The Global Industrial Revolution*

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Abstract

Standards of living were roughly constant before 1800, almost everywhere. In the early nineteenth century, English-speaking countries began a profess off sustained growth that continues today. This rise in living standards—first observed in Great Britain—is now a global phenomenon. Across countries, four regularities mark the development path: (i) reallocation of labor out-of-agriculture, (ii) demographic transition, (iii) rising human capital accumulation, and (iv) a U-shaped female labor force participation pattern. We develop a unified growth model to account for these facts. Two parents allocate time between work and child-rearing, and between a traditional land-based technology and a modern, human-capital–intensive one. A gradual increase in traditional productivity relaxes subsistence constraints, enabling investments in children quality. These investments, in turn, are the mechanism that triggers the transition to modern economic growth. Calibrated to Great Britain (1400–2000), the model captures the salient features of the industrial revolution.

Keywords: Industrial Revolution, Unified Growth Theory, Female Labor Force Participation
JEL codes: J11, J21, O11, O14, O40

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Suppose Galileo were here and we were to show him the world today and try to make him happy, or see what he finds out.

— Richard P. Feynman, 1964.

1 Introduction

The disparities in living standards across the globe are striking. As of 2018, the average individual in the United States enjoyed a level of affluence approximately eleven times greater than that of a typical resident of the African continent. While the deep historical origin of these differences remains passionately debated among economic historians, the proximate cause is clear, and roughly mechanical: sustained growth in income per capita in some regions, contrasted with stagnation in others. This divergence began in the early nineteenth century, initially with English-speaking countries taking off, and has since diffused—unevenly—across cultures and geographies around the globe. This process continues today, as more nations embark on their own transformations, replicating, in varying forms, the transition from stagnation to sustained growth and the patterns associated with this revolution.

Before the era of modern economic growth, differences in living standards across nations varied by a factor of two, perhaps three if one is generous. All citizens of the planet were more equal than dissimilar in terms of material prosperity. This is not to deny the existence of kings, pharaohs, or prime ministers, but such characters are not representative if one is interested in understanding the standards of living of ordinary people.

By the end of the twentieth century, the gap between the richest and poorest regions had reached staggering proportions. However, evidence from recent decades shows that this distance stopped growing and reversed its trend even in the poorest regions of the world. To illustrate this, Figure 1 plots the ratio of GDP per capita between the United States—the productivity frontier since the late 1800s—and the African continent as a whole.[1] By 1950, the gap was already elevenfold. It peaked at sixteenfold in the 1990s. Remarkably, this distance has since collapsed. By 2018, the entire gap created between 1950 and the 1990s had vanished. This reversal is not occurring at the expense of rich nations. Growth miracles are happening often and everywhere. Modern economic growth is not a zero-sum game.

To understand the global transformation from stagnation to sustained economic growth, one must begin with the nature of production before the industrial revolution. For millennia,

[1] Unless otherwise specified, the data is from the Maddison Project Database, available at https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2020.

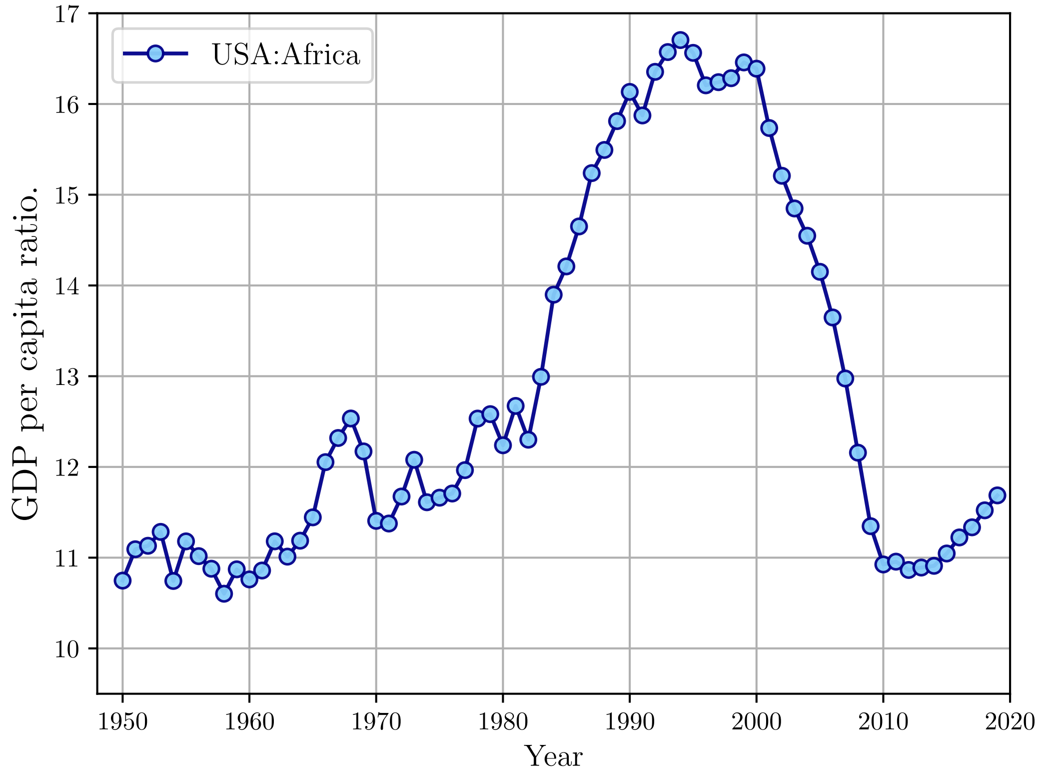

Figure 1: USA: Africa. GDP per capita ratio. 1950–2018.

output was generated through the combination of land and unskilled labor. This mode of production dominated the fate of societies since the advent of agriculture, which marked the transition from nomadic subsistence to settled life. As Diamond (2005) marvelously documents, hunter-gatherers relied on collection, not production. The discovery of farming enabled settlement and the genesis of production and civilization. As societies adopted new crops, irrigation systems, and tools, they built their livelihoods around farming, expanding their capacity to procreate and survive, in line with the population dynamics predicted by Malthus. One reads in the Bible that Joseph's wisdom is anchored in grain management, as he administers an Egyptian settlement with divine insight during seven years of famine. Agriculture was everything for Egypt then, and for some poor countries today, agriculture is still everything.

Pre-industrial, land-based modes of production impose a fundamental constraint on living standards. As productivity gains are repeatedly offset by population growth, living standards remain perpetually near subsistence. For centuries, this stagnation defined the economic condition of societies, with no foreseeable escape from this Malthusian trap. With the arrival of the industrial revolution—the transition from stagnation to sustained growth in living standards, the "Great Escape" in the words of Deaton (2013), continues unfolding globally. The emergence of this new growth regime, and the broad development patterns that accompanied it, are the subject of this paper.

The purpose of this paper is twofold. First, we document the industrial revolution across the world, and four key empirical regularities associated with this process: (i) reallocation of labor out of agriculture, (ii) demographic transition, (iii) rising human capital accumulation, and (iv) a U-shaped female labor force participation pattern. We emphasize that the recent experience of historically poor countries reflects an ongoing broad pattern, as country after country embarks on its own revolution. Thanks to the evidence collected since the 1990s, we show that poor countries are emerging from stagnation, following the same patterns observed in advanced nations more than 150 years ago. In line with Kremer, Willis, and You (2022), we show that the trend observed in the last three decades highlights a fundamental change, as many countries that were at subsistence levels during the second half of the twentieth century began their processes of sustained growth and development.

Second, to think about the choices that brought about the industrial revolution, we develop a unified growth model with parents optimally allocating their time between production and child-rearing. Following Lucas (2002), who builds on Hansen and Prescott (2002) and Becker, Murphy, and Tamura (1990), our model economy produces a single good under two technologies: a traditional sector that uses land and unskilled labor, and a modern sector that is linear in human capital.[2] We extend this framework by introducing intra-household and production specialization in two-parent households. The model combines Becker's (1981) insight on mothers' comparative advantage in child-rearing with Goldin's (1990) observation that early industrial technologies favored physical strength. Men's "brawn" therefore provides a temporary advantage at modest levels of human capital, while modernization ultimately erodes this gap as quality bequests become the engines of growth. These modeling choices are essential to account for the salient facts of the industrial revolution, including the U-shaped female labor force participation.

In the model, the transition from Malthusian stagnation to modern growth does not rely on exogenous progress in the modern sector. Instead, as productivity in the traditional sector begins to grow at a steady exogenous rate, living standards rise slowly as population expands, opening a window for the industrial revolution. The escape from stagnation, however, is not inevitable: it occurs only if productivity gains in the traditional sector are significant enough to offset the effects of land dilution before population dynamics trigger stagnation, albeit at modestly higher income levels. When this condition is met, households can afford

[2] In contrast to Hansen and Prescott (2002), Lucas (2002) endogenizes fertility by incorporating the Beckerian quantity–quality trade-off, as formalized in Becker (1960), Becker and Barro (1988), and Becker et al. (1990).

to reallocate time from production to child-rearing. Once parents begin investing in child quality, modern technology will inevitably become profitable, and the economy will transition to sustained growth.

Modernization proceeds asymmetrically. The parent with a comparative advantage in brawn—reflecting the physical demands of early industrial production—enters the modern sector first. This reallocation raises the marginal product of labor in the traditional sector, temporarily delaying the other parent's transition. As human capital accumulates, the brawn advantage disappears, both parents adopt the modern technology, and the household faces the quantity–quality trade-off, opting for fewer and more educated children. During this transition, mothers temporarily withdraw from the labor force to invest in child quality, reentering once their quality bequests accumulated across generations render the operation of modern technology profitable. This is only possible due to a drop in fertility, as parents do not compromise on quality. These mechanisms reproduce key empirical patterns of the industrial revolution: a shift out of traditional production, declining fertility, rising education, and a U-shaped path of female labor force participation.

The model admits analytical solutions at both ends of the development spectrum—the Malthusian steady state and the modern growth regime—but not during the transition, when production is dual. We therefore compute numerical solutions along the transition path by calibrating the model to Great Britain from 1400 to 2000. Parameters are calibrated to match the evolution of income per capita and the long-run female labor force participation rate. The model reproduces key features of the industrial revolution: the movements out-of-agriculture as countries grow richer, the demographic transition, the acceleration of human capital accumulation, and the U-shaped trajectory of female labor force participation. These outcomes are roughly consistent with both historical time series and cross-country evidence.

Theoretically, our model belongs to the literature of Unified Growth Theory. These theories seek to account for the transition from traditional modes of production toward modern economic growth while addressing the demographic transition witnessed in the development path to riches. Galor (2011) is the best source to understand the nature and scope of these theoretical developments. Besides the motivating facts and theoretical models mentioned above when discussing the building blocks of our theory, this paper is closely related to Galor and Weil (1996), who address the connection between fertility and growth in a model with two-parent households whereby women are exclusively in charge of child-rearing by assumption. A version of their model (section III) also generates the female U-shaped labor force pattern. Still, unlike our paper, their mechanism is not examined quantitatively to address its plausibility.

Another closely related paper is Ngai, Olivetti, and Petrongolo (2024). They propose an adjustment of the U.S. census data regarding female employment between 1870–2019 and show that after adjusting for unpaid family work, female employment displays a U-shaped pattern over time instead of the rising pattern that one obtains from the unadjusted U.S. census data. Then, they propose a structural transformation model where the female role in the marketization of home production generates the U-shaped female labor force participation.

The treatment of the facts presented in this paper is similar to Lucas (2002). Many articles have done similar expositions of the trends discussed here, including Kremer (1993), Parente and Prescott (1993) and Galor (2011), especially the second chapter. More recently, Kremer et al. (2022) presented data on the global trends for income and growth, showing more growth miracles than disasters if one observes the last 25 years of data, and their evidence suggests that poor countries are growing faster *unconditionally*.

The rest of the paper is organized as follows. Section 2 illustrates some facts about the Global Industrial Revolution. Section 3 presents a unified growth theory with two-parent households. Section 4 shows the model calibration and illustrates the main quantitative results of our paper. Section 5 concludes.

2 Facts of the Global Industrial Revolution

During the last 200 years, total world production has grown steadily at about 2.4 percent annually, whereas the population of our planet grew by one percent. These numbers imply a world income per person growing by about 1.3 percent in the last two centuries. Figure 2 plots the world production and population from 1820 to 2018. We use a logarithmic scale to interpret deviations from linearity as accelerations. From Figure 2, one can see production outpacing population since 1820 and accelerating in the aftermath of World War II. One also observes a deceleration in population growth during the last two decades. Whereas the average annual growth rate for world production between 1820 and 1950 was 1.6 percent – and further accelerated significantly since 1950 to about 3.9 percent – the population growth rate went from 0.7 percent between 1820 and 1950 up to 1.6 percent between 1950 and 2018. Since the start of the twenty-first century, output growth experienced a slight deceleration to about 3.6 percent annually, but the population growth also dropped its rate to 1.2 percent. The recent slowdown in population growth suggests that we are starting to reach the stabilization process in population anticipated by demographers and economists.

Figure 3 plots the world income per capita between 1820 and 2018. The world has steadily increased its production per person over the last 200 years, from an income per capita of about 1,100 U.S. 2011 dollars in 1820 to over 15,000 Dollars in 2018. These

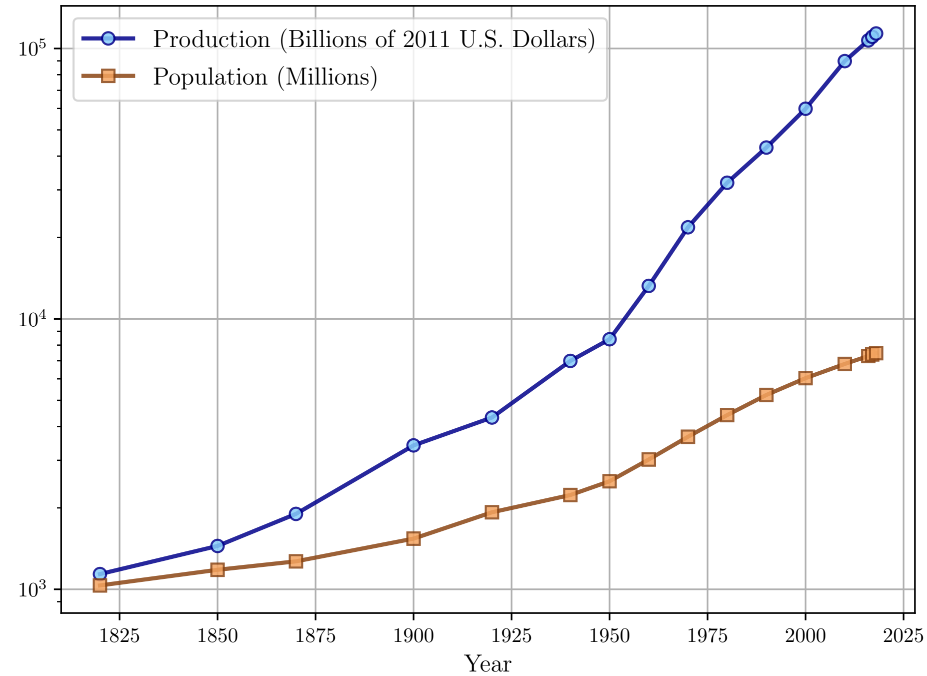

Figure 2: World production and population. 1820–2018.

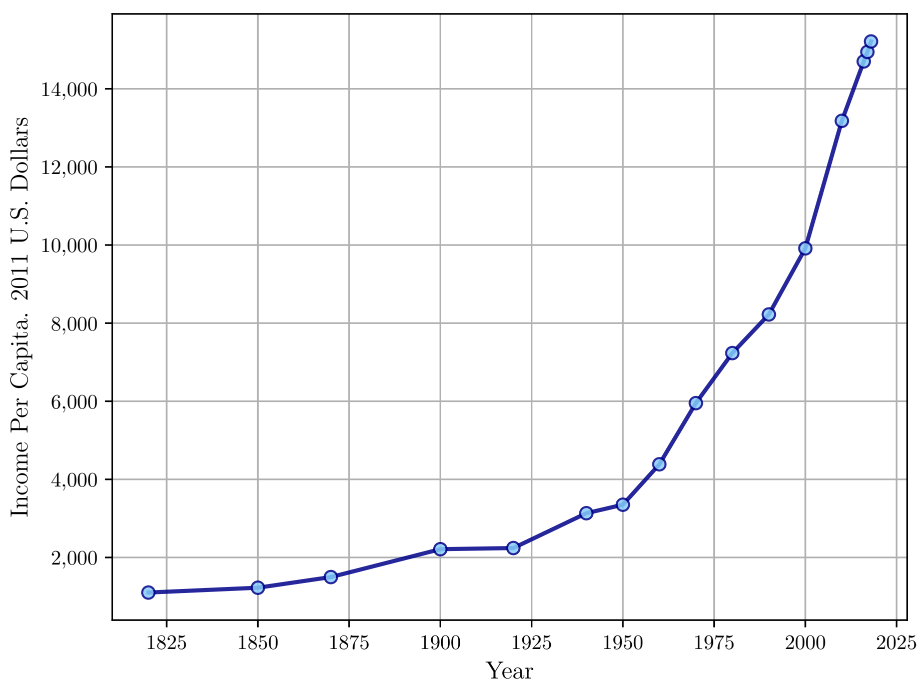

Figure 3: World income per capita (2011 U. S. Dollars). 1820–2018.

numbers translate into a 13-fold increase in the standards of living worldwide. No period in history has witnessed more economic prosperity around the globe. It took 80 years since

1820 for the world to duplicate the production per person. In contrast, income per capita multiplied about 4.5 times during the twentieth century, and between 2000 and 2018, the world witnessed an additional 50 percent increase in living standards.

2.1 The Industrial Revolution in Different Parts of the World

The changes documented above have no precedent in history. Still, one must ask whether a handful of success stories drives these rapid improvements in material prosperity or if the Industrial Revolution – namely, the sustained increase in the standard of living without foreseeable limit – is a global phenomenon. To address this question, Figure 4 plots the evolution of GDP per capita since 1900 for eight groups of countries, ordered by their 2019 income levels. These aggregations follow Lucas (2002), with some notable distinctions. In addition to Japan, we highlight two distinct development experiences: the Asian Tigers (Group III), composed of Hong Kong, Singapore, South Korea, and Taiwan; and China (Group VI). Their extraordinary growth over the last few decades has been nothing short of miraculous, and we pay special attention to them. Finally, we divide the poorest countries into two groups: the rest of Asia (Group VII) and Africa (Group VIII).

- I: U.K., USA, Canada, Australia, and New Zealand.
- II: France, Germany, Netherlands, Scandinavia.
- III: Hong Kong, Singapore, South Korea, Taiwan.
- IV: Japan.
- V: Rest of Western Europe, Latin America, Eastern Europe, Soviet Union
- VI: China.
- VII: Rest of Asia.
- VIII: Africa.

The English-speaking countries (Group I) were the pioneers of the Industrial Revolution. By 1900, they had already established a substantial lead over the rest of the world, with living standards about twice as high as those in much of continental Europe. Since then, they have grown steadily at about two percent per year, maintaining their position at the productivity frontier.

The major continental European economies — France, Germany, the Netherlands, and Scandinavia (Group II) — industrialized soon after the leaders. Their GDP per capita trends

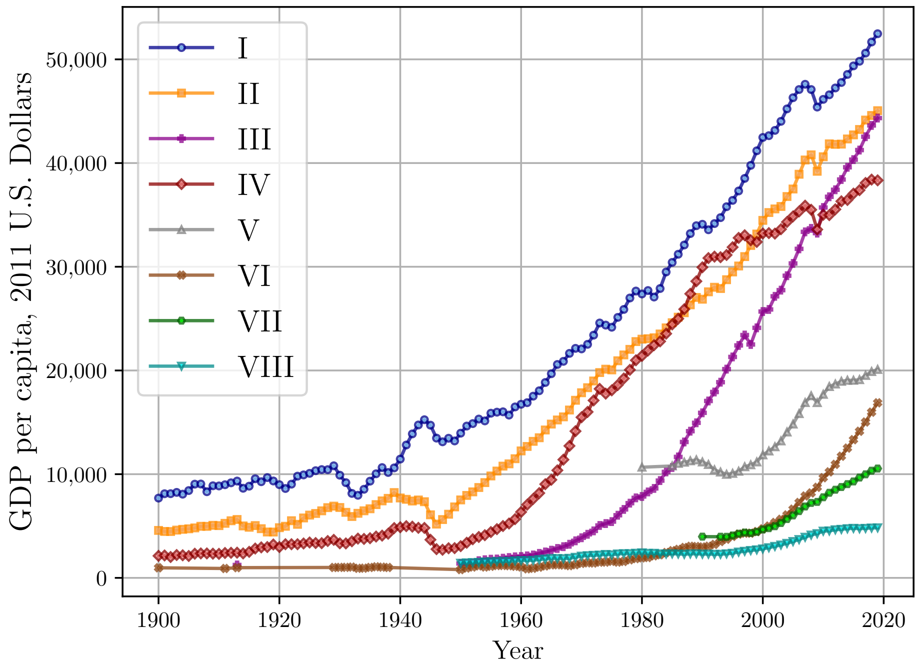

Figure 4: GDP per capita, eight groups. 1900-2018.

are essentially parallel to Group I, with no evidence of stagnation. By the 1960s, they had reached the same standards of living as the leaders, though with a lag of roughly a decade.

The Asian Tigers (Group III) — Hong Kong, Singapore, South Korea, and Taiwan — began their rapid ascent in the second half of the twentieth century. Since the 1980s they have grown at extraordinary rates, converging steadily toward Group I, and by the late 2000s they had already surpassed Japan in income per capita. Their trajectory is often taken as emblematic of latecomer success: fast, sustained growth without evident signs of stagnation.

Japan (Group IV) provides a useful contrast. At the start of the twentieth century, Japan lagged far behind the English-speaking countries. The gap widened until the late 1950s, when Japan entered a period of rapid convergence. By the 1970s and 1980s, Japan had closed much of the distance, but this catch-up came to an end in the mid-1980s. Since then, Japan has stagnated, and the gap with Group I has widened again. The Tigers thus followed a trajectory similar to Japan's, but with a delay of about two decades and with stronger momentum that carried them beyond Japan's income levels.

The countries in Group V — the rest of Western Europe, Latin America, Eastern Europe, and the Soviet Union — followed the leaders with a modest lag. By the 1960s, many had attained standards of living close to Groups I and II, though they have never fully closed the gap. Their growth has been steady but less spectacular, showing no stagnation but also no sustained catch-up beyond the frontier.

China (Group VI) represents yet another latecomer's path. In 1960, China's income per capita was just over 1,000 USD, essentially a subsistence level. Between 1960 and 2000, however, China multiplied its income by nearly five, and between 2000 and 2018 it nearly tripled again. This dramatic acceleration mirrors the earlier trajectories of Japan and the Tigers, but with delays of three to five decades, and at a substantially faster pace than the pioneers in Group I.

The recent take-off in China is similar to the Japanese development experience and the Asian Tigers but with five decades and three decades of delay, respectively: The significant acceleration witnessed during the 2000s, with no parallel in China's modern history, is typical to recent "Asian miracles" portrayed in groups II and III, and substantially *faster* compared to what we know from historical records for the take-off in Group I.

Last, one can observe that the poorest countries of the world, represented here by Group VII (the rest of Asia) and Group VIII (Africa), have already started their Industrial Revolution. By 2018 the Asian countries from Group VII had an income per capita slightly above 10,000; This still represents a substantial development gap relative to Group I, of the order of 5:1. However, in 1950, the distance was 14:1. This is a significant reduction in development gaps within seven decades. The poorest countries today are in Africa (Group VIII): By 2018, they still had a sizable development gap relative to Group I of the order of 10:1, and although this gap is similar to the 1950 levels, between 1960 and 2000 the gap widened up to a ratio of 15:1. The process of "catch-up" for Africa is an ongoing young process that started at the turn of the twentieth century.

Lucas (2002, p. 122) writes, "In group V [groups VII and VIII here] – most of Asia and Africa – the curve has only leveled off, but does anyone doubt that these regions will follow the path that the rest of us have already worn?" So far, this conjecture has been proven right by the evidence of growth and development in the poorest countries during the last 70 years.

Figure 5 plots the European-dominated economies' share of production and population worldwide. The European-dominated economies are represented by groups I, V, and VI, the regions of the world most heavily influenced by European culture. According to Lucas (2002, p. 174) "At the beginning of the twentieth century, many sophisticated observers were persuaded that northern Europeans must have a racial or cultural advantages that accounted for the income differentials that had emerged during the nineteenth century." The shares of income and population where virtually the same in 1750. By 1820, driven mainly by Group I, the "European" production share was 0.41 percent. In contrast, their weight in the worldwide population was about 0.25. This gap continued to widen until 1950, with a negligible interruption in 1920. By 1950, the share of European production was 0.8, and the

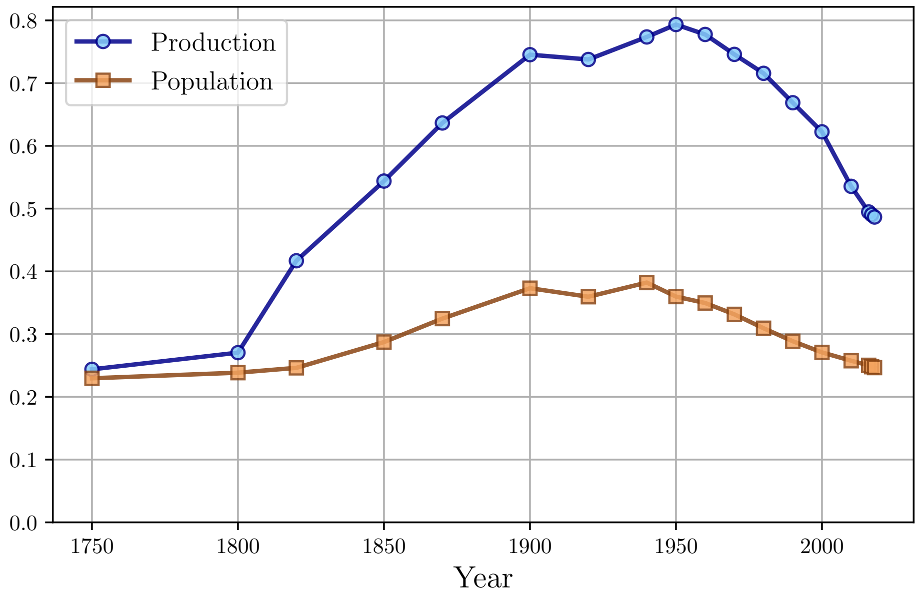

Figure 5: European-dominated economies' share of world production and population.

population share was 0.36, down from its historical peak of 0.38 in 1940. Since 1950, the gap between the weights of production and population has been closing quickly. By 2018, the European population share returned to 1820 levels, whereas the production share had fallen to 0.49. It is clear that the Industrial Revolution is open to everyone and everywhere, and the historical transient of inequality between European-dominated economies and the rest is vanishing at a rapid pace.

2.2 Growth "Miracles"

Table 1 presents the results of an exercise that aims to document which countries had a growth miracle, defined as a period when a country doubled its GDP per capita within ten years. We report the starting and ending dates of the miracle. We use a five-year rolling GDP per capita average to avoid picking up recoveries after deep troughs in the business cycle. The capacity to duplicate GDP per capita in a decade is recent. Except for Australia in 1851, growth miracles were absent in English-speaking countries. One also observes growth miracles in France and Germany in Group II. The post-war II recovery was nothing short of a miracle, but the warfare destruction drives this kind of growth that one cannot smooth out with a 5-year GDP per capita rolling average.

Groups III and IV represent Asian growth miracles, excluding China. Japan's miracle lasted 15 years, followed in calendar time by the South Korean miracle that started in 1971 and lasted almost a quarter of a century. Singapore and Taiwan also witnessed miracles

	Start	End		Start	End
Group I			**Group VII**		
Australia	1851	1854	Afghanistan	2007	2014
Group II			Azerbaijan	2005	2015
France	1953	1954	Bahrain	1998	2010
Germany	1957	1960	Iran*	1970	1976
Group III			Iraq*	2003	2015
Singapore	1971	1980	Kazakhstan	2006	2011
South Korea	1971	1995	Kuwait	1996	2010
Taiwan	1971	1980	Lebanon	2000	2001
Group IV			Mongolia	2003	2016
Japan	1959	1974	Myanmar	2005	2010
Group V			Oman	1966	2008
Albania	2006	2009	Qatar*	1997	2014
Austria	1956	1958	Saudi Arabia*	1967	1976
Bosnia–Herzegovina	2003	2005	Thailand	1994	1994
Ireland	1999	2004	Turkmenistan	2005	2016
Italy	1955	1956	United Arab Emirates*	2005	2005
Latvia	2006	2006	Yemen	1977	1979
Lithuania	2006	2006	**Group VIII**		
Malta	1974	1983	Angola*	2003	2015
Montenegro	1965	1966	Botswana	1971	1982
Montenegro	1968	1968	Botswana	1987	1993
Montenegro	2004	2013	Chad	2008	2013
Romania	1954	1956	Equatorial Guinea*	1966	1968
Romania	1968	1974	Equatorial Guinea*	1996	2011
Romania	2006	2012	Gabon*	1975	1977
Russian Federation	2006	2010	Libya*	1962	1974
Serbia	1964	1966	Libya*	2003	2011
Slovenia	1964	1965	Nigeria*	2008	2012
Ukraine	2007	2009	Swaziland	1968	1969
Venezuela*	1952	1953	Swaziland	1974	1979
Group V			Zambia	2009	2012
China	2010	2012			

Table 1: Growth miracles around the globe. A country is considered to have had a growth miracle if its GDP per capita is duplicated within ten years. We use a five-year rolling GDP per capita average to avoid computing ratios driven by the business cycle. The dates represent the first and last year when the five-year rolling GDP per capita was at least two times the five-year rolling GDP per capita of the previous decade.*Countries members (including former members) of the Organization of the Petroleum Exporting Countries (OPEC). The list of OPEC countries is available at https://www.opec.org/opec_web/en/about_us/25.htm.

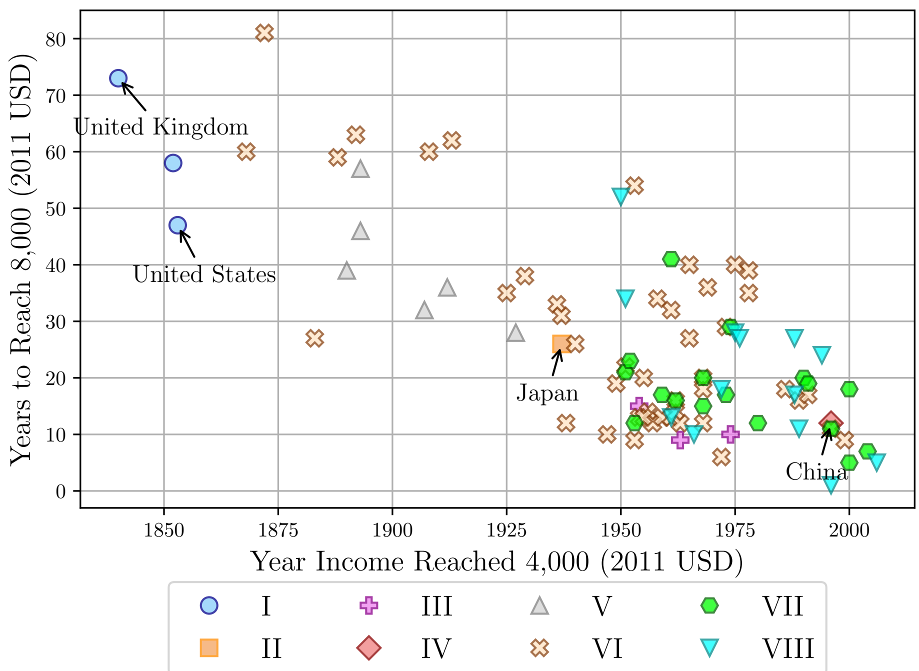

Figure 6: Years for income per capita to double (from 4,000 to 8,000 USD of 2011). Eight regions.

in the 1970s. Our methodology does not count Hong Kong as a miracle, but this economy increased its GDP per capita by over 95 percent within a decade in the 1970s.

Table 1 labels the development experience in China (Group VI) as a miracle that took place at the beginning of the 21st century. Despite the size of the Chinese economy, this economy duplicated its standards of living within ten years.

Groups VII and VIII show that the "elusive quest" for growth is over in many poor countries. In the last 30 years, the planet has had miracles happening everywhere. Ireland, for instance, did experience a miracle by the end of the millennium, whereas Nigeria and Myanmar started the new millennium with miracles. Table 1 includes an asterisk for countries that one may label as "oil-dependent" based on membership to the OPEC cartel. Although miracles driven by natural resources might not be what one has in mind when thinking about the Industrial Revolution, oil-dependent countries are not the sole driver of the recent progress in groups VII and VIII, and they are even less important for understanding several episodes we consider as miracles in Group V.[3]

[3]Venezuela is the only country in Group VI (and the only country in Latin America) that have had experienced a growth miracle according to our definition. However, about half of the miraculous growth in Group VIII corresponds to OPEC member countries. Although one cannot dismiss the importance of oil discoveries in Venezuela during the first half of the twentieth century, there is ample evidence of the substantial expansion of non-petroleum industries between 1936 to the 1970s. See Salazar-Carrillo (1976) for an overview of the historical role played by oil in the industrial development of Venezuela.

In short, Table 1 illustrates that escaping the Malthusian trap is not just a European story, even if one considers the broad notion of ”European-dominated” countries throughout history. Also, this escape, with a miraculous speed, is not a success story secluded to the Asian miracles in the 1970s or the most recent Chinese miracle. The fast pace of these escapes out of Malthus is common for countries starting their transformation process relatively late. In regards to the timing of miracles, Parente and Prescott (2000, p. 20) document the number of years a country takes to double its income per capita (from 2,000 to 4,000 dollars of 1990), and they find that late starters to the process of growth and development double their income per capita *faster*, in line with the absence of miracles for Group I in Table 1.

With almost three decades of new evidence on growth and development, one can now trace more doubling times for countries considered poor in the second half of the twentieth century. Figure 6 replicates the exercise proposed by Parente and Prescott (2000) using the latest version of the Maddison data. We employ 4,000 and 8,000 USD of 2011 as the metrics one wants to follow with these data. Figure 6 shows an agglomeration of data points from groups VI, VII, VIII, and China (Group V) around the east-south corner; This means that poorer regions, by definition, are starting their processes of development later, and they are doubling their income per capita faster, at a similar speed that the Asian Tigers did a few decades ago. What was once considered a fearless anomaly, worthy of an epithet resembling a big cat, is now a more typical pattern among groups VI, VII, and VIII.

2.3 (Out of) Agriculture and Development

One can think of the process of development as an inexorable transformation of production out of agriculture. Figure 7 plots the employment share in agriculture for groups I–VIII against both calendar time (left panel) and Log GDP per capita in 2011 USD. For computing the agricultural employment share, we use the data on agricultural workers from the U.S. Department of Agriculture in the numerator.[4].Agricultural workers are defined as people over 15 years who are economically active in agriculture. For the denominator, we use the number of persons engaged in economic activity from the Penn World Tables 10.0.[5] These data are comprehensive across the world but only started in 1961. Other data sources go far back in time, but are only available for a handful of countries that are not part of groups VII and VIII for the most part.

The left panel of Figure 7 shows that by 1961, Group I already had less than 10 percent

[4]The data on agricultural employment is available at https://www.ers.usda.gov/data-products/international-agricultural-productivity/

[5]The Penn World Table 10.0 database is available at https://www.rug.nl/ggdc/productivity/pwt/?lang=en.

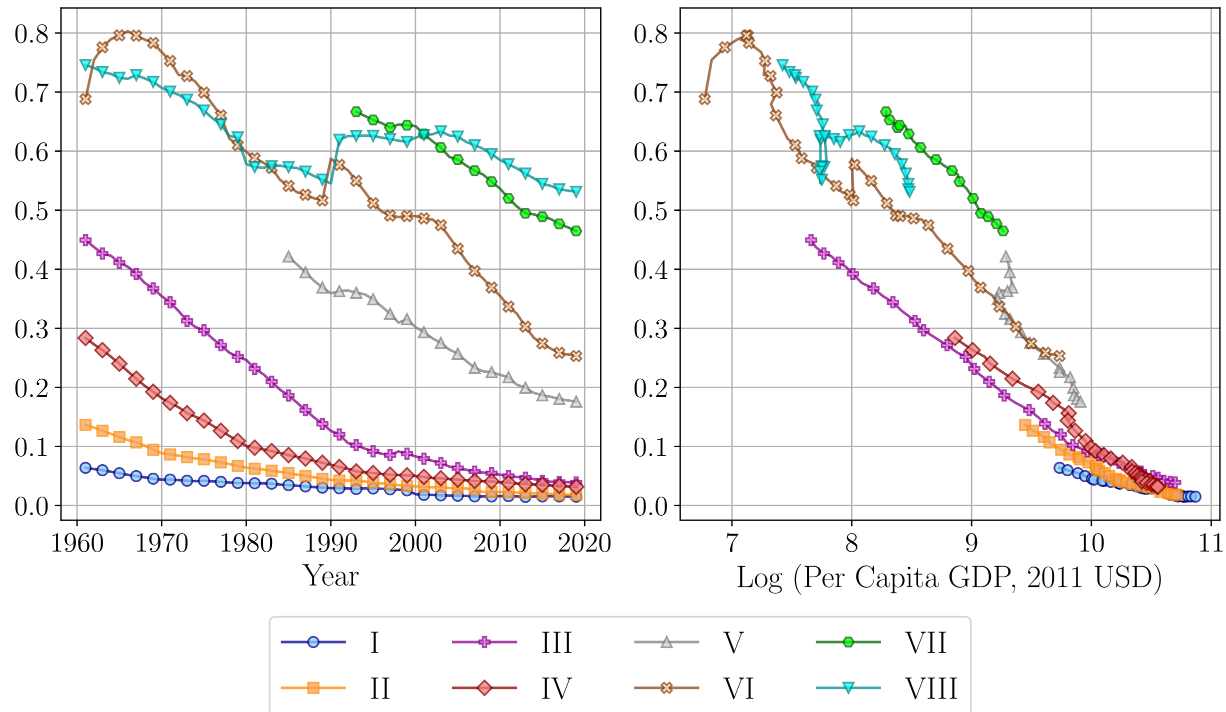

Figure 7: Employment share in agriculture, eight groups. 1961-2018.

of the labor force in agriculture. This share continued to decline to levels below 2 percent in 2018. Group II had about 15 percent of its labor force also in agriculture by 1961, and by 2018, this Group also had less than 2 percent of its labor force working on farms. On the contrary, the Asian Tigers (Group III) and Japan (Group IV) started this race out of agriculture in 1961, with 45 and 28 percent of the labor force in this sector, respectively. These are not negligible fractions. However, despite the significant initial differences, one observes that the transformation in these countries was faster compared to groups I and V. By 2018, Japan and the Asian Tigers had already achieved agricultural employment shares below 4 percent. One also observes that the series in groups IV (China), VI, and VIII fall almost parallel, with initial agricultural employment shares between 0.7 and 0.9. However, the slope for Group VIII (Africa) is less steep compared to the rest of the late starters when plotted against calendar time. However, even for this Group one observes a sizable reduction over time in the fraction of agricultural workers.

The right panel of Figure 7 plots these same employment shares for each group against their income per capita (in logs). The bulk of the initial differences by 1961 documented above reflects the differences in income per capita across calendar time. Of course, some differences in levels remain, but one can see a common pattern *everywhere*. As economies grow wealthier, the employment share in agriculture drops steadily. The poorest countries

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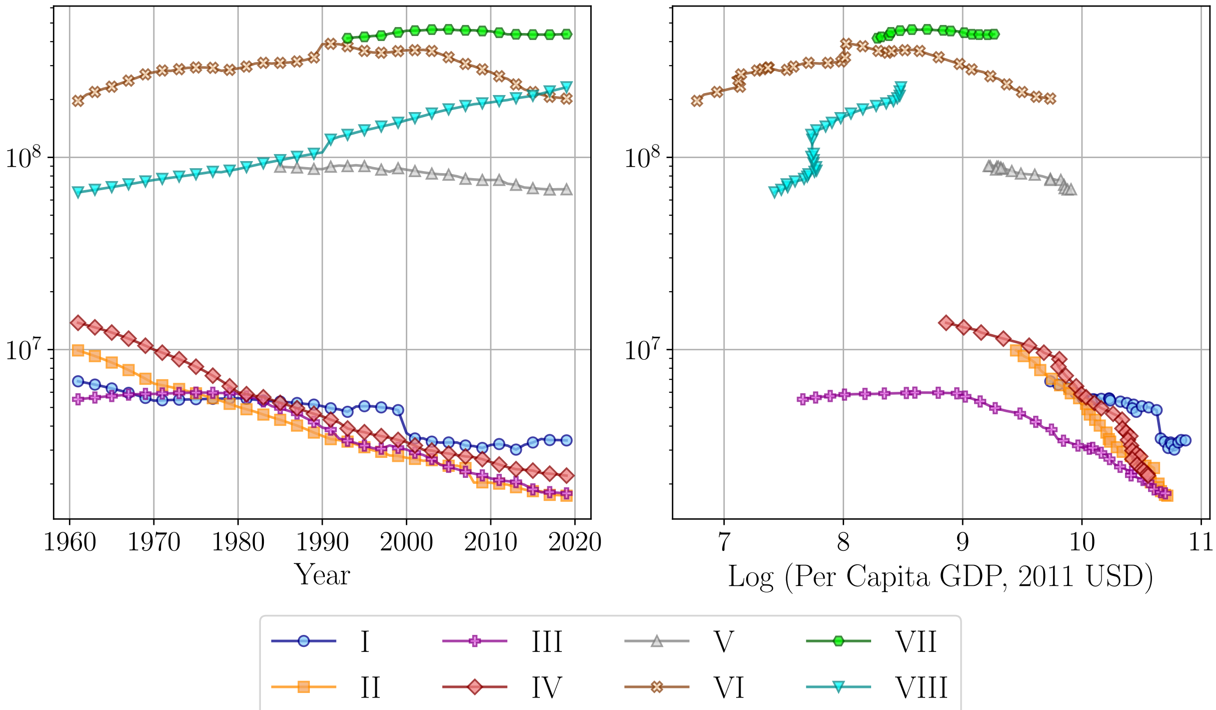

Figure 8: Employment in agriculture (thousands of workers), eight groups. 1961-2018.

are now entering similar paths that already followed by China, Japan, and the Asian Tigers follow. Alternative data sources that go back well into the nineteenth century portray a similar pattern out of agriculture for the early starters of the Industrial Revolution.

Now consider only the numerator of Figure 7, i.e., the absolute number of workers in agriculture. We use a logarithmic scale to observe the whole world in one plot. From Figure 7, one cannot infer if the drop in agricultural shares worldwide is because the growth in the agricultural labor does not compensate for the total growth in the labor force or if it is due to fewer workers in agriculture. To address this, Figure 8 plots the total employment in agriculture for groups I–VIII. The left panel plots the number of workers in agriculture as a function of calendar time for 1961–2018. The right panel plots the number of agricultural workers as a function of log income per capita.

The left panel of Figure 8 shows Groups I–VI already shedding people out of agriculture, not just as a percentage of the population (See Figure 7) but in absolute value. Consider Group II. In 1960, Japan had about twice as many workers in the agricultural sector compared to all the countries of Group I. By 2018, Japan already had about one million fewer people compared to Group I, even though the English-speaking countries shed nearly 3 million workers out of agriculture in the past 60 years.

Notably, the bulk of agricultural workers today are outside groups I–IV. China (Group

VI) was still growing the number of workers in agriculture, similar to the poorest countries in the 1960s through the 1980s, but early in the 1990s, China also started to employ fewer workers in farms. China still has over 200 million workers in agriculture, although, in 1991, the number of agricultural workers in China peaked at about 400 million, twice as many as in both 1961 and 2018.

Group VII displays a modest decrease in its number of farmers, not at the speed one observes in China but trending downwards. Africa is the only region still trending upward in terms of agricultural workers (Group VIII). Between 1961 and 2018, the number of agricultural workers in Africa increased by about 160 million people, from 65 million in 1961 to 225 million in 2018. å

The right panel of Figure 8 plots the time series of employment in agriculture for all groups as a function of their (log) GDP per capita. There are two messages worth highlighting from this exercise. First, one observes that for income levels below exp(9) $\sim$ 8,000 USD, the number of workers in agriculture rises. In the case of Africa, this threshold has not been achieved yet. One can observe the stagnation documented in Figure 4 reflected in a vertical rise in the agricultural workers. Second, the logarithmic scale allows us to see a significant acceleration in the movement out of agriculture as each Group achieves an income per capita above exp(9.5) $\sim$ 13,300 USD. Importantly, this acceleration happens everywhere. Poor countries are undergoing structural transformation, with a decline in the number of people employed in agriculture. Poor countries are beginning their industrial revolutions.

2.4 Demographic Transition

Figure 9 plots the demographic transition across groups I–VIII. Each point represents the Group's annual population growth against the GDP per capita for a given year. The pattern that emerges is one of high population growth rates in every group at low-income levels, followed by a steady reduction in population growth as each group grows wealthier. The initial steep rise (almost vertical) is consistent with the population dynamics expected by Malthus and Ricardo. However, population growth rates peak for every group within a narrow income window, between 2,000 and 3,000 USD. No group in our sample period continues to display positive population growth associated with gains in income per capita.

For Group I, one cannot observe in Figure 9 the population growth peak since the data started in 1900, several decades after the genesis of their Industrial Revolutions. However, one can see how their declining rates of the population overlap with the rest of the groups that are catching up.

Groups I, II, and have stabilized their population growth at positive rates below 1 percent.

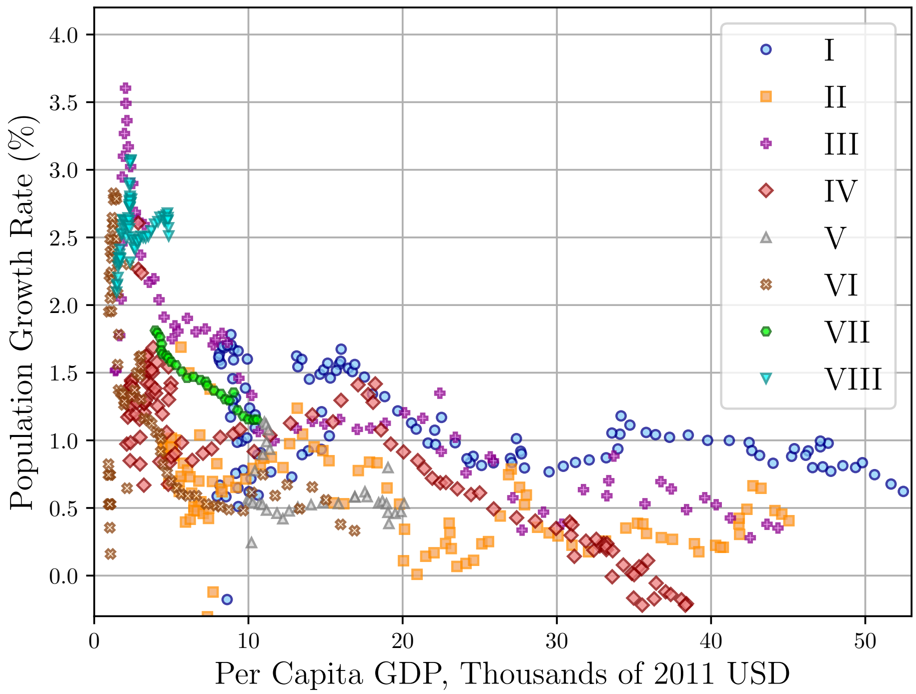

Figure 9: Demographic transitions, eight groups. 1900-2018.

China (Group VI) also has a stable population growth of around 0.5 percent. However, one can also see in Figure 9 the sharp decline in population growth during Mao's famine, well before the "one-child" policy took place. This reduction has no parallel in any of the groups from Figure 9. Another anomaly that one observes is the steady decline in population growth rates for Japan (Group IV) down to negative growth rates for incomes between 30 and 40 thousand dollars. Historically, there have been a few years with negative population growth in these data (notably China in 1961). Still, the case for Japan (Group IV) is different, as these observations are beyond a one-year anomaly.

The African countries (Group VIII) already peaked and have embarked on similar demographic transition paths compared to the groups that started this process earlier in calendar time but around the same income levels. Among the poor regions, the steep rise and steady decline for Group VII overlap with the rest of the regions. As expected, we have observations for groups VII and VIII with high population growth and low-income levels, but as these economies continue to develop, one can conjecture that they will continue following the trajectory of groups I, II and III back when they had similar income levels, to the point that it will continue to be hard to distinguish among these time-paths across countries.

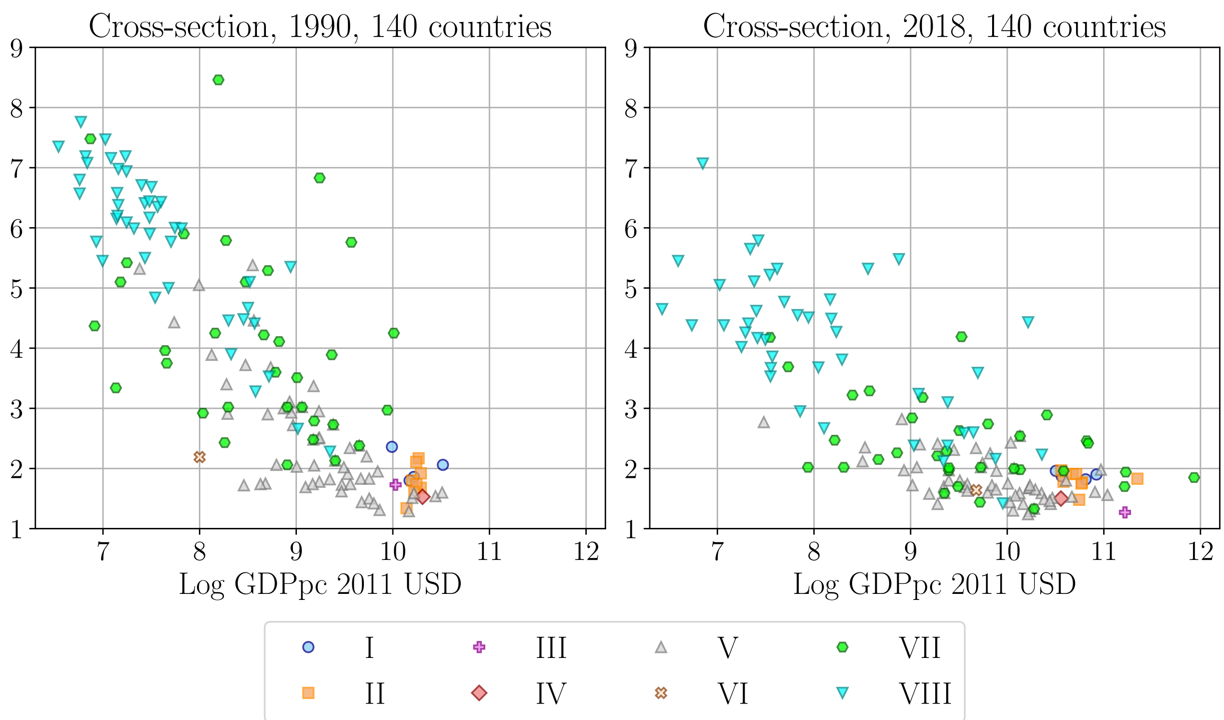

Figure 10: Fertility. Cross-section data for 140 countries in 1990 and 2018. Fertility is proxied by the number of children per woman. Eight groups

2.4.1 Fertility

Figure 10 plots the number of children per woman – a proxy of fertility – as a function of income. The relation observed in the cross-section of 1990 (left panel) is steeper than the cross-section of 2018 (right panel). One can see that groups VII and VIII overlapped in 1990 around the low-income-big-family zone. In contrast, in 2018, the demographic transition in Group VII is evident, to the point that one only observes these pairs of poor and big families in Africa 2018. It is also noticeable that the well-defined relation in 1990 for Group VI is almost flat by 2018. There is not much to say on the demographic transitions of advanced countries in these two cross sections, as the fertility rates were already substantially low by 1990, but the pattern that emerges from Figure 10 is that the relation between income and fertility is becoming weaker over time: For any given income level, one can expect smaller families today compared to just three decades ago. However, perhaps more importantly, the relationship between economic development and demographic transition is alive and well, transforming country after country everywhere.

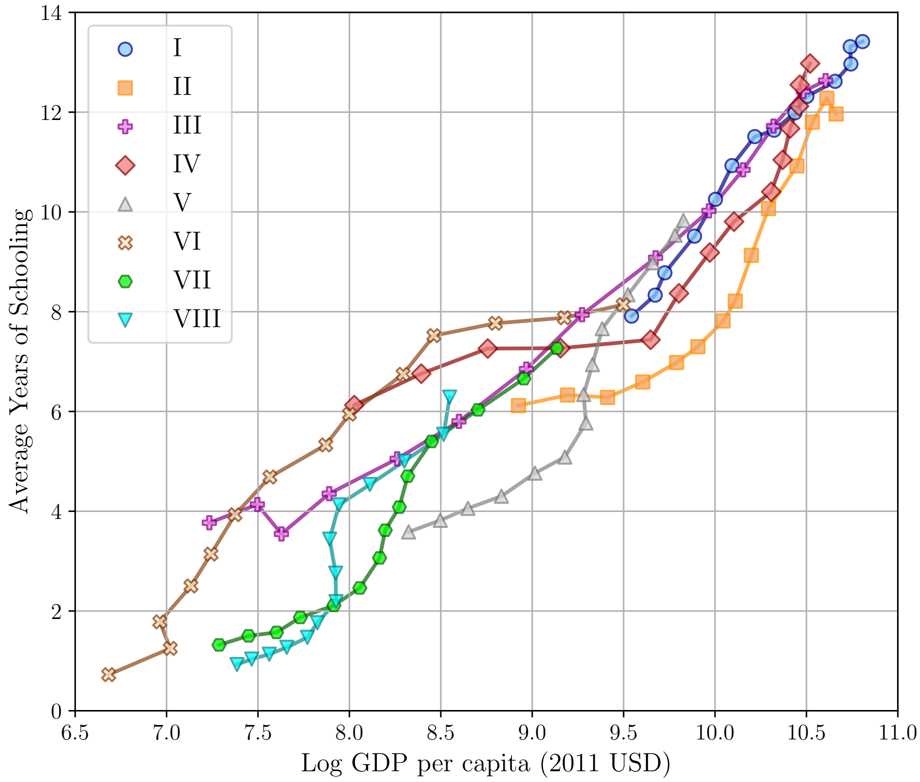

Figure 11: Average years of schooling, eight groups. 1950–2015.

2.5 Human Capital

Figure 11 plots average years of schooling from the Barro and Lee dataset, which measures the mean schooling of the population aged 25 and above. Despite vast differences in geography and history, all regions align along a remarkably similar path when human capital is plotted against income per capita. At low levels of development, represented by Africa (Group VIII) and the rest of Asia (Group VII), schooling averages fewer than three years. As income rises, education expands rapidly: the Asian Tigers and Japan (Groups III–IV) follow almost identical trajectories, closing the gap with the early industrializers. The advanced economies—the English-speaking offshoots and continental Europe (Groups I–II)—occupy the upper end of the curve, where schooling levels exceed twelve years.

The smooth alignment across all groups suggests that the accumulation of human capital follows a broad income–education relationship, independent of initial conditions or timing. In this sense, Figure 11 provides perhaps the clearest visual evidence that education and income coevolve along the same development path traced by every region since the onset of the industrial revolution.

2.6 U-Shaped Female Labor Force Participation

Goldin (1994) documents a U-shaped relation between economic development and female labor force participation using cross-country evidence for 1980. She finds that female labor force participation is high at low stages of development. It reaches a bottom low at the middle stages of development and rises back as the economy grows richer. She argues that one would observe this pattern for the United States if one has data before 1890, and conjectures that for the United States, female labor participation was above 15% before 1890 and probably reached a bottom around the 1920s.[6] Moreover, Ngai et al. (2024) focus on the role of unpaid family work in the evolution of female employment through the lenses of structural transformation. They argue that once one adjusts the historical U.S. Census data taking into account the historical prevalence of the "family enterprise," whereby all family members contributed to farm production and some non-farm family businesses, one observes the U-shaped pattern of female employment participation between 1860 and 2019.[7]

We are in a position to test whether one observes this U-shaped relation using recent cross-country evidence. The data we use on female labor force participation comes from the International Labour Organization, available at `gapminder.org`. Unfortunately, we only have these data since 1989. Figure 12 plots two cross-sections for the labor force participation rate of women aged between 15 and 65 as a function of (log) income per capita of 2011. The left panel is a cross-section of 140 countries in 1990. The right panel is a cross-section for 2018 for the same countries. We also fit a polynomial of second order in each panel where we regress income per capita (in logs) and its square on the female labor force participation for each cross-section. The coefficients obtained to plot the fitted line and their associated standard errors are presented in each panel.

Both panels in Figure 12 show the U-shaped relation between the female labor force and development documented by Goldin (1994). One observes that African countries (Group VIII) have similar participation rates to the advanced countries represented by groups I and II in 1990 and 2018. One would expect lower participation rates for African countries as they embark on their Industrial Revolutions; This is what one observes for Group VII:

[6]Goldin (1994) explains the U-shaped female labor force participation as follows: When modernization starts, an income effect leads women to consume more leisure. In addition, a stigma in "blue-collar" manufacturing dissuades women from entering the labor force. Later, due to a combination of women's human capital accumulation and the rise of a "white-collar" sector, women seek jobs where the stigma is no longer present; This happens when the substitution effect dominates and is reinforced with the absence of stigma in clerical tasks and other "white-collar" occupations for women.

[7]For instance, Ngai et al. (2024) show that whereas the Census data shows a female labor force participation of merely 15 percent in 1860, this participation rises to about 65 percent after adjusting for unpaid work. The adjusted time series shows that the lowest fraction of female employment was 45 percent in 1960. Since then, one observes a steady rise in the female labor force participation rate above 70 percent in 2019.

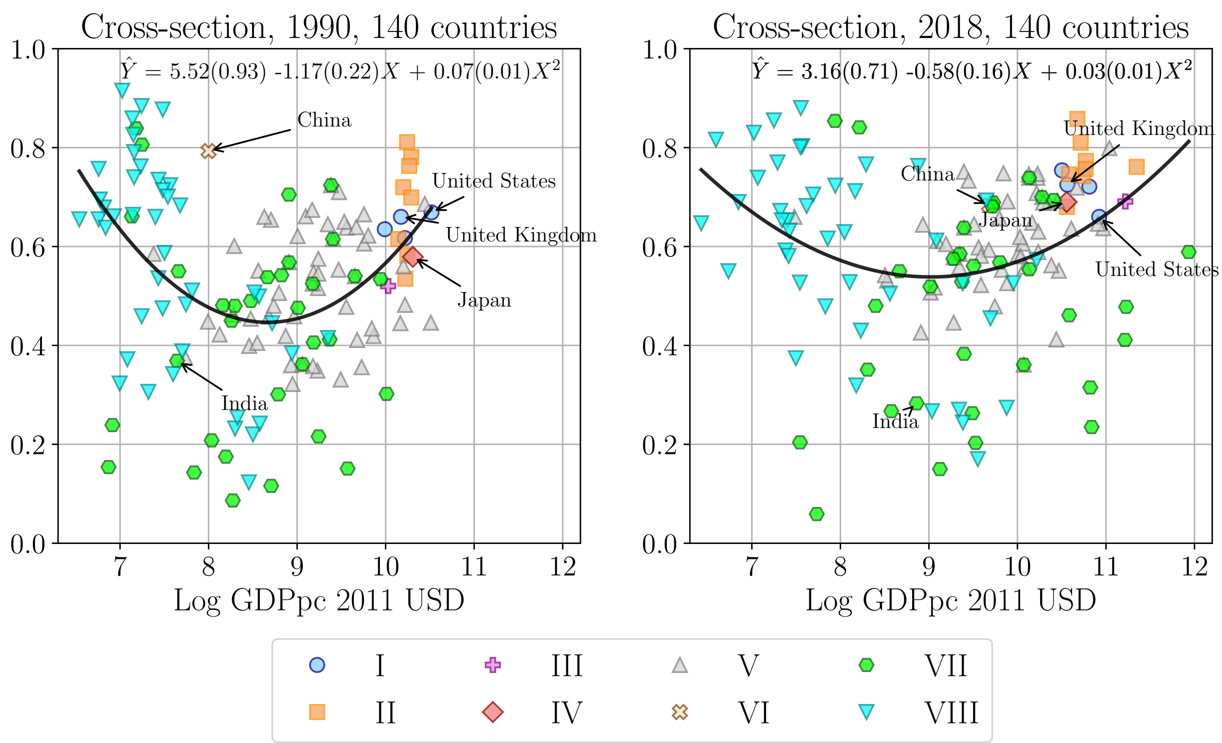

Figure 12: Female's U-shaped labor force participation over development. Cross-section data for 140 countries in 1990 and 2018. The labor force participation rate is for women aged between 15 and 65. Eight groups

More points at the northwest side of the U-shape belong to Group VII in 1990 (left panel) than in 1990 (right panel). These countries are growing wealthier and have lower labor participation rates for women. Goldin (1994) finds that Latin American countries in 1980 were the ones representing the bottom of the U. By 2018, the participation rates for women in Latin America had grown, and now these countries are in the rising part of the U-shape. The place at the bottom of the U now belongs predominantly to Group VII.

Appendix B uses panel data to address if the relationship documented in the cross-sections of Figure 12 vanishes once one controls for the idiosyncratic effect that each country might have, or if these phenomena simple reflect general time-trends rather than an inherent relation between development and female labor force participation. In other words, we want to see if the coefficients of log income and its square are robust to controlling for country and time-fixed effects. Table B.1 shows that the U-shaped relation between female labor force participation and development is robust to these controls, suggesting that countries that grow rich do experience a reduction in female labor force participation at early stages of development followed by a rise after about $\exp(9) \sim 8,000$ USD of income per capita.

3 A Unified Growth Theory with Two Parents

This section builds on the analysis in Lucas (2002), particularly Section 7, which integrates two defining features of the Industrial Revolution: the role of land as a fixed input in traditional production, and the emergence of a quality–quantity trade-off in fertility decisions that gives rise to both a demographic transition and rising human capital, trigering modern economic growth. Hansen and Prescott (2002) study a transition, from a stagnant regime—characterized by low productivity growth and decreasing returns to scale—to a modern growth regime with higher productivity and constant returns to scale. However, in their framework, the demographics are imposed, and there is no explicit role for parental investment in child quality. By contrast, Becker et al. (1990) develop a model of endogenous human capital formation during the Industrial Revolution, but omit the role of land in their "Malthusian" equilibrium.[8] In what follows, we attempt to unify these perspectives by embedding both features—land-based constraints in traditional production and endogenous human capital investment in a modern economy—into a unified framework..

Relative to Lucas (2002), our model incorporates two key departures. First, households are composed of two parents who jointly choose fertility and the time allocated to the formation of child quality. This setup, following Becker (1981), allows for specialization within the household. While parents are symmetric in most respects, we allow them to differ along two dimensions that define their type. We label these as Type xx and Type xy. The xx parent has a comparative advantage in child-rearing—a simplification meant to capture biological and social predispositions often associated with women. As Becker (1981, p. 37) notes, "[w]omen not only have a heavy biological commitment to the production and feeding of children, but they also are biologically committed to the care of children in other, more subtle ways." Conversely, the xy possesses a comparative advantage exclusively in modern production, intended to capture men's advantage in physical strength and suitability for brawn-intensive work—what Goldin (1990, p. 7) describes as "differences among industries ... [in] their initial requirement of strength and the hazardous and dirty nature of work."

Second, we adopt the preference structure proposed by Galor and Weil (2000), departing from the dynastic framework used in Lucas (2002). In this formulation, parents value the quality of their children for its own sake, rather than as a means to maximize the children's lifetime utility. This allows tractability when studying the transition to modern economic growth as a sequence of static choices updated each period in response to exogenous technological progress *exclusively* in the Malthusian sector. Importantly, the escape

[8]In their framework, the Malthusian regime corresponds to an equilibrium with zero investment in human capital.

from Malthusian stagnation in our model is entirely driven by human capital investments. This contrasts with Hansen and Prescott (2002), where an exogenous productivity growth in the modern sector is critical for the transition out of Malthus.

As in Lucas (2002), we work with functional forms that permit analytical solutions, allowing us to characterize both the Malthusian equilibrium—marked by low growth and high fertility—and the modern growth regime—defined by sustained gains in income and declining fertility. The model captures the essential features of the Industrial Revolution as a shift from stagnation to growth, driven by decisions made at the household level. We now turn to a formal description of the economic environment.

3.1 Endowments

In each period, every parent is endowed with one unit of time and h_t units of human capital, inherited from the previous generation's investment in child quality. The economy consists of a continuum of households of measure one. Each household comprises two parents and thus has a total time endowment of two units per period. Households also have access to x_t units of land, where X denotes the fixed aggregate land supply and N_t is the total number of adults alive in period t, born in period $t-1$.

To simplify the analysis, we assume that no property rights are defined over land; thus, land is held in common and cannot be transferred across generations. This assumption reflects the communal land use typical of hunter-gatherer societies, as discussed in Lucas (2002, Section 3), and aligns with the modeling approach of Galor and Weil (2000). In this setting, land does not enter into household optimization through bequests or asset accumulation. Instead, each household receives access to a per capita share of the fixed land stock in each period, determined by the prevailing population level. This treatment preserves Ricardo's classical insight: land is a fixed, non-reproducible factor of production. As population increases, the land-to-labor ratio declines, generating diminishing returns and constraining the possibility of sustained growth in living standards—an essential feature of the Malthusian regime.

Finally, each parent is endowed with a fixed comparative advantage. For each type $i \in xx, xy$, let ξ_i denote efficiency in child-rearing, and let δ_i represent a "brawn" advantage relevant when operating modern production technologies exclusively. We assume $\xi_{xx} \geq \xi_{xy}$ and $\delta_{xy} > \delta_{xx}$, so that Type xx is relatively more efficient in producing child quality, while Type xy has a comparative advantage in operating the modern technology production. These differences generate intra-household specialization and, crucially, are fixed over time—these advantages *do not* evolve.

3.2 Preferences

Household preferences, jointly representing both parents, are given by the utility function

$$\mathcal{U}(c_t, n_t, h_{t+1}) = (1-\beta)\log(c_t) + \eta\log(n_t) + \beta\log(h_{t+1}), \tag{1}$$

where c_t denotes consumption, n_t the number of children, and h_{t+1} the human capital level of each child. This specification captures parental trade-offs over material consumption, fertility, and the quality of offspring.

The term $\beta\log(h_{t+1})$ introduces Gary Becker's "quantity-quality" trade-off, emphasizing that parents derive utility not only from the number of children but also from the human capital invested in each child, thereby requiring an optimal allocation of resources between these two dimensions. The parameter η captures the weight parents assign to fertility, while β reflects the relative importance of children's quality. We assume that $\eta > \beta$; otherwise, parents may regard the number of children as a "bad"—a point emphasized by Lucas (2002, p. 137). Finally, note that the utility function is homogeneous of degree one in (c_t, h_{t+1}).

3.3 Human Capital Accumulation

In this model, the formation of human capital takes place entirely within the household. There is no formal schooling sector. All investments in child quality are made through the allocation of parental time, and no role is assigned to market institutions in the education process.

Let h_t denote the stock of human capital in period t. The law of motion for human capital across generations is specified as

$$h_{t+1} = h_t \cdot \varphi(r_{xx,t}; r_{xy,t}), \tag{2}$$

where $r_{xx,t}$ and $r_{xy,t}$ denote the fractions of time devoted to child-rearing by Type xx and Type xy parents, respectively. The function $\varphi(\cdot)$ satisfies $\varphi(0;0) = 1$, and is strictly increasing in each argument. Human capital is augmented only through active investment by parents. Thus, in the absence of time spent in child-rearing, human capital remains unchanged.

We assume $\xi_{xx} > \xi_{xy}$, so that Type xx parents are more productive in human capital formation per unit of time invested. This assumption is in line with the empirical findings of Del Boca, Flinn, and Wiswall (2014), which show that maternal time inputs have a disproportionately large effect on child cognitive development, even after controlling for education, labor supply, and other household characteristics. The model captures this asymmetry by assigning a higher marginal product to the xx parent's time, generating efficiency gains through specialization, in the spirit of Becker (1981).

3.4 Technology

At each point in time, two technologies are available for producing the final good: a traditional technology that combines land and unskilled labor, and a modern technology that uses skilled labor.

The traditional production function is

$$y_{Tt} = A_t x_t^{\alpha} (l_{xx,t} + l_{xy,t})^{1-\alpha}, \tag{3}$$

where x_t denotes land input, l_{it} is the labor supply of type $i \in xx, xy$, and A_t is total factor productivity.[9]

This land-intensive technology does not employ human capital. Labor is homogeneous, and productivity does not vary by type. The traditional sector represents the low-productivity farming systems that dominated economic life from the Neolithic Revolution to the Industrial Revolution—economies in which output was constrained by fixed land and unskilled labor, and where the returns to education were negligible. These are the "thankless" forms of cultivation that Quesnay distinguished from surplus-generating agriculture.[10]

The modern production technology is given by

$$y_{Mt} = B \left[(h_t + \delta_{xx})\, l_{xx,t} + (h_t + \delta_{xy})\, l_{xy,t} \right], \tag{4}$$

where B is a constant productivity parameter in the modern sector, assumed to be relatively low compared to A_t at any point in time.

Human capital plays a central role in the modern sector. Each unit of labor is augmented by h_t, reflecting productivity gains from acquired skills. In addition, raw labor productivity differs across types: $\delta_{xy} > \delta_{xx} \geq 0$, capturing an initial "brawn" advantage of Type xy in physically demanding tasks. This advantage is more relevant when human capital is scarce. As h_t rises, however, the contribution of raw strength to output diminishes, and production becomes increasingly reliant on cognitive skills. The modern economy thus transitions from one where physical attributes dominate to one where acquired knowledge drives productivity.

[9] Throughout, we use "sectors" and "technologies" interchangeably.

[10] As he wrote in the Tableau Économique, "[w]e are not speaking here of small-scale cultivation carried on with the aid of oxen, in which more than a million ploughs and about two million men would be required to work 40 million arpents of land" Quesnay (1758/1965, p. vi).

3.5 Household Problem

In each period, the household chooses the number of children, how each parent allocates time between child-rearing and work, and how work time is divided across production technologies. Since households do not value leisure, all non-work time is allocated to child-rearing. Letting n_t denote the number of children, the labor supply of a parent of type $i \in xx, xy$ is

$$l_{i,t} = 1 - r_{i,t} n_t.$$

between current production and future human capital. Conditional on this choice, households allocate work effort across sectors. Let $\theta_{it} \in [0, l_{it}]$ denote time allocated to traditional production; the remainder, $l_{it} - \theta_{it}$, is allocated to the modern sector.

Figure 13 summarizes the structure of time use.

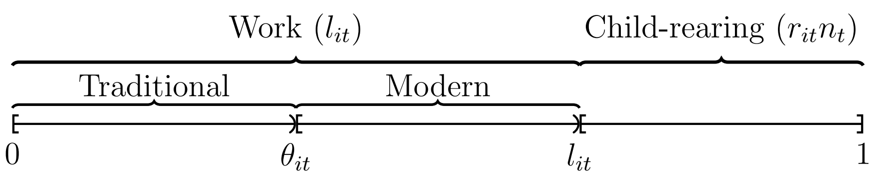

Figure 13: Time allocation for parent type $i \in xx, xy$.

Since output is maximized by reallocating labor across production technologies conditional on labor supply, it is convenient to treat l_{it} as given when analyzing production decisions. We then return to the fertility and time investment choice to solve the household's quantity–quality trade-off.

3.5.1 Output Maximization

Conditional on labor supply, the household allocates time across technologies to maximize output. Each parent devotes $\theta_i \in [0, l_i]$ units of labor to the traditional sector, with the remainder, $l_i - \theta_i$, used in modern production. Since the problem is static, we omit time subscripts. Total output maximization is given by

$$F(h, l_{xx}, l_{xy}) = \max_{\theta_{xx}, \theta_{xy}} \left\{ A x^{\alpha} (\theta_{xx} + \theta_{xy})^{1-\alpha} + B \left[(h + \delta_{xx}) (l_{xx} - \theta_{xx}) + (h + \delta_{xy}) (l_{xy} - \theta_{xy}) \right] \right\}. \quad (5)$$

The household chooses each θ_i to maximize 5. For parent type $i \in xx, xy$, the first-order condition is

$$(1 - \alpha) A x^{\alpha} (\theta_{xx} + \theta_{xy})^{-\alpha} \geq B(h + \delta_i), \text{ with equality if } \theta_i < l_i.$$

This condition governs the optimal allocation of working time across technologies by equating the marginal product of labor in the traditional sector to that in the modern sector. Solving for θ_i, the optimal allocations to the traditional sector are

$$\theta_i = \min\left\{\max\left\{\left(\frac{A(1-\alpha)}{B(h+\delta_i)}\right)^{\frac{1}{\alpha}} x - \theta_{-i}, 0\right\}, l_i\right\}, \quad i \in \{xx, xy\}. \tag{6}$$

The movement out of Malthus depends on the household's land endowment, the land intensity in traditional production, the relative productivity of the two technologies, the stock of human capital, and the partner's allocation of time. Each of these factors shapes the incentives for reallocating labor as the economy evolves.

Crucially, while A_t grows exogenously over time, B remains constant. Unlike in Hansen and Prescott (2002), where the escape from Malthus is driven by exogenous TFP growth in the modern sector, here the transition is entirely the result of endogenous human capital accumulation. Rising A_t increases traditional output, relaxing the household's resource constraint. At some point, this *may* enable parents to start investing in their children's quality, initiating a process that ultimately triggers modernization.

In light of (6), the transition to modern production unfolds asymmetrically across parent types. A parent with a comparative advantage in physical labor—reflected in a higher δ_i—exits the traditional sector earlier. This reallocation raises the marginal product of remaining labor in traditional production, thereby delaying the partner's exit. Modernization thus proceeds in stages: Type xy reallocates first, while Type xx remains in traditional production until $\theta_{xy} = 0$. Even beyond this point, diminishing returns imply that some fraction of Type xx labor remains in traditional use, though this share vanishes as h becomes large.

Given the differential timing in exit out of the traditional sector across types, there is an indirect production function during this transition, given by

$$y = \begin{cases} Ax^{\alpha}(l_{xx}+l_{xy})^{1-\alpha}, & \text{if } \theta_{xx} = l_{xx} \text{ and } \theta_{xy} = l_{xy}; \\ B(h+\delta_{xy})(l_{xx}+l_{xy}) + \alpha(1-\alpha)^{\frac{1}{\alpha}-1}A^{\frac{1}{\alpha}}[B(h+\delta_{xy})]^{1-\frac{1}{\alpha}}x, & \text{if } \theta_{xx} = l_{xx} \text{ and } \theta_{xy} < l_{xy}; \\ B(h+\delta_{xy})l_{xy} + Ax^{\alpha}l_{xx}^{1-\alpha}, & \text{if } \theta_{xx} = l_{xx} \text{ and } \theta_{xy} = 0; \\ B[(h+\delta_{xx})l_{xx} + (h+\delta_{xy})l_{xy}] + \alpha(1-\alpha)^{\frac{1}{\alpha}-1}A^{\frac{1}{\alpha}}[B(h+\delta_{xx})]^{1-\frac{1}{\alpha}}x, & \text{if } \theta_{xx} < l_{xx} \text{ and } \theta_{xy} = 0. \end{cases} \tag{7}$$

3.5.2 Utility Maximization

Each period, the household faces the resource constraint

$$c_t + kn_t \leq y_t, \tag{8}$$

where k represents a fixed cost per child, in terms of units of the final good.

Preferences, as defined in (1), satisfy the following boundary conditions:

$$\lim_{c_t \to 0} \mathcal{U}_{c_t} = \infty; \quad \lim_{n_t \to 0} \mathcal{U}_{n_t} = \infty; \quad \lim_{h_{t+1} \to 1} \mathcal{U}_{h_{t+1}} = \beta. \tag{9}$$

The first two are standard Inada conditions, ensuring interior solutions for consumption and fertility. The third is a boundary condition: if no time is allocated to child-rearing, human capital remains at its baseline level, $h_{t+1} = 1$, absent any prior investment. In this case, the marginal utility of quality equals β. At low levels of development, households choose quantity and consumption over quality. Quality investments begin only when the marginal returns to the former fall below β.

Once the household is rich enough to afford quality investments, who provides them? This is where Becker's insight on intra-household specialization becomes relevant. Under (2), the marginal benefit of time spent in child-rearing is always higher for the Type xx parent. The marginal cost, by contrast, is the same across parents: their time contributes equally to output at the margin. It follows that all quality investments are made by the Type xx parent, with $r_{xy,t} = 0$, as long as her time suffices to meet the household's demand. If not, the household moves to the opposite corner: $r_{xx,t} = 1$ and $r_{xy,t} > 0$.

Given (1), the resource constraint (8) binds with equality. Substituting directly into the utility function and using the law of motion for human capital from (2), along with the corner solution $l_{xy} = 1$, the household's problem reduces to:

$$\max_{r_{xx,t},\ n_t} \mathcal{U}(n_t, r_{xx,t}) = (1-\beta)\log(y_t - kn_t) + \eta\log(n_t) + \beta\log\left[h_t\varphi(r_{xx,t})\right]. \tag{10}$$

This formulation collapses the household's choice to two variables: fertility, and the share of Type xx's time allocated to quality investment. Although parents care about the quality of their children, there are no genuine intertemporal decisions. Labor supply affects the quality endowment directly, through time forgone from current production. We drop time subscripts to simplify notation. The first-order conditions for (10) are

$$\frac{1-\beta}{y-kn}\left(y_{l_{xx}} r_{xx} + k\right) = \frac{\eta}{n}, \tag{11}$$

and

$$\frac{1-\beta}{y-kn}\left(y_{l_{xx}} n\right) = \beta\frac{\varphi'(r_{xx})}{\varphi(r_{xx})}. \tag{12}$$

Solving (11) and (12) jointly yields the optimal fertility choice

$$n = \frac{y}{\left(\frac{1-\beta}{\beta}\frac{\varphi(r_{xx})}{\varphi'(r_{xx})}\right) y_{l_{xx}} + k}, \tag{13}$$

and the time allocated to each child

$$r_{xx} = \max\left(\frac{\eta}{\beta}\frac{\varphi(r_{xx})}{\varphi'(r_{xx})} - \frac{k}{y_{l_{xx}}}; 0\right) \tag{14}$$

To continue making progress, one needs explicit functional forms for both y and $\varphi(r_{xx})$. We already know that the indirect production function is implicitly defined in (7) when total output is produced using both traditional and modern technologies. However, from (7) one cannot solve analytically the system defined by (13) and (14) even with an explicit functional form for $\varphi(r_{xx})$ along the transition out of Malthus. Nevertheless, before proceeding to solve the equilibrium path numerically, it is insightful to characterize the equilibria at both ends of the development path: a Malthusian stagnant equilibrium and an equilibrium path with large human capital levels and constant growth in living standards. These endpoints provide insight into the forces driving the transition.

3.6 Malthusian Stagnation

The Malthusian model provides a compelling account of the long epoch spanning from the Neolithic Revolution to the dawn of the industrial revolution. As Diamond (2005) recounts, the shift from nomadic subsistence to settled agriculture furthered the ongoing domestication of plants and animals, enabling the development of transformative innovations, such as irrigation systems, crop rotation, and eventually, the foundations for more complex technologies: *guns, germs, and steel*. These advances supported larger, denser societies.

Yet from the Neolithic Revolution to the clash of civilizations at the time of the discovery of the Americas, there is little evidence of sustained gains in material living standards. How can this be? How can societies invent so much and remain so poor?

The Malthusian framework resolves this paradox, and its explanation aligns well with the facts. In a world where resources are finite and population responds to material conditions, technological progress does not raise income per capita—it raises population. Productivity gains are offset by demographic expansion, leaving average incomes near subsistence levels.

The model described above is consistent with this "date-zero" fact. The Malthusian economy is characterized by a traditional productivity level $A > B$ such that employing modern technology, while available, is not profitable. Material conditions are near subsistence. From (9), one learns that parents will not invest in children's quality if the marginal utility of consumption or fertility is above β – a defining characteristic of subsistency. In this case $r_{xx} = 0$ and the only relevant choice is around fertility. The optimal number of children is given by

$$n = \frac{\eta}{k(1-\beta+\eta)} A2^{1-\alpha}. \tag{15}$$

In line with Malthus, equation (15) shows that positive income shocks lead to higher fertility as long as these shocks occur within a neighborhood small enough that investing in child quality remains unaffordable. Naturally, rising costs of children (k) suppress fertility. Furthermore, the first-order conditions combined with (15) show that parents devote no time to child-rearing when

In line with Malthusian insights, (15) suggests that positive income shocks will lead parents to have more children, provided that these shocks occur within a subsistence neighborhood, where quality is still unaffordable. The cost per child, k, discourages fertility. From the first-order conditions, time investment in child-rearing remains zero as long as

$$\frac{k}{A2^{1-\alpha}} > \frac{\beta}{1-\beta+\eta}. \tag{16}$$

By solving for k consistent with $n = 1$ in (15) and substituting into (16), one finds that the optimal time investment in child quality is zero in steady state if

$$\eta > \beta. \tag{17}$$

The assumption imposed on preferences—namely, that the number of children is not a "bad" relative to each child's quality—appears directly in condition (17), and is sufficient to guarantee a Malthusian steady state with no quality investments. While parents derive utility from bestowing quality bequests on their children, regardless of the production possibilities of their time, condition (17) implies that in low-productivity, traditional economies, they cannot afford to allocate time to child quality. Instead, they must devote all available time to producing material goods for the subsistence of themselves and their offspring, despite their preference for children's well-being.

The steady state consumption of both parents in steady state is given by

$$c = \frac{1-\beta}{1-\beta+\eta} A2^{1-\alpha}.$$

3.7 Post-Malthusian Dynamics

The equilibria described in Section 3.6 reflect a stagnant living condition that remains near subsistence and is dynamically stable. This is what Ricardo (1817/2004, p. 93) refers to as "the natural price of labour," defined as the level "necessary to enable the labourers, one with another, to subsist and to perpetuate their race." It includes not only food and shelter but also the "conveniences required for the support of the labourer and his family." As Ricardo notes further (p. 94), in line with Malthus, this stability is maintained through endogenous responses: "by the encouragement which high wages give to increase of population, the number of labourers is increased, wages fall again to their natural price, and indeed from a re-action sometimes fall below it."

Is there any path out of this trap? Hansen and Prescott (2002) suggest one: exogenous growth in the productivity of modern technology, which at some point makes its adoption inevitable. In their framework, both traditional and modern technologies grow exogenously, while population growth adjusts mechanically to maintain constant living standards until the transition occurs.

In contrast, in this model, productivity growth in the traditional sector does not, by itself, trigger a transition. It raises income per capita temporarily, but these gains are checked by endogenous population growth. Yet the improvement opens a window. If living standards rise sufficiently, parents may find it optimal to invest in their children's human capital. They can now afford a quality bequest. These investments render the modern technology profitable, endogenously initiating the transition.

This mechanism gives rise to an intermediate regime that is neither fully Malthusian nor modern, marked by rising incomes and continued population growth. Galor (2011, pp. 17–18) calls this the "Post-Malthusian Regime," noting the irony that, just as Malthus formalized his population theory, the world was already beginning to escape his trap, though fertility still responded in the very way Malthus and Ricardo had described.

Consider Section 3.6 as a description of the dynamics that governed economic life from the dawn of civilization. There are no long-run dynamics in this regime, and that is precisely the point. Shocks—famine, wars, barbarian invasions, bubonic plagues—produce only *temporary* deviations. The system converges back to its steady-state values.

Now treat these conditions as the "initial state," or "date-zero," in a dynamic system. Let the level of technology, A, begin to grow at a constant, exogenous rate. Crucially, this is not a shock. Is this exogenous force in traditional technology sufficient to trigger a transition to modern economic growth? As we shall see, the answer hinges on whether this new source of growth alters parental incentives – specifically, whether it induces a shift from fertility to quality bequest.

First, normalize $h = 1$. Think of this as the basic level of dexterity and cognitive ability a child acquires in the absence of parental investment. With A growing, one can rearrange terms in (16) to express the child-rearing condition as

$$Ax^{\alpha} > \frac{\eta}{\beta}. \tag{18}$$

As A rises, the resource constraint becomes less binding. Parents may now afford to invest in quality, but they also raise fertility, diluting the land endowment x and offsetting the gains from technological progress.

3.7.1 Post-Malthusian Convergence

Assume that, at some point in history, productivity in the traditional sector begins to grow at a constant exogenous rate $\gamma > 0$, but condition (18) never holds because land dilution offsets the gains from rising productivity. In this case, the economy achieves higher output per capita and population *levels*, but no sustained growth in living standards.

To see this, begin from a steady state where $n_0 = 1$ and $l_{0,i} = 1$ for $i \in \{xx, xy\}$, with normalized values $h_0 = 1$ and $A_0 = 1$. Initial output per capita is then $y_0 = 2^{1-\alpha}$. Since (18) never holds, there are no investments in child quality, so human capital remains constant at $h_t = 1$ for all t. Population evolves according to $N_{t+1} = n_t N_t$, where n_t is determined by (15), and land per household is $x_t = 1/N_t$. Solving the system recursively, one obtains

$$n_t = (1+\gamma)^{P_t}, \quad y_t = 2^{1-\alpha}(1+\gamma)^{P_t},$$

where P_t is a polynomial resembling a Pascal's triangle with alternating signs, namely

$$P_t = \sum_{k=0}^{t-1} (-1)^k \binom{t}{k+1} \alpha^k = \frac{1}{\alpha}\left[1 - (1-\alpha)^t\right].$$

As $t \to \infty$, $P_t \to 1/\alpha$, and both n_t and y_t converge to $(1+\gamma)^{\frac{1}{\alpha}}$ and $2^{1-\alpha}(1+\gamma)^{\frac{1}{\alpha}}$ respectively. This is level convergence. Absent human capital formation, growth in traditional productivity alone cannot sustain rising living standards.

3.7.2 The Onset of a Human Capital Economy

On the other hand, if rising productivity allows condition (18) to be satisfied, the trajectory of development changes fundamentally. The gains from technology allow parents to reallocate time away from subsistence toward child quality. These choices bring about a transition from stagnation to sustained growth. With the quantity–quality trade-off now active, fertility and the time allocated to each child adjust accordingly, as given by (13) and (14), respectively.

Regarding modernization, the timing of the transition differs across parent types. Type xy begins to operate the modern technology when

$$(1-\alpha)Ax^{\alpha}(l_{xx}+l_{xy})^{-\alpha} < B(h+\delta_{xy}).$$

This first-mover characteristic arises solely from the initial "brawn" advantage in modern production, where physical effort dominates. This edge, however, is only relevant during the early stages of development, when human capital accumulation remains relatively low.

From this point onward, θ_{xy} declines steadily to equalize the marginal product of labor (of both parent types) across sectors. Here, diminishing returns in traditional production play a critical role: it is not optimal for Type xx parents to reallocate labor until $\theta_{xy} = 0$. Only then does modernization become profitable, provided that

$$\frac{(1-\alpha)A}{B}\left(\frac{x}{l_{xx}}\right)^{\alpha} < h+\delta_{xx}. \tag{19}$$

Once (19) is satisfied, Type xx parents begin reallocating time toward the modern sector. At this point, $\theta_{xx} < l_{xx}$ to equalize marginal returns across technologies.

3.8 Modern Economic Growth

At the other end of the development spectrum, we seek a solution in which n and c/h are constant, and living standards grow at a constant rate. In this regime, traditional technology becomes negligible as human capital accumulates without bound. From (7), one obtains the following limiting condition:

$$\lim_{h\to\infty}\frac{y}{h} = B(l_{xx}+l_{xy}). \tag{20}$$

As (20) shows, comparative advantages in modern production vanish in a human-capital-based economy with large h. Similarly,

$$\lim_{h\to\infty}\frac{c}{h} = \lim_{h\to\infty}\frac{c+kn}{h} = B(l_{xx}+l_{xy}). \tag{21}$$

Finally,

$$\lim_{h\to\infty}\frac{\partial y/\partial l_{xx}}{h} = B. \tag{22}$$

To derive closed-form solutions, we require a functional form for $\varphi(r_{xx})$ such that $\varphi'(r_{xx}) >$

0, $\varphi(0) = 1$, and $\frac{\varphi'(r_{xx})}{\varphi(r_{xx})}$ is independent of r_{xx}.[11] A function satisfying these properties is given by

$$\varphi(r_{xx}) = e^{\xi_{xx} r_{xx}}, \quad \xi_{xx} > 0, \tag{23}$$

with $\frac{\varphi'(r_{xx})}{\varphi(r_{xx})} = \xi_{xx}$. Using (20) and (22) in (13) and (14), together with the labor force participation of each parent ($l_{xx} = 1 - r_{xx}n$ and $l_{xy} = 1$ as long as $r_{xx} < 1$) one obtains the optimal number of children as

$$n = \frac{\beta \xi_{xx}}{1 - \beta + \eta} 2, \tag{24}$$

and the optimal fraction of time devoted to child-rearing is

$$r_{xx} = \frac{\eta}{\beta \xi_{xx}}. \tag{25}$$

Thus, in a regime of modern growth driven by human capital accumulation, Type xx's labor force participation is given by

$$l_{xx} = \frac{1 - \beta - \eta}{1 - \beta + \eta} < 1, \tag{26}$$

and per capita human capital grows at a constant rate given by

$$\frac{h_{t+1}}{h_t} - 1 = e^{\left(\frac{\eta}{\beta}\right)} - 1. \tag{27}$$

Finally, combining optimal labor supplies in (21), the consumption-to-human-capital ratio is

$$\frac{c}{h} = \left(\frac{1 - \beta}{1 - \beta + \eta} \right) 2B, \tag{28}$$

which is constant.

The allocations we just found in the Malthusian equilibrium and in the modern growth regime mirror those described in Lucas (2002), who notes (p. 166):

> [t]hese are the long-run possibilities for this two-state-variable system: a Malthusian steady state with a constant income level, and an"Ak"-like system in which income is perpetually growing, driven by the accumulation of human capital. Little is known about the possibilities for this model away from these two steady

[11] $\frac{\varphi'(r_{xx})}{\varphi(r_{xx})}$ represents the relative growth rate of φ at r_{xx}.

states, and since the intended subject of this section is the transition from one to the other, this is a serious limitation.

The next section calibrates the model to explore numerically such possibilities, those governing the dynamics between these two extremes, away from Malthus' stagnant equilibria and toward sustained growth driven by human capital accumulation.

4 Calibration

Can the theory presented above generate the key features of the industrial revolution? That is, can it quantitatively account for the transition from stagnation to sustained growth, marked by labor reallocation out of agriculture, a demographic transition, rising human capital, and the U-shaped path of female labor force participation? This section addresses this by calibrating the model to historical data. We parameterize the model to account for Great Britain's development path between 1400 and 2000.

The calibration proceeds in three steps. First, we target the equilibrium allocations in Sections 3.6 and 3.8 to characterize a Malthusian steady state at one end of the development spectrum, and a human capital regime at the opposite extreme under balanced growth conditions. Second, we introduce exogenous TFP growth in the traditional sector to account for the slow gains during the post-Malthusian phase. Third, we set the TFP *level* in the modern sector to match the observed transition path.

Table 2 summarizes the parameterization. Labor intensity in the traditional sector is set to $\alpha = 0.4$, consistent the labor intensity in traditional production from Hansen and Prescott (2002). The child cost parameter, $k = 0.168$, is chosen so that $n = 1$ in the Malthusian steady state, implying that the fertility is at the replacement level: each adult produces one child, so a two-parent household yields two children. The preference parameters η and β are calibrated jointly to match two targets: the modern long-run growth rate of income per capita (1.9 percent per year, compounded over 60 years), and the female-to-male labor force participation ratio in Great Britain in 2000, reported by Olivetti (2014, Appendix A.1).[12] The restriction $\eta > \beta$ is not imposed but emerges from the calibration.

TFP in the traditional sector is normalized to $A_{t=0} = 1$ and grows at a rate of $\gamma = 0.473$ from 1640 onward, corresponding to approximately 0.65 percent per year. This modest rate is sufficient to match the gradual rise in living standards during the post-Malthusian era, once land dilution "checks" the gains from technical progress. The transition from the post-Malthusian regime to the modern growth regime is dated to 1820, when a marked change

[12]Since the model abstracts from leisure, Type xy's labor supply is his full time endowment. The relevant data moment is therefore the female labor supply in 2000 as a fraction of the male's.

Parameter	Value	Target / Comment
α	0.4	Consistent with labor intensity in Malthus (Hansen & Prescott, 2002).
η, β	0.112, 0.099	Chosen jointly to match the female labor participation in (26) and the long-run growth rate in (27) to: (i) the observed long-run income per capita growth rate (1.9%, compounded over 60 years), and (ii) the female-to-male labor force participation ratio in Great Britain in 2000 from Olivetti (2014, Appendix A.1.).
$A_{t=0}$	1	Normalization.
k	0.168	Consistent with $n = 1$ in the Malthusian steady state.
γ	0.473	Chosen to match Post-Malthusian income per capita growth net of land dilution.
B	0.164	Modern TFP *level* to match modern growth.
ξ_{xy}	1	Normalization.
ξ_{xx}	1.56	Consistent with productivity differences between mothers' and fathers' time at the mean child age of 6.32 years from Del Boca et al. (2014).
δ_{xx}	0	Consistent with normalized basic human dexterity.
δ_{xy}	0.5	"Brawn" advantage; female strength $\approx$ 67% of male, from Peolsson, Hedlund, and Öberg (2001).

Table 2: Parameterization summary.

in the growth rate is observed. The TFP level in the modern sector is set to $B = 0.164$, chosen to minimize the distance between the model and the observed per capita growth in the modern regime.

Within-household specialization requires either $\xi_{xx} > \xi_{xy}$ or $\delta_{xx} < \delta_{xy}$. We set $\delta_{xx} = 0$ and $\delta_{xy} = 0.5$, consistent with the normalization of basic dexterity and with estimates of relative physical strength between women and men from Peolsson et al. (2001). For child-rearing productivity, we draw on the findings of Del Boca et al. (2014), which show that mothers' active time is substantially more effective in producing child quality when children are younger, while fathers' active time becomes slightly more effective as children age. Specifically, we use the estimated elasticities from the one-child model evaluated at the median child age (6.32 years). This yields $\delta_{xy} = 1.56$ (consistent with the normalization of ξ_{xy}), meaning that at the mean child age, an hour of a mother's active time is about 56 percent more effective than an hour of a father's in child-rearing.[13]

[13] Note, however, that with the presence of B in the calibration to match the modern growth path, the precise values of these parameters are not crucial. What matters is that $\xi_{xx} > \xi_{xy}$ or $\delta_{xx} > \delta_{xy}$ to generate

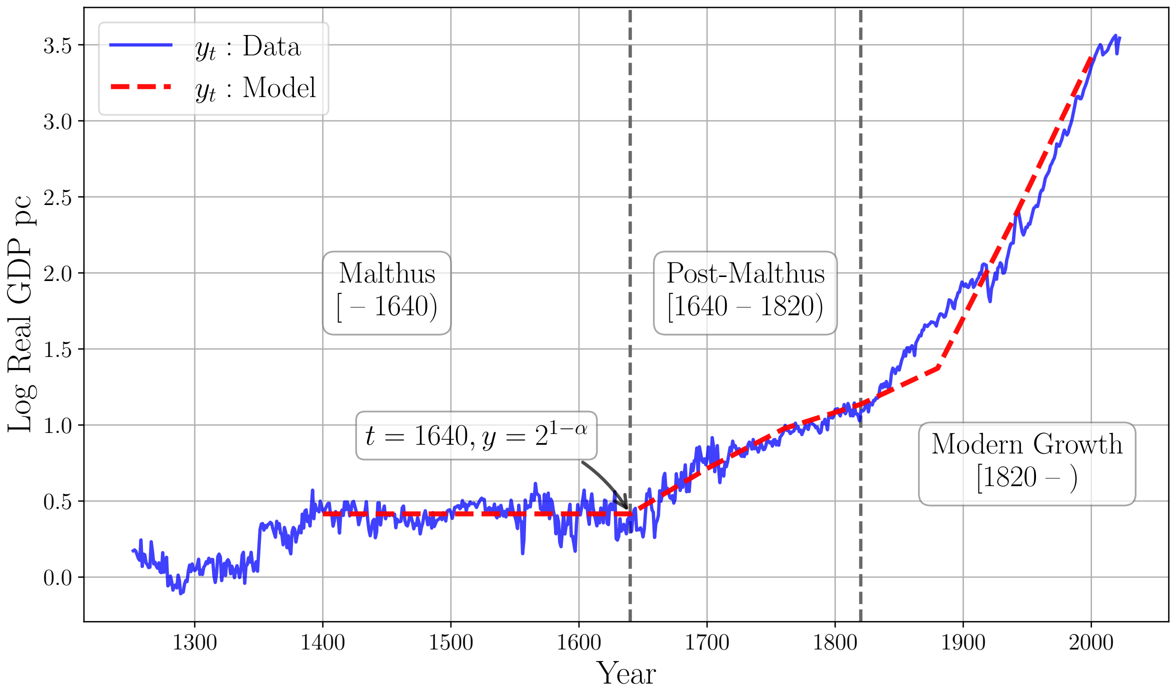

Figure 14: Development stages in Great Britain: 1252–2022.

Figure 14 displays the path of real income per capita in Great Britain from 1252 to 2022, in logarithmic scale and normalized so that $y = 2^{1-\alpha}$ in 1640—the last period of the Malthusian regime. The log scale is useful, as it allows one to observe growth rates as slopes. There is a short-lived growth period after the Black Death, but between 1400 and 1640 income is essentially flat at about $1,500 (2011 USD), or four dollars a day. From 1640 to 1820, the slope increases slightly, corresponding to a growth rate of about 0.4 percent per year.[14] After 1820, the slope steepens to 1.9 percent, the modern growth rate.

Figure 14 compares the model's predictions to observed data, with one model period corresponding to 60 years. This horizon is implied by equation (27), given $\eta > \beta$ and the empirical fact that modern income per capita grows at approximately 1.9 percent annually. The model replicates the evolution of income per capita in Great Britain reasonably well. This is expected, as the calibration is constructed to match this path. Prior to 1640, income per capita remains stagnant, consistent with the Malthusian dynamics described in Section 3.6. Strictly speaking, the steady-state condition requires both income and population to remain constant. Historical evidence suggests that this condition is not fully satisfied: substantial population growth occurred in the aftermath of the Black Death. One could rec-

division of labor and specialization within the household and across production technologies.

[14]Within this period, one can locate the years that are often identified with the early stages of the Industrial Revolution. Clark (2001) cites 1770 as a conventional starting point, associated with innovations such as the spinning jenny and the water frame.

oncile this fact within the model by introducing a shock before 1400, corresponding to the plague, and tracing the subsequent convergence toward the Malthusian steady state. We do not pursue this extension, as it yields no additional insight beyond what is already captured by the baseline calibration.

From 1640 onward, exogenous growth in traditional-sector productivity accounts for the gradual rise in income per capita characteristic of the post-Malthusian regime. By 1820, human capital accumulation renders modern technology viable, marking the final period in which traditional production is operated exclusively. Given the model's 60-year period length, the first simulated period in which modern technology is adopted is 1880, introducing a visible lag relative to the historical record. From that point forward, the model closely tracks the observed path of British income per capita through 2000.

The take-off is generated by an exogenous source of growth in traditional productivity, interacting with endogenous household choices around human capital formation. This mechanism ultimately makes the adoption of modern production profitable, consistent with the British development experience. This stands in contrast to the calibration strategy in Hansen and Prescott (2002), where the take-off is accounted for by an exogenous growth rate in a Solow-type technology, consistent with productivity growth in the modern sector.

Figure 15 presents the model's main predictions. Panel (i) shows the reallocation of labor away from traditional production—the Malthus sector—across types, as determined by equation (6). Modernization begins in 1820. The parent with the "brawn" advantage enters modern production first, while the other remains tied to traditional methods. Eventually, both exit Malthus and enter modern growth. For Type xy, the transition to modern production occurs within a single model period (60 years), as $\theta_{xy} = 0$ by 1880. This condition allows him to allocate a fraction of labor to modern production starting in 1880.[15]

Type xx follows a two-step path out of Malthus: first withdrawing from traditional production to invest in child quality, then gradually reallocating labor toward modern technology. Unlike θ_{xy}, this transition is slowed by diminishing marginal returns, which continue to operate in the traditional sector.

Panel (ii) shows the evolution of fertility. In the Malthusian era, fertility remains at replacement level. During the Post-Malthusian period, productivity gains in traditional technology induce rising fertility, as shown in (13). Children are a normal goods, and families want more of them as they grow wealthier. What is new here is the persistence of technical improvements, which create new *possibilities*. These gains lead to quality investments, made exclusively by mothers. Families can now afford more and "better" children!

[15] In alternative simulations, a slower adjustment of θ_{xy} delays the transition for Type xx, keeping her labor confined to traditional production until $\theta_{xy} = 0$.

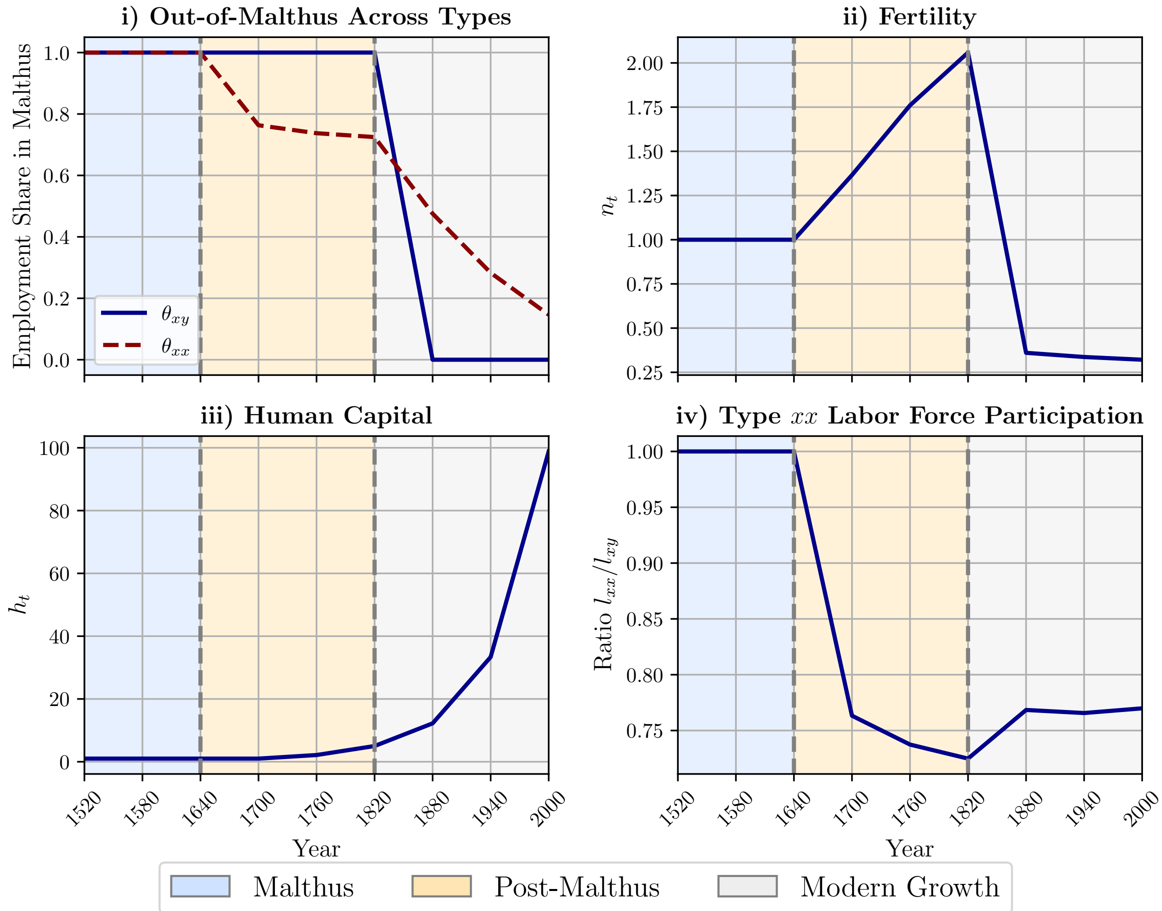

Figure 15: Main predictions.

The Beckerian quantity–quality trade-off emerges only with modernization. In traditional technology, time spent on child-rearing is time away from traditional production, which raises the marginal product of the remaining labor due to diminishing returns. No such compensation exists in the modern sector. Thanks to generational investments in human capital, Type xx's labor becomes profitable in modern production. Modern technology exhibits *linear* returns, and so do the opportunity costs of child-rearing—the "substitution" effect in (13). In the modern era, families choose fewer children to invest further in quality. This adjustment is sharp: within one period, fertility falls below replacement and gradually converges to its balanced-growth level, as shown in (24).

Panel (iii) illustrates the evolution of human capital per person. In the Malthusian era, h_t is normalized to 1, capturing the basic ability and dexterity of children when parents cannot afford to invest in their quality. During the Post-Malthusian era, technical improvements enable families to bequeath human capital to their children *before* modernization occurs, as parents value child quality for its own sake—despite its irrelevance to traditional production methods. Human capital not only precedes modernization. It is the cause of it.

Panel iv) plots the labor supply of Type xx, interpreted as female labor force participation relative to males, as the model abstracts from leisure. In the Malthusian era, men and women work side by side in subsistence farming to sustain life. Although families wish to invest in quality, they cannot afford fewer arms in the fields. Historical evidence suggests these constraints were so binding that even children were drawn into farm work—though the model abstracts from child labor. Female labor force participation declines sharply during the post-Malthusian era, driven entirely by child-rearing. This Beckerian division of labor accounts for the downward part of Goldin's "U." Panel iv) then depicts the rising segment of the "U," associated with modern growth and falling fertility. The opportunity cost of time, augmented by human capital, does not yield a compromise in quality, but rather in quantity.

Taken together, the model reproduces the key stylized facts of the historical transition: a decline in fertility, a rise in human capital, and a reallocation of labor across sectors and genders. This completes the model's core predictions. We now turn to the data to evaluate how well these mechanisms account for the salient patterns of the industrial revolution.

4.1 Model and Data

First, we compare the model's equilibrium paths with historical time series for Great Britain drawn from several sources, to contrast the model's predictions with data before and after the British Industrial Revolution. Second, we superimpose the model's predictions for Great Britain at different income per capita levels, rather than calendar time, onto a cross-section of countries in the year 2000, to assess how the predictions align with several economies at different stages of development.[16]

Figure 16 compares the model's predictions for the transition out of traditional agriculture with the data.[17] The available evidence tracks employment shares in agriculture rather than in traditional modes of production, though the overlap between the two is substantial. The model, however, applies more broadly to pre-industrial or semi-feudal modes of pro-

[16] There is nothing inherently "British" in our model calibration. The productivity path A_t and the TFP level B are chosen to replicate Britain's income per capita series, but any country's trajectory could be matched with three parameters: $\{A_0, \gamma, B\}$. If this were to be done, one would obtain a mapping between income per capita and the model's main predictions. Since the British calibration already establishes this correspondence, further country-specific calibrations would offer limited additional insight. However, it would be interesting to think of cross-country variations in B as "barriers", in the sense of Parente and Prescott (2000). Viewed this way, development gaps would reflect delays in modernization. These barriers could, in principle, be disciplined outside the calibration to shed light on the mechanics of growth miracles and their implications for fertility and human capital formation. We do not pursue this direction here, as our focus is on characterizing the salient patterns of the industrial revolution within a unified framework.

[17] In the model, the share of employment in traditional agriculture is given by $\frac{\theta_{xx}+\theta_{xy}}{l_{xx}+l_{xy}}$.

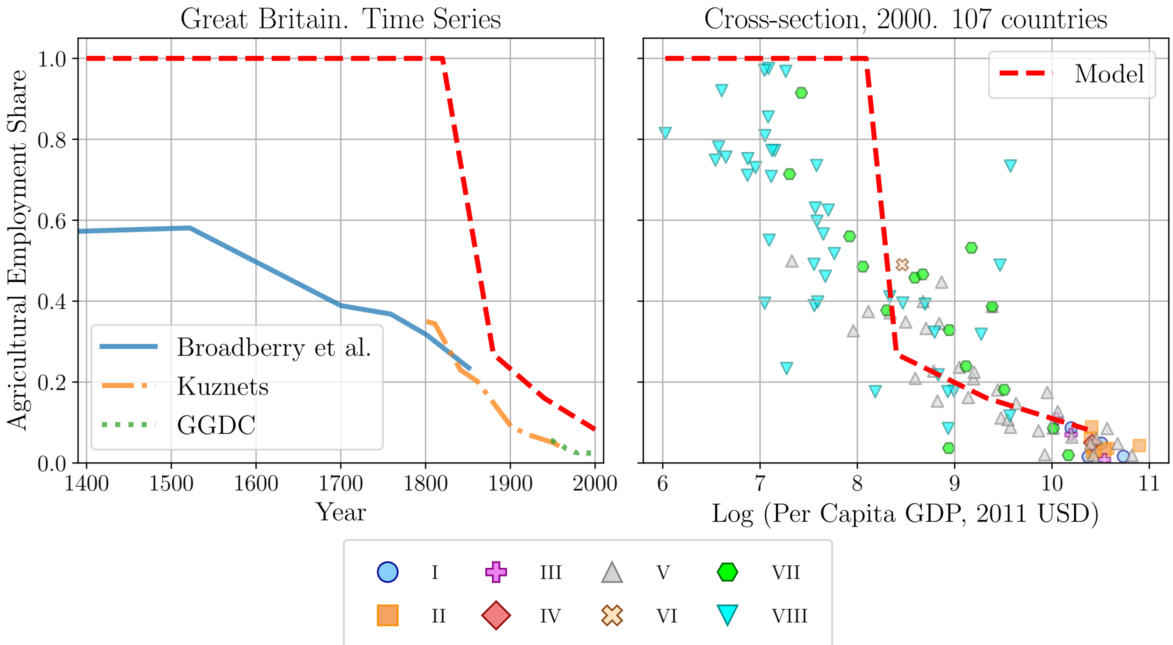

Figure 16: Out-of (traditional) agriculture.

duction—still present in parts of the developing world—where land and raw labor are the primary inputs. Not all agricultural employment is Malthusian, and some non-agricultural activity may still reflect traditional production relations. Agro-industrial employment, in particular, should not be regarded as part of the Malthusian sector.

The left panel of Figure 16 compares the model with British historical data from three sources: Broadberry, Campbell, Klein, Overton, and van Leeuwen (2015), Kuznets (1966, 1971), and the GGDC 10-Sector Database of Timmer, de Vries, and de Vries (2015). Before modernization (1820), the model assigns all available labor to traditional agriculture. Historical series for Great Britain, however, show that less than 40 percent of the labor force remained in agriculture at that date. Even by the fifteenth century, the British share in agriculture was unusually low, under 60 percent. This early transformation may reflect Britain's role as the pioneer of modern growth, and the model is silent on this. Yet, from 1820 onward, the model predicts a sharp contraction in traditional employment, which moves nearly in parallel with the historical decline after 1880. This correspondence is decent: once modernization begins, the model and the data trace the same downward path.

The right panel of Figure 16 compares the model with agricultural employment in a cross-section of countries in 2000, using the same data sources documented in Section 2. Here, the model captures the broad development pattern: the poorest economies—mainly in Africa and Asia—show agricultural employment shares well above 60 percent (some even close to 100 percent as in the model) at low income levels, while richer economies across

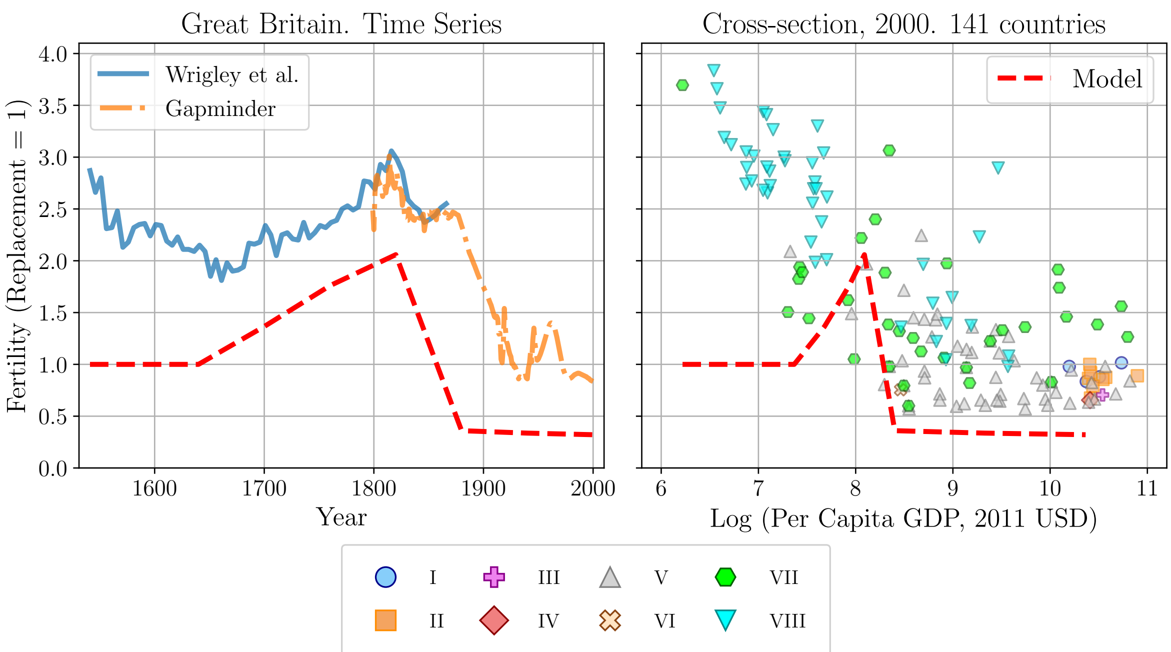

Figure 17: Fertility

all regions, including Africa and Asia, exhibit low agricultural shares consistent with the model's projection. The transition out of agriculture is therefore not a peculiarity of early industrial Europe but a general feature of economic development.

Figure 17 compares the model's predictions for fertility with the data. The left panel plots the historical time series of the Gross Reproduction Rate (GRR) from Wrigley, Davies, Oeppen, and Schofield (1997), which measures the average number of daughters born to a woman over her childbearing years, together with fertility data from Gapminder, which reports the number of children per woman by cohort. These are the same data used in Section 2 and in the cross-country panel.

Ignoring mortality, GRR is sufficient to account for the replacement level, which is 1, while for the Gapminder series, it is 2. To make the two measures comparable and consistent with the model, we divide the Gapminder series by 2. The British historical evidence in the left panel of Figure 17 shows that the GRR during the Malthusian era remained well above replacement, although it declined gradually until the early post-Malthusian period. As argued by Clark (2007), these high fertility rates are consistent with Malthusian dynamics: they reflect the long demographic recovery following the Black Death, a process that stabilized only as the population approached its pre-plague size.

Fertility never returned to those earlier levels, however. As the British economy entered its post-Malthusian phase, population growth accelerated once again. Clark (2007) also suggests that "Darwinian" selection mechanisms may have favored behavioral traits that later positioned Britain to exploit the "temporary" high wages and new opportunities emerging in the aftermath of the plague.

The model calibration does not compute "plagues" or any kind of deviation from the Malthusian equilibrium. Fertility is held at replacement throughout the Malthusian era, consistent with a steady-state interpretation of stagnation. Then, as the economy enters and exits the post-Malthusian era, the timing of the fertility rise and decline in the data aligns with the model's prediction. Fertility increases until roughly 1820, marking the onset of modern growth and the demographic break from Malthus. Differences in levels are expected: the model omits mortality and other demographic feedbacks that shaped historical fertility. Even so, it captures the essential dynamics, predicting a sharp decline below replacement about one period earlier than observed, while remaining silent—as intended—on later phenomena such as the post–World War II British baby boom.

The right panel of Figure 17 compares the model's predictions with cross-country data. At low income levels, the model predicts lower but rising fertility levels, a pattern not observed in the cross-country data. Poor countries—mainly in sub-Saharan Africa—display instead high but gradually declining fertility rates that the model cannot reproduce. Being poor in Africa in 2000 is not the same as being poor in pre-industrial Britain. Broad improvements in public health, nutrition, and sanitation have allowed populations in today's poorest countries to expand far beyond the limits that once constrained Europe, where Malthusian checks on mortality and subsistence were binding. During the modern growth era, the empirical relationship between income and fertility is smoother than in the model, but the income levels at which the model predicts fertility to fall below replacement correspond roughly to several observations across regions I–VII. In all African countries, by contrast, fertility remained above replacement in 2000 at income levels where the model predicts it should already have fallen below replacement.

Figure 18 compares the model's predictions for human capital accumulation with both historical and cross-country data. The historical series are taken from Lee and Lee (2016), who report average years of schooling from 1870 onward and construct human capital stocks based on Mincerian returns. Because the model and data are not expressed in the same units, we construct indices using observed growth rates and anchor them to the model's human capital stock in 1880. In the model, human capital is normalized to one in the Malthusian steady state; by 1880, it is twelve times higher, and by 2000, it rises by an additional factor of eight. Over the same period, the schooling index increases roughly tenfold, while the human capital index from Lee and Lee (2016) grows more than fourfold. Both model and data display a steep upward trajectory during the modern growth era, with the model falling between the two empirical series. It is important to note, however, that while the

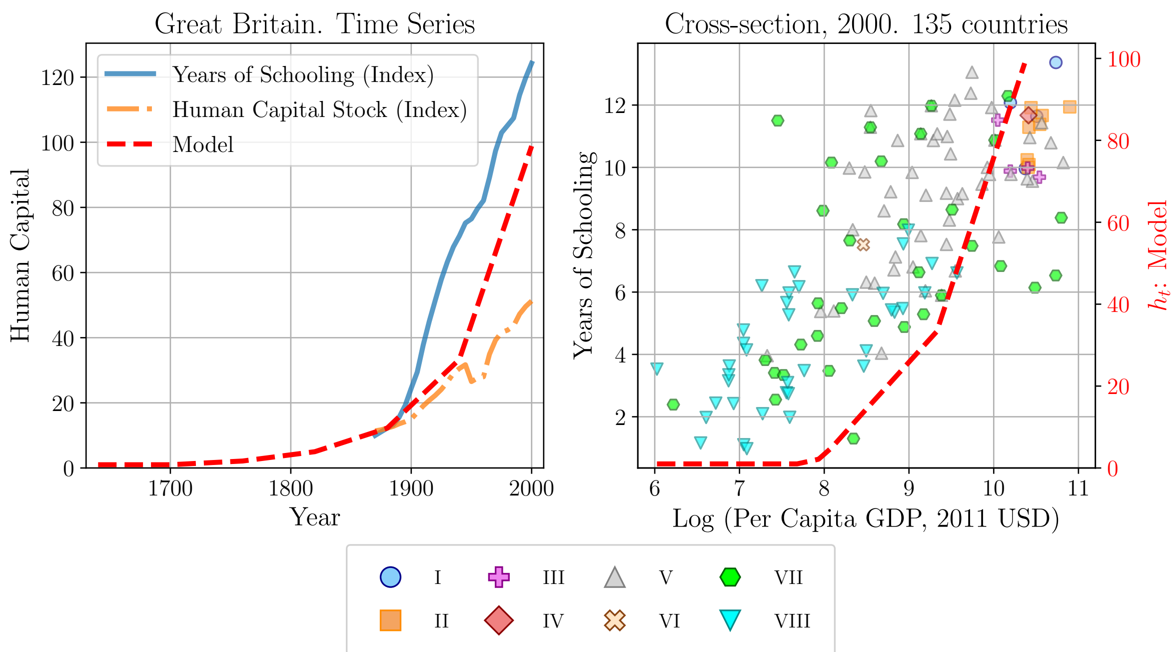

Figure 18: Human Capital.

model encompasses all parental investments in child quality, the data primarily reflect formal education—a narrower dimension of human capital.

The right panel of Figure 18 extends the comparison to cross-country evidence. Here, we plot model and data on independent y-axes, sharing a common x-axis for income per capita. We also experimented with indexed versions for each country, anchoring either to a reference year or to a level of GDP per capita. These alternatives, though visually noisier, yield the same conclusion: human capital rises sharply with development. A notable feature of the figure is that even at income levels consistent with Malthusian stagnation, many African and Asian countries exhibit years of schooling well above what the model would predict for that stage. Is this a novel feature of late development? Unfortunately, comparable data for pre-industrial Britain are unavailable, leaving the question unresolved.

Figure 19 traces the U-shaped relationship between female labor force participation and economic development, originally documented by Goldin (1994), using a cross-section of countries in the year 2000. Ideally, one would compare the model's long-run predictions with historical series for Great Britain extending back to the Malthusian era. The earliest systematic evidence we are aware of comes from Olivetti (2014), who reports data beginning in the late nineteenth century—too recent for our purposes, as these data capture only the rising portion of the "U."[18] Even for the United States, the longest available census series

[18] Moreover, we have already used these data in calibrating the model's long-run female labor force participation parameter.

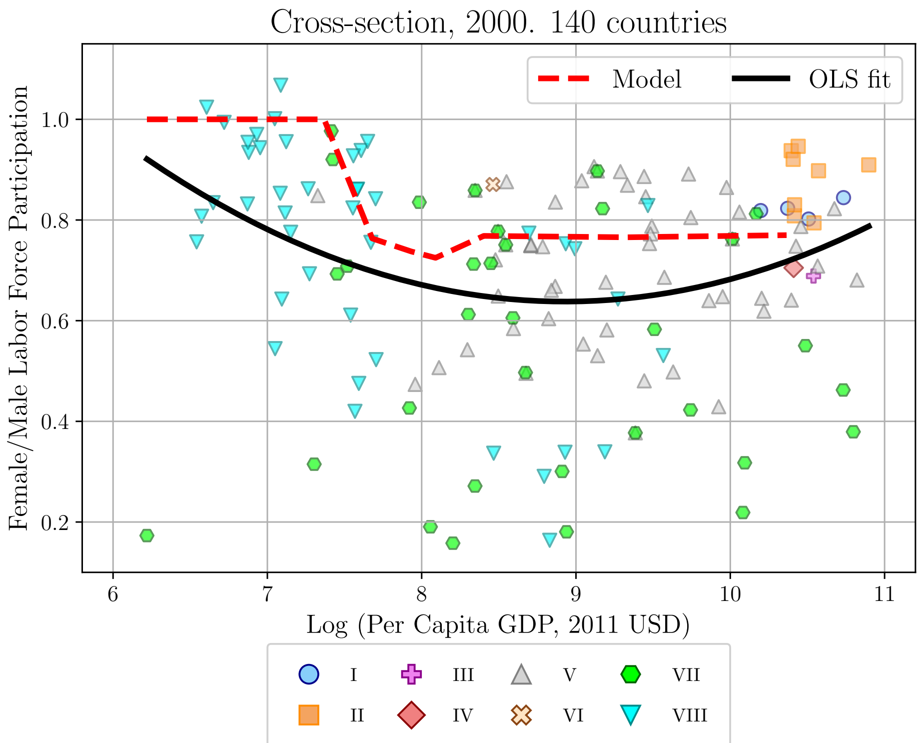

Figure 19: Female to male labor force participation ratio.

requires substantial adjustment. As shown by Ngai et al. (2024), when census data are corrected to include unpaid work in family farms and small enterprises, the U-shape emerges as early as the late nineteenth century. This finding confirms that the cross-sectional pattern observed by Goldin reflects a deeper development process—one that can be inferred from cross-country evidence in the absence of long-run historical data. Furthermore, Appendix C.1 shows that the within-country variation, rather than country or time fixed effects, is what drives the shape of the "U."

To remain consistent with the model, which abstracts from leisure, Figure 19 plots the ratio of female to male labor force participation. Since male participation remains relatively stable across levels of development, variation in this ratio is driven almost entirely by changes in female participation. For comparison, the figure includes a cubic OLS fit to the cross-sectional data. Given the dispersion, this fit provides a useful benchmark. The model, despite its simplicity, roughly follows the estimated curve: it begins near one and converges to its long-run value, $\frac{1-\beta-\eta}{1-\beta+\eta}$, through a U-shaped trajectory that mirrors the declining and rising pattern in the data. At low income levels—where most African economies are located—female participation is high, as both men and women engage in subsistence production. As income rises and the traditional sector contracts, female participation falls, reaching its trough among middle-income countries such as those in Latin America. At higher income levels, represented by the advanced industrial offshoots, female participation rises again as education expands and services dominate. The model thus explains this U-shaped pattern as part of the same mechanism that drives fertility decline and the accumulation of human capital during the transition to modern growth.

5 Conclusion

The patterns of sustained growth in income per capita, the reallocation of labor out of agriculture, rising educational attainment, and the U-shaped trajectory of female labor force participation are not isolated phenomena. They are recurring features of the development process, observed across countries and historical periods. These patterns suggest that the mechanisms underlying the Industrial Revolution are not specific to particular cultures or institutions but reflect general responses to changing economic conditions.

The evidence presented here supports the view that the Industrial Revolution is a global process, not a historical accident confined to a few nations. As more countries embark on their industrial revolutions, the gap between poor and rich nations begins to close, and perhaps these binary labels are becoming obsolete. The forces driving this convergence—technological change, human capital accumulation, and demographic adjustment—are universal but unfold unevenly and remain subject to persistent barriers.

The model presented here is a parsimonious fable that brings together the pieces of this process in a simple, unified way. Yet much remains to be understood. Formal schooling, largely absent from the framework, may amplify human-capital accumulation through complementarities and externalities not captured by parental investment alone. Economies of scale in child-rearing—arising from networks, social infrastructure, or shared caregiving—could alter the trade-off between child quantity and quality in ways our model abstracts from. Finally, the role of land congestion in driving fertility down, so central to the historical escape from Malthus, may be weaker in economies that continue to expand their arable frontiers. These are not flaws but frontiers for future research: understanding how these forces interact will determine whether the global Industrial Revolution now underway achieves its full promise.

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Appendix

A Alternative Preferences: Dynastic Approach

Lucas (2002) describe the preferences of single-parent households in terms of their children's utility rather than h_{t+1} directly and without the subsistence constraint parameter $\bar{c}$. This is

$$\mathcal{W}(c_t, n_t, u_{t+1}) = (1-\beta)\log(c_t) + \eta\log(n_t) + \beta\log(u_{t+1}). \tag{A.1}$$

Using these preferences, Lucas (2002) solves a Bellman equation to describe two equilibria: One Malthusian, where households are not investing in children's quality because it is not profitable to accumulate human capital, and another equilibrium where large values of human capital levels with perpetual growth driven by human capital accumulation. Consider the following Bellman equation:

$$\mathcal{V}(h) = \max_{c,n} \mathcal{W}\left(c, n, \mathcal{V}(\Omega h)\right). \tag{A.2}$$

In this large human capital system, where human capital accumulation is a relevant choice, one can show that the solution to (A.1) takes the form $\mathcal{V}(h) = \Omega h$ due to the homogeneity of degree one of (A.1) in (c_t, u_{t+1}).[19]

[19] See the proof of this claim in Lucas (2002, p. 154)

B OLS Estimation: Female Labor Force Participation over Development.

	(1)	(2)	(3)	(4)
Log GDP pc (2011 USD)	-0.669***	-0.653***	-0.223***	-0.238***
	(0.033)	(0.033)	(0.056)	(0.052)
Square of Log GDP pc (2011 USD)	0.038***	0.037***	0.015***	0.014***
	(0.002)	(0.002)	(0.003)	(0.003)
Constant	3.470***	3.394***	1.309***	1.576***
	(0.145)	(0.147)	(0.254)	(0.253)
Time FE	NO	YES	NO	YES
Country FE	NO	NO	YES	YES
Obs.	4758	4758	4758	4758
Adjusted R^2	0.11	0.12	0.27	0.34

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.1: Female's U-shaped labor force participation over development. Panel data evidence. Dependent variable: Labor force participation rate for women aged between 15 and 65.

C OLS Estimation: Female to Male Labor Force Participation Ratio over Development.

	(1)	(2)	(3)	(4)
Log GDP pc (2011 USD)	-0.729***	-0.704***	-0.188***	-0.208***
	(0.042)	(0.042)	(0.068)	(0.056)
Square of Log GDP pc (2011 USD)	0.041***	0.039***	0.015***	0.012***
	(0.002)	(0.002)	(0.004)	(0.003)
Constant	3.906***	3.776***	1.213***	1.622***
	(0.184)	(0.185)	(0.301)	(0.273)
Time FE	NO	YES	NO	YES
Country FE	NO	NO	YES	YES
Obs.	4758	4758	4758	4758
Adjusted R^2	0.09	0.10	0.31	0.44

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.1: Female to male U-shaped labor force participation ratio over development. Panel data evidence. Dependent variable: Female to male labor force participation rate, for ages between 15 and 65.

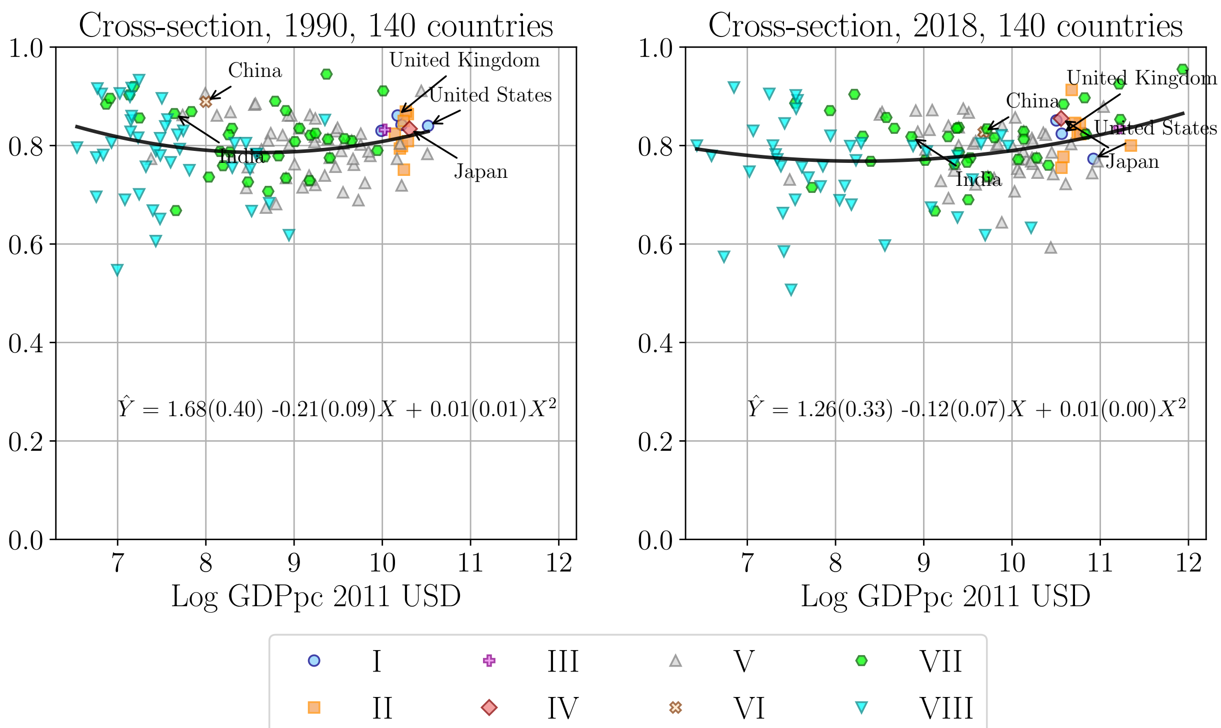

Figure C.1: Male's labor force participation over development. Cross-section data for 140 countries in 1990 and 2018. The labor force participation rate is for men aged between 15 and 65. Eight groups