure 2 is that informal firms are small-scale enterprises compared to formal firms. This result can be intuitively explained by the combined effects of two main forces. First of all, informal enterprises operate in a financial autarky regime, and entrepreneurs can thus only use their own savings to finance productive capital. The maximum production scale is then bounded by the total amount of wealth accumulated by the entrepreneur. Second, the probability of being audited by the government, $p(k_i)$, is an increasing function of the capital used in production and thus informal firms have strong incentives to produce on a low scale. The role of these two complementary forces is nicely apparent in the pattern of informal capital for individuals with middle managerial abilities. As Figure 2 illustrates, during the period of time in which these individuals are informal entrepreneurs, capital moves initially in step with savings, as a result of the financial autarky constraint (i.e. Equation (7)). This pattern continues until the capital reaches its optimal level. After this point, while saving continues to grow, capital stays constant as a result of the entrepreneurs' optimal decisions to keep their firms on a small scale to prevent detection.

Another key property of the framework with endogenous occupational choices is that financially constrained entrepreneurs, i.e. those for whom constraints (6) and (7) are binding, face a strong incentive to save in order to expand the scale of production through capital (see, e.g., Quadrini (2000)). The reason is the marginal productivity of capital, which is higher than the market interest rate in financially constrained firms,[22] and therefore self-financing their own firms is more profitable for entrepreneurs than borrowing resources from financial intermediaries. In our framework, this effect is amplified in the case of informal entrepreneurs that are not audited by the government, which, because of tax evasion, are able to accumulate wealth faster as their incomes grow at a higher rate.

As a final remark, note that once informal entrepreneurs have accumulated enough assets, they optimally decide to comply with business regulations and become formal entrepreneurs. This pattern is common to all formal entrepreneurs in the benchmark economy, in the sense that none of them start the entrepreneurial activity directly in the formal sector. In the next section we will show that this result crucially depends on the tightness of the borrowing constraint, which strongly affects the incentives of immediate formalization by talented entrepreneurs. In this respect, note that in the benchmark equilibrium, informal entrepreneurs in the highest-ability group save more than those with middle ability and, therefore, move into the formal sector earlier, as shown in Figure 2. This feature highlights that the rationale

[21] Intuitively, by producing in the formal economy, these individuals would be inefficiently constrained by the borrowing limit and, thus, they would have to produce on a very small scale compared to what would be their optimal scale under perfect credit markets.

[22] This is true as the amount of capital employed in production is below the optimal level with perfect capital markets.

for talented individuals to operate informally is precisely to accumulate faster the amounts of assets that, given the existing collateral constraints, make formalization a profitable option.

4.2 Long-run impact of financial frictions in Brazil

We now asses the aggregate implications of financial frictions in the Brazilian economy. This assessment is performed counterfactually, by evaluating the effects of a hypothetical reform that permanently improves the enforceability of credit contracts in Brazil and, consequently, brings the long-run credit-to-GDP ratio to the level of more advanced economies. Specifically, we will evaluate how the steady state equilibrium changes once we relax the tightness of borrowing constraints, which in the model is controlled by parameter λ. In this respect, note that in our framework the collateral constraint can be motivated as arising from a limited enforcement problem, in which formal entrepreneurs may renege on the contracts with financial intermediaries and keep a fraction $1/\lambda$ of the rented capital.[23] As such, parameter λ can also be interpreted as a proxy of the degree of financial efficiency of an economy, in the sense that it captures the extent of financial frictions due to limited enforcement of credit contracts.[24]

Results are reported in Table 3, which provides several key statistics for both the benchmark and the post-reform economy, and in Figure 3, which shows how individual decisions are affected by loosening borrowing constraints in Brazil. The steady state equilibrium in the post-reform economy is computed from the benchmark model by setting parameter λ to a value implying a credit-to-GDP ratio of 1.43, while all of the remaining parameters are kept fixed to their calibrated values. According to the financial structure database of the World Bank, the targeted credit-to-GDP ratio corresponds to average value among high-income countries over the 1990-2016 period, and implies $\lambda = 4.7$ in the model. In what follows, we will refer to the post-reform economy as the counterfactual economy.

4.2.1 The informal economy

As Table 3 illustrates, relaxing collateral constraints has a substantial impact on all the informal sector aggregates. We find in particular that − relative to the benchmark − both the percentage of informal to total firms and the share of informal entrepreneurs in the population decline in the counterfactual economy, leading to an overall decrease in the aggregate informal production of 30%. By contrast, although the share of formal entrepreneurs in population raises by only 2 percentage points, we find that the official GDP increases by 28%. Hence, our model predicts that improving the functioning of financial markets would have a huge impact on the size of the informal economy in Brazil, with the latter declining by around 17 percentage points. This result is particularly interesting as it shows that at least 40% of the current size of the informal economy in Brazil is explainable by credit market imperfections.

[23] See Buera and Shin (2013) and Moll (2014) for a detailed discussion on this point.

[24] Describing the financial development of a country to the extent to which its legal system can enforce domestic financial contracts is common in the theoretical literature (see e.g., Mendoza, Quadrini and Ríos-Rull (2009)).

Table 3: Experiment Results. Easing financial frictions

	Benchmark	**Counterfactual**
Credit-to-GDP ratio	0.41	1.43
Size informal economy (% Official GDP)	37.9%	20.7%
Share of Informal firms (% Total firms)	65.1%	49.1%
ΔInformal production		-30.2%
ΔOfficial GDP		28.0%
– Contribution of Extensive Margin		35.7%
– Contribution of Intensive Margin		64.3%
ΔTotal production		12.0%
ΔCapital		18.5%
ΔMeasured TFP		11.2%
– Contribution of Extensive Margin		76.4%
– Contribution of Intensive Margin		23.6%
ΔWage		4.6%
Interest Rate	1.7%	9.6%
Total workers (% Population)	78.0%	82.0%
Total informal entrepreneurs (% Population)	14.3%	8.8%
Total formal entrepreneurs (% Population)	7.7%	9.2%
ΔFiscal revenues		13.4%
ΔTax evasion		-35.6%
Wealth Distribution		
– Workers (% Total Wealth)	13.1%	49.5%
– Informal entrepreneurs (% Total Wealth)	27.7%	22.3%
– Formal entrepreneurs (% Total Wealth)	59.3%	28.2%
Firm dynamics		
– Share of firms that never become formal	29.6%	11.9%
– Share of firms that never become informal	0%	25.7%
– Average firm growth (total)	5.4%	1.9%
– Average firm growth (formal)	2.6%	4.3%
– Average firm growth (informal)	5.8%	1.2%
– Entry rate (total)	8.0%	6.1%
– Entry rate (formal)	6.3%	4.6%
– Entry rate (informal)	8.9%	7.7%
Static misallocation (Hsieh and Klenow (2009))		
– MPK dispersion	1.3	0.1
Extensive margin misallocation		
– MPK dispersion	1.4	0.1

Note: Δ stands for percentage deviations from the benchmark values, while MPK denotes marginal productivity of capital.

To grasp an intuition for the above results, it is worth pointing out that relaxing the collateral constraint (6) has an asymmetric impact on formal and informal entrepreneurs, in that the former now have better access to credit, while the latter remain in financial autarky. This asymmetry strengthens the incentive of producing in the formal sector, particularly for those agents that are more productive. In order to see this, note that in the benchmark economy with a higher degree of financial frictions, high-ability entrepreneurs without significant financial wealth choose between operating a small business in the formal sector (because of the existence of collateral constraints) and operating a small business in the informal sector, in which they could profitably escape taxation (because of financial autarky and because the probability of being audited is a function of the amount of capital rented). Thus, if the probability of detection and the fine are small enough, the gains from tax evasion easily overcome the advantages of having access to credit, and therefore it is more likely that these entrepreneurs decide to be informal producers. By contrast, with looser borrowing constraints, highly productive entrepreneurs now choose between operating a small informal business, and operating a relatively large formal business. In this circumstance, the gains from the potential high scale of production can be large enough to induce agents to comply with business regulations while starting up their own enterprises. This feature is apparent in Figure 3, which shows that highly productive entrepreneurs in the counterfactual economy open their businesses directly in the formal sector.[25] Given this property, we find that reducing financial frictions has quantitatively important implications for the life-cycle dynamics of firms, with the share of enterprises that never become informal moving from 0 in the benchmark economy to 26% in the counterfactual one as shown in Table 3. At the same time, the share of firms that never become formal declines by about 17 percentage points. Moreover, the average yearly growth rate of formal firms increases substantially, while that of firms operating in the informal sector declines.[26] In the aggregate, these effects induce a redistribution of resources from the informal producers to the formal ones (toward the more productive entrepreneurs), which eventually increases total formal production and decreases the informal production.

In addition to the above resources-reallocation mechanism involving the more productive individuals, removing financial frictions also dampens the gains from informal entrepreneurship for relatively low-skilled agents. The reason is the increase in the wage rate (+4.6%) driven by the higher demand for formal labour (see Table 3). Due to this effect, the opportunity cost of entrepreneurship increases, thereby making it less attractive for workers with relatively low managerial abilities to switch occupation and become informal entrepreneurs. This feature is illustrated in Figure 3, which shows that the amount of time in which middle-skilled individuals remain workers in the counterfactual economy is more than twice as long

[25]Interestingly, Figure 3 also shows that these entrepreneurs are able to accumulate enough collateral over the life-cycle to operate at the financially unconstrained scale of production, i.e. the one they would choose with a frictionless financial market.

[26]In Appendix D, we further characterize firm dynamics in the model by assessing how the age-size profile for firms changes once we relax the tightness of the borrowing constraint. The results we get confirm that easing financial frictions has a substantial boosting effect on formal firms' growth. The opposite is true for the informal sector, where with a more efficient financial market firms tend to grow less in comparison with the benchmark scenario. Interestingly, because of this last effect, we show that the impact of a financial reform on firm growth in the entrepreneurial sector is substantially muted when informality is taken into account in the model.

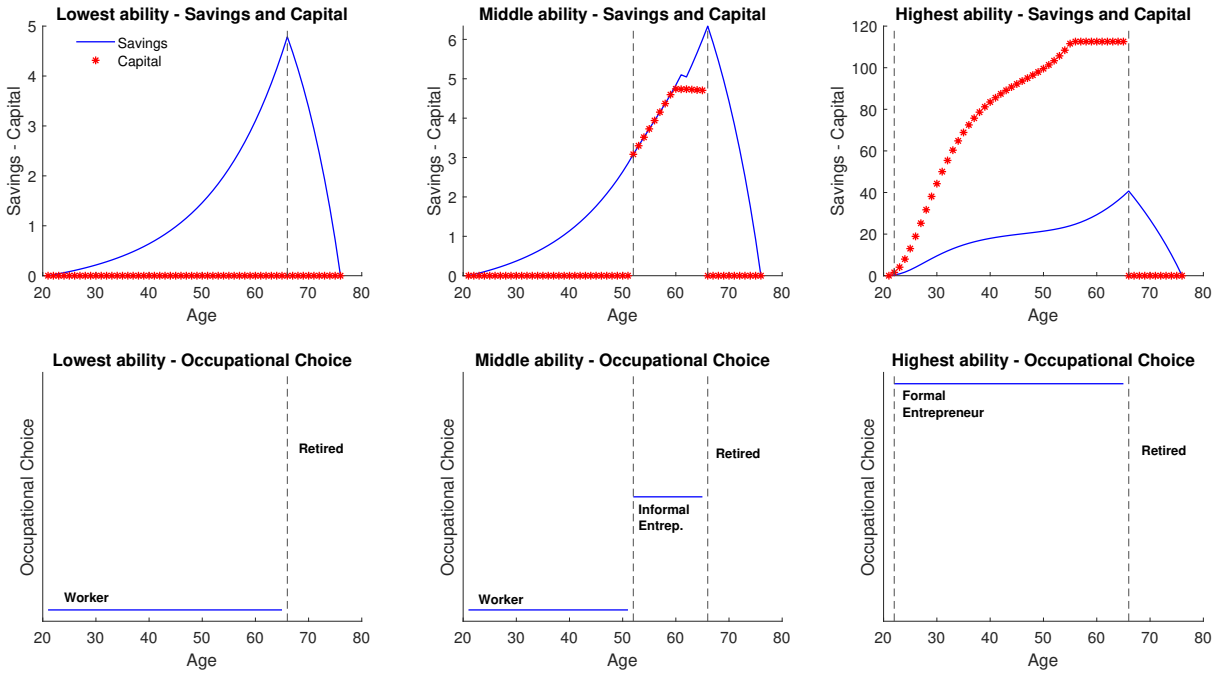

Figure 3: Individual decisions in the counterfactual economy

Notes: The picture shows optimal decisions for agents with the lowest (first column), middle (second column) and highest (third column) abilities. The first row shows the evolution across ages of capital (discontinuous red line) and savings (continuous blue lines). Occupational choices are reported in the second row.

as in the benchmark economy (see Figure 2).[27]

To summarize, there are two main forces behind the decrease in the size of the informal economy after the removal of financial frictions: first, the increase in the potential size of the operating firms that makes it more attractive to operate in the formal sector at a higher scale; and second, the increase in wages that makes it more profitable for many potential informal entrepreneurs to remain workers. The combined effects of these two mechanisms reduce the share of individuals that optimally decide to become informal entrepreneurs, declining from a high of 14.3% of total working age population in the benchmark to a low of 8.8% in the counterfactual economy with a lower degree of financial frictions (see Table 3). Consequently, the percentage of individuals under-reporting income to fiscal authorities declines after relaxing the collateral constraints, inducing a decrease in tax evasion of 35.6%. Table 3 shows that this last effect, jointly with the expansion in the formal production, results in quantitatively important gains for the government, with total fiscal revenues increasing by 13.4%.

[27]It is also interesting to note that easing collateral constraints has no effect on the occupational choice of agents in the lowest-ability group, who again remain workers until retirement. However, the saving behaviour of these individuals in the counterfactual economy is different with regard to the benchmark. The reason is the interest rate, which increases substantially in the economy with less tight borrowing constraints as shown in Table 3. This effect strengthens the agents' incentives to postpone consumption into the future, thus leading to higher savings rates throughout the life-cycle.

Overall, the results of the counterfactual experiment show that relaxing financial frictions is not enough to completely eliminate informality among firms. We find, in fact, that the share of informal firms is still high in the counterfactual economy (49%), implying an aggregate amount of informal production that accounts for about 21% of the official GDP. These firms are run by agents with heterogeneous managerial abilities that are producing at a relatively low scale, and find it optimal to become informal entrepreneurs − rather than workers or formal entrepreneurs − because of tax evasion. In this respect, the results of our experiment suggest that informality among firms can be further reduced only through specific fiscal policy interventions. We will return to this point in Section 4.3.

4.2.2 Official GDP and measured TFP: the amplification effect

One interesting result of the above experiment is that the removal of financial frictions has a huge impact on the Brazilian official GDP (+28%). A related finding is that measured TFP would also increase substantially (+11.2%) as shown in Table 3. There are two main mechanisms behind such results: first, entrepreneurs that were already formal in the benchmark economy increase their productions once we relax the collateral constraint (i.e. intensive margin); and second, resources for informal production − i.e. entrepreneur abilities and productive factors − are partially reallocated to the formal sector (i.e. extensive margin). The first channel is a conventional mechanism that is well known in the literature of occupational choice and financial market imperfections (see, e.g., Buera et al. (2011) and Buera and Shin (2013)). The second is driven by the assumption on imperfect tax enforcement, and represents an unconventional channel that amplifies the effects of the first mechanism. In Table 3, we break down changes in official GDP and measured TFP into the contributions of the extensive and intensive margins to give a sense of the quantitative importance of this second channel. The results reported show that the reallocation of entrepreneurial talent from the informal to the formal sector has a significant impact on measured TFP, explaining 76.4% of the overall variation.[28,29] By contrast, although quantitatively important, the impact is milder in the case of official GDP, where the adjustment occurs mostly at the intensive margin (64.3%).

A more direct way to assess the amplification effect is to compare the results reported in Table 3 with those obtained by means of an alternative version of the model with perfect tax enforcement where, by definition, the informality channel is shut down. Simulated results based on this model are reported in Table 4, which provides statistics for *pre-* (benchmark) and *post*-reform equilibrium (counterfactual).[30] As the table illustrates, the removal of financial frictions in the perfect tax enforcement economy increases GDP by 12.7% and TFP

[28]The statistics reported for the extensive margin refer to the overall contribution, which also includes the effect driven by individuals that were workers in the benchmark equilibrium and become formal entrepreneurs once we relax the borrowing constraints. We find that the importance of this component in driving changes in GDP and TFP is quantitatively negligible. Therefore, all of the adjustments that occur at the extensive margin can be virtually imputed to entrepreneurs that move from the informal to the formal sector.

[29]The importance of the extensive margin to explain changes in TFP is in line with the findings of Buera et al. (2011) and Midrigan and Xu (2014).

[30]The equilibrium in the economy with perfect tax enforcement is computed from the benchmark model by setting the penalty surcharge factor $s = \infty$. To make the results comparable, parameter λ has also been adjusted so that steady state credit-to-GDP ratio in the *pre-* and *post*-reform equilibrium coincide with those

by 3.4%. In comparison with the benchmark results, we can then conclude that firms informality substantially amplifies the effect driven by a looser collateral constraint as we find that (i) the increase in official GDP in the benchmark model is more than twice as large as its counterpart with perfect tax enforcement, and (ii) TFP gains from removing financial frictions are more than three times higher when informality is accounted for in the model.

It is interesting to further investigate sources of the amplification effect on TFP. To this end, we follow Hsieh and Klenow (2009) by measuring the dispersion in the marginal product of capital (MPK) across firms in the model with and without informality. The statistics reported are computed across all active entrepreneurs (static misallocation) as well as across occupational switchers (extensive margin misallocation).[31] As Tables 3 and 4 illustrate, there is substantial more MPK dispersion in the model with informality in the *pre*-reform equilibrium (benchmark), meaning that distortions induced by financial frictions on the allocation of capital across active firms − as well as on the allocation of entrepreneurial talent − are amplified by the presence of informality. As a result, the potential gains from a more efficient allocation of resources resulting from a financial market reform are larger when firms can also operate in the informal sector. This explains why the impact of easing financial frictions on measured TFP, and as a consequence on official GDP, is magnified when informality is accounted for in the model.

4.2.3 Wealth distribution

As is well known, the removal of financial frictions has important effects in terms of wealth redistribution in models with endogenous occupational choices (see e.g., Quadrini (2000)). Table 3 shows that this property holds true in our framework, where financial wealth held by workers moves from a low of 13.1% of total wealth in the benchmark to a high of 49.5% in the counterfactual economy. The table further illustrates that this effect is mostly driven by a reallocation of resources from formal entrepreneurs to workers, given that relaxing collateral constraints has a much milder impact on the share of wealth held by informal producers. Thanks to this feature, we find that entrepreneurs hold 50.5% of total wealth in the counterfactual economy, a value which is about 19 percentage points higher than its counterpart in the model with complete tax enforcement (see the second column of Table 4). Hence, our model predicts that firms' informality actually dampens the overall wealth redistribution effect induced by a reduction in financial frictions.

The reason for this finding lies in the mechanism that strengthens − through the marginal productivity of capital − the incentive of financially constrained entrepreneurs to accumulate wealth. While milder for formal entrepreneurs with better capital markets, this mechanism still holds true for informal entrepreneurs, as they have no access to credit regardless of the degree of financial market imperfections. Compared to the unconstrained entrepreneurs, these individuals accumulate financial wealth at a higher rate to self-finance capital, a mechanism that is further amplified by the increase in the interest rate (see Table 3).

reported in Table 3. All of the remaining parameters are kept fixed to their calibrated values.

[31] We compute MPK dispersion as the variance of $log(MPK)$. Extensive margin misallocation in the benchmark refers to the distribution of MPK across all individuals that were entrepreneurs in the *pre*-reform equilibrium and switch sector or occupation in the *post*-reform equilibrium. We follow the same strategy to compute extensive margin misallocation in the counterfactual economy.

Table 4: Experiment Results. Model with perfect tax enforcement

	Benchmark	**Counterfactual**
Credit-to-GDP ratio	0.41	1.43
ΔGDP		12.7%
ΔCapital		19.0%
ΔWage		4.6%
ΔTFP		3.4%
Total workers (% Population)	80.0%	85.9%
Total entrepreneurs (% Pop.)	20.0%	14.1%
Fiscal Pressure (% Total Income)	31.5%	31.0%
Wealth distribution		
– Workers (% Total Wealth)	14.4%	68.5%
– Entrepreneurs (% Total Wealth)	85.6%	31.5%
Firm dynamics		
– Average firm growth	5.0%	2.4%
– Entry rate	6.4%	2.4%
Static misallocation (Hsieh and Klenow (2009))		
– MPK dispersion	0.9	0.0
Extensive margin misallocation		
– MPK dispersion	0.8	0.1

Note: Δ stands for percentage deviations from the benchmark values.

4.3 Development and financial frictions

A large strand of literature has highlighted the important role of financial markets in economic development (see, e.g., Buera et al. (2011), Buera and Shin (2013) and Allub and Erosa (2019)). This section analyses this linkage through the lens of our model to assess, in particular, the role of informality. The model's predictions are also tested by means of cross-country data on macro aggregates and financial market reforms episodes. The section concludes with a discussion on the potential policy implications of our findings.

4.3.1 Predictions of the model

Figure 4 illustrates how key macro aggregates in the model respond to changes in the degree of financial frictions. The reported results are computed from the steady state equilibrium for different values of the parameter controlling for the tightness of collateral constraints, λ, while keeping all of the remaining parameters fixed at their calibrated values. We use 16 values of λ ranging from 1 (financial autarky) to ∞ (perfect credit markets). Since λ does not have a direct empirical counterpart, the results are presented in a way that lends itself to data comparisons. Specifically, we plot steady state statistics against the implied credit-to-GDP ratio, which is monotonically increasing in λ and observable in the data.

Two main results are worth emphasizing. First, as in Buera and Shin (2013), we find

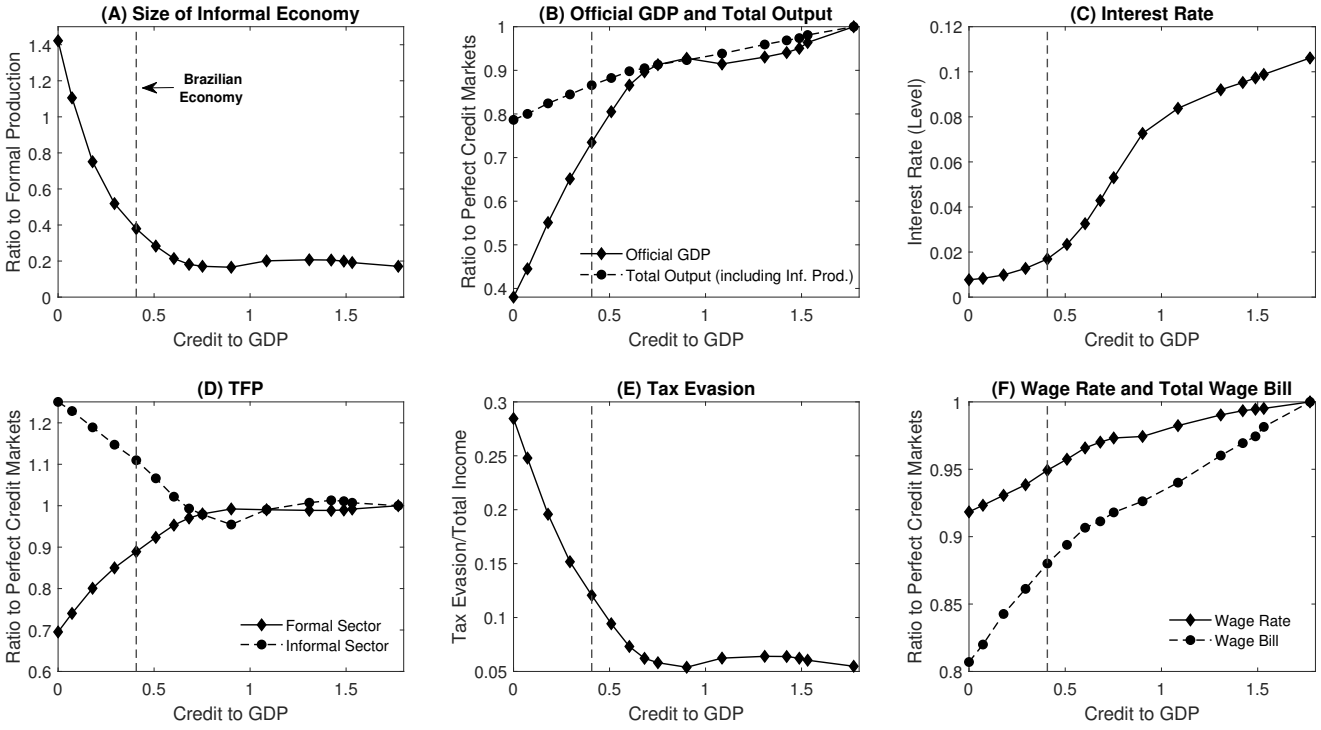

Figure 4: Development and financial frictions in the long-run

that financial frictions may entail quantitatively important costs in terms of both GDP and TFP losses. In our model, further tightening collateral constraints may bring down official GDP and measured TFP to, respectively, 38% and 69.5% of their counterpart levels with perfect credit markets.[32] Second, consistently with the data, the model predicts a negative correlation between financial development (as measured by the credit-to-GDP ratio) and informality. As we move from financial autarky to perfect credit, the size of the informal economy declines, moving from a maximum of 142% of official GDP to a minimum of around 20%. This pattern is mirrored by the response of tax evasion, which also decreases when we relax the collateral constraints, declining from a high of 28.5% of total output in the pure self-financing economy to a low of 5.5% with perfect credit.

The above results complement the findings of Antunes and Cavalcanti (2007) and Quintin (2008), who also report a negative correlation between firms' informality and financial development in models with imperfect contract enforcement. In terms of policy implications, however, our findings extend the existing literature by showing that the role of lowering financial frictions in reducing informality and tax evasion is only relevant (and is also extremely important in quantitative terms) at high levels of credit market imperfections. As Figure 4 illustrates, there is a non-linear relationship between financial development and informality: easing the collateral constraints has a significant impact on the size of the informal

[32]In the picture, figures for GDP and TFP, as well as those for wage rate and total wage bill, are normalized by the perfect-credit level and can therefore be read as a measure of costs (or gains) induced by financial frictions.

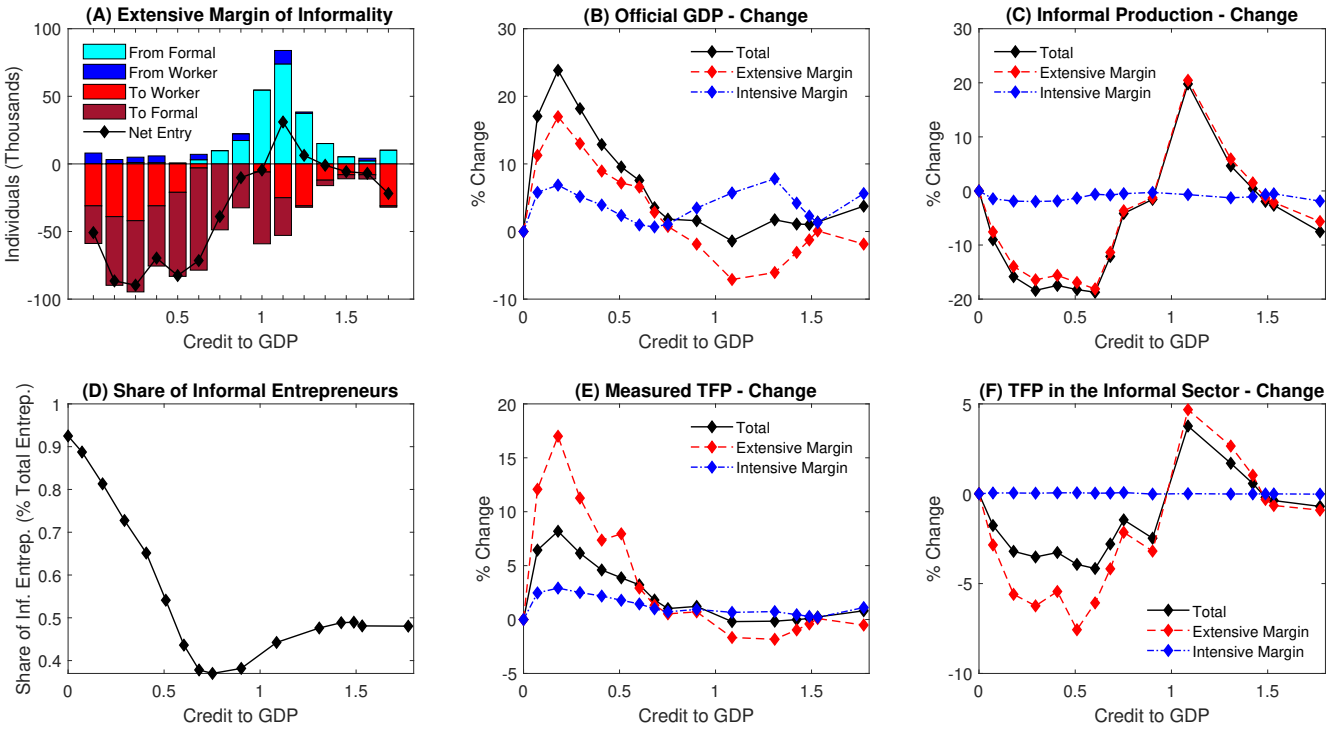

Figure 5: Development and financial frictions in the long-run: changes in the composition of the economy

Notes: In Panel A, bars refer to the number of individuals that, in each occupational category, have moved into or from the informal sector after the borrowing constraints were relaxed. A positive bar represents entry and a negative bar exit. The continuous line refers to net flows, i.e. entry - exit. In Panels E and F, the reported results are computed according to the breakdown $TFP = \omega_{in} TFP_{in} + \omega_{ex} TFP_{ex}$, where TFP_{in} and TFP_{ex} respectively denote aggregate TFP across incumbent firms (intensive margin) and across new entrants (extensive margin), while ω_{in} and ω_{ex} are weights reflecting the allocation of resources in the two margins. In the graph, the black line (i.e. total) refers to the overall adjustment of TFP (i.e. including the effects of ω_{in} and ω_{ex}), while the blue and the red lines show changes in TFP_{in} and TFP_{ex}, respectively.

economy for countries with credit-to-GDP ratios below around 60%, while no quantitatively important effect is found above this threshold. As a consequence of this feature, we also find a non-linear effect of financial frictions on tax evasion, official GDP and TFP, with the implication that poor countries may gain substantially from financial market reforms both in terms of growth in the official GDP and in fiscal revenues (because of the significant impact on tax evasion).

To shed light on the mechanisms driving the above non-linear effects, in Figure 5 we show, among other variables, the pattern of net entry into the informal sector (extensive margin of informality) across the stages of financial development, along with the breakdown of entry/exit flows into types of occupational switch.[33] Analyzing these flows is particu-

[33]For a given λ, we first compute the cross-sectional distribution of individuals across occupational status. Then, we increase parameter λ to assess how the distribution changes in the new equilibrium with a lower degree of financial frictions. This allows us to identify flows of individuals that, compared to the previous

larly useful, because it provides information on the role of three different types of informal firms that, in our model, coexist in equilibrium because of the heterogeneity in managerial abilities. The first type consists of enterprises that are run by highly productive but poor entrepreneurs, who stay out of formality because of the collateral constraints.[34] A second group involves the so-called *parasite* firms, which are productive enough to produce formally but decide to remain informal to take advantage of the higher earnings induced by tax evasion. Finally, the third group is characterized by firms owned by low-skilled agents who are too unproductive to ever become formal entrepreneurs.[35] With these different types of firms, relaxing the collateral constraints produces two countervailing effects on informal production. On the one hand, easing financial frictions brings firms out of informality, because (i) highly productive entrepreneurs have greater access to credit, so they can produce formally at a profitable scale; and (ii) given that the wage rate increases, low-skilled individuals exit from informality to become workers. On the other hand, the induced higher production costs[36] also increase the relative gains from tax evasion, thereby bringing more *parasite* firms into the informal sector.[37]

As Panel A of Figure 5 illustrates, the first effect drives substantial exit from the informal sector at a high level of financial frictions, where an increase in credit availability boosts the gains from the formalization of firms owned by talented entrepreneurs. By moving to the formal sector, these entrepreneurs operate at a larger scale, thereby inducing in the aggregate a sizeable increase in the official GDP and a quick decline in the size of the informal sector, together with a substantial growth in measured TFP. The importance of this effect is apparent in Figure 5, where we break down variations in official GDP and informal production into contributions of the extensive (i.e. changes due to occupational switchers) and intensive margins (i.e. changes due to existing entrepreneurs). At a low level of financial development, the extensive margin explains virtually all of the decline in the informal production and most of the increase in the official GDP. In this last case, moreover, the extensive margin moves in step with the intensive margin, thereby amplifying the response of the official GDP to an increase in parameter λ.[38]

The strength of this scale effect, however, decreases as the credit-to-GDP ratio becomes larger. With more developed financial markets, highly productive entrepreneurs become less financially constrained and, hence, informal production is mostly carried out by small *parasite* and *survival* firms. In this respect, note that an increase in parameter λ at higher levels of

equilibrium, have moved into and from the informal sector. We use the same technique to break down variations in output and TFP into contributions of the extensive and intensive margins.

[34]This type of firms can be somehow related to the so-called De Soto's view, which refers to the informal sector *as an untapped reservoir of entrepreneurial energy, held back by government regulations*. Evidence in this regard is provided by Dabla-Norris, Gradstein and Inchauste (2008), who document that financial market development and the quality of the legal system are among the determinants of firms' informality in developing countries.

[35]The literature usually refers to this case as *survival* firms.

[36]In this respect, Figure 4 shows that the interest rate and the wage rate increase in equilibrium by easing the collateral constraint.

[37]This feature is also apparent in Figure 3, which shows that agents in the middle-ability group never engage in formal entrepreneurship once the borrowing constraints are relaxed. For these individuals, the gains from having a better access to credit are offset by the increase in production costs. Consequently, they prefer to produce informally at a much lower scale and take advantage of tax evasion.

[38]A similar pattern characterizes changes in TFP in both formal and informal sectors.

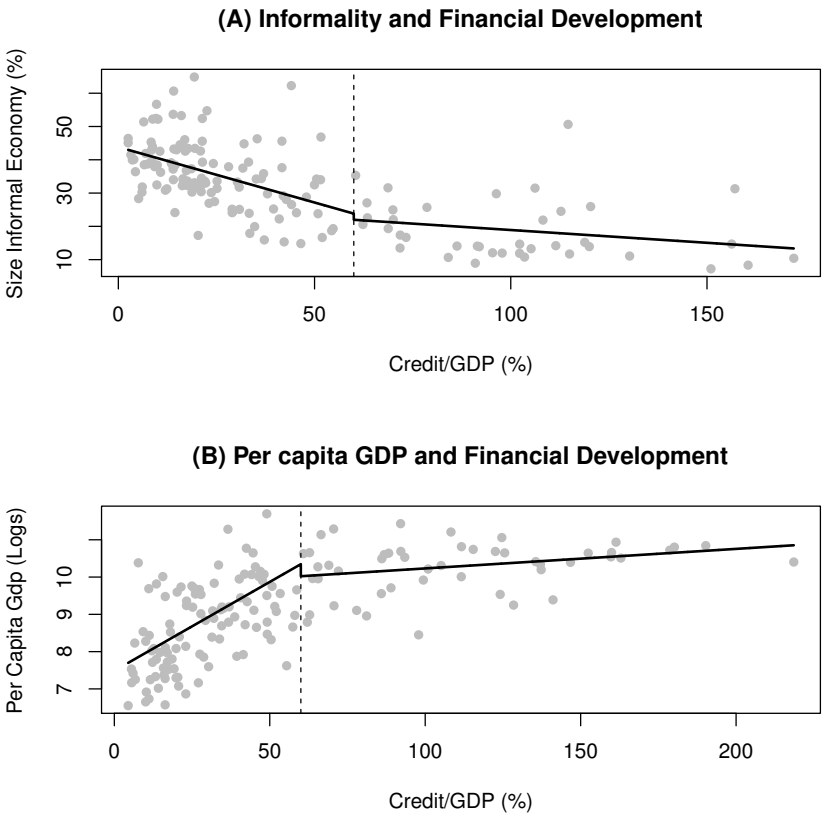

Figure 6: Testing for the non-linear effects of financial market frictions

Notes: The grey dots represent average values for cross-country data over the 1991-2015 period. Regression lines are depicted with solid lines, while the vertical dashed line refers to the threshold level for the credit-to-GDP ratio of 60.8%.

financial development (i.e. starting around a credit-to-GDP ratio of 60%) induces flows of entrepreneurs from the formal to the informal sector as shown in Figure 5. These are clearly *parasite* firms that in the new equilibrium move to the informal sector because the increased production costs strengthens the gains from tax evasion.[39] As a result, production resources are partially reallocated from the formal sector to the informal sector. In addition, the exit of firms from the formal sector also implies that the adjustment at the extensive margin in formal production partially compensates for the intensive margin, thereby dampening the increase in the official GDP induced by a lower degree of financial frictions. In these economies, therefore, further relaxing the collateral constraints has little impact on the size of the informal sector. This explains why we find a threshold level for the credit to GDP ratio above which increasing λ has a much smaller effect on informality and official GDP.

[39]Depending on the stage of financial development, this effect can be strong enough to trigger a positive net entry of informal firms. This feature explains why we find a range of values for the credit-to-GDP ratio where an increase in λ leads to a larger share of informal entrepreneurs and to an expansion in aggregate informal production (see Panels C and D of Figure 5).

Table 5: Testing for the non-linear effects. Regression coefficients.

Dependent variable:	**(A) Size of informal sector**	**(B) GDP per capita (logs)**
constant	24.48^{***}	9.86^{***}
	(4.10)	(0.27)
credit	-0.06	0.00
	(0.04)	(0.00)
credit* dummy	-0.26^{**}	0.04^{***}
	(0.08)	(0.01)
dummy	19.01^{***}	-2.34^{***}
	(4.30)	(0.32)
R^2	0.48	0.53
Adj. R^2	0.47	0.52
Num. obs.	153	153
RMSE	9.12	0.87

Note: Standard Errors in parenthesis below the coefficients. Asterisks denote significance levels (i.e. $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$). Coefficients are estimated with OLS using sample averages over the 1991-2015 periods.

4.3.2 Empirical evidence

We test the empirical plausibility of the above non-linear effects by running the following cross-country regression

$$y_i = \gamma_0 + \gamma_1 D_i + \gamma_2 credit_i + \gamma_3 D_i credit_i + \epsilon_i \tag{11}$$

where y_i denotes either the ratio of informal to formal production or the logs of per capita GDP, $credit_i$ stands for credit-to-GDP ratio, and D_i is a dummy variable that takes value 1 if $credit_i$ of country i is below a certain threshold. Data for the size of informal production are taken from Medina and Schneider (2018), while the source for GDP per capita and credit-to-GDP ratio is the financial structure database of the World Bank.[40] The dummy variable and its interaction with *credit* in equation (11) allow regression coefficients to differ across regions characterized by countries with advanced and less developed financial markets. The threshold level that distinguishes the two regions is set to the minimum value for the average credit-to-GDP ratio over the period 1991-2015 among advanced economies, which corresponds to the Belgian economy with an average ratio of 60.8%.[41]

Estimated results are reported in Figure 6, which displays scatter plots of the data along with the regression lines, and in Table 5, which provides regression coefficients and other

[40] We use *private credit by deposit money banks and other financial institutions over GDP* (code: pcrdbofgdp) as a measure of the credit-to-GDP ratio in each country. More details on data sources and further analysis are provided in technical Appendix G.

[41] The value 60.8% also corresponds to the 75th percentile of the cross-country distribution of 1991-2015 averages for the credit-to-GDP ratio among all countries. In Appendix G.2, we show that our empirical results still hold when we change the threshold value for the 1991-2015 period average credit-to-GDP ratio from 60.8% (minimum value among advanced economies) to 86% (median value among advanced economies).

statistics. We find that the coefficients for the interaction term are statistically significant in both regressions. The sign of these two coefficients is also coherent with the predictions of the model: negative when informality is included in the regression as a dependent variable and positive in the case of per capita GDP. These findings, and the fact that the coefficients on *credit* are not statistically significant, imply that, in the data, a higher credit-to-GDP ratio is associated with lower informally and a higher GDP per capita only in countries with less developed financial markets. As nicely apparent in Figure 6, this result is clearly consistent with a non-linear effect of financial frictions.

We next complement our empirical analysis with difference-in-difference regressions which allow us to control for possible endogeneity between the size of the informal economy and the quantity of credit given to the private sector, hence addressing an important limitation of the regression model given by Equation (11). To this end, we use data on financial reforms from Abiad et al. (2010), and estimate the following difference-in-difference model to evaluate the impact of credit reforms on informality across countries during the period 1991-2005:

$$\Delta y_i = \beta_0 + \beta_1 CreditReform_i + \beta_2 \Delta controls_i + \epsilon_i. \tag{12}$$

In the above specification, Δ denotes changes over our period of study, namely 1991-2005, which is the longest period of overlap between our different data sources.[42] As in Equation (11), y_i is the ratio of informal to formal production. $CreditReform$ is an indicator variable which is equal to 1 if country i experienced a credit reform during the 1991-2005 period, and to 0 otherwise. We define credit reforms as positive changes over the 1991-2005 period in the credit controls index taken from the Financial Reforms database of Abiad et al. (2010) described in great detail in technical Appendix G. Finally, $Controls$ denotes the set of control variables, if any are included. We include two control variables for our full sample estimations, namely $Credit$, which is the credit-to-GDP ratio, and $FiscalPressure$ which is the ratio of government tax revenues-to-GDP and is meant to capture the impact of tax policy on informality.

Our empirical methodology consists of assessing whether credit reforms, which we view as exogenous to the size of informal production, are on average associated with reductions in the relative size of informal production, and how this relationship differs for countries at different levels of financial development.[43] We answer the first question by estimating regression Equation (12) on our full sample. Results are reported in Table 6. We answer the second question by estimating regression Equation (12) on subsets of countries, by dividing our full sample above and below specified threshold values for the credit-to-GDP ratio in the pre-treatment year, namely 1991. In Table 7, we report results for three specified threshold values for the private credit-to-GDP ratio in 1991, namely the median 1991 private credit-to-GDP ratio among developing countries (19 percent), the mean 1991 private credit-to-GDP

[42] Regression model (12) produces identical results as the standard difference-in-difference specification $y_{i,t} = constant + \gamma_i + \beta_0 PostTreatment_t + \beta_1 did_{i,t} + \beta_2 controls_{i,t}$. In this alternative specification, γ_i would be the country's fixed effect. $PostTreatment_t$ would be a time indicator taking on value 0 if t is the pre-treatment year, namely year 1991, and taking on value 1 if t is the post-treatment year, namely year 2005. The difference-in-difference variable $did_{i,t}$ would be an indicator taking on value 1 if two conditions are satisfied: country i experienced a credit reform during the 1991-2005 period; and year t would be year 2005.

[43] In Appendix G.4, we provide a formal test for the exogeneity of credit reforms.

Table 6: Estimation Results (Full Sample)

Dependent Variable Δy	(1)	(2)	(3)	(4)
$CreditReform$	-2.680***	-2.845***	-2.733***	-2.760***
	(0.703)	(0.648)	(0.606)	(0.606)
$\Delta credit$		0.0138		-0.003
		(0.0158)		(0.011)
$\Delta FiscalPressure$			0.150**	0.151**
			(0.066)	(0.067)
Constant	-2.483***	-2.818***	-3.031***	-2.967***
	(0.408)	(0.420)	(0.382)	(0.445)
R^2	0.143	0.222	0.270	0.271
Adj R^2	0.133	0.200	0.248	0.237
Num. obs.	86	73	68	68
Num. Reforms	45	37	34	34
RMSE	3.312	2.973	2.503	2.521

Note: Standard Errors are in parenthesis below the coefficients. Asterisks denote significance levels (i.e. $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$). Num. Reforms is the number of countries which have undergone a credit reform defined as a positive overall change in the credit controls index between 1991 and 2005.

ratio among all countries (43 percent), and the threshold value identified in the previous regression analysis (61 percent), which is also roughly the 90th percentile value for the 1991 private credit-to-GDP ratio among developing countries. Due to the relatively small sizes of the subsamples, we do not include control variables in the estimations reported in Table 7.

Our difference-in-difference analysis corroborates our earlier empirical finding that greater access to credit lessens informality for initially low levels of financial development. We find the regression coefficient for $CreditReform$ in our full sample estimation to be negative and statistically significant at the 1 percent level (Column (1) of Table 6). This result is robust to the inclusion of $FiscalPressure$ and $Credit$ as control variables (Columns (2), (3), and (4) of Table 6). We interpret this finding to imply that for the average country in our sample with an initial credit-to-GDP ratio of 28 percent in 1991, credit reforms alone over the 1991-2005 period contributed to reducing the ratio of informal-to-formal production by about 3 percentage points.[44] Estimation results for the different subsamples of countries (Table 7) show that for each of the three specified threshold values, the impact of credit reforms on informality is significant and stronger for the pool of countries with a 1991 value for the credit-to-GDP ratio that is below the specified threshold, compared to the impact estimated for countries above the threshold. Moreover, reading the table form left to right, and focusing on the first, third, and fifth columns, it is nicely apparent that the impact of credit reforms on informality among the least financially developed countries (or countries

[44] Additional regression results presented in technical Appendix G.3 show that these findings are robust to a more stringent definition of credit reforms, and to the exclusion of potential outliers.

Table 7: Results for Subsamples Above and Below Initial Credit Thresholds

Initial Credit Threshold	19 Percent		43 Percent		61 Percent	
Dependent Variable Δy	Below 19	Above 19	Below 43	Above 43	Below 61	Above 61
$CreditReform$	-3.384**	-2.889***	-3.211***	-3.007***	-3.002***	-2.683**
	(1.638)	(0.925)	(1.045)	(0.772)	(0.856)	(0.962)
Constant	-2.135	-2.650***	-2.261***	-2.728***	-2.569***	-2.499***
	(1.464)	(0.336)	(0.782)	(0.438)	(0.592)	(0.555)
Num. Obs.	27	47	44	30	54	20
Num. Reforms	19	18	29	8	33	4
R^2	0.173	0.224	0.165	0.324	0.174	0.224
Adj. R^2	0.140	0.207	0.145	0.299	0.158	0.181
RMSE	3.508	2.672	3.510	1.990	3.252	2.108

Note: Standard Errors in parenthesis below the coefficients. Asterisks denote significance levels (i.e. $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$). Num. Reforms is the number of countries that have undergone a credit reform defined as a positive overall change in the credit controls index between 1990 and 2005.

with 1991 credit-to-GDP values that are below the specified threshold values) softens as we consider a larger threshold value for the credit-to-GDP ratio in 1991. In other words, the inclusion of more financially developed countries into the pool of countries below a given threshold value for the initial credit-to-GDP ratio lowers the average impact of credit reforms on informality. We interpret these findings as suggestive evidence that credit reforms are more effective at reducing informality in the least financially developed countries.

4.3.3 Policy implications

One of the main conclusions that can be drawn from the above analysis is that developing countries may effectively fight against informality and tax evasion by implementing reforms aimed at improving access to credit for firms. This conclusion is further corroborated by Figure 7, where, for economies at different stages of financial development, the *long-term* gains from the removal of collateral requirements for formal firms are compared to those resulting from a policy that introduces a flat tax on personal income (i.e. $\tau = 0$). As the figure illustrates, the removal of financial frictions in economies with very tight borrowing constraints dominates the flat tax policy not only in terms of lower informality and tax evasion, but also in terms of higher official GDP, measured TFP and fiscal revenues, while these results are generally reversed in advanced economies, where the role of financial frictions becomes of secondary importance.

Another interesting finding of the above experiment is that the lower the level of financial development in the country, the higher the gains are from adopting a tax flat. As an intuition for this result, note that a decrease in tax progressivity results in lower income taxation for the highly productive individuals. In our model, this affects the economy through two main margins. First, the scale of more productive firms already operating in the formal sector

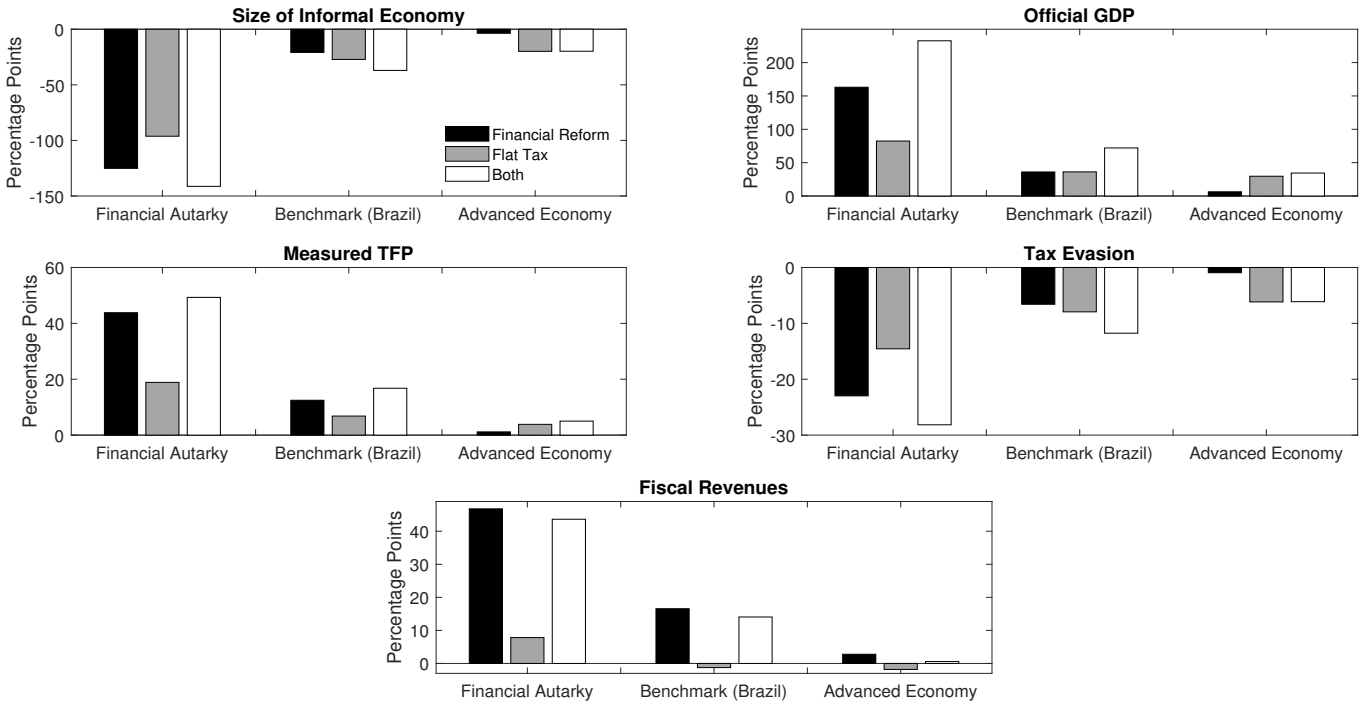

Figure 7: The effects of a flat-tax reform for different degree of financial frictions.

Notes: Results are computed for 3 economies at different stages of financial development: financial autarky ($\lambda = 1$), Brazil ($\lambda = 1.37$) and advanced economy ($\lambda = 4.7$). All of the remaining parameters remain fixed to their calibrated values. In each economy, results from a flat-tax reform are obtained by setting parameter $\tau = 0$, while the effects of financial reform are computed from an alternative version of the model with perfect credit. Bars refer to percentage deviations with respect to the pre-policy steady state. For comparison, the picture also reports results when both policies are taken simultaneously (white bars).

increases (*intensive margin*). Second, the relative gains from tax evasion decline for the richer individuals, thereby pushing talented agents that run informal firms to switch occupation and become formal entrepreneurs (*extensive margin*). The impact of this latter margin is magnified when the policy is adopted in the presence of financial frictions, given that, as we have seen in the previous section, high-ability entrepreneurs with insufficient wealth decide to operate in the informal sector when borrowing constraints are tight. By contrast, the effect of flat-tax reform in more developed financial markets is entirely driven by the intensive margin, as the more talented entrepreneurs do not need to operate informally to overcome the borrowing constraints. This is why we find that the gains of a flat tax are amplified when the financial markets are less developed.

5 Conclusions

In this paper, we build a model of occupational choice with progressive income taxation and informal production, in which informal entrepreneurs have no access to credit and face an endogenous probability of detection by fiscal authorities. We calibrate our model to the Brazilian economy and evaluate the long-run impact of a policy intervention oriented at improving the functioning of the financial market in this country. We find that easing credit market imperfections triggers a significant drop in the size of the informal economy (from 37.9% to 20.7% of GDP) and in tax evasion (-35.6%), together with an important increase in the official GDP (+28%), in fiscal revenues (+13.4%) and in measured TFP (+11.2%).

We then assess the implications of removing financial frictions for any level of financial development and find a non-linear relation between credit-to-GDP ratio and, both the size of the informal economy and official GDP. In particular, we find that, beyond a specific threshold value of the credit-to-GDP ratio, easing financial frictions has a more limited effect on the size of informality and GDP. We test the model's predictions with cross-country data on financial reform episodes, reporting supporting evidence in favour of both non-linearities. These empirical results document a new stylized fact: GDP per capita and financial development across-countries are related in a non-linear fashion. The predictions of our model show that accounting for informality is crucial to explain this feature of the data.

In terms of policy, the main message of our analysis is clear: the role of lowering credit frictions in reducing informality and tax evasion is only relevant at high levels of credit market imperfections. In these circumstances, the gains in terms of GDP per capita of financial market reforms are amplified by firms moving from the informal to the formal sector. This argument is further supported by the results of alternative policy intervention, where we evaluate the effect of changing the degree of progressiveness in personal income taxation under different levels of financial development.

In terms of policy advising, however, the theoretical results reported in this paper should be taken with some caution, given that they should first be complemented by a proper welfare analysis that assesses the transitional effects from one policy scenario to the other.[45] We see this task as a potentially interesting avenue for future research. In this respect, it would be ideal to extend our model to an environment where formal firms may also employ informal factors of production, a margin of informality whose empirical relevance has recently been

[45]In our model, a reform aimed at improving the functioning of the financial market has countervailing effects on prices and macro aggregates. On the one hand, the increase in the amount of credit induced by the reform triggers an increase in the production by the formal sector, which provides upward pressure on factor prices (i.e. wage and interest rate) and a positive impact on total GDP. On the other hand, however, higher wages push some informal entrepreneurs to switch occupation and become workers (i.e. extensive margin of informality) and, at the same time, higher factor prices induce active informal entrepreneurs to decrease their production (i.e. intensive margin of informality). Both margins of informality decrease aggregate informal production, with a resulting negative impact on total GDP (and also on factor prices). This last effect counteracts the impact driven by a larger formal production, with an ambiguous net effect on welfare. In this respect, while the results provided in section 4.2 of the paper show that the benefits of the reform overcome the costs in the very long-run (i.e. steady-state), our analysis is instead completely silent on the short- and medium-term effects. As a result, without studying the transition, the results provided in the paper are not informative on whether the reform is welfare enhancing or not.

documented by Ulyssea (2018). At the same time, certain assumptions of the model could be relaxed or modified with the aim of making the theoretical framework more consistent with the data. For example, to simplify the analysis the current version of the model assumes that entrepreneurial productivity is invariant over the life-cycle. Conditionally to a given managerial ability, this assumption implies that occupational switches over time only depend on wealth accumulation, a feature that makes it difficult for the model to match the life-span and exit rate of firms observed in the data. In this respect, the model could be extended by assuming, as in Buera et al. (2011), that in each period the agent has a probability to draw a new entrepreneurial productivity, with the distribution of the latter parameterized to match exactly relevant moments characterizing firm dynamics in the data. Along the same line, the assumption that agents have a deterministic life-span can be relaxed by assuming instead that, in each period, individuals have a survival probability. This modification would help the model to replicate the demographic structure observed in the data.

References

Abiad, Abdul, Enrica Detragiache, and Thierry Tressel, "A New Database of Financial Reforms," *IMF Staff Papers*, June 2010, *57* (2), 281–302.

Adda, Jrme, Christian Dustmann, and Katrien Stevens, "The Career Costs of Children," *Journal of Political Economy*, 2017, *125* (2), 293–337.

Allingham, Michael G. and Agnar Sandmo, "Income Tax Evasion: a Theoretical Analysis," *Journal of Public Economics*, 1972, *1* (3-4), 323–338.

Allub, Lian and Andrés Erosa, "Financial Frictions, Occupational Choice and Economic Inequality," *Journal of Monetary Economics*, 2019, *107* (C), 63–76.

Amaral, Pedro S and Erwan Quintin, "A Competitive Model of the Informal Sector," *Journal of Monetary Economics*, 2006, *53* (7), 1541–1553.

Antunes, Antonio and Tiago Cavalcanti, "Start Up Costs, Limited enforcement, and the Hidden Economy," *European Economic Review*, 2007, *51* (1), 203–224.

___ , ___ , and Anne Villamil, "The Effect of Financial Repression and Enforcement on Entrepreneurship and Economic Development," *Journal of Monetary Economics*, 2008, *55* (2), 278–297.

___ , ___ , and ___ , "The Effects of Credit Subsidies on Development," *Economic Theory*, January 2015, *58* (1), 1–30.

Buera, Francisco, "A Dynamic Model of Entrepreneurship with Borrowing Constraints: Theory and Evidence," *Annals of Finance*, June 2009, *5* (3), 443–464.

___ and Yongseok Shin, "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 2013, *121* (2), 221–272.

___ , **Joseph P Kaboski, and Yongseok Shin**, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 2011, *101* (5), 1964–2002.

Cagetti, Marco and Mariacristina De Nardi, "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 2006, *114* (5), 835–870.

Castaneda, Ana, Javier Diaz-Gimenez, and Jose-Victor Rios-Rull, "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, August 2003, *111* (4), 818–857.

Cavalcanti, Tiago and Marcelo Santos, "(Mis)Allocation Effects of an Overpaid Public Sector," CEPR Discussion Papers 15173, C.E.P.R. Discussion Papers August 2020.

Dabla-Norris, Era and Andrew Feltenstein, "The Underground Economy and its Macroeconomic Consequences," *The journal of policy reform*, 2005, *8* (2), 153–174.

___ , **Mark Gradstein, and Gabriela Inchauste**, "What Causes Firms to Hide Output? The Determinants of Informality," *Journal of development economics*, 2008, *85* (1-2), 1–27.

Davies, James B, Susanna Sandström, Anthony Shorrocks, and Edward N and Wolff, "The level and distribution of global household wealth," *The Economic Journal*, 2011, *121* (551), 223–254.

de Paula, Áureo and Jose A. Scheinkman, "Value-Added Taxes, Chain Effects, and Informality," *American Economic Journal: Macroeconomics*, October 2010, *2* (4), 195–221.

___ **and Jose A Scheinkman**, "The Informal Sector: an Equilibrium Model and Some Empirical Evidence from Brazil," *Review of Income and Wealth*, 2011, *57* (s1), S8–S26.

D'Erasmo, Pablo, "Access to Credit and the Size of the Formal Sector in Brazil," Research Department Publications IDB-WP-404, Inter-American Development Bank, Research Department April 2013.

D'Erasmo, Pablo N and Hernan J Moscoso Boedo, "Financial Structure, Informality and Development," *Journal of Monetary Economics*, 2012, *59* (3), 286–302.

Di Nola, Alessandro, Georgi Kocharkov, Almuth Scholl, and Anna-Mariia Tkhir, "The Aggregate Consequences of Tax Evasion," *Review of Economic Dynamics*, 2020.

Diaz, Juan Jose, Juan Chacaltana, Jamele Rigolini, and Claudia Ruiz, "Pathways to Formalization: Going beyond the Formality Dichotomy," IZA Discussion Papers 11750, Institute of Labor Economics (IZA) August 2018.

Erosa, Andrés, "Financial Intermediation and Occupational Choice in Development," *Review of Economic Dynamics*, 2001, *4* (2), 303–334.

Fajardo, José, José Renato Haas Ornelas, and Aquiles Rocha de Farias, "Estimating Risk Aversion, Risk-Neutral and Real-World Densities using Brazilian Real Currency Options," *Economia Aplicada*, 12 2012, *16*, 567 – 577.

Fernandez-Bastidas, Rocio, "Entrepreneurship and Tax Evasion,," Mimeo, University of Alicante, Department of Economics 2018.

Hassan, Mai and Friedrich Schneider, "Size and Development of the Shadow Economies of 157 Countries Worldwide: Updated and New Measures from 1999 to 2013," *IZA discussion paper*, 2016.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante, "Optimal Tax Progressivity: An Analytical Framework," *The Quarterly Journal of Economics*, 2017, *132* (4), 1693–1754.

Holter, Hans A., Dirk Krueger, and Serhiy Stepanchuk, "How do Tax Progressivity and Household Heterogeneity Affect Laffer Curves?," *Quantitative Economics*, 2019, *10* (4), 1317–1356.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.

Jung, Juergen and Chung Tran, "The Extension of Social Security Coverage in Developing Countries," *Journal of Development Economics*, 2012, *99* (2), 439 – 458.

Kuehn, Zoe, "Tax Rates, Governance, and the Informal Economy in High-Income Countries," *Economic Inquiry*, 2014, *52* (1), 405–430.

La Porta, Rafael and Andrei Shleifer, "Informality and Development," *Journal of Economic Perspectives*, Summer 2014, *28* (3), 109–126.

Leal Ordoñez, Julio Cesar, "Tax Collection, the Informal Sector, and Productivity," *Review of Economic Dynamics*, April 2014, *17* (2), 262–286.

Levenson, Alec R. and William F. Maloney, "The Informal Sector, Firm Dynamics, and Institutional Participation," Policy Research Working Paper Series 1988, The World Bank September 1998.

Lucas, Robert E., "On the Size Distribution of Business Firms," *Bell Journal of Economics*, Autumn 1978, *9* (2), 508–523.

Medina, Leandro and Friedrich Schneider, "Shadow Economies Around the World: What Did We Learn Over the Last 20 Years?," IMF Working Papers 18/17, International Monetary Fund January 2018.

Meh, Cesaire, "Entrepreneurship, Wealth Inequality, and Taxation," *Review of Economic Dynamics*, July 2005, *8* (3), 688–719.

Mendoza, Enrique G., Vincenzo Quadrini, and José Víctor Ríos-Rull, "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, June 2009, *117* (3), 371–416.

Midrigan, Virgiliu and Daniel Yi Xu, "Finance and misallocation: Evidence from plant-level data," *American Economic Review*, 2014, *104* (2), 422–58.

Moll, Benjamin, "Productivity Losses from Financial Frictions: Can Self-financing undo Capital Misallocation?," *American Economic Review*, 2014, *104* (10), 3186–3221.

Orsi, Renzo, Davide Raggi, and Francesco Turino, "Size, Trend, and Policy Implications of the Underground Economy," *Review of Economic Dynamics*, 2014, *17* (3), 417–436.

Paes, Nelson and Noemi Bugarin, "Parãmetros tributários da economia brasileira," *Estudos Economicos (São Paulo)*, 12 2006, *36*, 699 – 720.

Perry, Guillermo E., William F. Maloney, Omar S. Arias, Pablo Fajnzylber, Andrew D. Mason, and Jaime Saavedra-Chanduvi, *Informality : Exit and Exclusion* number 6730. In 'World Bank Publications.', The World Bank, Juni 2007.

Quadrini, Vincenzo, "Entrepreneurship, Saving and Social Mobility," *Review of Economic Dynamics*, January 2000, *3* (1), 1–40.

Quintin, Erwan, "Contract Enforcement and the Size of the Informal Economy," *Economic Theory*, 2008, *37* (3), 395–416.

Ulyssea, Gabriel, "Firms, Informality, and Development: Theory and Evidence from Brazil," *American Economic Review*, August 2018, *108* (8), 2015–47.

Williams, Colin C., Muhammad S. Shahid, and Alvaro Martnez, "Determinants of the Level of Informality of Informal Micro-Enterprises: Some Evidence from the City of Lahore, Pakistan," *World Development*, 2016, *84*, 312 – 325.