Inputs delays, Firm Dynamics, and Misallocation in Sub-Saharan Africa

Idossou Marius Adom and Immo Schott*

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Abstract

We quantitatively analyze the macroeconomic consequences of border delays in Sub-Saharan Africa. Delays in imported capital goods lower aggregate output through factor misallocation and an insufficient number of firms that use foreign capital in production. Our model economy features heterogeneous firms that endogenously differ in the degree to which foreign capital goods are used. The model is calibrated to micro-level data from Sub-Saharan Africa. We find that the existing delays are akin to a 37% tax on imported capital goods. Reducing border delays can increase aggregate output by up to 14%. The gains are mainly due to a reallocation of economic activity towards more productive firms.

Keywords: Firm dynamics, heterogeneous firms, misallocation, border delays, Sub-Saharan Africa

*Department of Economics, Université de Montréal and CIREQ. E-mail: immo.schott@umontreal.ca and idossou.marius.adom@umontreal.ca

1 Introduction

In this paper we argue that border delays constitute a key obstacle for development. Using micro-data from Sub-Saharan Africa (SSA) we show that a large number of firms uses directly imported foreign capital goods in production. These foreign capital goods are often significantly delayed after arriving at a country's border. Figure 1 shows the average time it takes from the moment goods arrive at a point of entry until the time the goods can be claimed from customs. These delays are considerable and are several orders of magnitudes larger than in richer countries, where delays are often negligible.[1]

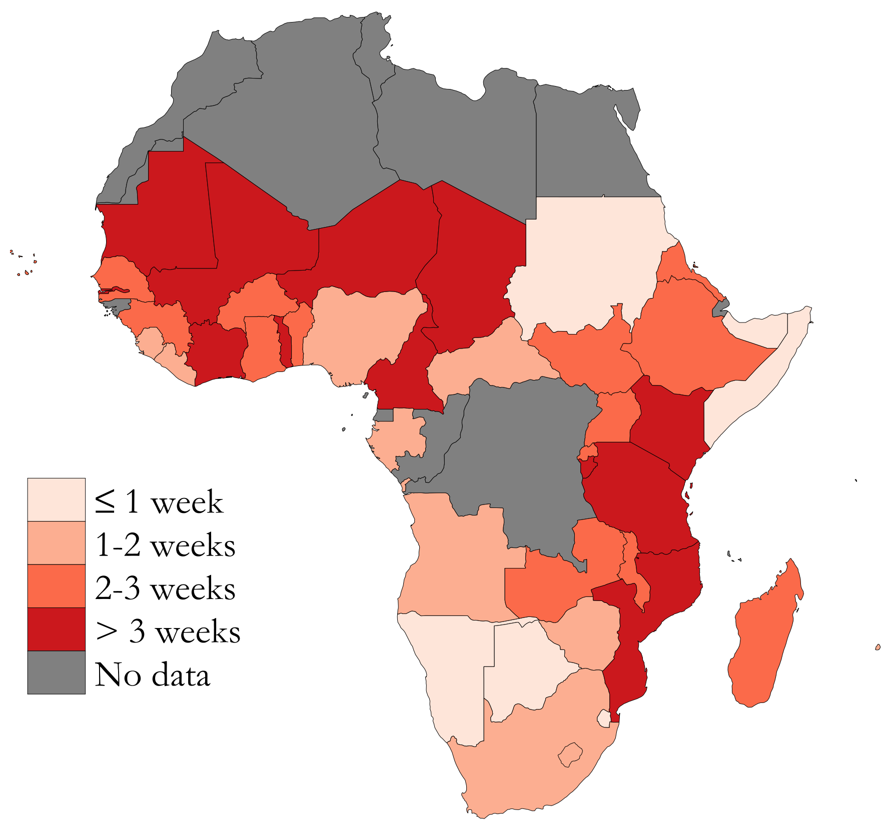

Figure 1: Border delays in Sub-Saharan Africa

Notes.– Darker colors represent longer average delays. Delays are measured as the time it takes from the moment that goods arrive at a point of entry until the goods can be claimed from customs. Source: World Bank Enterprise Survey and authors' calculations.

Delays at borders can result from a variety of sources, including customs and duty procedures, sanitary regulations, a lack of modern I.T., corruption, and the fact that border posts and customs offices are often physically separated (Barka, 2012). The resulting delays occur *after* a good arrives at a point of entry and are therefore distinct from factors such as poor roads, missing infrastructure, road blocks, etc., which may delay the delivery of goods to or

[1] Border delays across the world have increased due to the COVID-19 pandemic (see e.g. Bonadio, Huo, Levchenko, and Pandalai-Nayar (2021)).

from a border point. To the best of our knowledge, this is the first paper to study the role of border delays in the context of a macroeconomic model with firm heterogeneity.

Border delays increase firms' costs of production for at least three reasons. First, there are opportunity costs that arise from allocating goods to storage. The second reason is depreciation, especially when production processes rely on speedy delivery. Finally, the uncertainty about delivery times creates a source of idiosyncratic risk that is specific to firms which are importing foreign capital goods. Importantly, we find that such firms are larger in terms of revenue and employment. Therefore, border delays create a distortion that appears to be positively correlated with firm productivity (Restuccia and Rogerson, 2008). In this paper we quantify the macroeconomic consequences of these border delays.

We develop a model of heterogeneous firms that differ in productivity and their production technology. Firms optimally choose whether to produce using only local inputs or to also import foreign capital goods. Importing foreign capital allows firms to scale up their operation, but they must then face the risk of border delays. Our quantitative results imply that removing border delays would increase aggregate output by 14%. The positive output effects come from two channels. First, through a better allocation of resources, away from small firms using only local inputs, towards larger, more productive firms that also use foreign inputs. Importing firms expand their scale of operation because the gains to using foreign imports increase when delays are reduced, both because of the shorter expected average delays, and because the uncertainty surrounding the delays is eliminated. Second, aggregate output increases because of a selection effect: a larger fraction of firms decides to upgrade their production technology and use foreign inputs in production. This increases output and labor demand.

Our paper contributes to the literature on the sources and consequences of factor misallocation. Persistent factor misallocation has been shown to lead to large losses in aggregate output and measured productivity (e.g. Hsieh and Klenow, 2009). Restuccia and Rogerson (2008) find that distortions which take the form of firm-specific "taxes" have the largest negative output effects when they are positively correlated with firm-level productivity. However, identifying real-world counterparts of such frictions is difficult. Often the implied output losses from directly observed distortions are small (*cf.* Restuccia and Rogerson, 2017).[2] In the context of developing countries, potential distortions include credit frictions, formality status, tax systems, and institutions.[3] We offer a new explanation, based on the prevalence of significant border delays. We find that these delays are a potential source of "correlated distortions" (Restuccia and Rogerson, 2008) because the affected firms are on average larger and likely to be more productive than firms that rely exclusively on local inputs. Our results imply that the existing border delays are akin to a 37% tax on foreign capital investment.

Our model builds on Hopenhayn and Rogerson (1993), to which we add two sources of heterogeneity. First, firms can endogenously upgrade their production technology to include local capital, and later foreign capital. Second, firms that choose to produce using foreign

[2]Other recent examples include David and Venkateswaran (2019), Kaymak and Schott (2019), Bils, Klenow, and Ruane (2021), and Atkin and Donaldson (2021).

[3]Credit frictions are studied in Buera and Shin (2013), Midrigan and Xu (2014), Cole, Greenwood, and Sanchez (2016), and Itskhoki and Moll (2019). The literature on informality is summarized in La Porta and Shleifer (2014) and Ulyssea (2020). Tax systems are studied e.g. in Gordon and Li (2009). The role of institutions is summarized in Acemoglu, Johnson, and Robinson (2005).

capital are subject to idiosyncratic delivery risk. With a given probability imported foreign capital goods are not available for production and an order remains outstanding, possibly to be delivered at a future date.

We contribute to the literature that links firms' technology adoption decisions to micro-level frictions, similar to Midrigan and Xu (2014).[4] Different from that paper, our mechanism does not include binding financial constraints for the most productive firms. Instead, we consider firms that choose to operate a technology which uses foreign inputs to be affected by border delays. Other related papers include Bhattacharya, Guner, and Ventura (2013) and Bento and Restuccia (2017). In those papers, firms are prevented from undertaking the optimal amount of productivity-enhancing investments by the presence of (unspecified) distortions.[5] Our framework includes a discrete choice between three types of technologies that either use only labor, labor and local capital, or, additionally, foreign inputs. Border delays only affect those firms that rely on foreign inputs. The technology adoption margin thus amplifies the effects of frictions, similar to other models of technology adoption. Our contribution is to show how this novel type of friction, significant border delays, leads to misallocation and output losses.

The delays in our model are conceptually different from standard time-to-build models (e.g. Kydland and Prescott (1982)), which typically assume a deterministic amount of time before investment projects are completed. In contrast, delays of imported capital goods are stochastic in our model, implying that firms cannot circumvent the friction by investing in advance, and further generating an additional negative effect through uncertainty (*cf.* Bloom (2009)).[6] Our model generates rich joint distributions of firm sizes, border delays, and outstanding orders across firms with different production technologies which are broadly in line with what we find in the Sub-Saharan African micro-data.

There is an empirical literature that studies the macroeconomic effects of delays. Those papers include Hummels and Schaur (2013), who use US imports data to show that each additional day a good spends in transit is equivalent to a tariff of up to 2.1%. Djankov, Freund, and Pham (2010) document cross-country heterogeneity in time delays for exporters and conclude that each additional day that a product is delayed prior to being shipped reduces trade by more than 1%. Vijil, Wagner, and Woldemichael (2019) find that in developing countries, border delays are linked to lower survival rates for internationally operating firms. We contribute to this literature by focusing on the consequences of input delays through the lens of a quantitative heterogeneous firm model that allows for the evaluation of a range of counter-factual equilibria.

The rest of the paper is organized as follows. We present our data in section 2. Section 3 develops our heterogeneous firm model, which is used in our quantitative analysis of border delays in section 4. Section 5 concludes.

[4]An early example of endogenous technology adoption choices in the context of economic development is studied in Parente and Prescott (1994). Technology upgrading following the removal of an aggregate friction, trade liberalization, is studied in Bustos (2011),

[5]In an otherwise similar framework, Buera, Hopenhayn, Shin, and Trachter (2021) study the complementarity between firms' adoption decisions and find that this generates larger gains from removing micro-level distortions.

[6]In a related paper, Adom (2021) studies the effect of border delays in a model of ex-ante homogeneous firms. Meier (2020) studies stochastic time-to-build in U.S. business cycles due to supply chain disruptions.

2 Data

The data used in this paper comes from the most recent World Bank Enterprise Surveys (WBES). These surveys are representative of the universe of non-agricultural, formal, private sector firms for a large number of countries, especially developing countries. The data are deemed to be of high quality and have recently been used in related papers, including Ranasinghe and Restuccia (2018), Besley and Mueller (2018), and Atkin and Donaldson (2021). Our total sample consists of 10,830 firms across 42 Sub-Saharan African countries.[7]

Importantly for our purposes, the WBES not only provides information about firm size and revenue, but also includes, *inter alia*, three questions that determine to which degree firms use foreign inputs in production and to what extend they are affected by border delays. A complete list of the countries in our sample, the variables used, and the corresponding WBES survey questions can be found in Appendix A.

As a first step, we classify firms into three different *types* based on the degree to which foreign inputs are used in production. Firms which respond that no foreign inputs were used are classified as type 1 firms. Firms that use foreign inputs, but do not import these inputs directly, are classified as type 2 firms. Firms that use and directly import a positive amount of foreign inputs are defined as type 3 firms. The latter type is potentially subject to border delays.

Table 1 shows the distribution of firms in our sample. The resulting statistics are GDP-weighted averages across all countries in our sample. The table shows that almost 50% of firms do not use any foreign inputs in production. These *type 1* firms have fewer employees and are slightly younger compared to other firms. They make up about a third of total employment. We think of these firms as small, locally operating firms without a sophisticated production technology. An example from our dataset is a sewing shop in Lagos (Nigeria) with five employees.

Type 2 firms are older and larger than type 1 firms. They make up 27% of all firms, representing slightly under 20% of employment. Type 2 firms use foreign inputs, yet do not import them directly, but purchase them in local markets. We think of these firms as medium-scale production firms that produce output with a mix of different inputs. An example from our data is a 40-employee firm in Nairobi (Kenya) that produces plastic bags. This firm sources all of the required inputs locally, yet 50% of them are of foreign origin. To clearly delineate firm types and to render our estimates more conservative, we assume that there is no direct effect of border delays on type 2 firms.

Finally, *type 3* firms are the largest and oldest firms on average. The 90th percentile of type 3 firm sizes is 154 employees. These firms account for 23.4% of firms and have an over-proportional employment share of over 50%. We think of these firms as relatively large, internationally operating firms that use sophisticated production technologies. An example of a type 3 firm from our data is a leather manufacturing company that employs 251 workers in Abidjan (Côte d'Ivoire). This company directly imports a fraction of its inputs and reports an average delay of 45 days for imported goods to clear the border.

[7]The WBES from prior to 2009 do not include sampling weights, which is why we restrict our analysis to the most recent survey prior to 2020. We exclude South African data from the main part of our analysis because the survey was conducted in 2020 and 2021 and is thus potentially heavily affected by the global Covid-19 pandemic.

Table 1: Firms' characteristics by type

	Type 1	Type 2	Type 3	All
Uses inputs of foreign origin	No	Yes	Yes	–
Direct import of foreign inputs	No	No	Yes	–
Fraction of firms (in %)	48.6	27.0	24.3	100
Fraction of employment (in %)	30.5	18.8	50.6	100
Employment	23.3	27.7	89.3	37.3
Age (in years)	14.5	15.5	18.4	15.5
Share of foreign inputs	–	–	58.0	–

Notes.– Moments are GDP-weighted averages across countries. Source: World Bank Enterprise Survey, the 2010 World Bank national accounts data, and authors' calculations.

To measure the importance of foreign capital, we calculate an average share of foreign inputs in production of 58%. Importantly, type 3 firms import foreign inputs directly and are therefore subject to border delays. Using the pertinent survey response allows us to construct the distribution over the border delays experienced by firms. The country-specific average delays were used to construct Figure 1. The overall GDP-weighted average delay is 13.8 days.

There is substantial heterogeneity in average border delays both across and within countries. This is shown in Figure 2, which ranks Sub-Saharan African countries from left to right by their 75th percentile in the delay distribution.[8] Average delays range from four days in Eswatini (formerly officially entitled Swaziland) to 32.4 days in Burundi. For comparison, average delays are under five days in Germany (included on the right of Figure 2 for reference), Sweden, and Spain, and about 6–7 days in Thailand.[9]

Figure 2 also shows that in many countries there is a high probability to experience a long delay of over 90 days. For example, Cameroon has an average delay of 23.4 days, yet 10% of all orders experienced a delay of at least 90 days.[10]

We now develop our economic model to study the macroeconomic effects of border delays.

[8] Not all countries are surveyed in the same year, but country-averages are fairly stable over time for countries where two surveys are available. For example. the average border delay in Cameroon was 23.9 days in 2009 and 23.4 days in 2016.

[9] We show a distributional plot for a number of richer economies in Figure A.1 in Appendix A.

[10] These findings are confirmed by a large number of case studies. For example, Raballand, Refas, Beuran, and Isik (2018) document high average amounts of time that cargo spends within African ports as well as the frequent occurrence of extremely long delays. United Nations (2010) estimate delays in crossing borders for products in Sub-Saharan Africa to be the lengthiest in the world. According to Portugal-Perez and Wilson (2009), the number of import procedures in Sub-Saharan Africa is among the highest in the world. The World Bank (2012) finds that an average customs transaction in Africa involves 20 to 30 different parties, 40 documents, 200 data elements (30 of which are repeated at least 30 times) and the rekeying of 60 to 70 per cent of all data at least once.

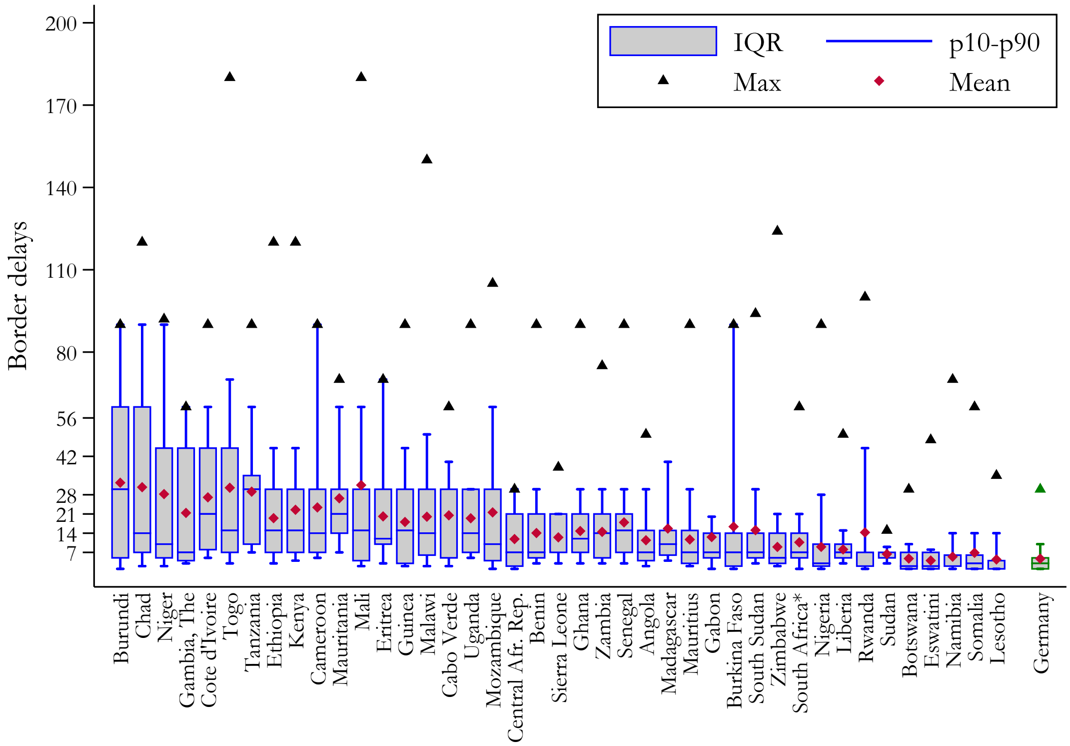

Figure 2: Border delays in Sub-Saharan Africa

Notes.– The figure shows country-specific distributions of border delays in days. The box plots cover the 25th to 75th percentiles of delays. Medians are indicated by the horizontal bar. The lines extending from the boxes show the 10th and 90th percentiles. Averages are shown as red diamonds. The black triangles show the maximum delays. Minimum delays were zero in all cases. Source: World Bank Enterprise Survey and authors' calculations. South Africa is shown here for purposes of comparison but is excluded from the calculation of cross-country moments because of the survey year 2020/21.

3 Model

Time is discrete and the horizon is infinite. A model period corresponds to one week.[11] The economy consists of a mass of heterogeneous firms and a representative household. At time t a positive mass of price-taking firms produces a homogenous output good using a combination of labor and capital. The price of the output good serves as the numéraire. Firms can accumulate foreign and local capital. All capital depreciates at rate $\delta \in (0, 1)$. Future profits are discounted with $\rho \in (0, 1)$.

While labor can only be hired on a domestic spot market at the wage rate w_t, capital can either be acquired locally or directly imported from international markets. The benefit of using foreign capital is that it complements domestic inputs in production. The downside is that this exposes firms to delivery risk.

There are three firm *types*, indexed by $j = \{1, 2, 3\}$. Firms start operating as type 1, but

[11]Because average border delays are in the order of two weeks, choosing a model period appropriately is important to gauge the effect of delays. Similar to the labor search literature, a model period must be chosen such that the probability of success (in this case a delivery) is below unity.

can choose to upgrade. Following the definitions used to analyze the data, firm types differ in the complexity of their production technology. Production of the final good is given by

$$y_t = \mu_j \varepsilon_t F^j(n_t, k_t), \tag{1}$$

where $F^j(\cdot)$ denotes the production function of a type j firm, and μ_j is a productivity shifter. The productivity parameter ε_t is idiosyncratic and follows a Markov chain, $\varepsilon \in \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_K\}$, where $\Pr(\varepsilon' = \varepsilon_k | \varepsilon = \varepsilon_i) \equiv \pi_{ik} \geq 0$ and $\sum_{k=1}^{K} \pi_{ik} = 1$ for each $i = 1, \ldots, K$.

There exists a large mass of potential entrants, of whom a constant mass $M > 0$ enters the economy each period by paying an entry cost $c_e \geq 0$. The initial productivity level ε_0 is drawn from the stationary distribution π_0. The subsequent evolution of productivity is governed by π_{ik}. A fraction x of incumbent firms exogenously exits the economy at the end of a period. Exiting firms cannot re-enter the market at a later period. Upon exit, a firm's remaining capital stock is liquidated, and a fraction θ of it can be distributed to its owners along with any income generated prior to exit. Outstanding foreign orders and a fraction $(1-\theta)$ of current capital are lost upon exit.

3.1 Firm types

There are three firm *types*. Firms enter the economy as type 1. Upon paying a fixed cost, denoted in terms of the consumption good, the firm can be upgraded to the next type, as explained below.

A type 1 firm uses labor as the only input in production, so that $F^1(n_t, k_t) = n_t^{\zeta}$, with $\zeta \in (0, 1)$. Type 2 and type 3 firms produce output using labor and capital with the production function $F^j(n_t, k_t) = n_t^{\beta} k_t^{\alpha}$ for $j \in \{2, 3\}$, with $\alpha \in (0, 1)$ and $\alpha + \beta = \zeta$.

The difference between type 2 firms and type 3 firms is that type 2 firms use only local capital, denoted as k_l, and no foreign capital. Therefore, $k_t = k_{lt}$ for type 2 firms. The accumulation of local capital is standard and follows the law of motion

$$k_{l,t+1} = k_{lt} \cdot (1 - \delta) + i_{lt}. \tag{2}$$

For type 3 firms the effective amount of capital used in production, k_t, is given by a combination of local (l) and foreign (f) capital:

$$k_t(k_{lt}, k_{ft}) = \left[\phi^{\frac{1}{\sigma}} k_{lt}^{\frac{\sigma-1}{\sigma}} + (1-\phi)^{\frac{1}{\sigma}} k_{ft}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

The parameter ϕ denotes a share parameter of local inputs, while σ determines the elasticity of substitution between local and foreign inputs. Foreign and domestic inputs are gross substitutes if $\sigma > 1$ and gross complements if $\sigma < 1$.

Investment into foreign capital is denoted as i_{ft}. Different from local capital, foreign capital needs to be ordered from international markets and because of possible delays at the border, these foreign investments have an associated delivery risk. This implies that a positive investment $i_{ft} > 0$ adds to a firm's stock of outstanding orders, which may or may not be delivered prior to the next production cycle. The total stock of outstanding orders after investment is denoted $\tilde{o}_{f,t+1}$, and is given by the sum of previously undelivered orders and new positive investment into foreign capital, denoted as i_{ft}^{+}:

$$\tilde{o}_{f,t+1} = o_{ft} + i_{ft}^{+} \tag{4}$$

This formulation implies that previously placed orders cannot be canceled.

To capture the idea that outstanding orders may be delayed at the border, we denote the probability that outstanding orders $\tilde{o}_{f,t+1}$ are delivered as ψ. The law of motion of foreign capital is given then by:

$$k_{f,t+1} = \begin{cases} k_{ft} \cdot (1-\delta) + i_{ft}^{-} + \tilde{o}_{f,t+1} & \text{with probability } \psi \\ k_{ft} \cdot (1-\delta) + i_{ft}^{-} & \text{with probability } 1-\psi \end{cases} \tag{5}$$

This equation says that outstanding orders only add to next period's foreign capital with probability ψ. Existing foreign capital can be liquidated with certainty. Negative foreign capital investments $i_{ft} < 0$ are denoted as i_{ft}^{-}.

Finally, the law of motion of outstanding orders of foreign capital at the beginning of the next period is:

$$o_{f,t+1} = \begin{cases} 0, & \text{with probability } \psi \\ \tilde{o}_{f,t+1} & \text{with probability } 1-\psi \end{cases} \tag{6}$$

This implies that an outstanding order is either delayed or fully delivered, i.e., there is no partial delivery.

To fix ideas, consider the special case of no delays, implying $\psi = 1$. From (6) outstanding orders prior to investment would always be zero and the accumulation of foreign capital in (5) would follow the same standard law of motion as local capital.

While firms start out as type 1, upon paying a fixed cost ξ_1, they can upgrade their production technology, i.e., become a type 2 firm that uses local capital in addition to labor. Every period, firms draw a realization of ξ_1 from a time-invariant uniform distribution $G_1(\xi_1)$, common across plants, as in Thomas (2002). The upper bound of the distribution is denoted Ξ_1. If the cost is paid, a firm can immediately start investing in local capital and becomes a type 2 firm next period. We interpret the fixed cost ξ_1 as a technology adoption cost which represents fundamental changes in a firm's scope, customer base, and production technology.

Similarly, type 2 firms can, upon paying a fixed cost ξ_2, upgrade to type 3 firms. This enables them to start importing foreign inputs. Draws of ξ_2 are taken from the uniform distribution $G_2(\xi_2)$ with upper bound Ξ_2. If the cost is paid, the firm may immediately start importing foreign inputs and becomes a type 3 firm next period. The fixed cost can be interpreted the costs of establishing international supply chains. Type 3 firms do not draw a fixed cost and cannot be downgraded to types 1 or 2.

The timing is shown in Figure 3. At the beginning of a period t, a firm's state consists of its current idiosyncratic productivity level ε_t, as well as its current levels of local and foreign capital. In addition, the state also includes the amount of outstanding, previously undelivered, orders of foreign capital, o_{ft}.[12] These stocks are determined through past accumulation decisions and histories of idiosyncratic realizations of delivery shocks.

Given a firm's state, it hires labor on a local spot market and produces output. After production, a firm learns its future productivity level. Each *type 1* and *type 2* firm draws a realization of the fixed cost required to upgrade the firm to the next type. If a current type 1 firm pays the fixed cost ξ_1, the firm becomes a type 2 firm next period. Such a firm may

[12]For type 2 firms, $k_{ft} = o_{ft} = 0$. For type 1 firms, additionally $k_{lt} = 0$.

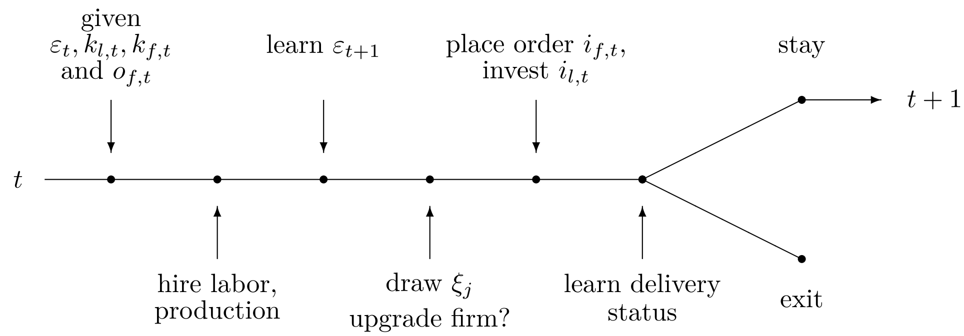

Figure 3: Timing

immediately begin investing in local capital. For current type 2 firms, paying the fixed cost ξ_2 allows the firm to immediately begin investing in foreign capital and start the next period as a type 3 firm.

Knowing its future type, firms make investment decisions into local and/or foreign capital. At the end of a period, firms learn about the status of their orders in foreign capital. If firms do not exogenously exit the economy, they continue operation in period $t+1$.

3.2 Firm problem

We now define the maximization problem of the firm recursively. Time subscripts are omitted in this formulation. Primes indicate future variables. A firm's type is $j \in \{1, 2, 3\}$.

Type 1 firm. The state vector of type 1 firms is given by $s_1 = \{\varepsilon, \xi_1\}$. First we define the beginning-of-period expected value of a firm. At this point, the firm knows its current value of ε, but has not yet learned the realization of its fixed cost draw.

$$V^1(\varepsilon) = \max_n \Big(\mu_1 \varepsilon n^\zeta - wn\Big) + \int_0^{\Xi_1} \sum_{k=1}^{K} \pi_{ik} \tilde{V}^1(\varepsilon_k, \xi_1) dG_1(\xi_1) \tag{7}$$

Given ε and the equilibrium wage rate w the firm chooses current employment n. The function $\tilde{V}^1$ is the end-of-period continuation value when future productivity and the fixed cost draw are realized. It is given by

$$\begin{aligned}\tilde{V}^1(\varepsilon, \xi_1) \quad = \quad \max\Bigg\{ & \underbrace{(1-x)\rho V^1(\varepsilon)}_{\text{do not upgrade}}, \\ & \underbrace{-\xi_1 + \max_{i_l}\Big(-i_l + (1-x)\rho V^2(\varepsilon, k_l') + x\rho V_x^2(k_l')\Big)}_{\text{upgrade \& invest}} \Bigg\}\Bigg).\end{aligned} \tag{8}$$

At the end of the period, the firm chooses whether to upgrade to a type 2 firm. The value of upgrading is given by the second term in the binary maximum choice in (8). The firm incurs the fixed cost ξ_1 and then makes a (local) capital investment i_l. The law of motion for local capital is given by (2).[13] In case of exogenous exit after upgrading, the value is given by $V_x^2(k_l) = \theta k_l$, i.e., the firm recuperates a fraction θ of the invested capital and then ceases operating.[14] We denote as $n^1(\varepsilon)$ and $\hat{i}_l^1(\varepsilon)$ the policy functions for employment and a potential initial investment in local capital.

From (8) a firm will pay the fixed cost ξ_1 and upgrade only if the value of doing so exceeds the continuation value as a type 1 firm. The upgrading decision is then governed by a threshold rule and will occur if ξ_1 is below some ε-specific value $\bar{\xi}_1$. Let $\tilde{\xi}_1$ denote the level of ξ_1 which makes a plant indifferent between upgrading or not:

$$(1-x)\rho V^1(\varepsilon) = -\tilde{\xi}_1 + \max_{i_l}\Big(-i_l + (1-x)\rho V^2(\varepsilon, k_l') + x\rho V_x^2(k_l')\Big) \tag{9}$$

This defines the threshold value as $\bar{\xi}_1(\varepsilon) \equiv \min\{\Xi_1, \max\{0, \tilde{\xi}_1\}\}$ and ensures that it is within the support of the the distribution. Firms which draw $\xi_1 < \bar{\xi}_1(\varepsilon)$ will upgrade to type 2 firms. The fraction of upgrading firms with productivity ε is given by $G_1(\bar{\xi}_1(\varepsilon))$. The upgrading probability is increasing in productivity ε.

Type 2 firm. The state vector of type 2 firms is given by $s_2 = \{\varepsilon, k_l, \xi_2\}$. Analogously to type 1 firms, the beginning-of-period expected value of a type 2 firm is given by

$$V^2(\varepsilon, k_l) = \max_n\Big(\mu_2\varepsilon\ F^2(n, k_l) - wn\Big) + \int_0^{\Xi_2}\sum_{k=1}^{K}\pi_{ik}\tilde{V}^2(\varepsilon_k, k_l, \xi_2)dG_2(\xi_2). \tag{10}$$

The firm maximizes static profits by choosing n. At the end of the period, after learning future productivity and the fixed cost ξ_2, the firm makes an investment decision and decides whether to upgrade the firm:

$$\begin{aligned}\tilde{V}^2(\varepsilon, k_l, \xi_2) \quad = \quad \max\Bigg\{ & \underbrace{\max_{i_l}\Big(-i_l + (1-x)\rho V^2(\varepsilon, k_l') + x\rho V_x^2(k_l')\Big)}_{\text{do not upgrade}}, \\ & \underbrace{-\xi_2 + \max_{i_l, i_f}\Big(-i_l - i_f + (1-x)\rho\mathbb{E}_\psi V^3(\varepsilon, k_l', k_f', o_f') + x\rho\mathbb{E}_\psi V_x^3(k_l', k_f')\Big)}_{\text{upgrade \& invest}}\Bigg\}\end{aligned} \tag{11}$$

If the firm does not upgrade to a type 3 firm, it only makes a local capital investment choice. The second row of (11) says that when the firm decides to upgrade the firm, it makes both a local and a foreign capital investment. Conditional on survival, the firm becomes a type 3 firm next period. Upon exit, it receives the value $V_x^3(k_l, k_f) = \theta\,(k_l + k_f)$. For an upgrading firm, both $V^3(\cdot)$ and $V_x^3(\cdot)$ depend on the probability that the foreign capital investment is delivered next period. If the investment is not delayed, it becomes the future level of foreign capital. If the investment is delayed, it increases instead the future stock of outstanding orders. The associated expectation operator is denoted as $\mathbb{E}_\psi$. We denote as $n^2(\varepsilon, k_l)$ and $i_l^2(\varepsilon, k_l)$ the policy functions for employment and investment in local capital. The policy functions for an upgrading firm are denoted $\hat{i}_l^2(\varepsilon, k_l)$ and $\hat{i}_f^2(\varepsilon, k_l)$.[15]

[13] The value of a type 2 firm is formally defined below.

[14] The value of exit for a type 1 firm is zero because such a firm has no capital.

[15] If the initial order is delivered, the firm has future foreign capital $k_f' = \hat{i}_f^2(\cdot)$ and $o_f' = 0$. If the order is not delivered $k_f' = 0$ and $o_f' = \hat{i}_f^2(\cdot)$.

As for type 1 firms, there exists a state-specific threshold value of $\bar{\xi}_2$ such that firms which draw $\xi_2 < \bar{\xi}_2(\varepsilon, k_l)$ will upgrade to type 3 firms. The fraction of upgrading firms in state (ε, k_l) is given by $G_2(\bar{\xi}_2(\varepsilon, k_l))$. The upgrading probability is increasing in productivity ε.

Type 3 firm. For type 3 firms the state vector consists of $s_3 = \{\varepsilon, k_l, k_f, o_f\}$. Different from the other firm types, a type 3 firm can no longer be upgraded and therefore does not face a discrete choice problem. At the beginning of a period a type 3 firm has learned whether any past investments have been delayed, i.e., it knows the amount of outstanding orders, o_f. The value is given by

$$V^3(\varepsilon, k_l, k_f, o_f) = \max_{n, i_l, i_f^+, i_f^-} \Big\{ \mu_3 \varepsilon F^3(n, k_l, k_f) - wn - i_l - i_f^+ - \theta i_f^- \\ + (1-x)\rho \cdot \sum_{k=1}^{K} \pi_{ik} \mathbb{E}_\psi V^3(\varepsilon_k, k_l', k_f', o_f') + x \cdot \rho \mathbb{E}_\psi V_x^3(k_l', k_f') \Big\}. \tag{12}$$

From (12), this firm begins a period with a given level of idiosyncratic productivity, levels of local and foreign capital in the firm, and a level of outstanding, previously undelivered orders of foreign capital. The firm solves a static labor optimization problem and invests into future capital. Existing capital in place can be sold at a markdown of $\theta < 1$, reflecting capital specificity.

The future stock of foreign capital k_f' is subject to delivery risk. In case of exogenous exit, the firm is liquidated and a fraction θ of the installed capital is sold at the beginning of the following period, captured by the value V_x^3. The policy functions for labor, and local and foreign capital investment are given by $n^3(\varepsilon, k_l, k_f, o_f)$, $i_l^3(\varepsilon, k_l, k_f, o_f)$, and $i_f^3(\varepsilon, k_l, k_f, o_f)$. The laws of motion for foreign capital and outstanding orders (5) and (6) imply two distinct possible future realizations of these variables, depending on whether outstanding orders are delayed. We denote future foreign capital and outstanding orders in case of delivery as k_{f1} and o_{f1} and in case of no delivery as k_{f0} and o_{f0}.[16]

3.3 Entrant's problem

New entrants start as type 1 firms. A potential entrant starts its operations if the value of entry exceeds the entry cost: $V^e \geq c_e$. The value of entry is the expected beginning-of-period value of a type 1 firm.

$$V^e = \sum_{k=1}^{K} \pi_0(\varepsilon_k) V^1(\varepsilon_k) \tag{13}$$

This formulation implies that upon paying c_e, entrants' initial productivity level is revealed. The entry costs reflect the cost of identifying a business idea, a target market, as well as all costs associated to setting up a business.

[16] More precisely, we have that $k_{f1} = k_f \cdot (1-\delta) + i_f^3(\varepsilon, k_l, k_f, o_f) \cdot (i_f^3(\varepsilon, k_l, k_f, o_f) < 0) + \tilde{o}_f'$, and $o_{f1} = 0$, while $k_{f0} = k_f \cdot (1-\delta) + i_f^3(\varepsilon, k_l, k_f, o_f) \cdot (i_f^3(\varepsilon, k_l, k_f, o_f) < 0)$, and $o_{f1} = \tilde{o}_f'$.

3.4 Distribution of firms

Let $\mathbf{\Gamma}^j(s_j)$ denote the beginning-of-period distribution of incumbent type j firms over the relevant firm states s_j for $j = \{1, 2, 3\}$. With $\mathcal{M}$ denoting the mass of entrants, the law of motion for the distribution of type 1 firms is given by

$$\mathbf{\Gamma}^{1\prime}(\varepsilon_k) = \sum_{i=1}^{K}(1-x)(1-G_1(\bar{\xi}_1(\varepsilon_i)))\pi_{ik}\mathbf{\Gamma}^1(\varepsilon_i) + \pi_0(\varepsilon_k)\mathcal{M}. \tag{14}$$

The first term represents the evolution of current type 1 firms that do not exit and do not upgrade. The second term represents entering firms.

The law of motion for incumbent type 2 firms is

$$\mathbf{\Gamma}^{2\prime}(\varepsilon_k, k_l') = \sum_{i=1}^{K}(1-x)(1-G_2(\bar{\xi}_2(\varepsilon_i, k_l)))\mathcal{I}_2(\varepsilon_i, k_l)\pi_{ik}\mathbf{\Gamma}^2(\varepsilon_i, k_l) + \sum_{i=1}^{K} G_1(\bar{\xi}_1(\varepsilon_i))\mathcal{I}_1(\varepsilon_i)\pi_{ik}\mathbf{\Gamma}^1(\varepsilon_i). \tag{15}$$

The first term represents current type 2 firms that do not exit and do not upgrade. The indicator function $\mathcal{I}_2(\varepsilon, k_l) = 1$ if $k_l' = (1-\delta)k_l + i_l^2(\varepsilon, k_l)$. The second term represents current type 1 firms that upgrade to type 2 firms. The indicator function $\mathcal{I}_1(\varepsilon) = 1$ if $k_l' = \hat{i}_l^1(\varepsilon)$.

Finally, the law of motion for incumbent type 3 firms is

$$\begin{aligned}\mathbf{\Gamma}^{3\prime}(\varepsilon_k, k_l', k_f', o_f') &= (1-x)\sum_{i=1}^{K}\mathcal{I}(\varepsilon_i, k_l, k_f, o_f)\pi_{ik}\mathbf{\Gamma}^3(\varepsilon_i, k_l, k_f, o_f) \\ &\quad + \sum_{i=1}^{K} G_2(\bar{\xi}_2(\varepsilon_i, k_l))\mathcal{I}_3(\varepsilon_i, k_l)\pi_{ik}\mathbf{\Gamma}^2(\varepsilon_i, k_l).\end{aligned} \tag{16}$$

The first term represents the current type 3 firms that do not exit. The indicator function $\mathcal{I}(\varepsilon, k_l, k_f, o_f)$ takes the value ψ if $k_l' = (1-\delta)k_l + i_l^3(\varepsilon, k_l, k_f, o_f)$, $k_f' = k_{f1}$, and $o_f' = o_{f1}$. It takes the value $1-\psi$ if $k_l' = (1-\delta)k_l + i_l^3(\varepsilon, k_l, k_f, o_f)$, $k_f' = k_{f0}$, and $o_f' = o_{f0}$. Using a law of large numbers, we assume that orders for a fraction $1-\psi$ of firms are delayed.

The second term in (16) represents current type 2 firms that upgrade to type 3 firms. The indicator function $\mathcal{I}_3(\varepsilon, k_l)$ takes the value ψ if $k_l' = (1-\delta)k_l + \hat{i}_l^2(\varepsilon, k_l)$, $k_f' = \hat{i}_f^2(\varepsilon, k_l)$, and $o_f' = 0$. It takes the value $1-\psi$ if $k_l' = (1-\delta)k_l + \hat{i}_l^2(\varepsilon, k_l)$, $k_f' = 0$, and $o_f' = \hat{i}_f^2(\varepsilon, k_l)$.

In the stationary equilibrium the distributions $\mathbf{\Gamma}^j(s_j)$ for $j = \{1, 2, 3\}$ will be constant over time. With $x > 0$ the mappings defined above have unique invariant distributions associated with them. Additionally, the distributions are linear in the mass of entrants $\mathcal{M}$.

3.5 Households

The representative household derives utility from consumption and leisure, with $U(C, N) = \log(C) - \frac{\lambda}{1+\frac{1}{\chi}} \cdot N^{1+\frac{1}{\chi}}$. The term λ determines the disutility from hours worked, while χ is the inverse Frisch elasticity. Consumption is given by total disposable income, which consists of labor earnings plus dividends: $C^d = D + w \cdot N^s$. Dividends D are the aggregate profits that are distributed from firms to households. The investment decision is handled by firms.

3.6 Competitive Equilibrium

A stationary recursive competitive equilibrium consists of type-specific beginning-of-period value functions $V^1(\varepsilon)$, $V^2(\varepsilon, k_l)$, $V^3(\varepsilon, k_l, k_f, o_f)$, a value of entry V^e, type-specific policy functions for labor, local, and foreign capital, cutoff values for upgrading $\bar{\xi}_1$ and $\bar{\xi}_2$, a wage rate w, distributions $\mathbf{\Gamma}^j$ for $j = \{1, 2, 3\}$, and a measure of entrants $\mathcal{M}$ such that:

1. The households optimizes: $N^s = \left(\frac{w}{\lambda \cdot C}\right)^{1/\chi}$.
2. For $j = \{1, 2, 3\}$, the value function V^j together with the associated policy functions solves the respective incumbent firm's problem.
3. The free entry condition holds: $V^e = c_e$.
4. The type-specific distributions over incumbent firms are stationary: $\mathbf{\Gamma}'^j = \mathbf{\Gamma}^j$ for $j = \{1, 2, 3\}$.
5. Markets clear:

$$\sum_{j=1}^{3} \int n^j(s_j) d\mathbf{\Gamma}^j \cdot \mathcal{M} = N^s = N^* \;\text{ and }\; C = Y - I^k - I^\xi - \mathcal{M} \cdot c_e$$

The last equation gives the resource constraint of the economy. Total output is $Y = \sum_{j=1}^{3} \int y(s_j) d\mathbf{\Gamma}^j \mathcal{M}$. Resources used for capital investment are given by I^k, which includes local and foreign capital investments by all firms, net of non-depreciated capital and the scrap value of exiting firms. Resources used for upgrading firms are $I^\xi = \frac{1}{2}\left[\frac{\bar{\xi}_1(\varepsilon)^2}{\Xi_1}\mathbf{\Gamma}^1 + \frac{\bar{\xi}_2(\varepsilon)^2}{\Xi_2}\mathbf{\Gamma}^2\right]$ The last term denotes the total resources spent on firm entry.[17]

4 Quantitative analysis of border delays

In this section, we use a quantitative version of the model developed in the previous section in order to assess the effects of border delays in Sub-Saharan African countries. We calibrate the model to match the firm size and employment distributions from African data, targeting average border delays of about 14 days. We then compare this economy to a counterfactual one in which border delays are eliminated. This allows us to quantify the impact of border delays on macroeconomic outcomes.

4.1 Computational strategy

To solve our model numerically, we use value function iteration with interpolation. The equilibrium wage is found via bisection.

To reduce the number of necessary state variables, we exploit the fact that local capital is not subject to adjustment costs and adopt an equivalent but computationally more convenient formulation. The algorithm to solve the model defines a computational period to begin after future productivity is realized, but before the investment decisions are made. By

[17] These equations imply that dividends paid to households are given by $D = Y - w \cdot N^* - I^k - I^\xi - \mathcal{M} \cdot c_e$.

adding non-depreciated local capital to current profits, we assume that the entire stock of local capital is repurchased every period. This implies that the policy functions for labor, as well as local capital can be derived analytically. The key challenge of the computational solution then consists in solving for the policy functions for foreign capital investments, which influences the evolution of the two endogenous state variables, foreign capital, and outstanding orders. Appendix B shows the modified firm problem in more detail.

4.2 Calibration

A number of parameters are preset, while others are calibrated to match data moments. The parameters set outside the model are shown in Table 2. We target an annual real rate of return of 5%, implying $\rho = 0.95$. Following Restuccia and Rogerson (2008), we set the span-of-control parameter ζ to 0.85, and split it into α and β according to their income shares of 2/3 and 1/3. implying $\alpha = 0.283$ and $\beta = 0.567$. The annual depreciation rate is set to 10%. From the WBES data we compute an average foreign capital share in production of type 3 firms of about 60%. This implies $\phi = 0.4$ in (3), the equation that defines effective capital. The elasticity of substitution between local and foreign capital is set to 0.5, following Chang (1995) and Boehm, Flaaen, and Pandalai-Nayar (2019). The exogenous exit rate is set to 10% annually. The recovery value of capital is $\theta = 0.9$.[18] We set the inverse Frisch elasticity to 1.2, which is in the range suggested by Chetty, Guren, Manoli, and Weber (2011). The value of λ is set to generate a labor supply of 1 in the baseline economy. Without loss of generality, we normalize the wage in the benchmark economy to 1, which pins down the entry cost c_e from the free-entry condition (13). A key parameter is the average delay of foreign capital investments. We compute an average delay for imported foreign capital goods of 13.8 days in the WBES data. This implies a weekly delivery probability of $\psi = 0.41$.

The six remaining parameters $\Theta = \{\mu_1, \mu_2, \Xi_1, \Xi_2, \rho_\varepsilon, \sigma_\varepsilon\}$ are found by matching six data moments that are informative about the joint distributions of firms, employment, and firm types. The parameters were chosen to minimize the average percentage difference between data and model moments. The calibrated parameter values are shown in Table 3, together with a comparison between data and model moments. The parameters are jointly identified, yet we briefly describe the intuition behind the identification and our choice of data moments.

The first two parameters determine the relative productivity of type 1 and type 2 firms. These parameters directly impact the average firm size and are used to match the fraction of total employment in firms of the respective type. The upper bounds of the distributions of upgrading costs are key in determining the distribution of firms across types. We therefore identify these parameters from the fraction of type 1 and type 2 firms in the data. The idiosyncratic productivity parameters ρ_ε and σ_ε shape the firm size and employment distributions, as well as the concentration of employment. The data is organized in employment bins. We match the fraction of firms in the largest size bin (100+ employees) as well as the employment share of small firms, defined as firms with less than 20 employees.

Table 3 shows that the model matches the data moments very well, both unconditionally and conditional on firm type. From the table, although the distributions of the number of firms and of total employment by firm types are close to the data, one shortcoming is that

[18] We present robustness exercises with respect to these parameter choices in Appendix C.

Table 2: Externally calibrated parameters

Parameter	Value	Meaning
ρ	0.95	Discount factor (annual)
ζ	0.85	Curvature of production function
α	0.28	Capital income share
β	0.57	Labor income share
δ	0.10	Depreciation rate (annual)
λ	0.86	Household disutility of labor
χ	1.2	Inverse Frisch elasticity
ψ	0.41	Average import delay of 13.8 days
ϕ	0.40	Local capital share in production of type 3 firms
σ	0.50	Elasticity of substitution between capital types
x	0.10	Exogenous exit rate (annual)
θ	0.90	Recovery value of capital
c_e	0.10	Entry cost

the model generates a too high share of large firms, and an unconditional employment share of small firms that is slightly below its data counterpart.

Nevertheless, the model is successful in replicating a number of untargeted moments. Figure 4 shows average firm size by type, which is well matched by the model. Figures 5 and 6 show the distributions of firms and employment by firm type and jointly for all firms. A key data feature that is replicated by the calibrated model is that most firms are small, while employment is concentrated among large firms: Roughly 75% of all type 1 and 2 firms have fewer than 20 employees, while more than 60% of total employment among type 1 and 2 firms is concentrated in large firms. The fraction of large type 3 firms is significantly higher, as is the employment share of large type 3 firms. Importantly, the model is able to generate both large type 1 firms and small type 3 firms. However, it does not generate as many small type 3 firms as we see in the data.

Overall, the model captures salient features of Sub-Saharan African economies. This implies that the quantitative model is a useful framework to study the consequences of a reduction in border delays for macroeconomic aggregates, to which we turn to next.

4.3 The effects of border delays

We now simulate a counterfactual economy in which border delays are eliminated, implying $\psi = 1$. A key finding is that reducing border delays from their current levels down to zero results in an increase of aggregate output of 14.0%.

The full results are presented in Table 4. The column 'Benchmark' shows results from the calibrated economy described above. To compute the moments from the economy without delays, shown in the second column, we set the probability that an order of foreign capital

Table 3: Internally calibrated parameters

Parameter	Value	Moment (in %)	Data	Model
μ_1	2.22	Employment share of type 1 firms	30.52	31.02
μ_2	0.78	Employment share of type 2 firms	18.83	18.83
Ξ_1	15.15	Fraction of type 1 firms	48.64	47.72
Ξ_2	35.83	Fraction of type 2 firms	27.04	26.70
ρ_ε	0.88	Fraction of large firms	6.67	7.50
σ_ε	0.07	Employment share of small firms	22.14	21.98

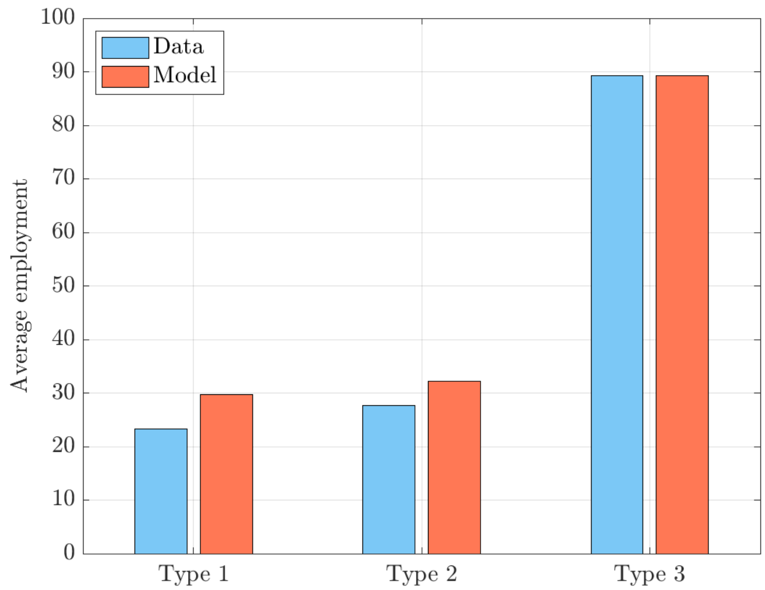

Figure 4: Untargeted moments: Average employment by firm type

is delivered to $\psi = 1$.

In the economy without delays, firms no longer face any delivery risk. This increases the expected return to foreign capital investments and encourages capital accumulation. Consequently, aggregate foreign capital increases, while undelivered orders fall to zero. Because foreign and local capital are complements in production, the removal of border delays also increases aggregate local capital.

The bottom half of Table 4 shows the effects on the distribution of firms. A key result is that the fraction of type 3 firms increases. This happens because the removal of border delays directly benefits type 3 firms, whose value consequently goes up. This in turn increases the threshold value $\bar{\xi}_2$, which pins down the maximum fixed cost that type 2 firms are willing to incur to upgrade. This then increases the value of type 2 firms and leads to more type 1 firms upgrading as well. In this way, the removal of border delays creates an extensive-margin effect that leads to an increase in the fraction of type 3 firms by almost 15 percentage

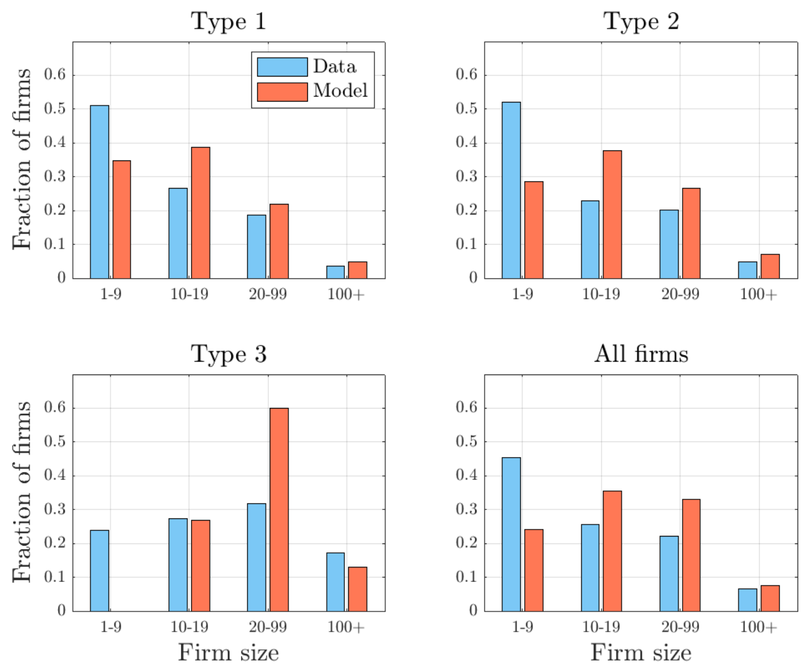

Figure 5: Untargeted moments: Distribution of firms by firm type

points. In terms of employment in type 3 firms, the effects are even stronger. Because type 3 firms now accumulate more capital than in the economy with border delays, they have a higher demand for labor. The employment share of type 3 firms increases from 50% to 83%.

This reallocation of labor across firm types is reinforced in general equilibrium. The increase in firm value leads to a higher real wage through the free-entry condition (13). Type 1 firms use labor as the sole factor of production. A given increase in the wage rate leads to a stronger reduction in labor demand for those firms than for firms that also use local and foreign capital. This channel further lowers the employment share of type 1 firms.

Finally, the higher real wage creates an income effect for the representative household, leading to a reduction of overall labor supply.

Decomposition. Aggregate output increases by 14.0% when border delays are eliminated. We show a decomposition of these gains in Figure 7. The increase in output is due to four distinct channels: a selection margin, a reallocation margin, a change in the mass of firms, and a general equilibrium effect. Figure 7 is organized as follows. On the left side, we normalize the size of the four areas, they correspond to the different channels. The overall size of the left bar is normalized to one, representing output in the baseline economy with input delays. The right bar corresponds to the economy without frictions. The size of the bar represents the overall gain in output compared to the baseline economy, +14%. The size of each area indicates the output gain from the associated channel. A change in the size of an area represents a proportional change in output from a specific channel.

To study the selection margin, we recompute total output using the distribution of firm

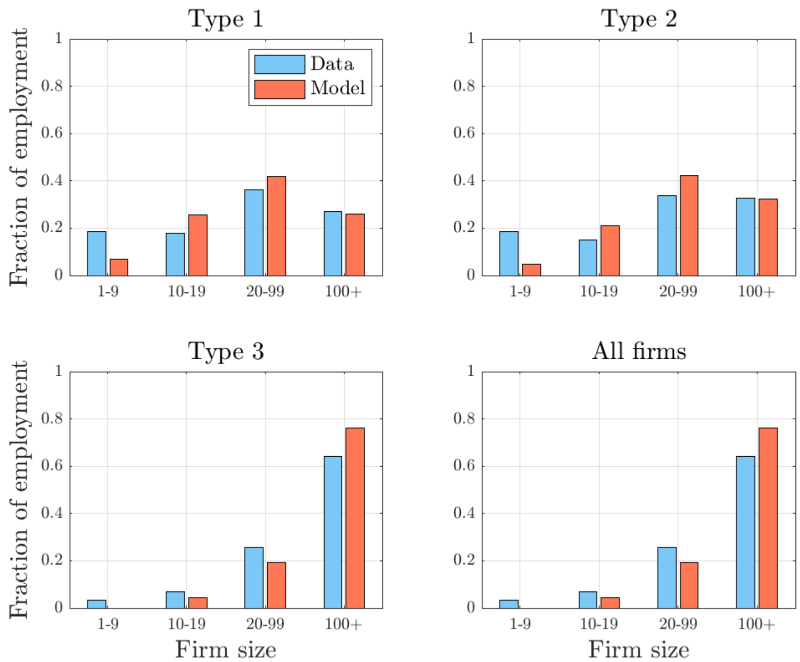

Figure 6: Untargeted moments: Distribution of employment by firm type

	Benchmark	No delays
Average border delays (in days)	13.8	0
Aggregate output	1.00	1.14
Local capital	1.00	1.17
Foreign capital	1.00	2.07
Outstanding orders	1.00	0
Labor supply	1.00	0.92
Wage rate	1.00	1.14
Mass of firms	1.00	0.95
Fraction of type 1 firms	0.48	0.35
Fraction of type 2 firms	0.27	0.24
Fraction of type 3 firms	0.26	0.41
Employment share type 1 firms	0.31	0.09
Employment share type 2 firms	0.19	0.09
Employment share type 3 firms	0.50	0.83

Table 4: Consequences of elimination of border delays

Note.– The column 'no delays' shows results for the stationary equilibrium of the economy with the same parameters as the benchmark model, except that $\psi = 1$.

types from the frictionless economy, holding all else constant at its baseline level, i.e., firms' policy functions and total mass of firms. The selection margin increases total output because more firms choose to pay the fixed upgrading costs to eventually become type 3 firms. Type 3 firms are larger and more productive on average, which implies that the higher fraction of type 3 firms increases output. This is represented by the light blue area labeled "Selection" in Figure 7. The selection margin is an important reason for the increase in aggregate output.

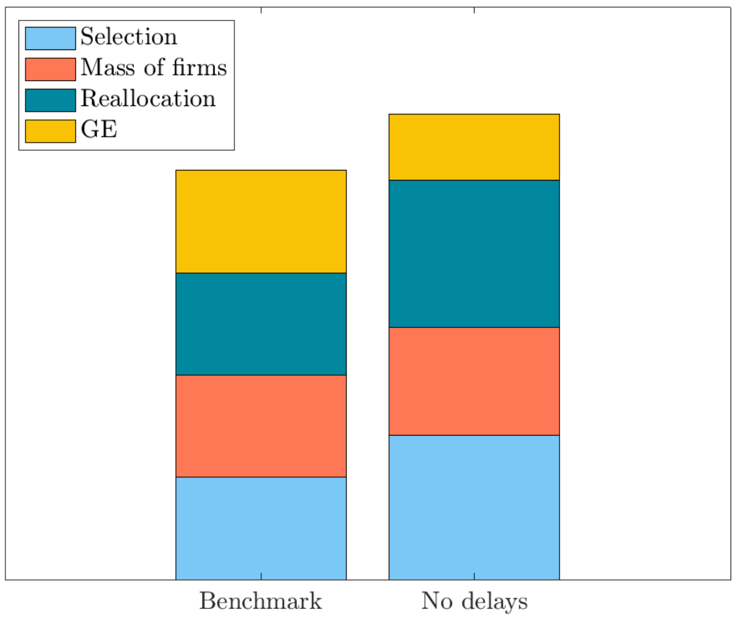

Figure 7: Decomposition

Notes.– The left bar is normalized to size one. All four areas are normalized to 1/4. The right bar represents total output in the economy without delays. The size of the areas compared to the left bar represents the gain in aggregate output from the respective channel, holding everything else constant.

The second channel, shown in red in Figure 7, is the total mass of firms in the economy. To compute this channel we hold all policies fixed from the baseline economy and compute aggregate output using the mass of firms from the economy without delays. When the border delays are removed, firms increase their labor demand, implying that the total mass of firms must fall in order to clear the labor market. This channel has a small negative effect on total output.

Similarly to the selection margin, the reallocation channel has a large positive effect on output. It is represented by the teal area in Figure 7. We compute its contribution by holding the fraction of each firm type constant from the baseline economy. Total output is then calculated using each firm type's average output from the economy without delays. The output gains are due to an improved allocation of factors of production within firm types. While the reallocation gains for type 1 and type 2 firms are negligible, output in type 3 firms grows significantly. This occurs primarily because in the economy without delays there are no firms with outstanding orders, i.e., firms that have received less than their chosen amount of foreign capital. The heterogeneity in the delivery status of previously ordered foreign capital disappears and average output for type 3 firms increases. Capital, labor, and output is reallocated from type 3 firms with delays to those without delays.

Finally, we quantify the general equilibrium effect of the removal of border delays. To do so, we recompute the baseline economy at the equilibrium wage rate that results from the removal of all delays. Compared to the baseline economy, the wage rate increases. This lowers labor demand and total output, especially for labor-intensive type 1 firms. The effects are represented by the yellow areas in Figure 7.

Delays as correlated distortions. Border delays lead to a misallocation of factors, More specifically, they create a distortion which is positively correlated with productivity. The literature on factor misallocation has identified such "correlated distortions" as being able to generate real effects of considerable magnitude (*cf.* Restuccia and Rogerson, 2008). In the context of our model, there are two reasons for a positive correlation of the distortion with firm-level productivity. First, only firms that import foreign inputs (type 3 firms) are directly affected by the friction. In addition, the fixed cost of upgrading is more likely to be paid by firms with higher idiosyncratic productivity. This implies that in equilibrium, type 3 firms are on average more productive. The import delay friction is therefore strongest for firms with higher average productivity. Figure 8 shows the distribution of output among type 3 firms in the economies with and without delays. In the baseline economy with border delays (blue bars), almost three quarters of output is produced by firms with very high productivity. Once the friction is removed, more firms upgrade to type 3 firms. The red bars in Figure 8 show the reallocation of production caused by this change. Because the critical productivity thresholds for upgrading fall when delays go down, more firms with relatively lower productivity levels now become type 3 firms.

The distortion also interacts with *changes* in firm-level productivity. Type 3 firms which receive a negative productivity shock and wish to shrink, can sell their installed capital with certainty. On the other hand, firms with positive productivity shocks that want to accumulate foreign capital will be subject to the delay friction. The impact of the friction is therefore higher when the gap between current and target capital is larger.

Border delays as a tax. The misallocation literature often represents distortions in the form of wedges, or 'taxes' on factor demand (*cf.* Restuccia and Rogerson (2008), Hsieh and Klenow (2009)). In the model described above, we explicitly modeled a concrete distortion and found it to be quantitatively important. An alternative approach would be to completely abstract from the exact type of friction and simply model border delays as a 'tax'. In our model, border delays increase the cost of foreign capital investments by creating uncertainty about the time of their delivery. We therefore ask: What would be the tax on foreign capital investments that generates the same aggregate output as the benchmark model with delays?

To answer this question, we start from our baseline model but eliminate all border delays (i.e. $\psi = 1$). We then introduce a tax τ on positive foreign capital investment and find the level of the tax which generates the same level of aggregate output we obtained in the baseline economy with border delays. The results show that the 13.8 days of average border delays we observe in the Sub-Saharan African data are equivalent to a 36.9% tax on foreign

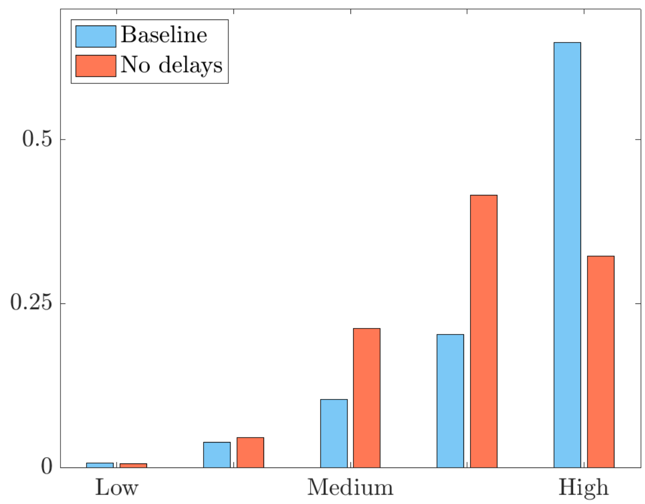

Figure 8: Distribution of output over productivity for type 3 firms

Notes.– The blue and the red bars each sum up to one. We group idiosyncratic productivity into five bins. The bars show the fraction of total output produced by type 3 firms in a given productivity bin. For each productivity level, the blue bars on the left show the baseline economy with delays, the red bars on the right show the economy without delays.

capital investment.

5 Conclusion

Using the WBES data, we documented that not only are border delays of inputs pervasive across Sub-Saharan Africa, they are also several orders of magnitudes larger than in richer countries. To assess their effects on firms and on the macroeconomy, we developed a dynamic general equilibrium model of heterogeneous firms. In the model, firms differ in productivity, their technology choice, and their levels of local and foreign capital. Importantly, not all firms are subject to delays because smaller, less productive firms opt for a production technology that only uses labor and local capital. Firms that choose to import foreign capital face a probability that those imports are delayed, leading to a suboptimal allocation of factors.

We find that when border delays are eliminated, aggregate output increases by 14%. These effects are mainly explained through selection and reallocation. The delays represent a distortion that is positively correlated with firms' productivity, affecting mainly the largest, most productive firms in the economy. Once the delays are removed, more firms choose to import foreign inputs and operate a more advanced technology. In this way, factors are reallocated towards more productive firms.

In terms of their effect on aggregate output, border delays are significant: we estimate that the observed average border delay is equivalent to a 36.9% tax on foreign capital investments. Our analysis suggests that simplifying and streamlining the border procedures for imported inputs in Sub-Saharan Africa should be a high-priority item on the region's agenda.

References

Acemoglu, D., S. Johnson, and J. A. Robinson (2005): "Institutions as a Fundamental Cause of Long-Run Growth," in *Handbook of Economic Growth*, ed. by P. Aghion and S. Durlauf, Elsevier, vol. 1 of *Handbook of Economic Growth*, chap. 6, 385–472.

Adom, I. M. (2021): "Time delays at the border: Macroeconomic Consequences for Sub-Saharan African Economies," Mimeo - Université de Montréal.

Atkin, D. and D. Donaldson (2021): "The Role of Trade in Economic Development," Tech. Rep. w29314, National Bureau of Economic Research, Cambridge, MA.

Barka, H. B. (2012): "Border Posts, Checkpoints, and Intra-African Trade: Challenges and Solutions," Working paper. African Development Bank.

Bento, P. and D. Restuccia (2017): "Misallocation, Establishment Size, and Productivity," *American Economic Journal: Macroeconomics*, 9, 267–303.

Besley, T. and H. Mueller (2018): "Predation, Protection, and Productivity: A Firm-Level Perspective," *American Economic Journal: Macroeconomics*, 10, 184–221.

Bhattacharya, D., N. Guner, and G. Ventura (2013): "Distortions, endogenous managerial skills and productivity differences," *Review of Economic Dynamics*, 16, 11–25.

Bils, M., P. J. Klenow, and C. Ruane (2021): "Misallocation or Mismeasurement?" *Journal of Monetary Economics*, 124, 39–56.

Bloom, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77, 623–685.

Boehm, C. E., A. Flaaen, and N. Pandalai-Nayar (2019): "Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Toōhoku Earthquake," *Review of Economics and Statistics*, 101, 60–75.

Bonadio, B., Z. Huo, A. A. Levchenko, and N. Pandalai-Nayar (2021): "Global supply chains in the pandemic," *Journal of International Economics*, 133, 103534.

Buera, F. J., H. Hopenhayn, Y. Shin, and N. Trachter (2021): "Big Push in Distorted Economies," *National Bureau of Economic Research Working Paper Series*, No. 28561.

Buera, F. J. and Y. Shin (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121, 221–272.

Bustos, P. (2011): "Trade Liberalization, Exports, and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinian Firms," *American Economic Review*, 101, 304–340.

Chang, L.-J. (1995): "Business cycles with distorting taxes and disaggregated capital markets," *Journal of Economic Dynamics and Control*, 19, 985–1009.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," *The American Economic Review*, 101, 471–475.

Cole, H. L., J. Greenwood, and J. M. Sanchez (2016): "Why Doesn't Technology Flow From Rich To Poor Countries?" *Econometrica*, 84, 1477–1521.

David, J. M. and V. Venkateswaran (2019): "The Sources of Capital Misallocation," *American Economic Review*, 109, 2531–67.

Djankov, S., C. Freund, and C. S. Pham (2010): "Trading on Time," *The Review of Economics and Statistics*, 92, 166–173.

Gordon, R. and W. Li (2009): "Tax structures in developing countries: Many puzzles and a possible explanation," *Journal of Public Economics*, 93, 855–866.

Hopenhayn, H. and R. Rogerson (1993): "Job turnover and policy evaluation: A general equilibrium analysis," *Journal of Political Economy*, 101, 915–938.

Hsieh, C.-T. and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124, 1403–1448.

Hummels, D. L. and G. Schaur (2013): "Time as a Trade Barrier," *American Economic Review*, 103, 2935–59.

Itskhoki, O. and B. Moll (2019): "Optimal Development Policies With Financial Frictions," *Econometrica*, 87, 139–173.

Kaymak, B. and I. Schott (2019): "Loss-offset provisions in the corporate tax code and misallocation of capital," *Journal of Monetary Economics*, 105, 1–20.

Khan, A. and J. K. Thomas (2003): "Nonconvex factor adjustments in equilibrium business cycle models: do nonlinearities matter?" *Journal of Monetary Economics*, 50, 331–360.

Kydland, F. E. and E. C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50, 1345–1370.

La Porta, R. and A. Shleifer (2014): "Informality and Development," *Journal of Economic Perspectives*, 28, 109–26.

Meier, M. (2020): "Supply Chain Disruptions, Time to Build and the Business Cycle," Mimeo. Universität Mannheim.

Midrigan, V. and D. Y. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 104, 422–58.

Parente, S. L. and E. C. Prescott (1994): "Barriers to Technology Adoption and Development," *Journal of Political Economy*, 102, 298–321.

Portugal-Perez, A. and J. S. Wilson (2009): "Why trade facilitation matters to Africa," *World Trade Review*, 8, 379–416.

Raballand, G., S. Refas, M. Beuran, and G. Isik (2018): *Why Does Cargo Spend Weeks in Sub-Saharan African Ports?*, Why Does Cargo Spend Weeks in Sub-Saharan African Ports?, The World Bank.

Ranasinghe, A. and D. Restuccia (2018): "Financial frictions and the rule of law," *Journal of Development Economics*, 134, 248–271.

Restuccia, D. and R. Rogerson (2008): "Policy distortions and aggregate productivity with heterogeneous establishments," *Review of Economic Dynamics*, 11, 707–720.

——— (2017): "The Causes and Costs of Misallocation," *Journal of Economic Perspectives*, 31, 151–74.

The World Bank (2012): *De-fragmenting Africa : Deepening Regional Trade Integration in Goods and Services*, The World Bank.

——— (2022): "Enterprise Surveys," http://www.enterprisesurveys.org. Accessed on January 15th, 2022.

Thomas, J. K. (2002): "Is Lumpy Investment Relevant for the Business Cycle?" *Journal of Political Economy*, 110, 508–534.

Ulyssea, G. (2020): "Informality: Causes and Consequences for Development," *Annual Review of Economics*, 12, 525–546.

United Nations (2010): *Assessing Regional Integration in Africa IV: Enhancing Intra-African Trade*, United Nations.

Vijil, M., L. Wagner, and M. T. Woldemichael (2019): "Import Uncertainty and Export Dynamics," The World Bank Policy Research Working Paper.

Appendix A Data

This appendix includes descriptive statistics and explains key variables in more detail.

WBES questions used. To measure the use of foreign inputs by firms, to classify firms into types, and to measure border delays and firm employment, we rely on the following questions from the World Bank Enterprise Surveys.

D.12: "*In the last fiscal year, what percentage of this establishment's purchases of material inputs or supplies were: D12.a.) of domestic origin? D12.b.) of foreign origin?*"

D.13: "*Were any of the material inputs or supplies purchased in fiscal year imported directly?*"

D.14: "*In the last complete fiscal year, when this establishment imported material inputs or supplies, how many days did it take on average from the time these goods arrived to their point of entry (e.g. port, airport) until the time these goods could be claimed from customs?*"

L.1 "*At the end of the last fiscal year, how many permanent, full-time individuals worked in this establishment?*"

We use the answer to question D.13 to identify type 3 firms, i.e., those firms that import foreign inputs directly. From the answers to D.12 we are able to infer the foreign input share. The answers to question D.14 constitute our measure of border delays. Firm-level employment is taken from question L.1.

Descriptive statistics. In Tables A.1 and A.2 we report summary statistics of our main variables. For each country in our sample, the tables include the fraction of firms that uses foreign inputs, the share of foreign inputs (in %), the average border delays, the number of employees, and the country's 2010-level of real GDP in logs.

Country	Uses foreign inputs			Share of foreign inputs			Border delay			Full time employees			Log real GDP
	mean	sd	N	mean	sd	N	mean	sd	N	mean	sd	N	(2010)
Angola	0.63	0.48	139	43.40	25.93	88	11.43	10.66	42	34.27	49.17	349	25.26
Benin	0.69	0.47	148	76.78	30.37	106	14.06	15.08	77	45.44	107.51	150	22.93
Botswana	0.87	0.34	87	71.08	30.90	77	4.76	6.23	61	66.78	231.55	267	23.16
Burkina Faso	0.76	0.43	92	68.05	29.52	70	16.38	26.68	29	28.69	121.11	391	22.93
Burundi	0.79	0.41	60	63.11	33.04	45	32.39	30.51	29	38.47	78.67	157	21.72
Cameroon	0.63	0.48	350	52.03	33.45	209	23.39	25.77	70	23.17	58.87	360	23.91
Cabo Verde	0.58	0.50	74	81.37	28.25	48	20.51	15.35	30	20.37	46.72	152	21.12
Central Afr. Rep.	0.72	0.46	29	74.53	26.20	21	11.86	11.88	9	27.13	47.88	149	21.57
Chad	0.58	0.50	145	71.46	31.38	89	30.76	31.76	50	13.51	27.89	153	22.88
Côte d'ivoire	0.53	0.50	341	64.40	34.66	170	27.06	21.77	83	32.60	91.62	349	24.24
Djibouti	0.69	0.47	52	89.90	20.16	36	5.09	2.66	26	36.25	86.66	261	21.47
Eritrea	0.43	0.50	90	53.44	34.82	38	20.12	22.42	11	17.49	20.17	177	21.45
Ethiopia	0.38	0.49	363	46.68	30.69	166	19.49	16.09	126	37.88	132.80	832	24.41
Gabon	0.85	0.36	168	74.23	29.59	145	12.59	23.38	52	43.29	135.84	170	23.13
Gambia	0.63	0.48	150	75.21	30.22	87	21.39	21.55	45	17.29	34.51	151	21.02
Ghana	0.69	0.46	372	69.65	31.95	251	14.79	13.04	103	34.71	78.94	718	24.24
Guinea	0.74	0.44	137	84.35	23.66	105	18.09	17.20	92	20.39	51.29	149	22.67
Kenya	0.48	0.50	989	58.28	30.99	451	22.52	20.59	250	33.53	154.86	1000	24.61
Lesotho	0.60	0.49	125	69.71	33.74	82	4.42	6.87	72	60.15	1437.07	147	21.41
Liberia	0.46	0.50	150	75.34	28.60	73	8.08	6.07	31	27.05	41.76	151	21.63
Madagascar	0.32	0.47	394	68.29	37.41	139	15.50	19.25	79	67.70	192.70	515	23.02
Malawi	0.81	0.39	149	60.71	30.83	119	20.01	22.83	67	68.97	260.31	498	22.38
Mali	0.76	0.43	185	62.42	32.90	143	31.52	44.85	58	46.42	134.75	183	23.12
Mauritania	0.73	0.45	47	66.81	33.61	35	26.72	19.06	27	40.60	64.64	146	22.32
Mauritius	0.57	0.50	142	76.95	29.99	90	11.70	14.72	62	42.96	152.06	398	23.00
Mozambique	0.46	0.50	597	61.14	34.31	259	21.58	25.85	177	40.45	98.61	601	23.15

Table A.1: Descriptive statistics 1/2

Notes.– See notes under Table A.2

Country	Uses foreign inputs			Share of foreign inputs			Border delay			Full time employees			Log real GDP
	mean	sd	N	mean	sd	N	mean	sd	N	mean	sd	N	(2010)
Namibia	0.45	0.50	153	51.25	38.23	89	5.44	9.27	66	22.83	50.82	568	22.90
Niger	0.57	0.50	132	74.80	26.04	78	28.25	31.66	44	35.98	82.35	150	22.71
Nigeria	0.34	0.47	972	48.19	26.95	398	9.00	14.27	142	20.78	107.67	2564	26.67
Rwanda	0.62	0.49	74	58.39	36.88	52	14.27	25.69	38	36.20	100.09	238	22.52
Senegal	0.41	0.49	234	44.73	31.81	79	17.93	17.84	31	58.08	214.85	596	23.40
Sierra Leone	0.48	0.50	152	62.31	33.69	90	12.48	8.60	29	14.48	21.37	152	21.96
Somalia	0.86	0.34	247	58.62	31.51	197	6.85	12.05	74	20.08	32.84	250	22.06
South Africa	0.08	0.27	1071	44.66	21.56	114	10.67	13.36	44	51.50	277.71	1093	26.38
South Sudan	0.87	0.33	89	65.51	27.25	77	15.10	19.99	40	13.09	22.84	733	23.83
Sudan	0.66	0.48	101	53.77	26.95	67	6.40	2.80	28	26.68	39.01	660	24.71
Eswatini	0.46	0.50	102	64.79	30.76	53	4.04	4.55	66	35.72	85.45	149	21.98
Tanzania	0.63	0.48	355	52.11	30.33	244	29.13	18.70	71	24.23	104.76	718	24.27
Togo	0.60	0.49	149	71.92	31.46	91	30.56	34.59	75	48.83	104.43	150	21.86
Uganda	0.40	0.49	368	33.86	28.84	145	19.44	16.53	56	19.53	116.60	740	23.93
Zambia	0.49	0.50	354	54.10	27.29	157	14.47	12.79	106	24.95	54.50	714	23.53
Zimbabwe	0.66	0.47	592	58.14	32.93	374	8.99	11.05	247	31.88	434.95	600	23.37

Table A.2: Descriptive statistics 2/2

Notes.– The columns labeled “Uses foreign inputs” report answers to *D12.b*, assigning a value of 1 to responses larger than zero and 0 otherwise. The columns “Share of foreign inputs” report answers to *D12.b* for firms with $D12.b > 0$. The shares are expressed in %. The columns “Border delay” and “Full time employees” respectively report answers to *D.14* and *L.1*. Source: World Bank Enterprise Survey, World Bank World Development Indicators, and authors’ calculations.

Border delays in non-SSA countries. – In Figure A.1 we replicate Figure 2 for Russia, Thailand, Italy, Germany, Spain, Turkey and Sweden. The differences are stark. Although the maximum delay in Russia is comparable to that observed in some Sub-saharan African countries, the overall distribution is more concentrated towards low delays in all these countries. For comparison, average delays are 2.6 days in Sweden and Turkey, under five days in Spain and Germany, and around 6 to 7 days in Thailand, Italy and Belgium. These are 4 to 15 times lower than the average delay in Burundi, Mali or Togo.

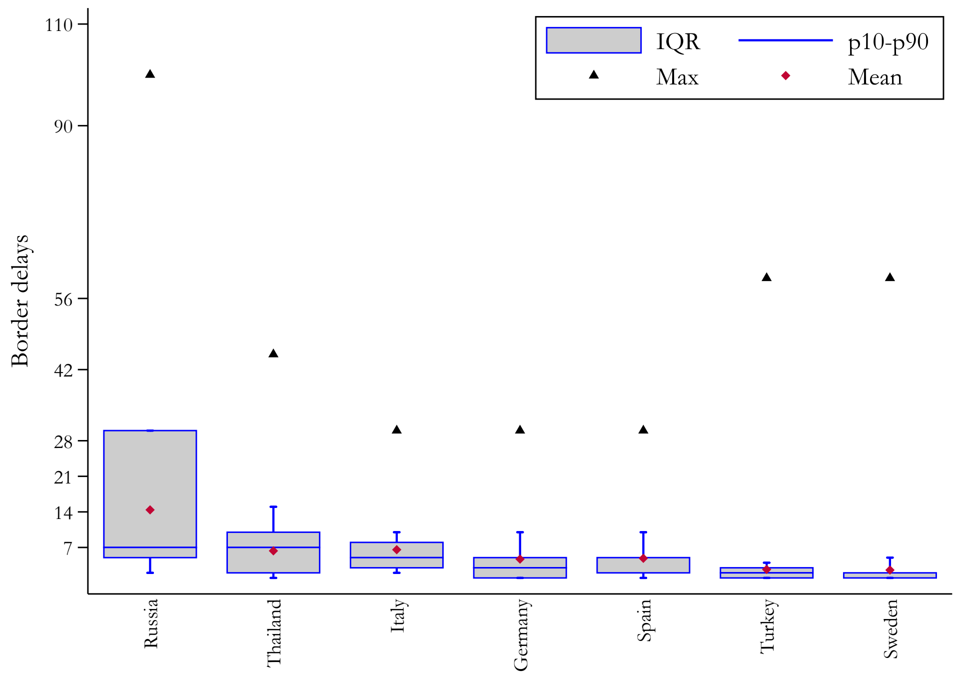

Figure A.1: Border delays in other countries

Notes.– The figure shows country-specific distributions of border delays in days. The box plots cover the 25th to 75th percentiles of delays. Medians are indicated by the horizontal bar. The lines extending from the boxes show the 10th and 90th percentiles. Averages are shown as red diamonds. The black triangles show the maximum delays. Minimum delays were zero in all cases. Source: World Bank Enterprise Survey and authors' calculations.

Appendix B Model derivations

For the computational solution, we define a firm's value function *after* production and *after* the firm learns its future level of productivity, but *before* the fixed cost of upgrading is drawn. At this point the firm decides on a threshold fixed cost level $\bar{\xi}_j$. If the fixed cost draw is below the threshold, the firm decides to upgrade to the next level, otherwise it does not. We derive the following terms to simplify the firm problem.

Labor demand. Firms' optimal labor demand is static, given the state s. Taking the first-order conditions of (7), (10), and (12) with respect to n yields

$$n^1(\varepsilon) = \left(\frac{\mu_1 \zeta \varepsilon}{w}\right)^{\frac{1}{1-\zeta}}, \qquad n^2(\varepsilon, k_l) = \left(\frac{\mu_2 \beta \varepsilon \cdot k_l^{\alpha}}{w}\right)^{\frac{1}{1-\beta}}, \quad \text{and}$$
$$n^3(\varepsilon, k_l, k_f) = \left(\frac{\mu_3 \beta \varepsilon \cdot k(k_l, k_f)^{\alpha}}{w}\right)^{\frac{1}{1-\beta}}. \tag{B.1}$$

For a type 2 firm, labor demand depends positively on local capital in place. For a type 3 firm, labor demand also depends on the amount of foreign capital, but not on any outstanding order of foreign inputs. The term $k(\cdot)$ represents the CES aggregate of k_l and k_f as described by equation (3).

Capital demand. Revenue minus labor costs for a type 1 firm, evaluated at the optimal labor choice given in (B.1), can be written as

$$\Pi^1(\varepsilon) = \varepsilon^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \cdot (1-\beta). \tag{B.2}$$

A type 2 firm's optimal labor choice implies that revenue minus labor costs becomes

$$\Pi^2(\varepsilon, k_l) = (\mu\varepsilon)^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} k_l^{\frac{\alpha}{1-\beta}} (1-\beta) \equiv A_2 k_l^{\hat{\alpha}}, \tag{B.3}$$

with $A_2 = (\mu\varepsilon)^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} (1-\beta)$ and $\hat{\alpha} = \frac{\alpha}{1-\beta}$. Because raising local capital is frictionless, k_l is not a state variable for a type 2 firm. This implies that the intertemporal investment problem of a future type 2 firm is independent of the continuation value.[19] Therefore, the investment problem net of the continuation value and any potential upgrading costs can be written as

$$\max_{k_l} -k_l + \rho\left[(1-x)\left(A_2 k_l^{\hat{\alpha}} + (1-\delta)k_l\right) + x\theta k_l\right], \tag{B.4}$$

with closed form solution $k_l^* = \left(\frac{A_2 \hat{\alpha}}{\tilde{r}}\right)^{\frac{1}{1-\hat{\alpha}}}$, where the effective user cost of local capital is $\tilde{r} = \frac{1}{\rho(1-x)} - 1 + \delta - \frac{x}{1-x}\theta$. Evaluating (B.4) at k_l^* yields

$$A_2^{\frac{1}{1-\hat{\alpha}}} \left(\frac{\hat{\alpha}}{\tilde{r}}\right)^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} \rho(1-x)(1-\hat{\alpha}) \equiv \rho(1-x)\Pi_2(\varepsilon). \tag{B.5}$$

A type 3 firm's static profits net of wage costs from (12), evaluated at the optimal labor decision can be written as

$$\Pi^3(\varepsilon, k_l, k_f) = \varepsilon^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \cdot k(k_l, k_f)^{\frac{\alpha}{1-\beta}} \cdot (1-\beta) \equiv A_3 k(k_l, k_f)^{\hat{\alpha}}, \tag{B.6}$$

[19]Implicitly, a firm liquidates all capital k_l after production and then raises the future desired capital stock k_l' as equity.

with $A_3 = \varepsilon^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} (1-\beta)$. Because total effective capital is a CES of local and foreign capital, no analytical solution exists for the optimal choice k_l. The local capital investment problem net of the continuation value, the foreign capital choice, and any potential upgrading costs can be written as

$$\max_{k_l} -k_l + \rho \left[(1-x)\left(A_3 \mathbb{E}_\psi k(k_l, k_f)^{\hat{\alpha}} + (1-\delta)k_l\right) + x\theta k_l\right], \tag{B.7}$$

with an implicit optimal level given by $k_l^* = \phi \left(\frac{\hat{\alpha} A_3 \mathbb{E}_\psi k(k_l^*, k_f)^{\hat{\sigma}}}{r_l}\right)^{\sigma}$, where the effective user cost of local capital is $r_l = \frac{1-\rho\cdot(1-x(1-\theta))}{(1-x)\rho} + \delta$, and $\hat{\sigma} = \frac{1+\sigma(\hat{\alpha}-1)}{\sigma}$.[20] The value of effective capital k in (B.7) is not known at the time of the local capital investment decision because the future level of foreign capital depends on the outcome of the delivery process.

Value functions. We now define each firm type's value function *after* production and the realization of future productivity, but *before* the fixed cost of upgrading is drawn. The known level of future productivity is denoted ε. The value of a type 1 firm at this point is denoted as $W^1(\varepsilon)$.

$$\begin{aligned} W^1(\varepsilon) &= \max_{\bar{\xi}_1} \int_{\bar{\xi}_1}^{\infty} \rho(1-x)\left[\Pi^1(\varepsilon) + \sum_{k=1}^{K} \pi_{ik} W^1(\varepsilon_k)\right] dG(\xi_1) + \\ & \int_0^{\bar{\xi}_1} \max_{k_l} -\xi_1 + \rho(1-x)\left(\Pi^2(\varepsilon) + \sum_{k=1}^{K} \pi_{ik} W^2(\varepsilon_k)\right) dG(\xi_1) \end{aligned} \tag{B.8}$$

From (B.8) the cutoff value $\bar{\xi}_1$ which makes firms indifferent between upgrading or not, has a closed form solution which does not depend on the particular assumption about the distribution of the fixed cost. The cutoff is given by the difference between the value of upgrading the firm or not:

$$\bar{\xi}_1(\varepsilon) = W_u^1(\varepsilon) - W_{nu}^1(\varepsilon), \tag{B.9}$$

where

$$\begin{aligned} W_{nu}^1(\varepsilon) &= \rho(1-x)\left[\Pi^1(\varepsilon) + \sum_{k=1}^{K} \pi_{ik} W^1(\varepsilon_k)\right] \\ W_u^1(\varepsilon) &= \rho(1-x)\left(\Pi^2(\varepsilon) + \sum_{k=1}^{K} W^2(\varepsilon_k)\right) \end{aligned}$$

This follows from taking the first order condition of B.8 with respect to $\bar{\xi}_1$, and applying the Leibniz formula for the derivative of an integral with functional bounds,

$$\frac{\partial}{\alpha} \int_{a(\alpha)}^{b(\alpha)} h(y,\alpha) dy = h\left(b(\alpha),\alpha\right) \cdot \frac{\partial b(\alpha)}{\alpha} - h\left(a(\alpha),\alpha\right) \cdot \frac{\partial a(\alpha)}{\alpha} + \int_{a(\alpha)}^{b(\alpha)} \frac{\partial h(y,\alpha)}{\alpha} dy.$$

[20] This derivative of (B.7) with respect to k_l is given by:

$$-1 + \rho(1-x)\left(A_3 \hat{\alpha} \mathbb{E}_\psi k(k_l,k_f)^{\hat{\alpha}-1} \cdot \phi^{\frac{1}{\sigma}} k_l^{-1/\sigma} \cdot \mathbb{E}_\psi k(k_l,k_f)^{1/\sigma} + 1 - \delta\right) + \rho x \theta$$

The exponents on $k(k_l, k_f)$ combine to $\hat{\sigma}$.

Using our functional form assumption for $G(\xi_1)$ the expected fixed cost paid becomes $\int_0^{\bar{\xi}_1} \xi_1 dG(\xi_1) = \frac{\bar{\xi}_1^2}{2} \cdot \frac{1}{\Xi_1}$.

The value of a type 2 firm is denoted as $W^2(\varepsilon)$. If the firm is upgrading, it must choose an investment into future foreign capital. Because a type 2 firm's existing level and outstanding orders of foreign capital are zero, from (4) we have that the firm chooses $\tilde{o}_f$ directly. We denote as $\tilde{k}_f$ the amount of foreign capital in place for future production prior to depreciation.

$$\tilde{k}_f = \begin{cases} k_{f,-1} + \tilde{o}_f, & \text{with probability } \psi \\ k_{f,-1} & \text{with probability } 1-\psi \end{cases} \tag{B.10}$$

The amount $k_f = (1-\delta)\tilde{k}_f$ in (B.11) denotes the amount of foreign capital in place with which a firm makes next period's investment decision. Following the same steps as above, we can write a type 2 firm's problem as

$$\begin{aligned} W^2(\varepsilon) &= \max_{\bar{\xi}_2} \int_{\bar{\xi}_2}^{\infty} \rho(1-x)\left(\Pi^2(\varepsilon) + \sum_{k=1}^{K} \pi_{ik} W^2(\varepsilon_k)\right) dG(\xi_2) + \\ &\int_0^{\bar{\xi}_2} \left[\max_{k_l,\tilde{o}_f} -\xi_2 - k_l - \tilde{o}_f + \rho(1-x)\mathbb{E}_\psi\left(\Pi^3(\varepsilon, k_l, \tilde{k}_f) + (1-\delta)k_l + \right.\right. \\ &\left.\left.\sum_{k=1}^{K} \pi_{ik} W^3(\varepsilon_k, k_f, o_f)\right) + \rho x\theta \mathbb{E}_\psi(k_l + \tilde{k}_f)\right] dG(\xi_2), \end{aligned} \tag{B.11}$$

subject to the non-negativity constraint on foreign capital investment[21]. Optimality requires that k_f be chosen such that the expected marginal benefit of k_f equal the marginal cost of raising foreign capital. If the newly ordered foreign capital is delivered, the marginal benefit is an increase in future production and a higher continuation value. If the order is not delivered, the marginal benefit is an increase in o_f, which increases the continuation value of the firm.

Following the same steps as above, the cutoff value $\bar{\xi}_2$ determines the firm's upgrading rule. It is given by

$$\bar{\xi}_2(\varepsilon) = W_u^2(\varepsilon) - W_{nu}^2(\varepsilon), \tag{B.12}$$

with

$$W_{nu}^2(\varepsilon) = \rho(1-x)\left(\Pi^2(\varepsilon) + \sum_{k=1}^{K} \pi_{ik} W^2(\varepsilon_k)\right),$$

and

$$\begin{aligned} W_u^2(\varepsilon) &= \max_{k_l,\tilde{o}_f} \left[-1 + \rho(1-x)(1-\delta) + \rho x\theta\right] k_l - \tilde{o}_f + \\ &\rho(1-x)\mathbb{E}_\psi\left(\Pi^3(\varepsilon, k_l, \tilde{o}_f) + (1-\delta)k_l + \sum_{k=1}^{K} \pi_{ik} W^3(\varepsilon_k, k_f, o_f)\right) + \rho x\theta \mathbb{E}_\psi(k_l + \tilde{o}_f). \end{aligned}$$

[21]For a type 2 firm, $k_{f,-1} = 0$

Finally, the value of a type 3 firm is denoted as $W^3(\varepsilon, k_{f,-1}, o_{f,-1})$, defined as

$$\begin{aligned} W^3(\varepsilon, k_{f,-1}, o_{f,-1}) &= \max_{k_l, i_f^-, i_f^+} -k_l - i_f^+ - \theta i_f^- + \rho(1-x)\mathbb{E}_\psi\Bigg(\Pi^3(\varepsilon, k_l, \tilde{k}_f) + \\ &(1-\delta)k_l + \sum_{k=1}^{K} \pi_{ik} W^3(\varepsilon_k, k_f, o_f)\Bigg) + \rho x \theta \mathbb{E}_\psi(k_l + \tilde{k}_f). \end{aligned} \tag{B.13}$$

This firm faces no upgrading decision.

Appendix C Robustness analysis

We present a robustness analysis of our main findings in Table C.3. The table has the same structure as Table 4 in the main text. We perform four robustness checks. For each case, we recalibrate the baseline economy and then perform our main experiment, in which we eliminate border delays. For every robustness check, Table C.3 shows two columns, (i) the recalibrated economy with an average import delay of 13.8 days, and (ii) the change in key moments when delays are eliminated. The values in the upper half of the table are normalized by the baseline economy results for each case.

The first robustness exercise is the case where labor is supplied exogenously. The increase in the wage rate following the removal of border delays does not exert an income effect in this model and labor supply remains unchanged. Relative to our result in the main text this implies that the output effects of eliminating border delays are larger.

The second robustness check concerns the elasticity of substitution between different types of capital. This was equal to $\sigma = 0.5$ in the baseline. In Table C.3 we consider the case of $\sigma = 1.5$. Our results are hardly affected by this change. The reason is that even with $\sigma = 1.5$ type 3 firms continue to use both types of capital. Only for very high levels of substitutability between types of capital could firms forfeit the use of foreign capital.

Third, we study the implications of making foreign capital investments fully reversible. In the main text, negative investments of foreign capital in place are penalized, there is a markdown for selling existing foreign capital in place, reflecting capital specificity. Eliminating this cost implies that the selection margin of the benchmark solution is much less distorted. Following the removal of border delays, there is only a relatively small increase in the number of type 3 firms and their employment share. At the same time, there is a significant decline in the total number of firms. Because the wage rate increases, total labor supply falls, yet ceteris paribus all firms increase their factor demand due to the removal of the friction. To clear the labor market, M^* and thus the total mass of firms must fall. This channel is more pronounced than in the baseline economy because when foreign capital is fully reversible, the distribution of output among type 3 firms is initially much less skewed towards high productivity firms than in the benchmark economy (this was shown in Figure 8). The large output gains implied from enabling more low to medium productivity firms to choose to become type 3 firms is largely already realized in the economy without irreversibility.

Finally, we study how changes in θ affect our main result. This parameter determines the fraction of capital which can be recuperated by firms in terms of exogenous exit. It also governs the degree of capital specificity and is used for the markdown when selling foreign capital in place and is thus related to the previous exercise. Increasing θ to 0.95 does not have a big impact on our quantitative results, as shown in the last two columns of Table C.3.

	$N^s = 1$		$\sigma = 1.5$		reversibility		$\theta = 0.95$	
Border delays	Yes	No	Yes	No	Yes	No	Yes	No
Aggregate output	1.00	1.23	1.00	1.12	1.00	1.01	1.00	1.12
Local capital	1.00	1.27	1.00	1.08	1.00	1.05	1.00	1.16
Foreign capital	1.00	2.24	1.00	2.36	1.00	1.52	1.00	2.00
Outstanding orders	1.00	0	1.00	0	1.00	0	1.00	0
Labor supply	1.00	1.00	1.00	0.93	1.00	0.92	1.00	0.92
Wage rate	1.00	1.14	1.00	1.11	1.00	1.04	1.00	1.12
Mass of firms	1.00	1.03	1.00	0.90	1.00	0.84	1.00	0.85
Fraction of type 1 firms	0.48	0.35	0.55	0.44	0.52	0.44	0.50	0.38
Fraction of type 2 firms	0.27	0.24	0.25	0.25	0.30	0.31	0.28	0.27
Fraction of type 3 firms	0.26	0.41	0.20	0.31	0.18	0.25	0.22	0.35
Emp. share type 1 firms	0.31	0.09	0.34	0.12	0.32	0.18	0.31	0.10
Emp. share type 2 firms	0.19	0.09	0.18	0.10	0.20	0.15	0.18	0.09
Emp. share type 3 firms	0.50	0.83	0.48	0.78	0.49	0.67	0.51	0.81

Table C.3: Robustness analysis